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Total cross section measurements with π^-, Σ^- and protons on nuclei and nucleons around 600 GeV/c

SELEX Collaboration

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Abstract

Total cross sections for Σ^- and π^- on beryllium, carbon, polyethylene and copper as well as total cross sections for protons on beryllium and carbon have been measured in a broad momentum range around 600 GeV/c. These measurements were performed with a transmission technique in the SELEX hyperon-beam experiment at Fermilab. We report on results obtained for hadron–nucleus cross sections and on results for $\sigma_{\text{tot}}(\Sigma^-\text{N})$ and $\sigma_{\text{tot}}(\pi^-\text{N})$, which were deduced from nuclear cross sections.

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1. Introduction

Hadronic total cross sections provide one measure of the strength of the hadronic interaction. They have been measured for a variety of reactions over a broad range of center of mass energies. These studies revealed that with increasing center of mass (CM) energy, hadron–hadron cross sections (generally) decrease to a minimum and then start rising again. An important current physics question is whether the rise of a specific hadron–hadron cross section is described by a power law in the CM energy. Addressing this question requires total cross-section experiments performed with a variety of hadronic projectiles, targets and energies covering the maximum possible range. However, for almost 20 years, there have been few new experiments in this field. Thus, important hadron–hadron cross sections such as $\sigma_{\text{tot}}(\pi p)$ and $\sigma_{\text{tot}}(\text{Kp})$ are measured only up to $380\,\text{GeV}/c$ and $\sigma_{\text{tot}}(\Sigma^- p)$ is only measured up to $137\,\text{GeV}/c$. At these maximum laboratory momenta only a first indication of the rise of these total cross sections is observed.

SELEX (Fermilab E781) is a fixed-target experiment at the Fermi National Accelerator Laboratory using a hyperon beam of about $600\,\text{GeV}/c$. The SELEX spectrometer, designed for spectroscopy of charm baryons, is well-suited to measure total cross sections with a transmission technique. It has excellent scattering-angle resolution, achieved by a system of silicon microstrip detectors.

SELEX does not have a liquid hydrogen target. Therefore, we measured the total hadron–nucleus cross sections $\sigma_{\text{tot}}(\pi^-\text{Be})$, $\sigma_{\text{tot}}(\pi^-\text{C})$, $\sigma_{\text{tot}}(\pi^-\text{CH}_2)$, $\sigma_{\text{tot}}(\Sigma^-\text{Be})$, $\sigma_{\text{tot}}(\Sigma^-\text{C})$, $\sigma_{\text{tot}}(\Sigma^-\text{CH}_2)$, $\sigma_{\text{tot}}(\text{pBe})$ and $\sigma_{\text{tot}}(\text{pC})$ with high precision. We then deduced the total cross sections $\sigma_{\text{tot}}(\Sigma^- p)$ and $\sigma_{\text{tot}}(\pi^- p)$ using both a CH_2–C subtraction technique and a method based on the Glauber model to derive hadron–nucleon cross sections from hadron–nucleus cross sections.

Further, as data on hadron–nucleus cross sections are extremely scarce for charged projectiles, we also measured $\sigma_{\text{tot}}(\pi^-\text{Cu})$ and $\sigma_{\text{tot}}(\Sigma^-\text{Cu})$. All measurements were done during dedicated run periods in July 1997. Laboratory momenta range from $455\,\text{GeV}/c$ to $635\,\text{GeV}/c$, the highest energy yet used for these studies.

2. Experimental setup

2.1. The hyperon beam

The hyperon beam is generated by selecting positively or negatively charged secondaries around $600\,\text{GeV}/c$ that emerge from interactions of an $800\,\text{GeV}/c$ primary proton beam with a beryllium production target. Its composition has not been completely measured. However, we have measured the main particle components of the event samples, which we selected to determine total cross sections (see Section 5.2.1). This analysis shows that at the position of the total cross-section target the negative beam samples consist in average of $(52.5 \pm 1.6)\%$ mesons and $(47.5 \pm 1.6)\%$ baryons. Further, we measured a Ξ^- fraction of $(1.18 \pm 0.06)\%$ in these samples. Other baryonic fractions ($\overline{\text{p}}$, Ω^-) were not measured, but empirical formulae (see [1]) predict they are less than 0.1%. Likewise, the K^- fraction of the negative beam is estimated with [1] to be $(1.6 \pm 1.0)\%$. Thus, we expect that the π^- fraction of the event samples is $(50.9 \pm 1.9)\%$ and the Σ^- fraction is $(46.3 \pm 1.6)\%$.

In the event samples for positive beam we measured a meson fraction of $(8.1 \pm 1.4)\%$ and a baryon fraction of $(91.9 \pm 1.4)\%$. The measured Σ^+ fraction was $(2.7 \pm 0.7)\%$. Using the empirical formula given in [1], we expect that the tiny meson fraction consists of 70% π^+ and 30% K^+. Thus, the samples for positive beam consisted to $(89.2 \pm 1.6)\%$ of protons.

From these compositions, one sees that as long as one can distinguish mesons from baryons (see Section 2.2), the SELEX hyperon beam offers a unique possibility to measure total cross sections for protons, π^-, and Σ^- in a low contaminant environment.

2.2. The section of the SELEX spectrometer used for total cross-section measurements

The SELEX spectrometer is a 60 m long, 3 stage spectrometer. In total cross-section measurements, only its upstream detectors, shown in Fig. 1, are used.

The beam spectrometer placed in front of the target, is equipped with 12 silicon microstrip detectors to track incoming particles. The first 4 microstrip detectors (HSDs) have a resolution (pitch/$\sqrt{12}$) of 14.4 μm and a maximum signal integration time of 100 ns. As this is the shortest integration time, but poorest spatial resolution, of all SELEX silicon microstrip detectors, the HSDs serve chiefly to reject out-of-time tracks. Always, two HSDs are housed in a single station. The average efficiency of the HSDs is 92%.

The remaining 8 silicon microstrip detectors of the beam spectrometer are grouped into 3 stations (BSSDs) mounted on a granite block inside a noise shielded cage (RF-cage). These detectors have a resolution of 5.8 μm and an average efficiency of 99.6%.

Incoming particles are identified by a transition radiation detector (BTRD) with 10 separate transition radiation detector modules (TRMs). Each module is build of a radiator followed by 3 proportional chambers (PCs). A radiator consists of a stack of 200 polypropylene foils, each 17 μm thick and spaced at 500 μm. The PC gas is a 70% Xe, 30% CO_2 mixture to optimize signal response time and to maximize absorption of transition-radiation photons. Each chamber has a single anode readout amplifier.

Each BTRD PC gives a digital output when it detects an energy deposition above a fixed threshold. The sum of all PCs detecting a signal above threshold is the TRD plane count k. A typical probability spectrum of TRD plane counts, a BTRD signal spectrum, is shown in Fig. 2. It shows the baryon and meson responses at low and high TRD plane counts, respectively.

The signal components are separated by fitting the function:

$$p_{\mathrm{fit}}(k) = \underbrace{\sum_{i=1}^{2} \kappa_i \binom{n}{k} p_i^k (1-p_i)^{n-k}}_{\text{baryon signal}} + \underbrace{\sum_{i=3}^{4} \kappa_i \binom{n}{k} p_i^k (1-p_i)^{n-k}}_{\text{meson signal}} \tag{1}$$

to the normalized BTRD signal spectrum. We used four binomials to account for the four main beam components as well as to obtain an excellent description of the

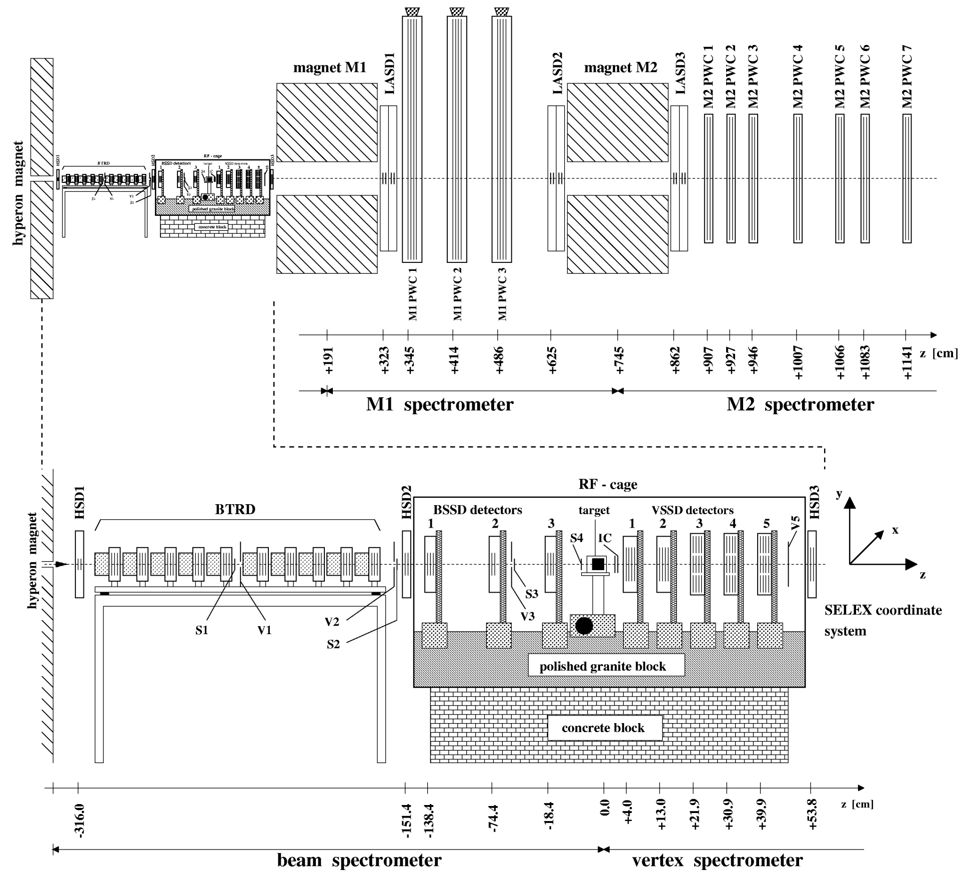

Fig. 1. Sections of the SELEX spectrometer involved in the measurement of total cross sections.

