Analyzing the Total Structural Intensity in Beams Using a Homodyne Laser Doppler Vibrometer

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ABSTRACT

The total structural intensity in beams can be considered as composed of three kinds of waves: bending, longitudinal, and torsional. In passive and active control applications, it is useful to separate each of these components in order to evaluate its contribution to the total structural intensity flowing through the beam. In this paper, a z-shaped beam is used in order to allow the three kinds of waves to propagate. The contributions of the structural intensity due to the three kinds of waves are computed from measurements made over the surface of the beam with a simple homodyne interferometric laser vibrometer. The optical sensor incorporates some additional polarizing optics to a Michelson type interferometer to generate two optical signals in quadrature, which are processed to display velocities and/or displacements. This optical processing scheme is used to remove the directional ambiguity from the velocity measurement and allows to detect nearly all backscattered light collected from the object. This paper investigates the performance of the laser vibrometer in the estimation of the different wave components. The results are validated by comparing the total structural intensity computed from the laser measurements with the measured input power. Results computed from measurements using PVDF sensors are also shown, and compared with the non-intrusive laser measurements.

keywords: structural power flow, vibration intensity, laser vibrometers, wave components, wave propagation

1. INTRODUCTION

Predicting and measuring elastic waves propagating through a structure can be of foremost importance in vibroacoustic problems. The prediction and the measurement of the propagating elastic waves within a structure is usually referred to as structural power flow or structural intensity. The energy flow to localized dampers or to neighboring structures and supports can be cardinal mechanisms through which structural vibration is damped out, which partly explains the practical difficulty in estimating internal damping coefficients in structures from ground vibration tests. It can also be a key for solving structure-borne noise problems, by channeling vibrations to where they do not radiate noise, instead of trying to suppress them.

Structural intensity refers to the active part of the vibration energy. As the active energy is usually only a small fraction of the total vibration energy, estimating it from measured vibration is not always a simple task. Measuring structural intensity is more elaborate than measuring acoustic intensity. Sound propagates through compression-type waves only, while vibration propagates through two basic types of waves: compression and shear waves. Depending on the geometry of the continuum, these two basic types of waves combine into different types of waves, such as bending, torsional, and longitudinal waves, which must be measured.

Similarly to acoustic intensity, vibration intensity measurements are also associated with cross measurements between closely-spaced transducers, which are microphones in the former case and, usually, accelerometers or strain sensors in the latter. Following the original work by Noiseaux [1] ,many authors have investigated different ways of computing the bending vibration intensity from measured accelerations in beams and plates [2,3,4]. Most authors use finite-difference approximations to compute the partial spatial derivatives that are necessary to compute the power flow, but the wave component approach[5] is more general and does not require closely spaced measurements.

In this paper, a z-shaped beam where the three kinds of waves - bending, longitudinal, and torsional - propagate is used to investigate the difficulties in estimating the total structural intensity. The contributions of the structural intensity due to the three kinds of waves are computed from measurements made over the surface of the beam with a simple homodyne interferometric laser vibrometer and with piezoelectric film sensor patches (PVDF). The optical sensor, which was developed in our laboratories, is briefly described.

2. STRUCTURAL INTENSITY COMPONENTS

The total structural intensity (SI) in a structure depends on the kinds of forces and moments that excite it . Each component of these forces and moments is associated with a different kind of wave propagating through the structure. In general, the total structural intensity in a straight beam segment along the x axis can be considered as being composed of the following kinds of waves: bending wave in the xy plane, bending in the xz plane, longitudinal wave in the x direction and torsional wave(angle θ in the yz plane). In vibration control applications, it can be useful to find a way to separate each of these components in order to calculate the total SI.

Berthelot et al.[6] used a Laser Doppler Vibrometer (LDV) to separate the contributions to the structural intensity carried by the longitudinal waves in the presence of strong flexural waves. In this paper, a z-shaped beam (see the figure 3) is used. Due to the geometry of the cross section of the beam and the low frequency range that was investigated, bending in the xz plane was assumed negligible. (The beam is much stiffer in this plane.) This configuration permits the three kinds of waves to propagate in the last segment of the beam. The purpose is therefore to separate the contributions of the SI due to the three kinds of waves using an LDV and compared the obtained results with those obtained using PVDF patches.

