

Generation of machined multiple flat surfaces on circular bar by its rotation and in synchronism with cutting tool

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Abstract - The operation of machining by lathe is one of the most common operations in automobile and machinery industries. The process is able to generate cylindrical bars under a wider variety of diameters / staggerings and using different materials, metallic's or non-metallic's (copper, wood, aluminum, brass, steel, etc...). However, when more complex geometries are required, additional machines are used, like milling machine and/or machines under numerical control (CNC) and multiple axles. The possibility of joining circles, polygons, angular motion and synchronism, open the mind to multiple figures (shapes) and movements, which can generate different geometries. This paper aims to demonstrate that both operations (turning and milling), can be made at the same time and at the same machine, special but simpler lathe. The studies of geometry, associated with calculus (trigonometry) and angular motion give basis and sustain this statement. Tests in bench materialize the idea and reinforce this thesis.

Keywords - geometry, machining, special machine.

1 Introduction

Geometry (Ancient Greek: γεωμετρία; geo-"earth", -metron "measurement") is a branch of mathematics concerned with questions of shape, size, relative position of figures, and the properties of space¹. Trigonometry (from Greek trigōnon "triangle" + metron "measure" [1]) is a branch of mathematics that studies triangles and the relationships between their sides and the angles between those sides. Trigonometry defines the trigonometric functions, which describe those relationships and have applicability to cyclical phenomena, such as waves [2]. The field evolved during the third century BC as a branch of geometry used extensively for astronomical studies [3]. It is also the foundation of the practical art of surveying. Trigonometry basics are often taught in school either as a separate course or as part of a pre-calculus course. Its functions are pervasive in parts of pure mathematics and applied mathematics such as Fourier analysis and the wave equation [4-6], which are in turn essential to many branches of science and technology. Spherical trigonometry studies triangles on spheres, surfaces of constant positive curvature, in elliptic geometry. It is fundamental to astronomy and navigation [7].

Trigonometry on surfaces of negative curvature is part of hyperbolic geometry [7-9].

Machining is the broad term used to describe removal of material from a workpiece; it covers several processes and a wide range of materials, metallic and non metallic. The machining is usually divided into the following categories:

- Cutting, generally involving single-point or multipoint cutting tools, each with a clearly defined geometry;
- Abrasive processes, such as grinding;
- Nontraditional machining processes, utilizing electrical, chemical, and optimal sources of energy.

Turning is the machining operation that produces cylindrical parts. In its basic form, it can be defined as the machining of an external surface:

- with the workpiece rotating;
- with a single-point cutting tool, and
- with the cutting tool feeding parallel to the axis of the workpiece and at a distance that will remove the outer surface of the work.

Taper turning is practically the same, except that the cutter path is at an angle to the work axis. Similarly, in contour turning, the distance of the cutter from the work axis is varied to produce the desired shape.

Even though a single-point tool is specified, this does not exclude multiple-tool setups, which are

often employed in turning. In such setups, each tool operates independently as a single-point cutter.

Milling is another process of cutting away material by feeding a workpiece past a rotating multiple tooth cutter. The cutting action of the many teeth around the milling cutter provides a fast method of machining. The machined surface may be flat, angular, or curved. The surface may also be milled to any combination of shapes. The machine for holding the workpiece, rotating the cutter, and feeding it is known as the milling machine [10]. Process geometry parameters are dependent on workpiece geometry, cutter run-out, assumption made for tooth trajectory i.e. circular of trochoidal and parametric machining direction. Estimation of the process geometry parameters is well established and trivial in case of machining straight and circular geometries [11-18]. This paper demonstrates that

both operations, when studied properly, can be made at the same time with a special and simpler lathe. The studies of the geometry, associated with calculus and circular motion, give basis and sustain this conclusion. Tests in bench materialize the idea and reinforce the thesis.

2 Materials and Methods

The bar may be a circle or polygon which, when projected along its own axis to an intended length / direction becomes a bar or shaft. The lathe, recalling the introduction, is responsible to change the bar by turning for a certain diameters and producing staggering when applicable. The process is sketched at Figure 1 below.