BTRD signal spectrum. In Eq. (1), p_i and κ_i are fit-parameters with the constraints $1 = \kappa_1 + \kappa_2 + \kappa_3 + \kappa_4$ and $p_1,\ p_2 < p_3,\ p_4$ and n is the maximum possible TRD plane count. The fit-parameters p_i have the meaning of a PC response probability, when a meson (light particle) or baryon (heavy particle) passes. Thus, we obtain from (1) the meson fraction $(\kappa_3 + \kappa_4)$ and the baryon fraction $(\kappa_1 + \kappa_2)$ of the beam.

The target is followed by the vertex spectrometer, which consists of 22 silicon microstrip detectors grouped into 6 stations (VSSD1, ..., VSSD5 and HSD3). All VSSDs have a resolution of 5.8 μm. Except for one plane, which has a reduced efficiency of 68%, all VSSDs have an average efficiency of 98.8%. At the end of the vertex spectrometer, station HSD3 is mounted to the RF cage.

Although the total cross-section measurements presented in this article are based only on detectors placed in the beam and the vertex spectrometer, we also use other parts of the SELEX apparatus to compute corrections. Further detectors involved in the analysis are situated in the M1 and the M2 spectrometer (see Fig. 1), which we describe briefly.

The M1 spectrometer starts at the center of the M1 magnet and ends at the center of

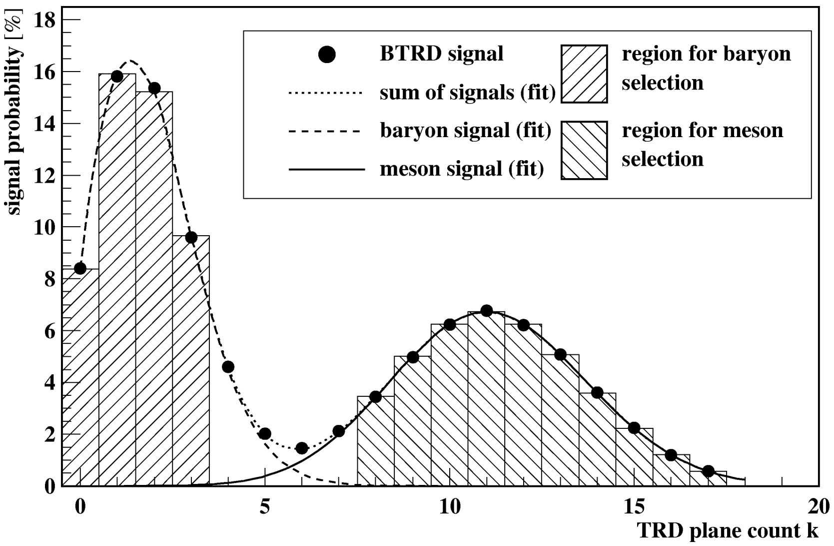

Fig. 2. A typical BTRD signal spectrum obtained for 600 GeV/c negatively charged secondaries.

the M2 magnet. For high resolution tracking of high energy particles in the central beam region, sets of 6 silicon microstrip detectors (LASD1 and LASD2) are mounted to the faces of the M1 and the M2 magnet. The LASD detectors have a resolution of 14.4 µm, and an average efficiency of 95.8%. For tracking outside the central beam region, 12 planes of wire chambers (PWCs) are installed.

The M2 spectrometer starts at the center of the M2 magnet. To enhance the momentum resolution for high energy particles, a third station of silicon microstrip detectors (LASD3) is mounted to the end face of the M2 magnet. This station is followed by 14 PWCs that are grouped into 7 stations (M2 PWC1, ..., M2 PWC7).

2.3. The targets

To optimize the precision, total cross-section measurements are done with special targets. Great care was taken in selecting and machining adequate target materials in order to obtain best chemical and mechanical properties (see Table 1). All targets are thin; multiple scattering, quantified by σ_θ of Molières' formula is significantly lower than the 25 µrad angular resolution provided by the beam and vertex spectrometer.

The carbon target is a stack of three pyrocarbon plates, each about 5 mm thick. Pyrocarbon is composed of thin carbon layers accumulated on top of each other in a high-temperature methane atmosphere. Compared to standard graphite it offers the advantages: no open porosity, a density close to that of a graphite monocrystal and less than 1 ppm (parts per million) non-carbon constituents. The beam faces of the carbon plates were milled with a diamond-powder liquid and oriented such that the beam faces of the stack are parallel to each other.

Table 1
Specifics of the targets used in total cross-section measurements. L: target thickness, ρ^*: density, σ_θ: expected spread in scattering angle due to multiple scattering calculated with Molières' formula for $p_{\mathrm{lab}} = 600\,\mathrm{GeV}/c$, X_{coll}: collision length

Target material	Thickness L [mm] z-direction	Transverse dimensions x [mm]	y [mm]	Density ρ^* [g/cm^3]	σ_θ [µrad]	X_{coll} [%]
beryllium	50.92	30.7	51.2	1.848 $\pm$ 0.002	8.3	16.86
carbon	15.46	30.0	30.0	2.199 $\pm$ 0.003	6.0	5.40
polyethylene	40.86	30.0	25.0	0.9291 $\pm$ 0.0008	6.3	6.66
copper	1.00	30.0	30.0	8.96 $\pm$ 0.009	5.7	1.05

The polyethylene target is build from a high-purity polyethylene granulate with less than 1000 ppm contaminants. Molten granulate was solidified in a vessel, where great care was taken that no air bubbles penetrated. The material was then carefully machined to a target block, and beam faces were flattened using a diamond pin.

For the beryllium and the copper target, standard industry products of high purity are used.

2.4. Trigger and data acquisition

The SELEX trigger is a programmable four-stage trigger, designed to select events involving decays of charm hadrons in a high-intensity beam environment. The first 3 levels: T0, T1 and T2 are hardware triggers, whereas level T3 is an online software filter. In this section, we describe only the trigger as programmed for total cross-section data-taking.

At data-taking, the trigger accepted all beam events defined by the minimum-bias condition:

$$\mathrm{T0} = \mathrm{S1} \wedge \mathrm{S2} \wedge \mathrm{S3} \wedge \overline{\mathrm{V1}} \wedge \overline{\mathrm{V2}} \wedge \overline{\mathrm{V3}}. \tag{2}$$

S1, S2 and S3 are scintillation counters, and V1, V2 and V3 are veto counters to reject beam halo (see Fig. 1). In definition (2), a T0-pulse indicates a particle traversing the beam spectrometer in the direction of the target beam face. The transverse trigger acceptance is constrained to the size of the hole in V2 (12.8 mm $\times$ 12.8 mm).

In order to keep the minimum bias condition provided by the definition of T0, no information from detectors placed downstream of the experiment target influenced the spectrometer readout. Thus, each T0-pulse passed the T1 trigger level unbiased, and generated a T2-pulse, which started the spectrometer readout. The online software filter (level T3) was not used for total cross-section data-taking. Pulses of all trigger levels were counted by scalers for each spill, and saved in a trigger log file.

The SELEX trigger controlled readout and reset of the silicon-detector system, the basic tool in our total cross-section measurements. Except for the HSDs, all other silicon detectors use an SVX-I chip technology for data readout [2]. SVX chips are controlled by a sequencer SRS (silicon readout sequencer) that interacts very closely with the trigger.

First, it keeps the silicon detectors sensitive (for about 5 μs effective integration time) and starts the chip readout when receiving a T2-pulse. Second, the SRS resets the SVX-chips, when a silicon-clear signal arrives. The silicon-clear signal is generated in the trigger logic through:

$$\text{Silicon clear} = \mathrm{V5}_{\text{mult}} \vee \mathrm{C}_{\text{pulser}} \vee (\mathrm{T1} \wedge \overline{\mathrm{T2}}). \tag{3}$$

Here, $\mathrm{C}_{\text{pulser}}$ are pulses from a gate generator running at a frequency of 20 kHz and $\mathrm{V5}_{\text{mult}}$ represents pulses generated, when the V5 veto counter (see Fig. 1) detects a high multiplicity event. The condition $(\mathrm{T1} \wedge \overline{\mathrm{T2}})$ was irrelevant for total cross-section data.