First, consider the bending waves using Bernoulli-Euler's theory, which is suitable for thin beams at low frequencies. The bending SI component may be expressed as[1]:

$$P_b = EI\left\langle \frac{\partial^3 w}{\partial x^3}\frac{\partial w}{\partial t} - \frac{\partial^2 w}{\partial x^2}\frac{\partial^2 w}{\partial x \partial t} \right\rangle_t \tag{1}$$

where E is the Young's modulus, I is the inertia moment of the cross section of the beam, $\langle \ \rangle_t$ denotes the time average, and w is the displacement in the y direction. Considering only the far field components and using PVDF sensors of length 2d spaced by Δx bonded to the surface of the beam, it is possible to show that[5]

$$P_{b\,\mathrm{ff}} = \frac{2EI\omega}{t^2 k_b r_b^2 sin(k_b \Delta x)} \Im\left\{\varepsilon_1^* \varepsilon_2\right\} \tag{2}$$

where $r_b = \dfrac{sin(k_b d)}{k_b d}$, $k_b^4 = \omega^2 \dfrac{S\rho}{EI}$ is the flexural wavenumber, t is half of the beam thickness, ε_i is the complex amplitude of the strain measured at position i, and the argument ω is suppressed for clarity.

The SI component due to longitudinal waves can be written as[7]

$$P_L = -SE\left\langle \frac{\partial u}{\partial x}\frac{\partial u}{\partial t} \right\rangle_t \tag{3}$$

where S is the cross-section area. Using again PVDF sensors, it is possible to show that

$$P_L = \frac{SE\omega}{2k_L r_L^2 sin(k_L \Delta x)} \Im\left\{\varepsilon_1^* \varepsilon_2\right\} \tag{4}$$

where $r_L = \dfrac{sin(k_L d)}{k_L d}$ and $k_L^2 = \omega^2 \dfrac{\rho}{E}$ is the longitudinal wavenumber.

When using the LDV, the out-of-plane and in-plane velocities are measured. Therefore, we will now review the expressions for the structural intensity in beams using velocities.

The flexural SI component can be written as[1]

$$P_{b\,\mathrm{ff}} = \frac{2EIk_b^3}{\omega\, sin(k_b \Delta x)} \Im\left\{\dot{W}_1^* \dot{W}_2\right\} \tag{5}$$

where $\dot{W}$ is the complex amplitude of the transverse velocity component associated with the bending.

The longitudinal SI component can be written as[1]

$$P_L = \frac{SEk_L}{2\omega\, sin(k_L \Delta x)} \Im\left\{\dot{U}_1^* \dot{U}_2\right\} \tag{6}$$

where $\dot{U}$ is the complex amplitude of the longitudinal velocity. Here, the correction in the velocity proposed by Berthelot et al.[6] was used in order to take into account that the measured longitudinal velocity has a component induced by the flexural waves.

Finally, the torsional SI component may be expressed as[7]

$$P_T = -T\left\langle \frac{\partial\theta}{\partial x}\frac{\partial\theta}{\partial t}\right\rangle_t \tag{7}$$

where $T = G\,bh^3/3$ is the torsional stiffness , b is the width of the beam, and G is the shear modulus. And, using the LDV measurements

$$P_T = \frac{T\omega k_T}{2b^2\omega\, sin(k_T \Delta x)} \Im\left\{\dot{V}_1^* \dot{V}_2\right\} \tag{8}$$

where $k_T^2 = \omega^2 \dfrac{b^2\rho}{16h^2G}$ and $\dot{V}$ is the complex amplitude of the transversal component associated with the torsion.

The input power was computed using the expression:

$$\mathrm{P_{in}} = \frac{1}{2}\Re\left\{\mathrm{F\dot{U}^*}\right\} = \frac{1}{2\omega}\Im\left\{\mathrm{F\ddot{U}^*}\right\} \tag{9}$$

where F is the force and $\dot{U}$ is the velocity in the direction of the applied force.

3. THE HOMODYNE LASER DOPPLER VIBROMETER

Laser Doppler Vibrometers are instruments used to measure velocities and/or displacements of points on a vibrating surface by focusing a laser beam onto the object. The operation principle of all LDVs relies on the detection of the Doppler shift in the frequency of the light backscattered by the surface. In most applications the Doppler shifts are too low to be measured by a spectrometer, so the light collected from the object is mixed with an optical reference beam and projected onto a detector. The detector is a nonlinear device with an output proportional to the power of the incident light, i.e., the square of the optical electric field. Thus, the coherent addition (interference) of the scattered beam collected from the object with the reference beam results in a voltage output varying at the difference of the two optical frequencies. The frequency ν_D (called Doppler frequency) of this alternate signal can be written as $\nu_D = 2u/\lambda$, where u is the component of the velocity in the direction of the incident beam and λ is the light wavelength. For the He-Ne laser light in the vacuum $\lambda = 0.6328\,\mu m$ and $\nu_D/u \approx 3{,}16\ MHz/m/s$.