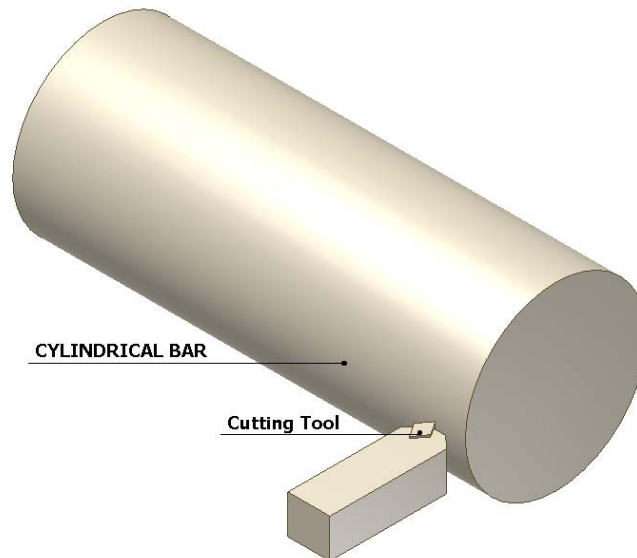


Fig. 1 Turning process sketch.

At the conventional turning operation the tool is fixed and static, while the bar turns around at its own axis (spins) and is driven by the lathe chuck. As known, the tool also has a movement longitudinal along the bar centre line. Imagining now the bar as fixed and the tool turning at its proper chuck, the resultant surface is a concave geometry – surface- , as seen in the Figure 2(a) below. Note that the tool longitudinal movement is kept present through all the analysis. Whether the bar is released from the chuck, turned around and

relocated to another position, for instance at π (radian) from the initial one, another concave geometry – surface shape - is obtained, as seen in the Figure 2(b).

Therefore and based on the last statement, multiple machined surfaces can be made on the bar, simply by positioning it properly, and afterwards removing the material by turning (Figure 3).

When the bar and the tool (both) turn around in their own axis, additional surfaces can also be

made in any angular position along the circle (cylinder). The unique exigency is to keep a certain calculated motion ratio (synchronism) between the bar and the cutting tool. This ratio is indeed related to the quantity of surfaces desired. For instance, when four surfaces are expected and only one tool is fixed at the chuck, the motion ratio (synchronism) between them (bar and cutting tool) must be 1:4. Another way to generate the same

polygon (four surfaces) is doubling the quantity of cutting tools (2 tools displaced at π radian at the chuck) and reducing the ratio, from 1:4 to 1:2. The ratio 1:4 with one cutting tool means that when the bar turns $\pi/2$ radian, the cutting tool turns completely (2π radian) and reached the bar again (cuts). The ratio 1:2 with two cutting tools fixed at the chuck means that when the bar turns $\pi/2$ radian each cutting tool turns π radian.

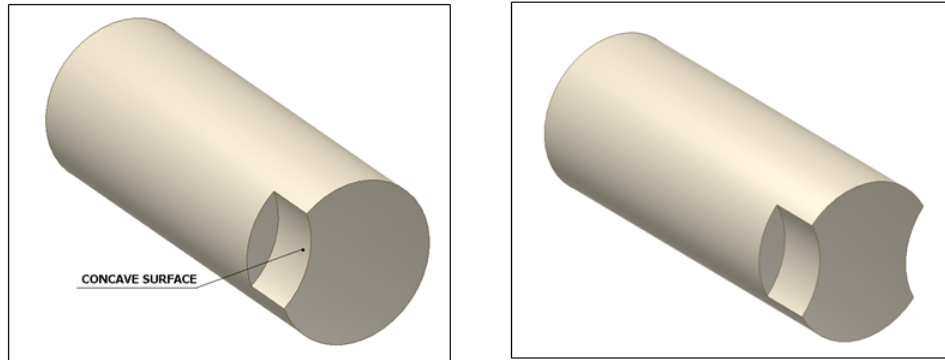


Fig. 2 Concave surfaces and multiple cuttings (a) 1x (left) and (b) 2x (right) (bar fixed and tool rotating)