2.5. Experimental conditions and recorded data

During the fixed-target run 1996/97, the TEVATRON was operated in 60 s cycles with a spill time of 20 s. Data for total cross sections were taken during dedicated periods, with optimized experimental conditions for this measurement.

By adjusting the flux of the 800 GeV/c proton beam, the T0-rate was optimized to run the SELEX DAQ near, but safely below its capacity limit of 5×10^4 particles per spill. The low hyperon-beam flux allowed a high silicon-clear rate, which resulted in a very low-noise condition for the silicon-detector system and a low probability for out-of-time tracks.

During data-taking, the M1 magnet was switched off to obtain a 2.5 m field- and material-free section, serving as fiducial region for precise reconstruction of hyperon decays. Magnet M2 was operated with a transverse momentum kick of $p_{\mathrm{T}}^{\mathrm{M2}} = 0.84\,\mathrm{GeV}/c$.

At data-taking start, after mounting an experiment target in the RF-cage, an alignment RUN was taken to account for eventual detector displacements caused during the target installation. Then, the position of the experiment target was alternated every 30 min between its out and in-beam position. Thus, almost equal amounts of data were taken with full and empty target. A RUN, started after each target-position change, comprised typically 10^6 events. A total of 9.8×10^7 minimum-bias events were recorded with negative beam for the targets Be, C, Cu and $\mathrm{CH_2}$. With positive beam, 3.0×10^7 minimum-bias events were written using the targets Be and C.

3. The principle of the transmission method

In contrast to scattering experiments, where σ_{tot} is deduced from a measured scattering angle distribution, in a transmission experiment σ_{tot} is deduced from the number of unscattered projectiles. Strictly, unscattered means zero scattering angle, but experimental resolution and Coulomb scattering limit this to a determination of the number of projectiles scattered by an angle θ, which is smaller than a maximum angle parameter $\theta_{\max}$ ($F_0(<\theta_{\max})$). Thus, one infers the number of unscattered particles by extrapolating $F_0(<\theta_{\max})$ to $\theta_{\max} = 0$.

A standard transmission experiment consists of three elements: beam monitor, target, and transmission counter. The number of projectiles hitting the target under full-target (empty-target) condition F_0 (E_0) is counted by the beam monitor placed in front of the target. A transmission counter, placed downstream of the target, counts the corresponding number of projectiles $F_{\rm tr}(<\Omega_i)$ ($E_{\rm tr}(<\Omega_i)$), leaving the target within the maximum solid angles $\Omega_1, \ldots, \Omega_N$. Recorded counts are combined to give a set of partial cross sections $\sigma_{\rm part}(<\Omega_i)$, defined as:

$$\sigma_{\rm part}(<\Omega_i) = \frac{1}{\rho L}\log\left[\frac{F_0}{F_{\rm tr}(<\Omega_i)}\frac{E_{\rm tr}(<\Omega_i)}{E_0}\right] \quad \text{with } \rho = \frac{N_{\rm A}\rho^*}{A}, \tag{4}$$

where ρ is the density of scattering centers in the target, A is the atomic mass and $N_{\rm A}$ is Avogadro's number.

Driving our choice of a transmission method is an important technical advantage of Eq. (4). We do not need to know absolute efficiencies of the beam and the transmission monitor. Their absolute values will cancel in (4) as long as they remain unchanged between and during the full- and the empty-target RUNs (stability condition).

Taking into account the event correlations between F_0 (E_0) and $F_{\rm tr}(<\Omega_i)$ ($E_{\rm tr}(<\Omega_i)$), the statistical error of a partial cross section is given by:

$$\delta\sigma_{\rm part}(<\Omega_i) = \frac{1}{\rho L}\sqrt{\frac{1}{F_{\rm tr}(<\Omega_i)} - \frac{1}{F_0} + \frac{1}{E_{\rm tr}(<\Omega_i)} - \frac{1}{E_0}}. \tag{5}$$

In a thin target approximation ($\rho L\sigma_{\rm tot} \ll 1$), a partial cross section $\sigma_{\rm part}(<\Omega_i)$ is related to the total hadronic cross section $\sigma_{\rm tot}$ (see, e.g., [3]) by:

$$\begin{aligned}\sigma_{\rm tot} = \sigma_{\rm part}(<\Omega_i) &\underbrace{- \int_{\Omega_i}^{4\pi}\left(\frac{d\sigma}{d\Omega}\right)_{\rm C} d\Omega - \int_{\Omega_i}^{4\pi}\left(\frac{d\sigma}{d\Omega}\right)_{\rm CN} d\Omega}_{\text{Correction for C and CN scattering}} \\ &+ \underbrace{\int_0^{\Omega_i}\left(\frac{d\sigma}{d\Omega}\right)_{\rm el}^{\rm hadr} d\Omega}_{\text{elastic term}} + \underbrace{\int_0^{\Omega_i}\left(\frac{d\sigma}{d\Omega}\right)_{\rm inel}^{\rm hadr} d\Omega}_{\text{inelastic term}}.\end{aligned} \tag{6}$$

In equation (6), $\sigma_{\rm tot}$ is inferred by first correcting partial cross sections for Coulomb scattering (C) and the Coulomb hadronic interference (CN) and then extrapolating to zero solid angle.

4. Data analysis

4.1. Data selection

In general, total cross-section data taken for a specific target were subject to varying experimental conditions: thresholds on silicon microstrip detectors, high voltages for trigger scintillators, and the inclination angle between primary proton beam and production target. Therefore, data belonging to a cross-section measurement with a specific target were divided into as many data sets as differing conditions had to be taken into account. This offered the possibility to calculate corrections and errors specifically for each experimental condition in a later stage of the analysis. To preserve the stability condition mentioned in section 3, a spill by spill data pre-selection was performed. Data of a spill or a whole run were rejected:

(1) When the experimental conditions concerning the functionality of the spectrometer (detector efficiencies, trigger performance and track reconstruction efficiencies) suddenly changed.
(2) When it was not possible to synchronize raw data with information in the trigger log file.
(3) When the BTRD showed instabilities or when the beam phase space lay outside the BTRD fiducial region.

4.2. Event selection for normalization

The total cross-section determination is made by counting how many good beam tracks are removed from the beam by interactions in the target. The normalization therefore depends only on the number of good beam tracks, which are identified by a software decision routine. This routine reconstructs tracks in the beam spectrometer using the HSD and BSSD hit information. It preserves the minimum-bias condition for the selected data by strictly avoiding event-selection rules that require information from detectors placed downstream of the target. An event is accepted when it is possible to reconstruct a “norm track” that satisfies the following criteria:

(1) Not more than a total of 150 hits in all BSSDs.
(2) At least 6 hits from BSSD planes along the track.
(3) At least one hit from an HSD plane along the track (HSD-tagging).
(4) A reduced track-fit χ^2 below 3.
(5) An extrapolated origin of the track at the known transverse position of the primary production target.
(6) Track intercept and slope parameters within the beam phase space accepted by magnetic collimation.
(7) A transverse track position at the longitudinal position of the experimental target, which is inside the trigger acceptance window and inside BTRD acceptance.
(8) A beam momentum assigned to the track, which is $\pm$ 100 GeV/c around the center of gravity value of the momentum spectrum.

Condition (3) rejects out-of-time tracks. The selection rules (4)–(6) remove events in which hyperons decay before reaching the experiment target or react with detector material in the beam spectrometer. Constraint (7) assures also that selected tracks point to the mid-part of the experiment target face, where the best mechanical accuracy is obtained.

About 50% of the selected events had a norm track. From the resulting set of norm tracks for full- and empty-target conditions, we establish classes of BTRD-tagged norm tracks. This is done by introducing cuts on the BTRD information as indicated in Fig. 2 to separate baryonic and mesonic norm tracks. We then determine the corresponding normalization counts F_0 and E_0 by summing the norm tracks over the appropriate signal region.

4.3. Transmission counting

When a norm track is found in the event, we try to reconstruct a single track in the vertex spectrometer at small angle to the norm track. The single-track algorithm was efficient and fast. It used hits of HSD3 to remove out-of-time tracks. With loose cuts on the track parameters 98% of the norm tracks got assigned a track in the vertex spectrometer. Such vertex tracks were finally accepted as "transmitted tracks", when:

(1) There are at least 15 hits from VSSDs found within a track search corridor.
(2) The reduced track-fit χ^2 is below 3.

For each transmitted track, the scattering angle θ between norm and transmitted track is calculated. Following the idea of [4], a four-momentum transfer t is assigned to the event using the small angle approximation $t \approx -p_{\text{beam}}^2\theta^2$, where p_{beam} is the momentum of the incoming particle. Transmitted tracks are assigned to t bins of width $5.0 \times 10^{-4}\,\text{GeV}^2/c^2$. Note that we count transmitted tracks in t-bins, rather than in bins of solid angle Ω as discussed in Section 3. Summing the events in the t-bins from zero up to a maximum t_i leads to sets of transmission counts $F_{\text{tr}}(<|t_i|)$ and $E_{\text{tr}}(<|t_i|)$.

