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A set of coupled nonlinear equations for dispersive Alfvén waves (DAWs) in nonuniform magnetoplasmas with two-ion species is derived by employing a multifluid model. The DAW frequency is assumed to lie between the gyrofrequencies of the light and heavy ion impurities. In the linear limit, a local dispersion relation (LDR) is derived and analyzed. The LDR admits a new type of DAW in two-ion plasmas. Furthermore, it is found that stationary solutions of the nonlinear mode coupling equations in two-ion plasmas can be represented in the form of different types of coherent vortex structures. The relevance of our investigation to space and laboratory plasmas is pointed out.
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I. INTRODUCTION

The Alfvén wave is one of the important normal modes of a two-component electron ion plasma that is embedded in a uniform magnetic field. The dynamics of nondispersive Alfvén waves is normally governed by ideal magnetohydrodynamic (MHD) equations. In the Alfvén wave, the restoring force comes from the equilibrium magnetic pressure, and the ion mass provides the inertia. The inclusion of nonideal effects,[1–3] such as the perpendicular (parallel) inertial force of the ions (electrons) and the Hall force, is responsible for dispersion of the Alfvén wave. The dispersive Alfvén wave (DAW), which is also referred to as the kinetic (or shear) and inertial Alfvén waves,[1,3] accompanies a finite parallel (along the ambient magnetic field lines of force) electric field, and the DAW dynamics is either governed by gyrokinetic equations or by two fluid equations that include the ion polarization drift and the parallel electron inertial force. The linear and nonlinear properties of the kinetic Alfvén and inertial Alfvén waves in a two-component electron-ion plasma have been discussed in depth by several authors.[2,4–7] It is widely thought that the DAW can energize both the electrons and ions, and that it can also be associated with numerous scale low-frequency (in comparison with the ion gyrofrequency) electromagnetic waves in both the laboratory and in space/cosmic plasmas.

However, most of the laboratory (such as the tokamak) as well as space and astrophysical (such as those in Earth's ionosphere and magnetosphere, the solar wind, cometary tails, etc.) plasmas contain multiple ion species[8–10] and inhomogeneities. Accordingly, it is of practical interest to examine the properties of linear and nonlinear DAWs in nonuniform multicomponent magnetized plasmas with equilibrium density gradients and sheared plasma flows.

In this article, we shall employ a multifluid model to derive a set of nonlinear equations for the DAW in a nonuniform magnetoplasma, by assuming that the frequency of the DAW is much smaller (either smaller, comparable, or larger) than the gyrofrequency of the heavier or inertial (lighter or inertialess) ions. The mode coupling equations consist of the electron continuity equation, the parallel component of the electron momentum equation, the conservation of the charge current density, as well as an equation which governs the dynamics of perpendicular velocity of the heavier ion component. In the linear limit, the four field equations are Fourier transformed and a general local dispersion relation is derived and analyzed in several limiting cases. It is found that sheared plasma flows can excite the DA-like waves in plasmas without the density gradient. On the other hand, the nonlinear coupling between finite amplitude DA-like waves can produce coherent vortex structures. Conditions under which the latter appear are given. The relevance of our investigation to space and laboratory plasmas is pointed out.

II. DERIVATION OF THE NONLINEAR EQUATIONS

We consider a nonuniform multicomponent plasma immersed in a homogeneous magnetic field $B_0\hat{\mathbf{z}}$, where B_0 is the strength of the external magnetic field, and $\hat{\mathbf{z}}$ is the unit vector along the z axis. The equilibrium density (n_{j0}) and velocity (v_{j0}) have gradients along the x axis. Here, the subscript j is e for the electrons and i for the ions. The equilibrium gradients are maintained by body forces and by noncontinuous injection of charged particles into plasmas. We assume that the strength of sheared magnetic fields, which are produced by the equilibrium parallel current, is negligibly small in comparison with the strength of the ambient magnetic field.

At equilibrium, the divergence of the equilibrium plasma current density is zero, and the charge neutrality condition reads

$$n_{e0}=Z_i^l n_{i0}^l+Z_i^h n_{i0}^h, \tag{1}$$

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where the superscript l (h) stands for the lighter (heavier) ion component, and Z_i is the ion charge. The negative ion is characterized by $Z_i<0$.