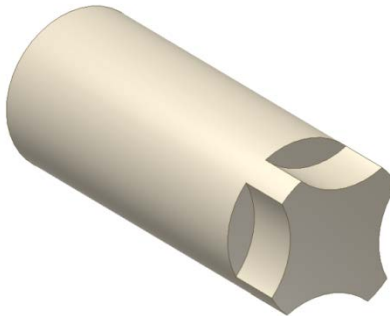


Fig. 3 Concave surfaces and multiple cuttings

The Figure 4 shows the geometrical configuration of this particular machining system which is purely based on trigonometry. The main purpose of this machining, recalling the introduction, is to have a flat surface at the bar in order to avoid additional operations and machines. This objective requires shrinking the dimension C as closer to Zero as possible (Figure 4 and 5) in order to generate such flat surface. It is clear that some characteristics must be fixed in some circumstances in order to stabilize and give basis to the system study, like; the distance between the centre of the bar and the centre of the chuck, (which fix the cutting tool (s)) and the

bar diameter. Beyond of these, the angular velocity and the motion ratio between the centre lines (bar and tool) must be fixed for each analysis. One question that might arise is the ideal diameter of the cutting tool (chuck). As expected, bigger radius R at the tool tip – fixed at the chuck – generates smaller distance C , which is the main focus to obtain a flat surface. Another way to decrease the distance C is fixing the tool radius and decreasing the bar diameter, diminishing, as a result, the dimension C and $2.H$, seen in Figure 5 and Equation (1).

In terms of geometry, the arc in the horizontal centreline in the Figure 4 is isolated in Figure 5

and gives basis to the calculations, as follow.

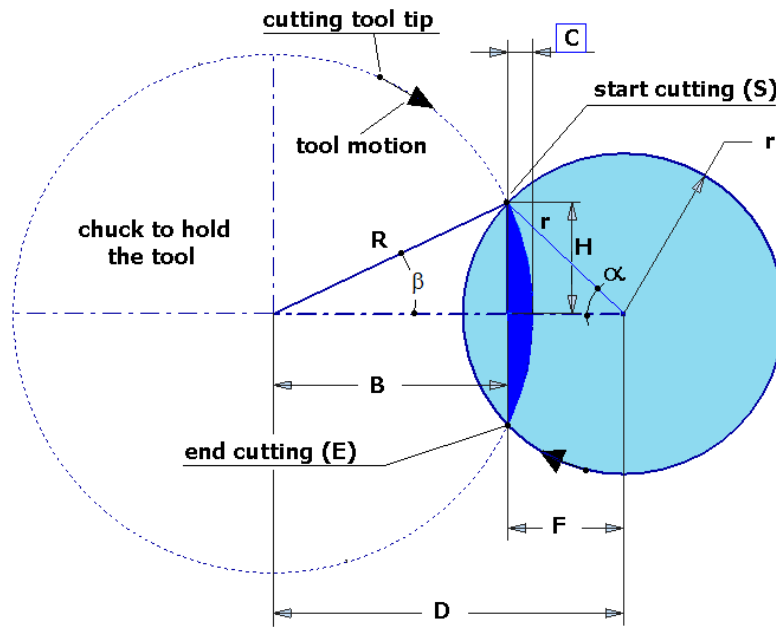


Fig. 4 Trigonometry of the machining

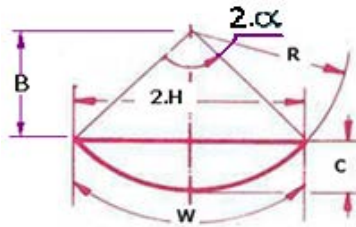


Fig. 5 Arc calculation – sketch

In order to calculate the dimension C the equation (1) can be written as follow;

$$C = R - B \tag{1}$$

where,

$$C = R - [(R^2 - H^2)^{1/2}] \tag{2}$$

or

$$C = R - [R^2 - (r^2 - F^2)]^{1/2} \tag{3}$$

to C = height of the arc on the cutting tool (mm), R = radius of the cutting tool (mm), B = distance between the centre of the chuck of the cutting

tool (mm), H = height of the arc at the appropriate angle β or α (mm), r = radius of the bar (mm), F = distance between the tangency of the arc at its angle β and the centre of the bar (mm).