4.4. Spectra of uncorrected partial cross sections

Using the counts F_0, E_0, $F_{\text{tr}}(<|t_i|)$, $E_{\text{tr}}(<|t_i|)$ and the mechanical properties of the targets, partial cross sections $\sigma_{\text{part}}(<|t_i|)$ are calculated according to Eq. (4).

Fig. 3 shows some spectra for uncorrected partial cross sections. The strong rise of $\sigma_{\text{part}}(<|t_i|)$ for $|t| < 0.002\,\text{GeV}^2/c^2$ is ascribed to multiple scattering in the target and the finite angular resolution of ≈ 25 µrad. Differing levels of partial cross-section spectra for beam particles of different kind indicate nicely the dependence of the total cross section on the projectile type.

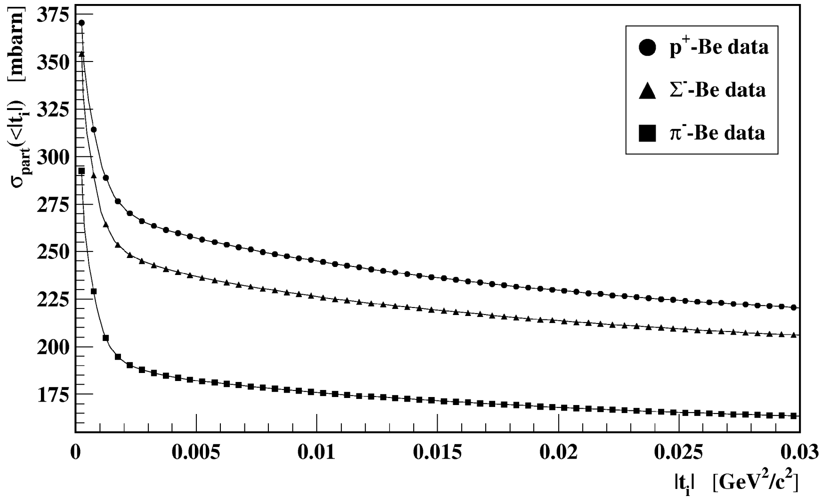

Fig. 3. Spectra of uncorrected partial cross sections resulting from beryllium target data sets.

4.5. Corrections for non-hadronic effects

Partial cross sections were corrected for single Coulomb scattering (C) and for the Coulomb–Nuclear interference effect (CN) evaluating the expression:

$$\sigma_{\mathrm{part}}^{\mathrm{corr}}\big(<|t_i|\big)=\sigma_{\mathrm{part}}\big(<|t_i|\big)-\underbrace{\int_{-\infty}^{t_i}\left(\frac{\mathrm{d}\sigma}{\mathrm{d}t'}\right)_{\mathrm{C}}\mathrm{d}t'}_{\text{C correction}}-\underbrace{\int_{-\infty}^{t_i}\left(\frac{\mathrm{d}\sigma}{\mathrm{d}t'}\right)_{\mathrm{CN}}\mathrm{d}t'}_{\text{CN correction}}. \tag{7}$$

Applying the Coulomb correction, a change in the extrapolated cross section of not more than 0.5% is observed for the light targets Be, C and CH_2. For the Cu target a change of up to 11% is noticed. The CN correction is roughly one order of magnitude smaller than the Coulomb correction and has negligible effect on the extrapolated cross section.

4.6. The extrapolation method

As $|t|$ approaches zero, the growth behavior of partial cross sections is governed ideally by the elastic term in Eq. (6). At small $|t|$, hadronic coherent elastic scattering off nuclei dominates. Thus, we obtain for the elastic term in Eq. (6), the expression:

$$\int_0^t\left(\frac{\mathrm{d}\sigma}{\mathrm{d}t'}\right)_{\mathrm{el}}^{\mathrm{hadr}}\mathrm{d}t'=\frac{\sigma_{\mathrm{tot}}^2}{16\pi B_{\mathrm{nuc}}}\big(1+\rho^2\big)\big[1-\mathrm{e}^{B_{\mathrm{nuc}}t}\big], \tag{8}$$

where B_{nuc} is the exponential slope observed in hadronic coherent elastic scattering off nuclei. Therefore, we choose the functional form

$$f(\alpha_1,\alpha_2,t)=\alpha_1\big[1-\mathrm{e}^{\alpha_2 t}\big] \tag{9}$$

to describe the variation of partial cross sections with respect to $|t_i|$.

The parameters α_1 and α_2 are determined in fitting function (9) to differences in corrected partial cross sections of adjacent t-bins in the range of $t_{\mathrm{min}}=-0.007\,\mathrm{GeV}^2/c^2$ to $t_{\mathrm{max}}=-0.03\,\mathrm{GeV}^2/c^2$. The limits t_{max} and t_{min} account for experimental sensitivity to hadronic coherent elastic scattering off nuclei. Their derivation is described in Section 4.6.1.

Starting from the partial cross section $\sigma_{\mathrm{part}}(<|t_{\mathrm{min}}|)$, the total cross section σ_{tot} is determined by extrapolating the t-variation of the partial cross sections from t_{min} to $t=0$ using the expression:

$$\sigma_{\mathrm{tot}}=\sigma_{\mathrm{part}}\big(<|t_{\mathrm{min}}|\big)+\alpha_1\big[1-\mathrm{e}^{\alpha_2 t_{\mathrm{min}}}\big]. \tag{10}$$

4.6.1. The limits t_{min} and t_{max} and the sensitivity of the SELEX experiment to coherent hadronic elastic scattering off nuclei

In measurements of hadron–nucleus cross sections, it is essential that the experiment is sensitive to hadronic coherent elastic scattering off nuclei. Further, one must be able to distinguish coherent from incoherent scattering processes off nucleons. In scattering off nuclei, the nucleus can break up when the energy transfer exceeds the binding energy of its nucleons. This leads to a contribution of incoherent scattering off nucleons for $|t| > 0.015\,\mathrm{GeV}^2/c^2$. In that case, the hadronic differential elastic cross section, entering the elastic term of Eq. (6), contains two parts:

$$\left(\frac{\mathrm{d}\sigma}{\mathrm{d}t}\right)_{\mathrm{el}}^{\mathrm{hadr}} = \underbrace{\frac{\sigma_{\mathrm{tot}}^2(\mathrm{hA})}{16\pi}\left(1+\rho'^2\right)\mathrm{e}^{B_{\mathrm{nuc}}t}}_{\text{coherent scattering}} + \underbrace{N(A)\frac{\sigma_{\mathrm{tot}}^2(\mathrm{hN})}{16\pi}\mathrm{e}^{B_{\mathrm{N}}t}}_{\text{incoherent scattering}}. \tag{11}$$

There is a term for coherent elastic scattering off the nucleus, in which $\sigma_{\mathrm{tot}}(\mathrm{hA})$ is the total nuclear cross section, and a term for incoherent scattering off nucleons, in which $\sigma_{\mathrm{tot}}(\mathrm{hN})$ is the corresponding hadron–nucleon cross section. B_{N} is the slope parameter for scattering off nucleons, and $N(\mathrm{A})$ is a factor describing the effective number of nucleons taking part in the incoherent process for target nuclei of mass A (see [5]).

The contribution of the incoherent term decreases the growth behavior of the elastic term in (6) because B_{N} is typically one or more orders of magnitude smaller than B_{nuc}. An extrapolation based on partial cross sections, selected in a $|t|$-range far above $0.015\,\mathrm{GeV}^2/c^2$ would lead to a systematically lowered cross-section result, because a fraction of the elastic processes would be ignored. Consequently, we looked for a t-interval $[t_{\mathrm{min}}; t_{\mathrm{max}}]$ to select partial cross sections where their growth is dominated by B_{nuc}.

The sensitivity of the SELEX spectrometer to hadronic coherent elastic scattering off nuclei was verified by looking at background-subtracted differential scattering spectra. These spectra, not acceptance-corrected, are defined by:

$$S(t) = \frac{1}{\rho L \Gamma}\left[\frac{F(t)}{F_0} - \frac{E(t)}{E_0}\right]. \tag{12}$$

Here, Γ is the width of the t-bins. $F(t)$ ($E(t)$) is the number of scattering events found in the full-target (empty-target) data sets that fall into the interval $[|t| - \Gamma/2; |t| + \Gamma/2]$.

Fig. 4 shows a typical example of an $S(t)$ spectrum obtained for Σ^- scattering off carbon nuclei. The spectrum shows three regions governed by apparently different exponential slopes, which can be explained by contributions of Coulomb scattering, coherent elastic scattering and incoherent elastic scattering comparable to measurements described in [5].

Determinations of the slope parameters B_{nuc} and B_{N} in $S(t)$ spectra showed the expected order of magnitude for all targets, and B_{nuc} agreed quite well with data presented in [6]. Furthermore, the magnitude of B_{nuc} is also reflected by the size of parameter α_2 in Eq. (10), when applying the extrapolation.