We assume that the frequency of the DAW is much smaller than the gyrofrequency ($\Omega_{cl}=Z_i^l eB_0/m_i^l c$) of the lighter ions, where e is the magnitude of the electron charge, m_i^l is the mass of the lighter ions, and c is the speed of light. Thus, the perpendicular (to $\hat{\mathbf{z}}$) components of the electron and lighter ion fluid velocity perturbations in the electromagnetic fields of the DAW are, respectively,

$$\mathbf{v}_{e\perp}\approx\mathbf{v}_{EB}+\mathbf{v}_{De}+(v_{e0}+v_{ez})\frac{\mathbf{B}_\perp}{B_0}, \tag{2}$$

and

$$\mathbf{v}_i^l\approx\mathbf{v}_{EB}+\frac{c}{B_0\Omega_{cl}}(\partial_t+v_{i0}\partial_z+\mathbf{v}_i^l\cdot\nabla)\mathbf{E}_\perp+\frac{v_{i0}\mathbf{B}_\perp}{B_0}, \tag{3}$$

where $\mathbf{v}_{EB}=c\hat{\mathbf{z}}\times\nabla\phi/B_0$, and $\mathbf{v}_{De}=-(cT_e/eB_0n_e)\hat{\mathbf{z}}\times\nabla n_e$ are the $\mathbf{E}\times\mathbf{B}_0$, and the diamagnetic drift velocities, respectively, $\mathbf{E}=-\nabla\phi-(1/c)\partial_t A_z\hat{\mathbf{z}}$ is the electric field vector, ϕ (A_z) is the electrostatic (z component of the vector) potential, and $\mathbf{B}_\perp=\nabla A_z\times\hat{\mathbf{z}}$ is the perpendicular component of the wave magnetic field. Furthermore, n_e is the electron number density and T_e is the constant electron temperature. The compressional magnetic field perturbation along the $\hat{\mathbf{z}}$ direction has been neglected in view of the low-$\beta(\ll 1)$ approximation. For simplicity, the motion of cold ions along the $\hat{\mathbf{z}}$ direction has been neglected.

The parallel (to $\hat{\mathbf{z}}$) component of the electron fluid velocity perturbation (v_{ez}) can be obtained from the z component of Ampère's law,

$$v_{ez}\approx(c/4\pi n_{e0}e)\nabla_\perp^2 A_z, \tag{4}$$

where $\nabla_\perp^2=\partial_x^2+\partial_y^2$.

The relevant equations for nonlinear dispersive Alfvén waves in plasmas with two-ion components can easily be derived by substituting Eqs. (1)–(4) into the continuity equations for the electrons and ions, and into the parallel component of the electron momentum equations. Thus, by substituting Eq. (2) into the electron continuity equation and by eliminating v_{ez} by means of Eq. (4), we obtain

$$D_t^e n_{e1}-\frac{c}{B_0}\hat{\mathbf{z}}\times\nabla n_{e0}\cdot\nabla\phi+\frac{1}{eB_0}\hat{\mathbf{z}}\times\nabla A_z\cdot\nabla J_{e0}+\frac{c}{4\pi e}D_z\nabla_\perp^2 A_z=0, \tag{5}$$

where $D_t^e=\partial_t+v_{e0}\partial_z+\mathbf{v}_{EB}\cdot\nabla+(c/4\pi n_{e0}e)\nabla_\perp^2 A_z\partial_z$, $D_z=\partial_z+B_0^{-1}\nabla A_z\times\hat{\mathbf{z}}\cdot\nabla$, $J_{e0}=-en_{e0}v_{e0}$ is the equilibrium electron current density, and $n_{e1}(=n_e-n_{e0}\ll n_{e0})$ is the perturbed electron number density.

Inserting Eqs. (2) and (4) into the parallel component of the electron momentum equation, and noting that $E_z=-\partial_z\phi-c^{-1}\partial_t A_z$, we readily obtain

$$(D_t-\lambda_e^2\nabla_\perp^2 D_t^e)A_z+\mathbf{v}_{D0}\cdot\nabla A_z+c(\partial_z+\mathbf{S}_{v0}\cdot\nabla)\phi-\frac{cT_e}{en_{e0}}D_z n_{e1}=0, \tag{6}$$

where $D_t=\partial_t+\mathbf{v}_{EB}\cdot\nabla$, $\mathbf{v}_{D0}=-(cT_e/eB_0n_{e0})\hat{\mathbf{z}}\times\nabla n_{e0}$ is the equilibrium electron diamagnetic drift, $\lambda_e=c/\omega_{pe}$ is the collisionless electron skin depth, $\omega_{pe}=(4\pi n_0e^2/m_e)^{1/2}$ is the electron plasma frequency, and $\mathbf{S}_{v0}=(\hat{\mathbf{z}}\times\nabla v_{e0})/\omega_{ce}$ is the electron shear parameter.