One of the simplest optical sensors for a LDV is the Michelson interferometer[8]. In its original configuration (using a fixed mirror in the reference arm), the Michelson interferometer can be employed for measuring vibrations when the displacement of the object is somewhat higher than the laser wavelength and when the knowledge of the direction of the velocity is of no importance. However, most applications require the discrimination of the direction of the velocity, which can't be solved by this scheme. In fact, although the Doppler shift in the frequency of light depends on the direction of the motion, the output of such interferometer is symmetric with respect to the sign of velocity. The most commonly used approach to discriminate the direction of the motion is by the use of a frequency shifting device in the reference laser beam to break the symmetry of the optical output. These are called heterodyned systems[9,10]. In our experiment, we have implemented an optical setup based on the scheme used by Jackson *et al.*[11] This homodyned system mixes the Doppler shifted laser beam from the object with a reference beam which is not frequency shifted, as in the case of the classical Michelson interferometer. The directional ambiguity problem is solved by generating two optical signals in phase quadrature.

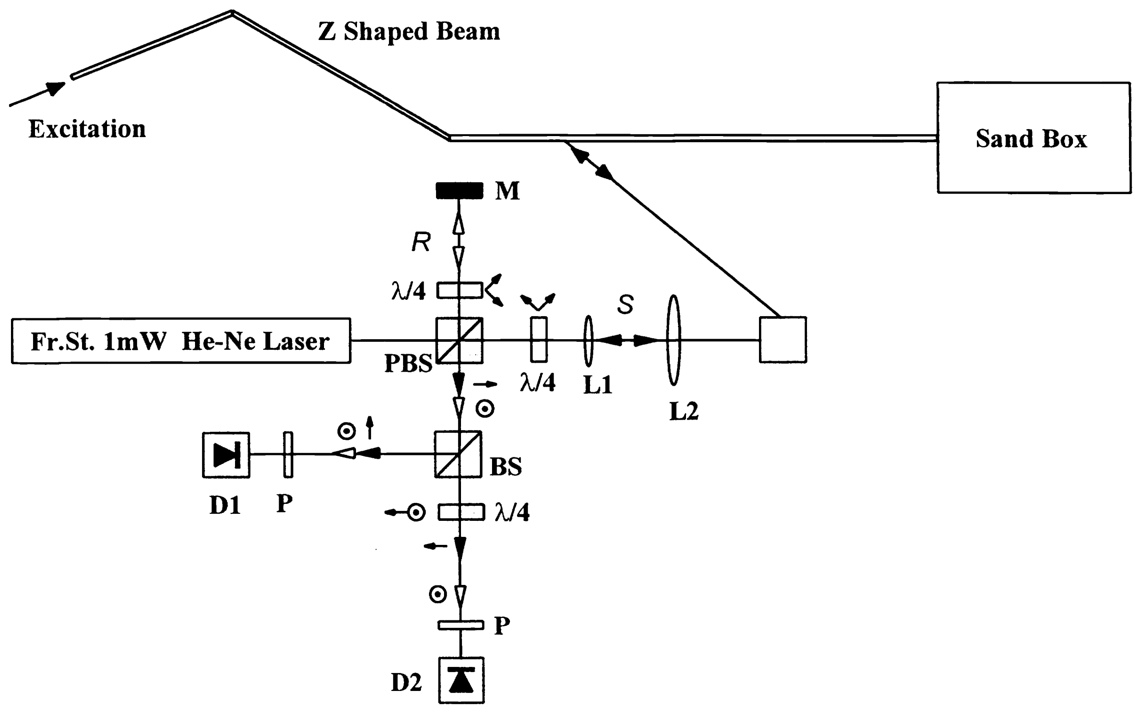

Figure 1. Simplified scheme of the homodyne LDV

A simplified scheme of the optical setup is shown in figure 1. The linearly polarized light from a 1 mW frequency stabilized He-Ne laser is split into orthogonal linear polarization components by the polarization beam splitter (PBS). The reference beam (R) is made circularly polarized using a quarter wave plate ($\lambda/4$). After passing again through the plate, the polarization of the returned light from the mirror (M) is rotated with to respect to the polarization of the incoming wave and it is transmitted by the PBS cube. A similar procedure is used in the S direction, where a beam expanding telescope is used to focus the laser beam onto the point of interest. A system composed by two sliding orthogonal mirrors allows easy selection of the measurement point and the angle of incidence. In this way, the light collected back from the mirror and from the object are reflected and transmitted by the PBS cube, respectively. These two orthogonal linear