From such studies, we choose $t_{\mathrm{max}} = -0.03\,\mathrm{GeV}^2/c^2$, as this value is well inside the region dominated by coherent hadronic elastic scattering off nuclei for all targets. The contribution of the integrated incoherent term at this t_{max} is much lower than the integrated coherent term.

To avoid large multiple-scattering corrections, we chose t_{min} of $-0.007\,\mathrm{GeV}^2/c^2$, so that the angular resolution has negligible effect on the extrapolated total cross section.

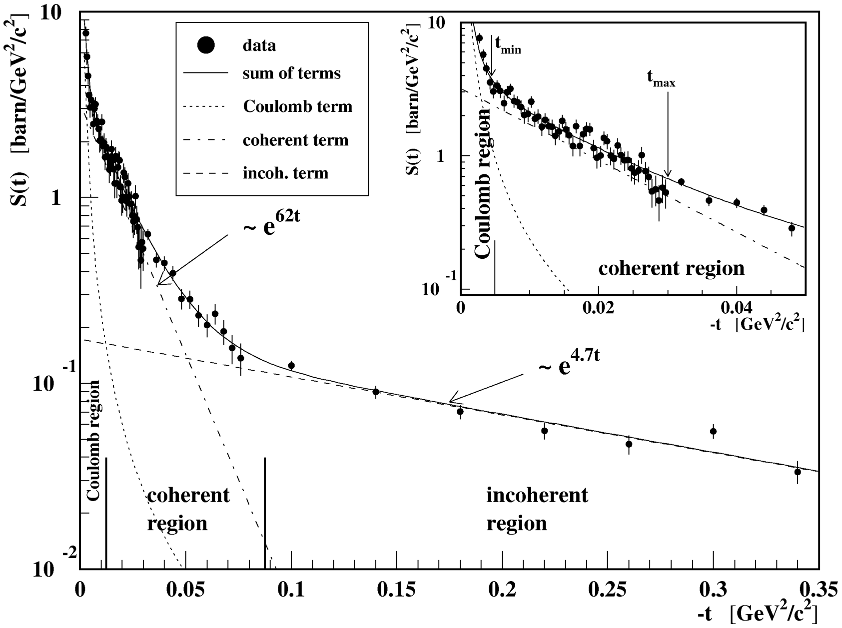

Fig. 4. Differential scattering spectrum obtained for Σ^- carbon reactions, showing the Coulomb, the coherent and the incoherent region.

5. Corrections

5.1. Trigger-rate corrections

The trigger rate influences the reconstruction efficiency for tracks and thus alters the transmission ratios T_{full} and T_{empty} per spill. Fig. 5 shows an instructive example of this effect.

Due to the rate effect, our extrapolated total cross-section experiences a shift Δ_{T0} when the average T0-counts, calculated for all empty and all full-target spills separately, differ.

To determine the shift Δ_{T0} we calculate full and empty-target transmission ratios per spill for $|t| < 0.01\,\text{GeV}^2/c^2$ and describe their rate dependency by fitting to the expression

$$\mathrm{T}_{\text{fit}}(\mathrm{T0}) = \tilde{\beta}_{1,k} + \tilde{\beta}_{2,k}\,\mathrm{T0}^k. \tag{13}$$

We have studied the effect of different powers ($k = 2, 3, 4$) to estimate systematic errors.

We choose the average T0-rate $\overline{\mathrm{T0}}$, comprising all full and all empty-target spills as reference rate for the rate correction. Thus, transmission ratios per spill are corrected by evaluating:

$$\underbrace{T_{j,k}^{\mathrm{T0}}}_{\text{corrected}} = \underbrace{T_j\big(|t| < 0.01\,\text{GeV}^2/c^2\big)}_{\text{uncorrected}} + \underbrace{\tilde{\beta}_{2,k}\big(\overline{\mathrm{T0}}^k - \mathrm{T0}_j^k\big)}_{\text{correction}}, \tag{14}$$

which results in a set of corrected transmission ratios $T_{j,k}^{\mathrm{T0}}$. Fit-function dependent offsets $\Delta_{\mathrm{T0},k}$ are deduced by:

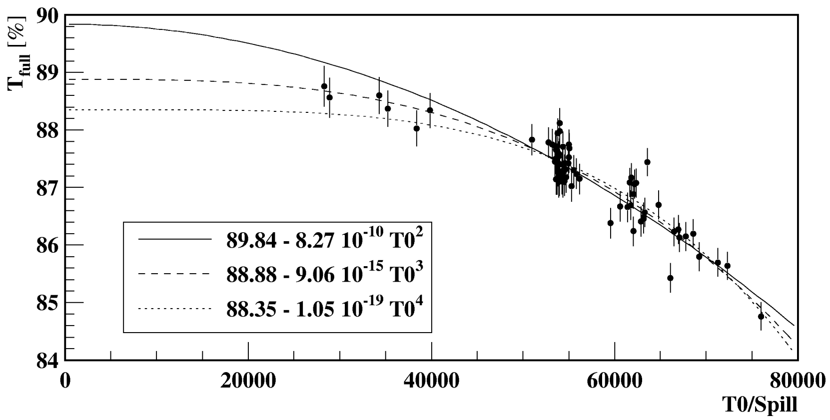

Fig. 5. Dependency of full-target transmission ratios on the T0-count.

$$\Delta_{\mathrm{T0},k} = \sigma_{\mathrm{part}}^{\mathrm{T0},k}\big(< 0.01\ \mathrm{GeV}^2/c^2\big) - \sigma_{\mathrm{part}}\big(< 0.01\ \mathrm{GeV}^2/c^2\big) \tag{15}$$

and averaged to a mean offset Δ_{T0}. Total cross sections are then corrected by:

$$\sigma_{\mathrm{tot}}^{\mathrm{T0}} = \sigma_{\mathrm{tot}} + \Delta_{\mathrm{T0}}. \tag{16}$$

Averaged sizes of the rate correction are presented in Table 2. We want to mention that the copper data were taken at higher rate, where the slope of function (13) is steeper. This, together with the small thickness of the copper target, causes large corrections.

5.2. Corrections for beam contaminants

A transition radiation detector does not make an exact particle identification because of statistical fluctuations in X-ray generation and background from various processes. Therefore, when selecting the baryon or the meson component of the hyperon beam by applying cuts on the BTRD plane count, we need to account for:

(1) The meson (baryon) contamination in the baryon (meson) sample and the effect on the total cross section.
(2) The baryon (meson) contamination in a specific sample for a measurement with protons or Σ^- (π^-) and the effect on the total cross section.

Once the contaminant fraction ε is determined, the experimental cross section $\sigma_{\mathrm{tot}}^{\mathrm{exp}}$ can be corrected by the term Δ_{cont} using:

$$\sigma_{\mathrm{tot}}^{(1)} = \sigma_{\mathrm{tot}}^{\mathrm{exp}} + \underbrace{\frac{1}{\rho L}\log\Big[1 + \varepsilon^{(2)}\big(\mathrm{e}^{-\rho L(\sigma_{\mathrm{tot}}^{(2)} - \sigma_{\mathrm{tot}}^{(1)})} - 1\big)\Big]}_{\text{Correction } \Delta_{\mathrm{cont}}}. \tag{17}$$

This formula was derived in [7] for a two component beam having a contamination fraction $\varepsilon^{(2)}$.

5.2.1. Beam contaminant determination

In a first step, total cross sections resulting from data sets are corrected for the fraction of mesons (baryons) in a baryon sample (meson sample) using (17). Therefore, we fit

Table 2
Average sizes of systematic errors and corrections. For explanation of symbols see text of Sections 5.1, 5.2 and 6.1

		Systematic errors						Corrections	
Cross section	p_{lab} [GeV/c]	δ^{extr} [mb]	δ^{BTRD} [mb]	δ^{fluc} [mb]	δ^{rate} [mb]	δ^{cont} [mb]	δ^{tgt} [mb]	Δ_{T0} [mb]	Δ_{cont} [mb]
$\sigma_{\mathrm{tot}}(\mathrm{pBe})$	536	0.91	0.70	0.25	0.35	0.06	0.30	−1.24	0.62
$\sigma_{\mathrm{tot}}(\Sigma^-\mathrm{Be})$	638	1.20	0.49	0.04	0.10	0.07	0.27	−0.93	0.65
$\sigma_{\mathrm{tot}}(\pi^-\mathrm{Be})$	638	0.50	0.17	0.22	0.05	0.61	0.21	−0.79	1.00
$\sigma_{\mathrm{tot}}(\mathrm{pC})$	457	0.90	2.11	0.54	0.38	0.09	0.47	11.22	0.91
$\sigma_{\mathrm{tot}}(\mathrm{pC})$	490	1.81	1.53	0.68	1.15	0.10	0.47	−3.87	0.86
$\sigma_{\mathrm{tot}}(\Sigma^-\mathrm{C})$	598	1.57	1.92	1.21	1.18	0.13	0.43	−6.42	1.12
$\sigma_{\mathrm{tot}}(\pi^-\mathrm{C})$	591	1.30	1.40	1.50	0.95	0.63	0.33	−3.11	1.03
$\sigma_{\mathrm{tot}}(\Sigma^-\mathrm{CH_2})$	589	2.10	2.55	0.69	0.16	0.16	0.30	3.67	1.44
$\sigma_{\mathrm{tot}}(\pi^-\mathrm{CH_2})$	585	1.26	0.96	0.54	0.12	0.75	0.23	2.90	1.21
$\sigma_{\mathrm{tot}}(\Sigma^-\mathrm{Cu})$	609	163	41	76	41	0.33	1.23	−754	3.1
$\sigma_{\mathrm{tot}}(\pi^-\mathrm{Cu})$	608	85	52	78	36	2.99	1.03	−649	4.7