From the conservation of the charge current density, viz., $\nabla\cdot\mathbf{J}=\nabla_\perp\cdot\mathbf{J}_\perp+\hat{\mathbf{z}}\cdot\nabla J_{ez}=0$, we obtain

$$\frac{cZ_i^h e}{B_0}\hat{\mathbf{z}}\times\nabla n_{i0}^h\cdot\nabla\phi+\frac{cZ_i^l en_{i0}^l}{B_0\Omega_{cl}}D_{tl}\nabla_\perp^2\phi-Z_i^h e\nabla\cdot(n_{i0}^h\mathbf{v}_{i\perp}^h)-\frac{1}{B_0}\hat{\mathbf{z}}\times\nabla J_0\cdot\nabla A_z+\frac{c}{4\pi}d_z\nabla_\perp^2 A_z=0, \tag{7}$$

where $D_{tl}=\partial_t+v_{i0}^l\partial_z+\mathbf{v}_{EB}\cdot\nabla$, $J_0=e(n_{i0}v_{i0}-n_{e0}v_{e0})$ is the unperturbed total plasma current density, and $\mathbf{v}_{i\perp}^h$ is the perpendicular component of the heavier (or inertial) ion fluid velocity perturbation. The latter is determined from

$$(D_{th}^2+\Omega_{ch}^2)\mathbf{v}_{i\perp}^h+\frac{Z_i^h e}{m_i^h}\partial_t\nabla_\perp\phi-\frac{c\Omega_{ch}^2}{B_0}\hat{\mathbf{z}}\times\nabla\phi=0, \tag{8}$$

where $D_{th}=\partial_t+v_{i0}^h\partial_z+\mathbf{v}_{i\perp}^h\cdot\nabla$ and $\Omega_{ch}=Z_i^h eB_0/m_i^h c$ is the gyrofrequency of the heavier ion component.

Equations (5)–(8) are the desired nonlinear equations for the study of dispersive Alfvén waves in nonuniform plasmas with two distinct groups of ions.

III. THE LOCAL DISPERSION RELATION

In Sec. III, we shall present the local linear dispersion relation for the DAW in a nonuniform plasma by assuming that the wavelength of the disturbance is much smaller than the scale lengths of the equilibrium inhomogeneities. Accordingly, we Fourier transform Eqs. (5)–(8) by assuming that the perturbed quantities $n_{e1},\mathbf{v}_{i\perp}^h,\phi$, and A_z are proportional to $\exp(i\mathbf{k}\cdot\mathbf{r}-i\omega t)$, where $\mathbf{k}(=\hat{\mathbf{y}}k_y+\hat{\mathbf{z}}k_z)$ and ω are the wave vector and the frequency, respectively. The unit vector along the y direction is denoted by $\hat{\mathbf{y}}$.

We first present a general dispersion relation for the DAW in the presence of an equilibrium density gradient and equilibrium sheared plasma flows. Accordingly, we Fourier transform Eqs. (5)–(8) by neglecting the nonlinear terms. From Eq. (5) we have

$$n_{e1}=-\frac{1}{\omega}\left[\frac{k_y cn_{e0}'}{B_0}\phi+\left(\frac{k_y J_{e0}'}{eB_0}+\frac{k_y^2 k_z c}{4\pi e}\right)A_z\right], \tag{9}$$

where we have assumed that $\omega\gg k_z v_{j0}$ and have denoted $n_{e0}'=\partial n_{e0}/\partial x$ and $J_{e0}'=\partial J_{e0}/\partial x$.

On the other hand, we Fourier transform Eq. (6) and eliminate n_{e1} by means of Eq. (9). The resultant equation reads

$$D_m A_z=k_z c\left[\omega\left(1+\frac{k_y S}{k_z}\right)+\omega_{ce}k_y\kappa_e\rho_e^2\right]\phi, \tag{10}$$

where $D_m=(1+k_y^2\lambda_e^2)\omega^2-\omega\mathbf{k}\cdot\mathbf{v}_{D0}-k_z^2c^2k_y^2\lambda_{De}^2-k_yk_z\rho_e^2\Omega_{ce}J_{e0}'/en_{e0}$, $\lambda_{De}=(T_e/4\pi n_{e0}e^2)^{1/2}$ is the electron Debye length, $\rho_e=v_{te}/\Omega_{ce}$ is the electron Larmor radius,

$v_{te}(\Omega_{ce})$ is the electron thermal velocity (the electron gyrofrequency), $\kappa_e=n'_{e0}/n_{e0}$, and $S=(\partial v_{e0}/\partial x)/\Omega_{ce}\equiv V'_{e0}/\Omega_{ce}$.