Recalling the equation (3) it considers only the geometric condition of the machining, supposing the bar static (fixed) and the unique angular motion due to the cutting tool.

In the proposed cutting system when the cutting tool starts at “start cutting (S)” Figure 4 the bar also initiate its movement, contrary (against) of the cutting tool and both clockwise. Finally at the “end cutting (E)” the cutting tool finalize its cutting motion (path) as well as the bar its angular displacement within a certain

time. If both, tool and bar were totally engaged, like gears, and there weren't any indirect motion between them, the bar surface would respect the original tool radius, what is not the case. Whether the bar displaces angularly, the tool during its movement removes additional material from the slight inclination θ of the bar (Figure 6). This slight displacement increases the original tool radius, a portion, generating a bigger concave surface due to the dimension C decrease. Note that the surface can be concave or convex; it means that beyond of geometry, the surface shape depends also on the motion ratio. The analysis is clearer when analysing the Figure 4. When the cutting tool travels from the "starting cutting (S)" to the "end cutting (E)" (Figure 4 and 6) as described, the time of the cutting tool must be projected (considered) in order to determine the slight angular displacement associated with the bar (angle θ) (Figure 6). In general view, the time depends directly on the angular velocity; then the total angular displacement α at the bar when machined can be described as follow,

$$\alpha = \omega \cdot T \tag{4}$$

where T = time for angular displacement (s), ω = angular velocity (rad/s), α angle of displacement of the bar when being cut (rad).

The equation 5 is applicable in the tool motion as well as in the bar motion. Considering the cutting tool radius much higher than the bar radius, which is applicable in most of the cases, the time spent by the cutting tool to travel half

side of the bar (from S to E – see Figure 4) is shorter than the time to the bar to rotate consequently. Considering the theoretical angles of the bar (α) and the cutting tool β , see Figures 4 and 7 and equation 5 we have,

$$T_C = \frac{\beta}{\omega_C} < T_E = \frac{\alpha}{\omega_E} \tag{5}$$

where T_C = time for angular displacement at cutting tool (chuck) (s), ω_C = angular velocity of cutting tool (rad/s), β = angle of displacement of the tool when cutting (rad). T_E = time for angular displacement at bar (s), ω_E = angular velocity of bar (rad/s), α = angle of displacement of the bar (rad).

The slight angular displacement at the bar in reference to the time of the cutting tool travelling is then determined at the following manner,

$$\theta = \omega_E \cdot T_C \tag{6}$$

being θ = angular displacement of the bar after the tool has finished its cutting travel (rad), T_C = time for angular displacement at cutting tool (chuck) (s) and ω_E = angular velocity of the bar (rad/s).

With the slight motion with the time, the bar displaces X and can be determined as follow; (Figures 6 and 7) and equation 7,

$$X = [P^2 - (H - H_1)^2]^{1/2} \tag{7}$$

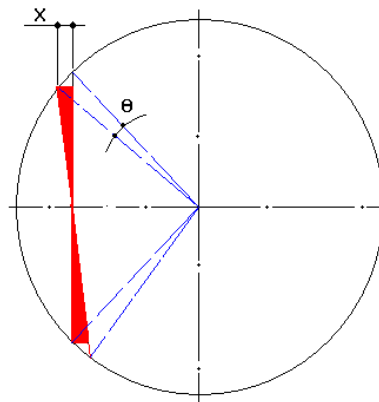


Fig. 6 Angular inclination θ – displacement - of the bar

The hypotenuse described as P in the equation 7 and shown in the figure 7 aims to simplify the calculations. Being the angle θ very small and the difference between the chord of the arc and a straight line negligible, the difference is not considered relevant for the purpose and P adopted in the calculations.