polarization beams are then splitted in two channels by the non-polarizing beam splitter cube (BS), resulting in two exactly similar samples of the reference and the scattered light having the same phase difference in each channel. A quarter wave plate is placed in one channel to produce an additional 90° phase shift between the beams. To obtain interference from these two beams, 45° components are taken with polarizers (P) and the mixed optical signals are projected onto the photodetectors (D). The alternate terms of the resulting signals can be written as $V_X = V_0\cos(2\pi\nu_D t + \varphi_0)$ and $V_Y = V_0\sin(2\pi\nu_D t + \varphi_0)$, where φ_0 is a constant phase term (neglecting noise effects introduced by thermal fluctuations and mechanical instabilities in the interferometer) and V_0 is a voltage depending, among other parameters, on the collected speckles[12].

The LDV used in this work is being developed in a collaboration between the Vibroacoustics Laboratory and the Optics Laboratory of the University of Campinas. The V_X and V_Y signals are acquired by a digital oscilloscope and numerically processed for displaying velocities using a procedure similar to that described by Koo *et al*[13]. Figure 2 shows the implemented setup used in our experiments.

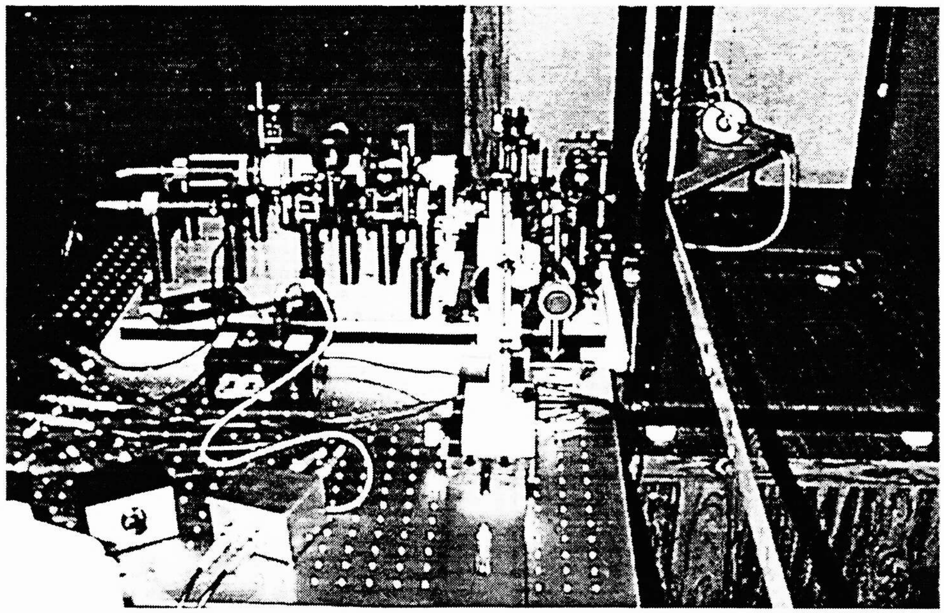

Figure 2. The implemented homodyne LDV and the z-shaped beam structure

4. THE EXPERIMENT

4.1 Experimental set-up

Using different combinations of the measurements made using the PVDF patches and the LDV, it is possible to separate the contribution of each one of the displacements associated with different types of waves and, therefore, with different SI components. Eight PVDF patches were used in the configuration shown in figure 5.

Combining the response measured at each of the PVDFs, it is possible to separate the contributions due to the different types of waves. The measured strain distribution can be decomposed into the sum of an anti-symmetric distribution - the bending component - about the center of the beam (e.g., PVDF1-PVDF3+PVDF2-PVDF4 in section 1) and a uniform strain distribution - the longitudinal component (e.g., PVDF1+PVDF3+PVDF2+PVDF4 in section 1). With this PVDF configuration it is not possible to measure the torsional contribution, which is expected to be small in this example, as it will be shown with the LDV measurements.