Finally, we combine Eqs. (7) and (8) and Fourier transform the resulting equation. The result is

$$\epsilon_l\phi=\left(\frac{4\pi}{B_0}J'_0+k_yk_zc\right)k_yA_z, \tag{11}$$

where

$$\epsilon_l=\left(\frac{\omega_{ph}^2}{\Omega_{ch}}+\frac{\omega_{ph}^2\Omega_{ch}}{\Omega_{ch}^2-\omega^2}\right)k_y\kappa_i+\left(\frac{\omega_{pl}^2\omega}{\Omega_{cl}^2}+\frac{\omega_{ph}^2\omega}{\Omega_{ch}^2-\omega^2}\right)k_y^2,$$

with ω_{ph} and ω_{pl} being the plasma frequency of the heavier and lighter ion components, respectively. Furthermore, we have denoted $J'_0=\partial J_0/\partial x$ and $\kappa_i=(\partial n_{i0}^h/\partial x)/n_{i0}^h$.

From Eqs. (10) and (11) we can eliminate A_z or ϕ, and obtain the general dispersion relation

$$D_m\epsilon_l=\omega k_yk_zc\left(1+\frac{k_yS}{k_z}\right)\left(\frac{4\pi}{B_0}J'_0+k_yk_zc\right). \tag{12}$$

In the absence of the density gradients and equilibrium sheared plasma flows Eq. (12) reduces to

$$[(1+k_y^2\lambda_e^2)\omega^2-k_z^2c^2k_y^2\lambda_{De}^2]\left(\omega^2-\Omega_{ch}^2-\frac{\omega_{ph}^2\Omega_{cl}^2}{\omega_{pl}^2}\right)-k_z^2V_{Al}^2(\omega^2-\Omega_{ch}^2)=0, \tag{13}$$

where $V_{Al}^2=B_0^2/(4\pi\rho_l)$ is the Alfvén velocity and $\rho_l(=n_{i0}^lm_i^l)$ represents the mass density of the light ions. Equation (13) shows that the dispersive Alfvén waves are linearly coupled with the ion-cyclotron waves involving the heavy ion component. For $\omega\ll\Omega_{ch}$, Eq. (13) yields

$$\omega^2=\frac{k_z^2V_A^2+k_z^2c^2k_y^2\lambda_{De}^2}{1+k_y^2\lambda_e^2}, \tag{14}$$

where $V_A=c/\sqrt{a}$ is the Alfvén velocity in two-ion plasma, and $a=\Sigma_{i=l,h}\omega_{pi}^2/\Omega_{ci}^2$. Equation (14) shows that the phase velocity of the usual kinetic/inertial Alfvén wave is decreased when an additional ion component is present in plasmas.

It can be readily shown from Eq. (12) that the DAW in two-ion plasmas can be driven by sheared plasma flows even in the absence of the density gradients. For $k_zv_{te}\ll\omega\ll\Omega_{ch}$, the instability occurs provided that $(k_z+k_yS)\times(k_yk_zc+4\pi J'_0/B_0)<0$. The latter is satisfied for $\partial v_{e0}/\partial x\equiv V'_{e0}<0$ and $|V'_{e0}|/\omega_{ce}>k_z/k_y$ provided that $k_yk_zc>4\pi J'_0/B_0$. The growth rate of that current convective instability is $k_zV_A|k_yV'_0/k_z\omega_{ce}|^{1/2}$. Finally, we would like to mention that when the density gradients and sheared plasma flows are present simultaneously, then one has to resort to a numerical analysis of Eq. (12) in order to deduce complete information regarding the growth of dispersive Alfvén-like waves in two-ion plasmas that are inhomogeneous.