function (1) to normalized BTRD signal spectra, which are recorded for norm tracks. For negative beam these fits yield an average baryon fraction ($\kappa_1 + \kappa_2$) of (47.5 ± 1.6)% and an average meson fraction ($\kappa_3 + \kappa_4$) of (52.5 ± 1.6)%. For positive beam, we measure a baryon fraction of (91.9 ± 1.4)% and a meson fraction of (8.1 ± 1.4)%. To deduce the meson (baryon) contaminant fraction ε, we sum the meson (baryon) component of (1) over the TRD plane count region shown in Fig. 2. Further, the difference $\sigma_{\text{tot}}^{(2)} - \sigma_{\text{tot}}^{(1)}$ is calculated in taking rate corrected extrapolated cross-section results obtained for the meson and the baryon beam component.

In a second step, we account for the main contaminant disturbing a specific measurement for protons, Σ^- and π^-. According to the expected hyperon-beam composition we correct:

(1) For the effect of Ξ^- particles in the baryon sample, when measuring Σ^-A cross sections.
(2) For the effect of Σ^+ particles in the baryon sample, when measuring pA cross sections.
(3) For the effect of K^- particles in the meson sample, when measuring π^-A cross sections.

For case (1), we measure the overall fraction of Ξ^- particles in each negative-beam data sample and for case (2) we measure the overall fraction of Σ^+ particles in each positive-beam data sample. Therefore, we count the decays $\Sigma^- \to n + \pi^-$, $\Xi^- \to \Lambda^0 + \pi^-$ and $\Sigma^+ \to n + \pi^+$, reconstructed for a known number of norm tracks within the field-free region of the M1 magnet. Fig. 6 shows some hyperon-mass spectra obtained by the decay reconstruction.

Particle decay counts are corrected for geometrical acceptance, branching ratio and decay losses after the target, to yield the overall hyperon contaminant fractions. Here, we find an overall Ξ^- fraction of (41.18 ± 0.06)%, and an overall Σ^+ fraction of (2.7 ± 0.7)%. These fractions are then divided by the baryon fraction ($\kappa_1 + \kappa_2$), known from the first step

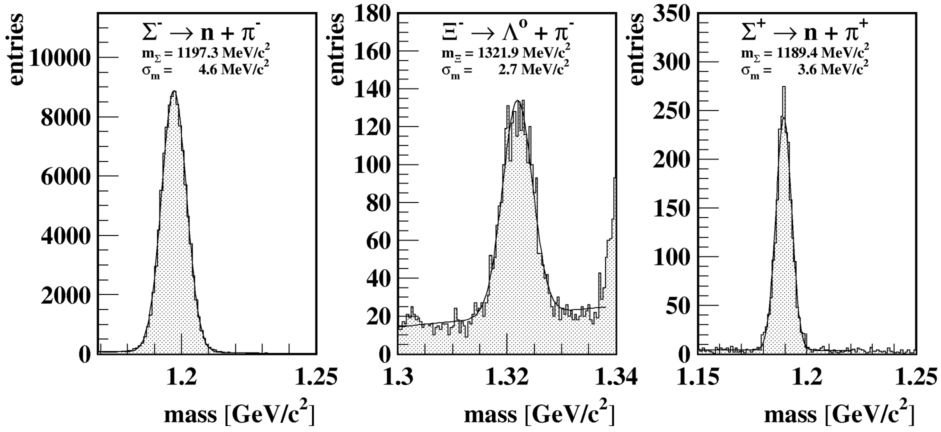

Fig. 6. Hyperon-mass spectra obtained from reconstructed Σ^-, Ξ^- and Σ^+ decays. The spectra are fit to a Gaussian plus a linear background function. m_X is the mean mass found for hyperon X and σ_m is the corresponding mass resolution resulting from the fit.

procedure to yield the hyperon contaminant fraction ε of the baryon component.

Case (3) requires knowledge of the number of K^- particles in the meson sample. The SELEX spectrometer cannot differentiate between $600\,\mathrm{GeV}/c$ π^- and K^- particles. We estimate the overall fraction of K^- particles in the sample using particle-flux parameterizations of [1]. This results in an overall K^- fraction of $(1.6 \pm 1.0)\%$, which divided by the meson fraction $(\kappa_3 + \kappa_4)$ yields the K^- contaminant fraction of the meson component.

Calculating the contaminant correction using Eq. (17) requires knowledge of the total cross sections $\sigma_{\mathrm{tot}}(\Xi^-\mathrm{A})$, $\sigma_{\mathrm{tot}}(\Sigma^+\mathrm{A})$ and $\sigma_{\mathrm{tot}}(\mathrm{K}^-\mathrm{A})$. As data on these cross sections are either scarce or do not exist, we estimate them using approximations like:

$$\sigma_{\mathrm{tot}}(\Xi^-\mathrm{A}) \approx \sigma_{\mathrm{tot}}(\Xi^- p)\frac{\sigma_{\mathrm{tot}}(\mathrm{pA})}{\sigma_{\mathrm{tot}}(\mathrm{pp})}, \tag{18}$$

and neglect weak energy dependencies. Necessary data for hadron–nucleon cross sections are taken from [4,8] and data for pA-cross sections are taken from [9].

Averaged sizes of the contaminant correction including both correction steps are shown in Table 2.

6. Results for hadron–nucleus cross sections

Total cross sections as well as their statistical and systematic errors were determined for each dataset separately. In order to calculate average total cross sections and average systematic errors, we use weighted means. We present the error contributions, the data averaging method and the final results.

6.1. Measurement errors

6.1.1. The statistical error

The dominant error contribution is the statistical error, which is governed by the statistical uncertainty of the partial cross section $\sigma_{\mathrm{part}}(< |t_{\mathrm{min}}|)$, used in the extrapolation. Further statistical error contributions, originating in other terms of the error propagated formula (10), are negligible. The statistical errors for each measurement are presented in Table 3.

6.1.2. Systematic errors

In this section we briefly describe the systematic errors found during the data analysis. Table 2 gives an overview of the average sizes of these errors as well as the rate correction and the contaminant correction.

Systematic error of the extrapolation δ^{extr}

A significant systematic error contribution is the choice of t_{min} for extrapolation of partial cross sections. This error is based on the RMS-spread (root mean square) of the extrapolated total cross section when t_{min} is varied from $-0.004\,\mathrm{GeV}^2/c^2$ to $-0.01\,\mathrm{GeV}^2/c^2$.

Table 3
Results for nuclear total cross sections. For explanation of symbols see text of Section 6.2

Cross section	p_{lab} [GeV/c]	$\overline{\sigma}_{\mathrm{tot}}$ [mb]	$\delta^{\mathrm{stat}}\overline{\sigma}_{\mathrm{tot}}$ [mb]	$\delta^{\mathrm{syst}}\overline{\sigma}_{\mathrm{tot}}$ [mb]	$\delta^{\mathrm{tot}}\overline{\sigma}_{\mathrm{tot}}$ [mb]
$\sigma_{\mathrm{tot}}(\mathrm{pBe})$	536	268.6	±0.7	±1.3	±1.5
$\sigma_{\mathrm{tot}}(\Sigma^-\mathrm{Be})$	638	249.1	±0.9	±1.3	±1.6
$\sigma_{\mathrm{tot}}(\pi^-\mathrm{Be})$	638	188.7	±0.8	±0.9	±1.2
$\sigma_{\mathrm{tot}}(\mathrm{pC})$	457	333.6	±3.1	±2.4	±3.9
$\sigma_{\mathrm{tot}}(\mathrm{pC})$	490	335.4	±3.6	±2.9	±4.6
$\sigma_{\mathrm{tot}}(\Sigma^-\mathrm{C})$	598	308.9	±2.1	±3.8	±4.3
$\sigma_{\mathrm{tot}}(\pi^-\mathrm{C})$	591	234.1	±1.5	±3.1	±3.5
$\sigma_{\mathrm{tot}}(\Sigma^-\mathrm{CH}_2)$	589	376.4	±2.0	±4.1	±4.5
$\sigma_{\mathrm{tot}}(\pi^-\mathrm{CH}_2)$	585	286.1	±1.3	±2.0	±2.4
$\sigma_{\mathrm{tot}}(\Sigma^-\mathrm{Cu})$	609	1232	±133	±192	±233
$\sigma_{\mathrm{tot}}(\pi^-\mathrm{Cu})$	608	1032	±77	±162	±179

Cut on the BTRD signal spectrum δ^{BTRD}

Although contaminant and rate corrections are applied for each specific cut on the BTRD signal spectrum, we still observe a variation of the cross section when varying the cut on the TRD plane count by ±1 unit around its nominal value. Therefore, we calculate a systematic error, which is the maximum spread in the cross sections found in the cut variation.