Now, using the LDV and measuring at 4 points in 2 different sections of the beam (corresponding to the center of the PVDFs in figure 5, at the edges of the beam), aimed at with different angles, it is possible to separate the contribution due to the three different waves: bending, longitudinal and torsional. In each one of the points two measurements were made at +45° and at -45° in the *xy* plane, relative to the y direction. The out-of-plane velocity was calculated by adding the measurements made at each location at +45° and -45° and dividing the result by $\sqrt{2}$. The in-plane velocity is simply the difference between the two measurements divided by $\sqrt{2}$. The torsional component is obtained by subtracting the out-of-plane components measured at the upper and lower edges of the beam section and dividing the result by the beam width. Thus, the separated velocity components associated with the different kinds of waves are obtained.

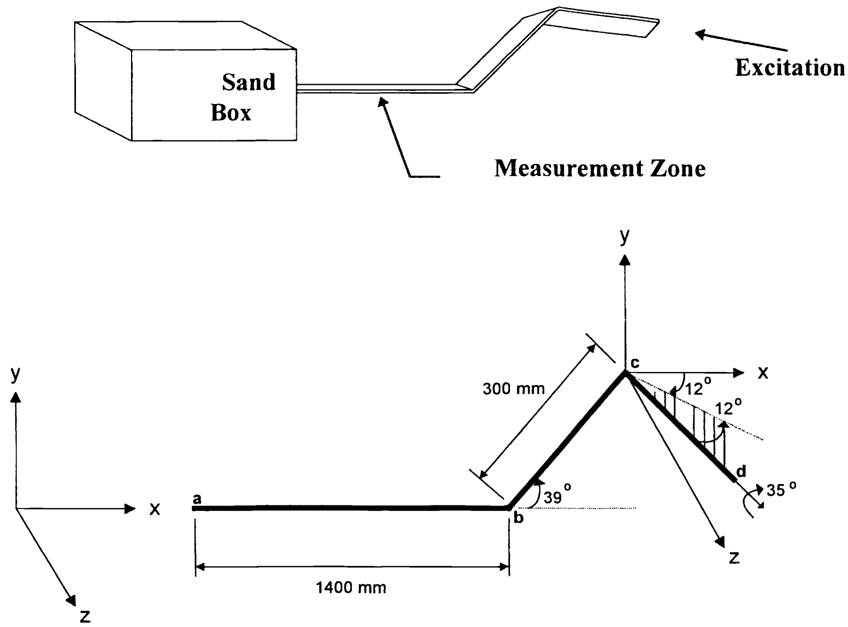

Figure 4. Z-shaped beam with a quasi-anechoic termination (sand box).

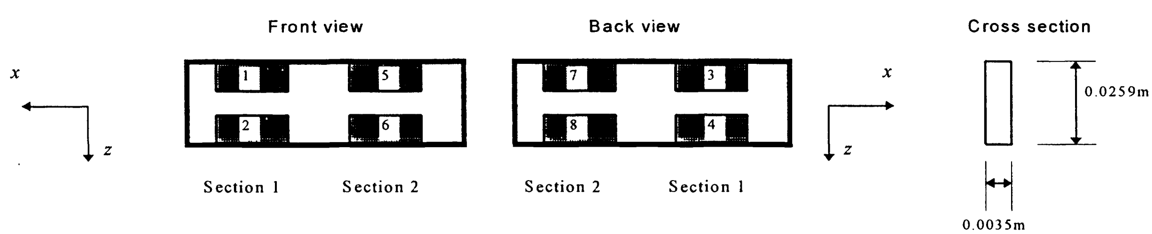

Figure 5. Configuration of the PVDF patches in the measurement zone of the z-shaped beam

4.2 Experimental results

In order to apply the techniques exposed previously, the z-shaped beam structure was excited by an electrodynamic shaker using a periodic chirp signal. The period of the chirp was 250ms in the case of the LDV test and 1s in the case of the PVDF test. This was necessary to accommodate the LDV signal processing requisites. Due to the fact that the LDV signal processing was done off-line, with digitized data, there was a compromise between the high sampling rate necessary for the high frequency Doppler signal and a sufficiently high signal length for the frequency resolution of the resulting low frequency velocity signal. The data acquisition was performed at a rate of 50kHz and 10 blocks with 15000 samples each were acquired. Also, because of this limitation (which can be overcome using an analog signal processing board to extract the velocity signal from the photodetector signals), the magnitude of the excitation force was adjusted so that the maximum

Doppler shift was approximately 3kHz (corresponding to a maximum velocity of about 1mm/s). Therefore, the active power input to the structure by the shaker was much smaller in the LDV test when compared with the PVDF tests.