IV. NONLINEAR SOLUTIONS

The nonlinear interaction between finite amplitude dispersive Alfvén waves in two-ion plasmas can be responsible for the formation of ordered structures. Although the general stationary and nonstationary solutions of Eqs. (5)–(8) cannot be found analytically, we discuss here stationary solutions in some limiting cases. Specifically, in the following, we shall present vortex solutions[2,4–7,11–15] of Eqs. (5)–(8) by assuming that $\partial_xn_{j0}=0$, $|\partial_t|\ll\Omega_{ch}$, $c\omega_{ce}|\nabla_\perp^2A_z\partial_z|\ll\omega_{pe}^2|\hat{\mathbf{z}}\times\nabla\phi\cdot\nabla|$ and $\partial_z^2\ll\nabla_\perp^2$. Accordingly, we introduce a new reference frame $\xi=y+\alpha z-ut$, where α and u are constants, and assume that ϕ and A_z are functions of x and ξ only. The introduction of the new reference frame ξ with constant α and u for an inhomogeneous medium is a well established fact for cases involving Rossby and gravity dipolar vortices in fluids,[2,13–15] as well as for drift-acoustic[11,12] and drift-Alfvén[2,4,5,7] vortices in nonuniform magnetized plasmas.

In the stationary frame $\xi=y+\alpha z-ut$, we can replace ∂_t by $-u\partial_\xi$, ∂_y by ∂_ξ, and ∂_z by $\alpha\partial_\xi$. In the absence of the density gradients, Eq. (5) becomes

$$D_{\xi\phi}n_{e1}=D_{\xi A}\left[\frac{n_{e0}V'_{e0}}{uB_0}A_z+\frac{c\alpha}{4\pi eu}\nabla_\perp^2A_z\right], \tag{15}$$

where $D_{\xi\phi}=\partial_\xi-(c/uB_0)(\partial_x\phi\partial_\xi-\partial_\xi\phi\partial_x)$ and $D_{\xi A}=\partial_\xi+(1/\alpha B_0)(\partial_\xi A_z\partial_x-\partial_xA_z\partial_\xi)$, and $u\gg v_{j0}$ has been assumed.

From Eq. (6), we have

$$D_{\xi\phi}\left[(1-\lambda_e^2\nabla_\perp^2)A_z-\frac{c\alpha_0}{u}\phi\right]+\frac{cT_e\alpha}{en_{e0}u}D_{\xi A}n_{e1}=0, \tag{16}$$

where $\alpha_0=\alpha+S$.

On the other hand, Eq. (7) gives

$$D_{\xi\phi}\nabla_\perp^2\phi=D_{\xi A}\left(\frac{en_{e0}V'_0}{auB_0}A_z+\frac{c\alpha}{au}\nabla_\perp^2A_z\right), \tag{17}$$

where $V'_0\approx\partial(v_{e0}-v_{i0})/\partial x$.

We now discuss analytical solutions of Eqs. (15)–(17) in some limiting cases. Let us focus on kinetic Alfvén waves which assume that the scale sizes of the vortices are much smaller than the collisionless electron skin depth. Here, we obtain from Eq. (16)

$$D_{\xi\phi}\left(A_z-\frac{c\alpha_0}{u}\phi\right)=-\frac{cT_e\alpha}{en_{e0}u}D_{\xi A}n_{e1}. \tag{18}$$

In the ideal MHD limit, the Alfvén waves have insignificant density perturbations, so we can approximate A_z by $(c\alpha_0/u)\phi$. Substituting the latter into Eq. (17) we obtain

$$\left(1-\frac{c^2\alpha\alpha_0}{au^2}\right)\partial_\xi\nabla_\perp^2\phi-\frac{en_{e0}c\alpha_0V'_0}{au^2B_0}\partial_\xi\phi-\frac{c}{uB_0}\left(1-\frac{c^2\alpha_0^2}{au^2}\right)J(\phi,\nabla_\perp^2\phi)=0, \tag{19}$$

where $J(f,g)=\partial_xf\partial_\xi g-\partial_xg\partial_\xi f$.

In the absence of sheared plasma flows, Eq. (19) assumes the form of a stationary Navier-Stokes equation

$$\partial_\xi \nabla_\perp^2 \phi - \frac{c\mu_s}{uB_0} J(\phi, \nabla_\perp^2 \phi) = 0, \tag{20}$$

where $\mu_s = (1 - c^2\alpha_0^2/au^2)/(1 - c^2\alpha\alpha_0/au^2)$.

Equation (20) is satisfied by

$$\nabla_\perp^2 \phi = \frac{4\phi_s K_s^2}{a_s^2} \exp\left[-\frac{2}{\phi_s}\left(\phi - \frac{uB_0}{\mu_s c} x\right)\right], \tag{21}$$

where ϕ_s, K_s and a_s are arbitrary constants. The solution of Eq. (21) is given by[7,11]

$$\phi = \frac{uB_0}{\mu_s c} x + \phi_s \ln\left[2 \cosh(K_s x) + 2\left(1 - \frac{1}{a_s^2}\right)\cos(K_s \xi)\right]. \tag{22}$$

For $a_s^2 > 1$ the vortex profile given by Eq. (22) resembles the Kelvin-Stuart ''cat's eyes'' that are chains of vortices.