Spill to spill fluctuations δ^{fluc}

Here, we compare the statistical error in $\sigma_{\mathrm{part}}(< 0.01\,\mathrm{GeV}^2/c^2)$, which we calculate from (5) with the error in $\sigma_{\mathrm{part}}(< 0.01\,\mathrm{GeV}^2/c^2)$ calculated from the experimentally observed RMS-spread of rate corrected transmission ratios per spill. The difference in these errors accounts for remaining non statistical spill to spill fluctuations.

Systematic error of the rate correction δ^{rate}

This error takes into account the error arising from different functional attempts to describe the rate effect presented in Section 5.1. Its value is given by the maximum spread of the $\Delta_{\mathrm{T0},k}$ with respect to their average value Δ_{T0}.

Systematic error of the contaminant correction δ^{cont}

This systematic error accounts for the uncertainty in the fit parameters of the four-fold binomial distribution (1) and for the uncertainty in the contaminant fractions for Σ^+, Ξ^- and K^-.

Uncertainty of the target density δ^{tgt}

The target densities were measured several times, using a pycnometer and a buoyancy method. Laboratory studies showed systematic discrepancies in the density measurement, which are included in the density errors shown in Table 1. These errors are propagated to an error contribution to the total cross sections, which are on a 0.1% level.

6.2. Data-averaging and results on hadron–nucleus cross sections

6.2.1. The average total cross section

Total cross-section results $\sigma_{\mathrm{tot},i}$, obtained from $i = 1, \ldots, N$ data sets, are combined to an average total cross section $\overline{\sigma}_{\mathrm{tot}}$ with a statistical error $\delta^{\mathrm{stat}}\overline{\sigma}_{\mathrm{tot}}$ and an average systematic error $\delta^{\mathrm{syst}}\overline{\sigma}_{\mathrm{tot}}$. The results are shown in Table 3.

We average the total cross-sections $\sigma_{\mathrm{tot},i}$ that correspond to a specific measurement using the weighted mean:

$$\overline{\sigma}_{\mathrm{tot}} = \frac{\sum_{i=1}^{N} \omega_i\, \sigma_{\mathrm{tot},i}}{\sum_{i=1}^{N} \omega_i}, \qquad \omega_i = \frac{1}{(\delta_i^{\mathrm{stat}})^2 + \sum_{j=1}^{M} (\delta_{j,i}^{\mathrm{syst}})^2}. \tag{19}$$

The weight ω_i includes the statistical error (δ_i^{stat}) of data set i and all $j = 1, \ldots, M$ systematic errors $\delta_{j,i}^{\mathrm{syst}}$ described in Sections 6.1.1 and 6.1.2.

The statistical error is supposed to decrease when adding more data to the evaluation. We calculate the statistical error in the averaged total cross section by:

$$\delta^{\mathrm{stat}}\overline{\sigma}_{\mathrm{tot}} = \sqrt{1 \Big/ \sum_{i=1}^{N} \frac{1}{(\delta_i^{\mathrm{stat}})^2}}. \tag{20}$$

In assigning a systematic error to an average total cross section $\overline{\sigma}_{\mathrm{tot}}$ we assume that the systematic errors of the single measurements can be just averaged. Therefore, we quote as average systematic error:

$$\delta^{\mathrm{syst}}\overline{\sigma}_{\mathrm{tot}} = \sqrt{\frac{\sum_{i=1}^{N} \omega_i \left[\sum_{j=1}^{M} (\delta_{j,i}^{\mathrm{syst}})^2\right]}{\sum_{i=1}^{N} \omega_i}}, \tag{21}$$

using the weights ω_i defined in (19).

Further, we quote a total error $\delta^{\mathrm{tot}}\overline{\sigma}_{\mathrm{tot}}$ of the average total cross section, which is calculated from:

$$\delta^{\mathrm{tot}}\overline{\sigma}_{\mathrm{tot}} = \sqrt{\left(\delta^{\mathrm{stat}}\overline{\sigma}_{\mathrm{tot}}\right)^2 + \left(\delta^{\mathrm{syst}}\overline{\sigma}_{\mathrm{tot}}\right)^2}. \tag{22}$$

6.3. Comparison to existing data on hadron–nucleus total cross sections

A literature survey showed that experimental data on hadron–nucleus total cross sections for charged projectiles at high energies are extremely scarce. Information for proton–nucleus and π^-–nucleus total cross sections is only provided by the Refs. [5,6,9] and displayed together with our results in the Figs. 7, 8, 9, 10 and 11. No data were found for Σ^-–nucleus total cross sections.

6.3.1. Comparison of nucleon–nucleus total cross sections

In Figs. 7 and 8 we display a compilation of proton–nucleus and neutron–nucleus cross sections extracted from [7,10–12] together with our results. As can be seen, the proton–nucleus cross sections of [5] at $p_{\mathrm{lab}} = 20\,\mathrm{GeV}/c$ and the neutron–nucleus cross sections are

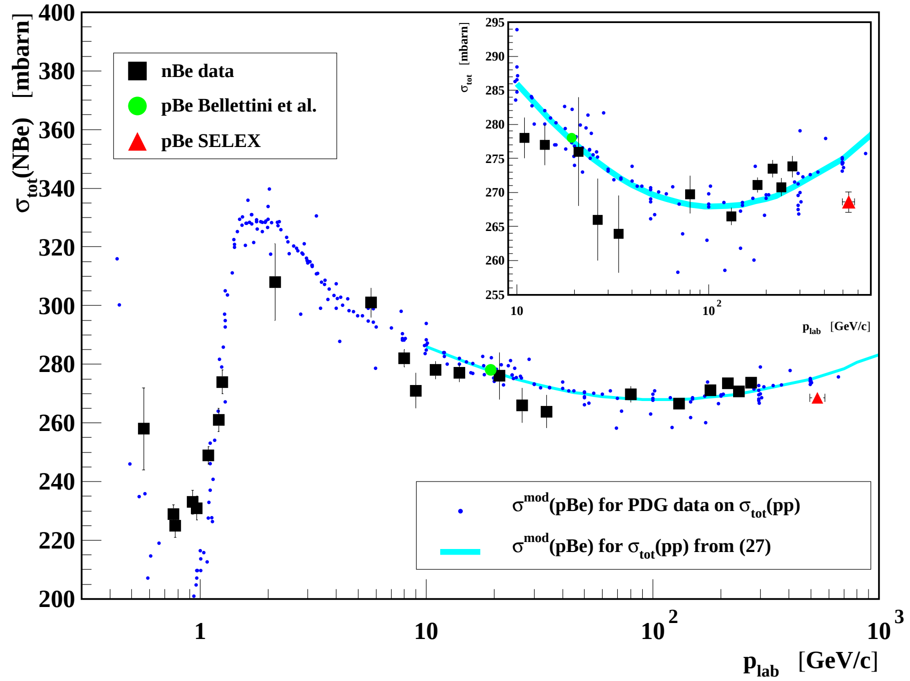

Fig. 7. Summary of experiment data on $\sigma_{\text{tot}}(\text{nBe})$ from [7,10–12] and on $\sigma_{\text{tot}}(\text{pBe})$ from [5] and SELEX (the colour version of this figure can be seen on the Nuclear Physics Electronic website: http://www.elsevier.nl/locate/npe). Overlaid are results from the model calculation (see Section 7).

similar. For this reason we assume that differences between neutron–nucleus and proton–nucleus cross sections are negligibly small above 20 GeV/c. This allows a comparison of our proton–nucleus cross sections with corresponding neutron–nucleus cross-section data available at much higher energy.

Comparing our results with neutron–nucleus cross sections at 131−273 GeV/c (data of [7]) shows that our measurements follow the trend of these data points. Averaging the neutron–beryllium total cross sections in this momentum range results in 271.0 ± 0.6 mb, which is close to our proton–beryllium cross section at 536 GeV/c of 268.6 ± 1.5 mb. A similar calculation for the neutron–carbon cross section gives a mean value of 331.0 ± 0.8 mb, which is close to our measurements of the proton–carbon cross section around 457 GeV/c of 333.6 ± 3.9 mb.