In order to improve the signal-to-noise ratio, the Frequency Response Functions between the force transducer signal and the LDV or PVDF signals were calculated. The structural intensity spectra were computed using the cross spectra between the input force and each measured signal, which is the FRF multiplied by the auto power spectrum of the force input.

Figure 5 shows a comparison of the input power and the total structural intensity computed from measurements made with PVDFs. It can be observed that at higher frequencies the computed SI is smaller than the input power. This can be explained partly by the influence of the PVDFs which, together with their cabling, absorb part of the incoming energy. Figure 6 shows the relative contributions of the bending and longitudinal waves to the total intensity. It can be observed that, at these test force levels, the longitudinal waves carry more active power than the flexural waves.

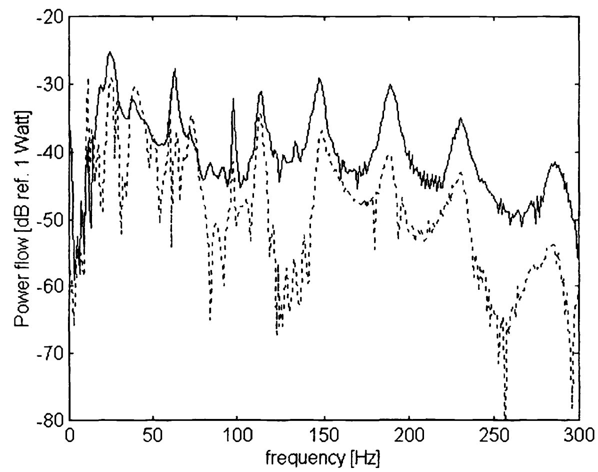

Figure 5. Input power versus active SI measured with PVDF patches. — Input power; ... active SI

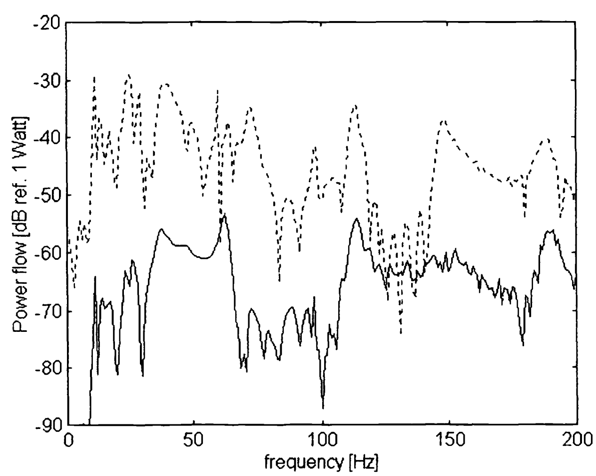

Figure 6. SI components measured using PVDF patches. — Flexural SI; ... longitudinal SI

Figure 7 shows the results obtained with the LDV measurements. There is a better agreement between input power and total structural intensity. The fact that the total SI does not decrease with frequency can be explained by the fact that the LDV is non-intrusive and, thus, the energy flow is preserved. Figure 8 shows the relative contributions of the different waves to the total SI. It can be observed that, in this test, with much smaller power levels (compare the magnitudes with figure 5), the flexural and longitudinal waves have contributions of similar magnitudes to the total SI. This can be explained by the different behavior of the sand box at different vibration levels. At lower levels most of the energy is dissipated through flexural motion while at higher amplitude levels most of the dissipation is due to longitudinal motion. To verify this hypothesis, another test was carried on at an intermediate force level. Results in figure 9 confirm that there is a tendency of decreasing the contribution of the longitudinal waves compared with the flexural waves at lower vibration levels.

The torsional contribution, as expected (due to the geometry of the z-shaped beam), is much smaller than the other two components.