In the presence of sheared plasma flows, Eq. (19) admits a double vortex solution, the profiles of which are similar to those given in Refs. 2 and 12.

Next, we consider the case when $u \gg \alpha v_{te}$. Here, the last term on the left-hand side of Eq. (16) can be neglected and Eq. (15) becomes redundant. Thus, a typical solution of Eq. (16) is

$$(1 - \lambda_e^2 \nabla_\perp^2) A_z - \frac{c\alpha_0}{u} \phi = 0. \tag{23}$$

Combining Eqs. (17) and (23) we obtain an equation whose solution is

$$\nabla_\perp^2 \phi + \beta_1 \phi - \beta_2 A_z = F_3\left(\phi - \frac{uB_0}{c} x\right), \tag{24}$$

where $\beta_1 = \alpha\alpha_0 c^2/au^2\lambda_e^2, \beta_2 = \alpha_0 c/au\lambda_e^2$, and F_3 is a constant. In deriving Eq. (24) we have assumed that $\alpha = \alpha_0 + \lambda_e^2 e n_{e0} V_0'/B_0$.

By substituting Eq. (23) into Eq. (24), we finally obtain

$$\nabla_\perp^4 \phi + C_1 \nabla_\perp^2 \phi + C_2 \phi - \frac{F_3 u B_0}{c\lambda_e^2} x = 0, \tag{25}$$

where $C_1 = \beta_1 - F_3 - 1/\lambda_e^2$ and $C_2 = [(F_3 - \beta_1)/\lambda_e^2] + c\beta_2\alpha_0/u\lambda_e^2$. Equation (25) is a fourth order differential equation, which admits spatially bounded dipolar vortex solutions. Specific forms of the latter are given in Refs. 4 and 7.

V. SUMMARY

In this article, we have investigated the linear as well as the nonlinear properties of dispersive Alfvén waves in a nonuniform multicomponent magnetized plasma. For this purpose, we have employed the multifluid plasma model and have derived a set of coupled nonlinear equations for low-frequency long wavelength (in comparison with the ion gyroradius) electromagnetic waves in plasmas that have equilibrium density and magnetic-field aligned velocity gradients. In the linear limit, we have derived a local dispersion relation. The latter is analytically analyzed in order to demonstrate the current convective instability of the DAW in plasmas without the density inhomogeneity. Physically, the current convective instability arises because a phase lag between the parallel electron velocity perturbation and the wave potential appears due to the equilibrium velocity gradients.

Furthermore, it has been shown that finite amplitude DA disturbances in two-ion plasmas interact nonlinearly, giving rise to the vortex street and the dipolar vortex as possible stationary states. This has been shown analytically by seeking stationary solutions of the governing nonlinear equations, Eqs. (5)–(8), in two limiting cases.

We have thus reported a possible mechanism for the generation of dispersive Alfvén-like fluctuations in the presence of sheared plasma flows in a magnetized plasma containing two-ion species. The nonlinear mode couplings between finite amplitude DAWs provide the possibility of the formation of solitary vortices. We note that a vortex chain arises in the absence of the equilibrium sheared plasma flows, whereas the latter are required for the formation of a dipolar vortex. Thus, a possible saturated state of a current convective instability could appear as a dipolar vortex. However, the existence of the vortex chain and the double vortex is only guaranteed if these nonlinear coherent structures are stable against two- or three-dimensional perturbations. In order to investigate the stability of our nonlinear vortex solutions, we have to perturb the dynamical equations, Eqs. (5)–(8), around the zero order (vortex) solutions, and subsequently study the vortex stability by employing the method of Refs. 16 and 17. Although a complete stability analysis of our nonlinear equations is truly tedious, we anticipate that the coherent nonlinear structures should remain stable, because the form of the Jacobean nonlinearity in our problem is similar to that in the hydrodynamic problem.[16,17]

In conclusion, we stress that the results of the present investigation should be useful in identifying the frequency and wave number spectra of low-frequency electromagnetic fluctuations and the salient features of associated coherent nonlinear structures which are produced by sheared plasma flows in a nonuniform, low-temperature, magnetized plasma containing two-ion components. The latter are frequently found in tokamak edges as well as in space and cosmic environments.

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