Once the effectiveness of the surface geometry – tendency to flatness – depends on the angle and time, the equation (7) can be re-arranged as follow;

$$X = [(r \cdot \sin\theta)^2 - (H - H_1)^2]^{1/2} \quad (8)$$

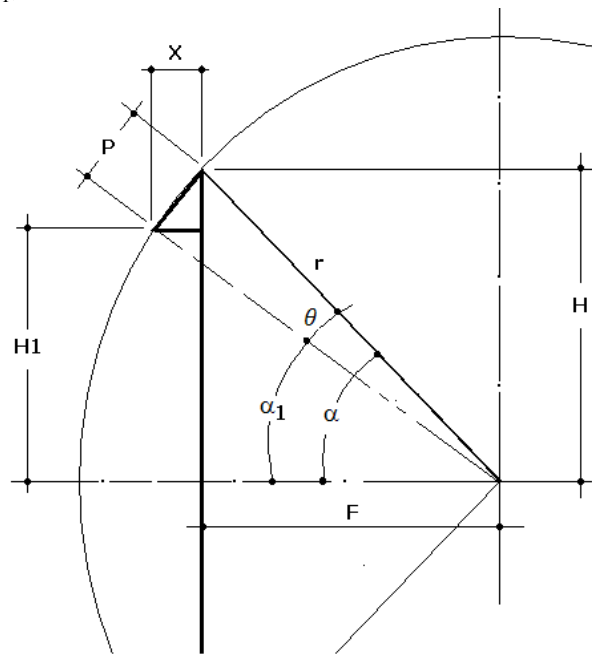


Fig. 7 Bar motion - variables

or better described,

$$X = \{ [r \cdot \sin(\omega_E T_C)]^2 - [r \cdot \sin\alpha - r \cdot \sin(\alpha - \theta)]^2 \}^{1/2} \quad (9)$$

where P = perimeter (mm), H = high of flatten surface (mm), H_1 = the high of flatten surface after the bar angular dislocation (mm), θ = the bar displacement angle (rad), α = angle of the bar (static) (rad), α_1 = angle of the bar (subtracting θ) (rad), r = radius of the bar (mm).

The smallest dimension of the curvature (concave or convex) called Δ is obtained by the subtraction of the values obtained in the equation (3) from the equation (9) which results in;

$$\Delta = C - X \quad (10)$$

$$\Delta = 2\{R - [R^2 - (r^2 - F^2)]^{1/2} - \{ [r \cdot \sin(\omega_E T_C)]^2 - (r \cdot \sin\alpha - r \cdot \sin\alpha_1)^2 \}^{1/2}\} \quad (11)$$

where θ = bar displacement angle (rad), r = radius of the bar (mm), H_1 = the high of flatten surface after the bar angular dislocation (mm), T_C = time to the tool displace angularly (s), ω_C = angular velocity of tool (rad/s), ω_E = angular velocity of bar (rad/s), α = angle of flatten surface (rad), α_1 = remaining angle after bar displacement (rad).

The equation 11 can predict the final surface reached in the machining process. In order to obtain a flat surface the characteristics are; bar diameter, motion ration and cutting radius. When

the result is positive a concave surface is reached and when negative a convex is found. The equation (11) has a multiplicative of 2 because the analysis was done for half of the machined surface.

3 Results and Considerations

The graphs that follow, from Figure 6 to 8, exemplifies cases where some parameters are fixed to analysis.