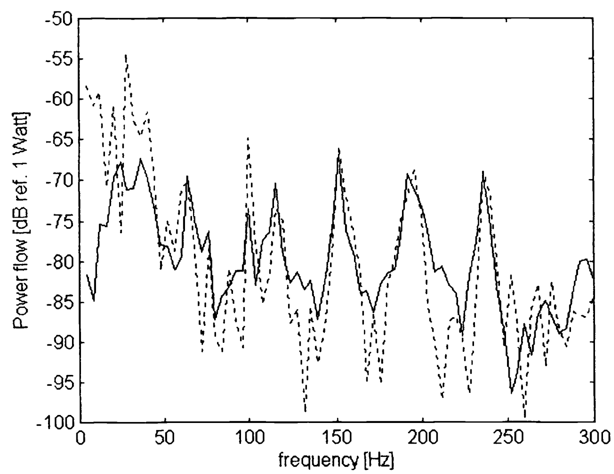

Figure 7. Input power versus active SI - LDV. — Input power; ... active SI

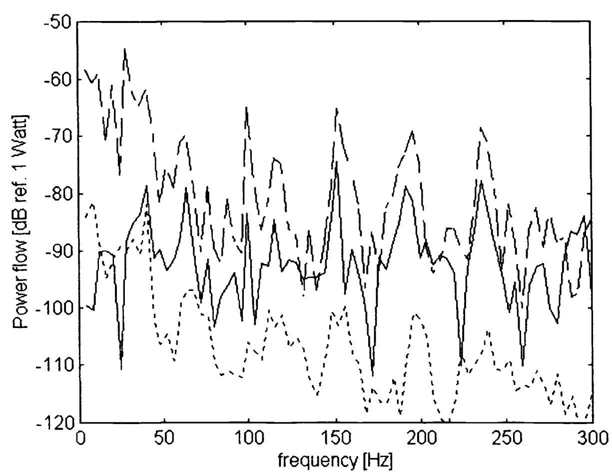

Figure 8. SI components measured using the LDV. --- Flexural SI ;— longitudinal SI; ... torsional SI

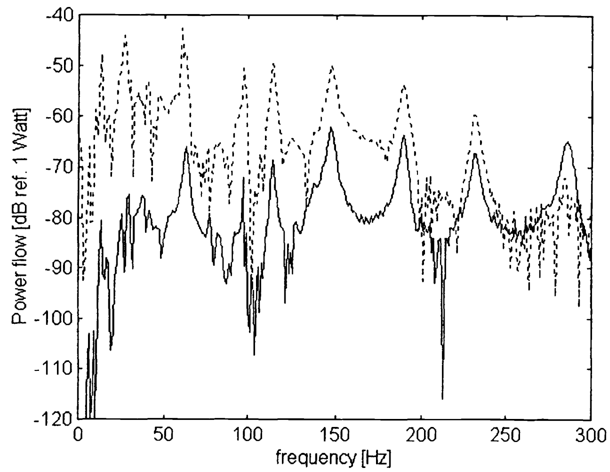

Figure 9 . SI components at a lower vibration level measured using PVDF patches. — Flexural SI; ... longitudinal SI

5. CONCLUSIONS

Structural intensity measurements are difficult to be carried out because the active part of the total vibration energy is usually very small. Furthermore, conventional instrumentation may alter the structural intensity pattern by absorbing energy locally. In this paper, the formulation of the structural intensity propagated via the three types of waves in beams (flexural, longitudinal, and torsional) is reviewed. Sensor placement and signal combinations that separate the three types of waves were exposed. Piezoelectric film sensor (PVDF) and LDV measurements were used.

A z-shaped beam was used to investigate experimentally the proposed techniques. With the LDV, the three types of waves were measured, while with the PVDF arrangement, only the longitudinal and flexural waves were obtained. It was shown that, in the frequency range investigated (DC-300 Hz), the torsional wave contribution was very small.

The LDV used in this work, which was developed at our laboratories, was briefly described. Due to the fact that the signal processing of the photodetector signals was done numerically, off-line, there was a compromise between the sampling frequency necessary to acquire the high frequency Doppler signals (which frequency is proportional to the velocity magnitude) and the low frequency vibration signals to which they are referenced. Thus, in the tests performed using the LDV the vibration levels were considerably lower then the tests performed using the PVDF sensors. This limitation will be overcome with the implementation of an analog signal processing circuit.

It was observed that, at higher vibration levels, the longitudinal waves were responsible for the propagation of most of the input energy, while, at lower levels, the flexural waves were as effective as the longitudinal waves in conveying the energy. A test at an intermediate vibration level confirmed this hypothesis. This tendency is linked to the behavior of the imperfect anechoic termination used in the experiments (sand box).

6. ACKNOWLEDGMENTS

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