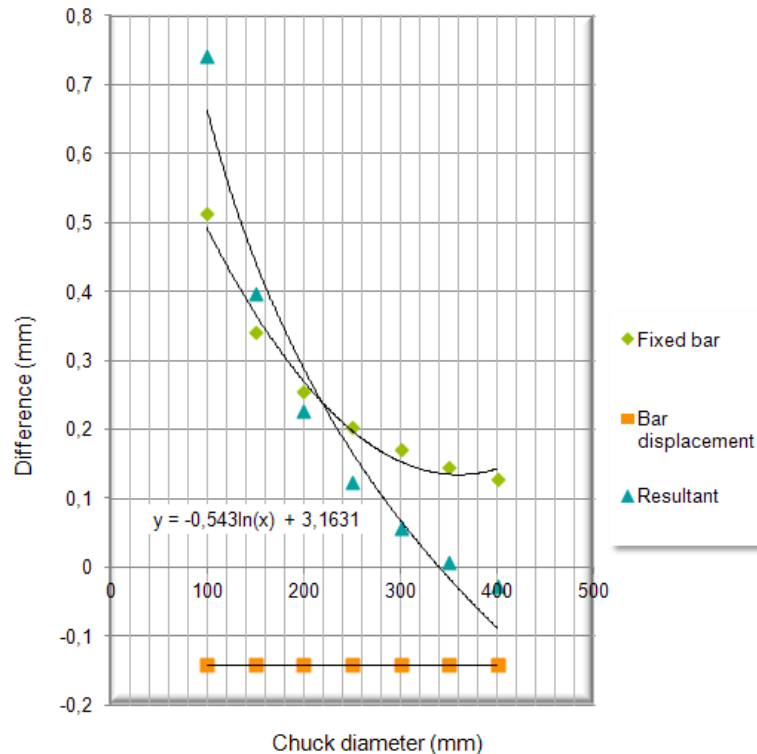


Fig. 6 Difference delta against cutting tool diameter at chuck

The “Fixed bar” value in the legend at the Figure 6 is for the case where the bar is fixed and only the chuck turns and cuts the material, like seen in Figure 2. The “Bar displacement” value, at the same Figure 6, shows that, once the synchronized motion is added the theoretical surface, called X at Figure 7 and calculated by the equation (9) is constant and equal of 0.142mm. It is noted in Figure 6 that the ideal chuck diameter of 350mm (cutting tool diameter) to a bar diameter of 20mm, with a synchronism of motion ratio of (1:2) reaches a surface close to zero. It is based on the “Resultant curve” logarithm equation best fit at the same Figure 6.

Figure 7 presents the values of delta when the cutting diameter is fixed at 350mm with the same motion ratio (1:2) and now changing

uniquely the bar diameter. The “Resultant” is a linear curve, meaning that with the increase of the bar diameter the surface only increases, being concave in most of the cases. The maximum concave value to the bar diameter of 40mm is 0.074mm, very small when compared with the bar dimension.

Figure 8 represents the values of delta when the cutting diameter is fixed at 350mm with the same bar diameter of 20mm and changing now uniquely the motion ratio. The “Resultant” shows that, with the increase of motion ratio the surface only decrease the delta, the flat surface is reached with a motion ratio of (1:2). This ratio coincides with the result found in the first graph of Figure 6. The best fit of the curve is a polynomial expression of third order (power 3), as seen in the same figure.

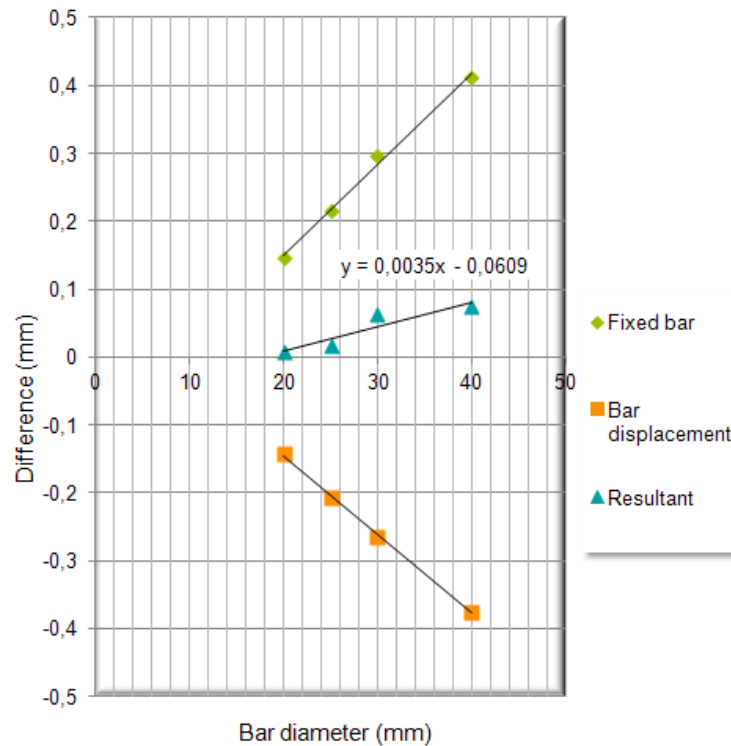


Fig. 7 Difference delta against bar diameter

Conclusions

Based on the experiments and the collected data, the following conclusions can be stated:

- The easier way to reach an almost flat surface, but concave, would be fixing the bar and apply a very big cutting tool diameter;
- the machining of flat surfaces in a system composed by both rotating, cutting tool and bar is possible and well succeed;
- the surfaces generated by a special machine is close to a perfect plane but can vary from concave or convex, depending on the machining parameters;
- due to limits of machine construction, the dimension of the cutting tool at the chuck, the angular movements, the bar diameter and the motion ratio (based on the quantity of flat surfaces) must be analysed in details;

- the idea of studying and gathering the geometry with angular motion deeply contributed to the concept development;
- the cutting tool diameter at the chuck is the most impacting characteristic on the surface shape result;
- the other parameters can also be adjusted in order to obtain the smaller delta and closer to a flat surface;
- the quantity of surfaces can be reached uniquely by changing the motion ratio between the chuck and the bar;
- one machine was built and demonstrated the theory in practice. The results will be shown afterwards in a specific paper ahead.

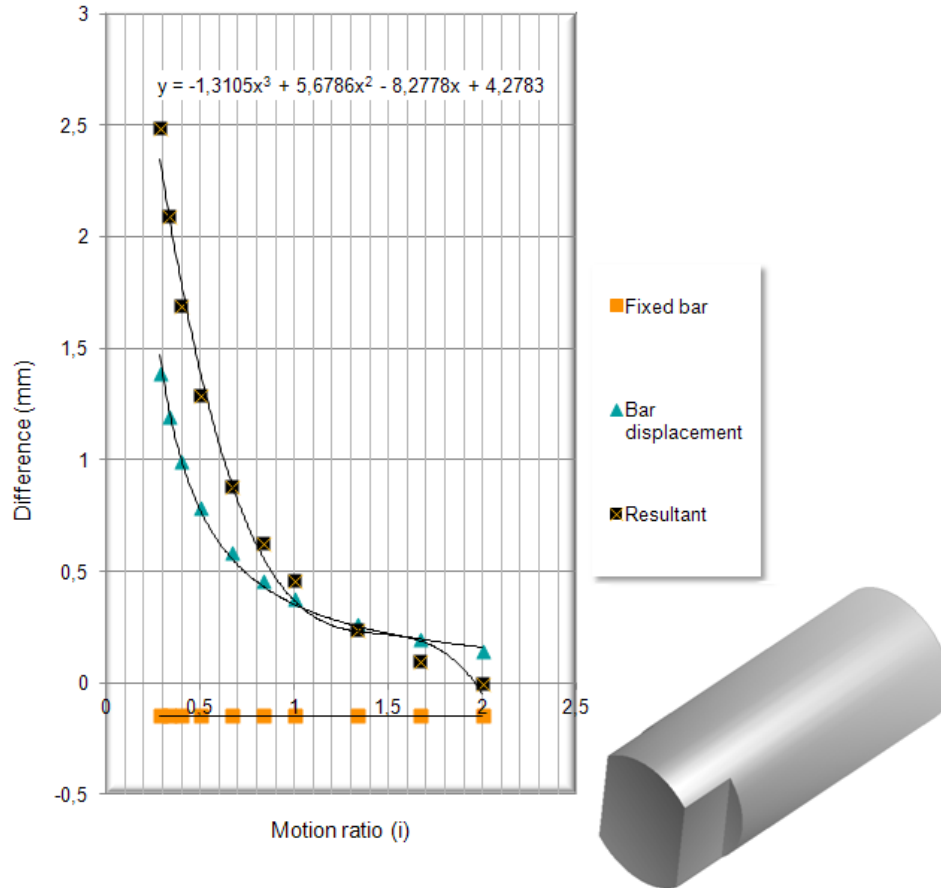


Fig. 8 Difference delta against motion ratio.

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