6.4. *Comparison of π^-–nucleus total cross sections*

High-energy data for $\sigma_{\text{tot}}(\pi^-\text{Be})$, $\sigma_{\text{tot}}(\pi^-\text{C})$ and $\sigma_{\text{tot}}(\pi^-\text{Cu})$ that were determined using a transmission technique are presented in the thesis of A. Schiz [9]. Unfortunately, the statistical errors quoted for the π^-A total cross-sections are quite large and mask other corrections. Luckily, on the basis of [9], a publication on hadron–nucleus elastic scattering appeared [6], where fits to elastic π^-–nucleus scattering data are performed. We extracted

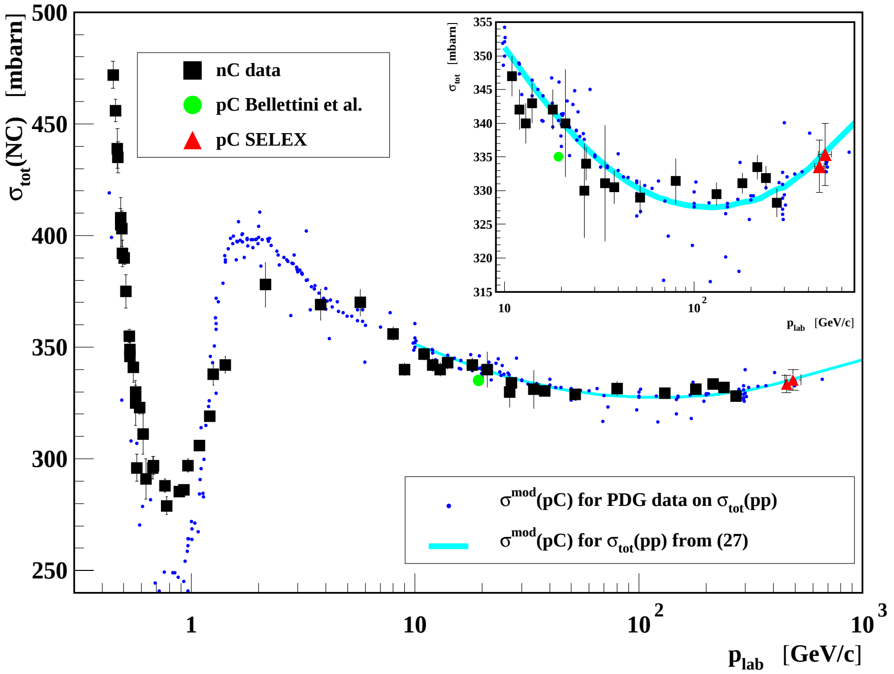

Fig. 8. Summary of experiment data on $\sigma_{\text{tot}}(\text{nC})$ from [7,10–12] and on $\sigma_{\text{tot}}(\text{pC})$ from [5] and SELEX (the colour version of this figure can be seen on the Nuclear Physics Electronic website: http://www.elsevier.nl/locate/npe). Overlaid are results from the model calculation (see Section 7).

π^-–nucleus total cross-sections from these fits and present them in the Figs. 9, 10 and 11.

The Figs. 9, 10 and 11 show that the SELEX results for π^-Be, π^-C and π^-Cu cross sections are quite comparable to the data from [6]. However, more precise data are needed to do a detailed comparison.

7. Model description of hadron–nucleus cross sections

In this section, we introduce a model calculation for hadron–nucleus cross sections and show how well it describes the data.

7.0.1. The Glauber model and the inelastic screening correction

As shown in [7], the Glauber model [13,14] including an inelastic screening correction [15], is very precise in describing neutron–nucleus cross sections at high energy. The Glauber model accounts for the elastic screening effect in nuclei via multiple elastic scattering between the incident hadron h and the nucleons N. As mentioned in [7], nuclear total cross sections calculated by the Glauber model exceed experimental data. This is compensated by taking into account the inelastic screening correction described in [15]. It accounts for inelastic reactions $\text{h} + \text{N} \to \text{N} + \text{X}$, which produce an inelastic screening effect. Consequently, a model cross section $\sigma_{\text{tot}}^{\text{mod}}$ comprises two parts:

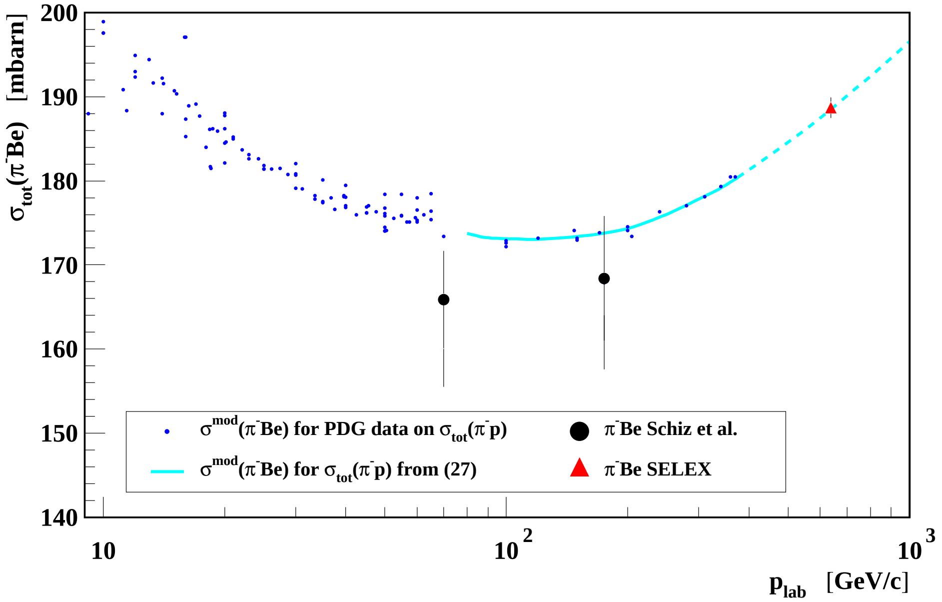

Fig. 9. Summary of experiment data on $\sigma_{\mathrm{tot}}(\pi^{-}\mathrm{Be})$ from [6] and SELEX (the colour version of this figure can be seen on the Nuclear Physics Electronic website: http://www.elsevier.nl/locate/npe). Overlaid are results from the model calculation (see Section 7).

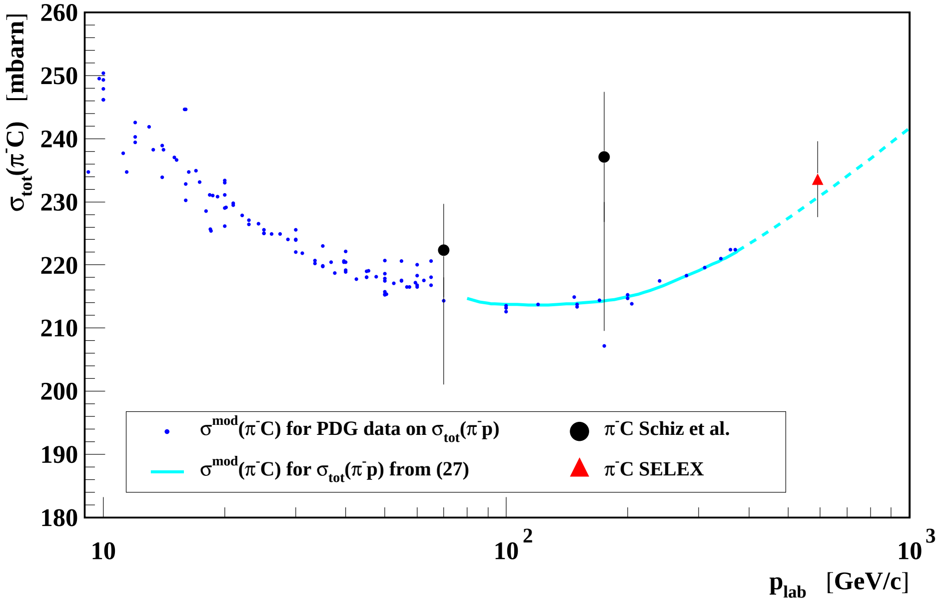

Fig. 10. Summary of experiment data on $\sigma_{\mathrm{tot}}(\pi^{-}\mathrm{C})$ from [6] and SELEX (the colour version of this figure can be seen on the Nuclear Physics Electronic website: http://www.elsevier.nl/locate/npe). Overlaid are results from the model calculation (see Section 7).

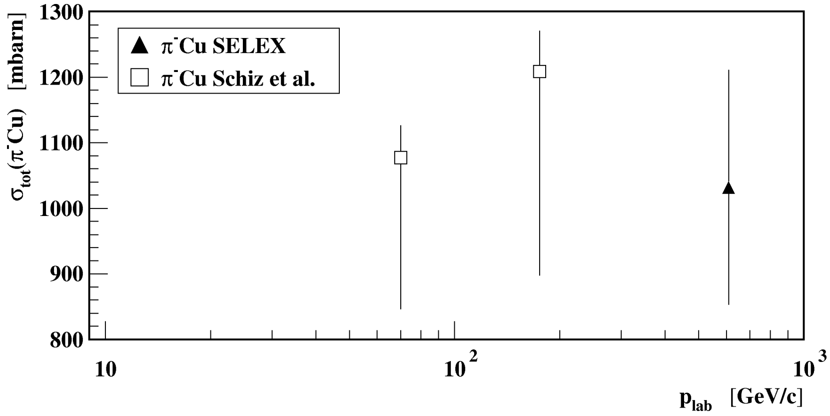

Fig. 11. Results for $\sigma_{\mathrm{tot}}(\pi^-\mathrm{Cu})$ from [6] and SELEX.

$$\sigma_{\mathrm{tot}}^{\mathrm{mod}}\big(A,\sigma_{\mathrm{tot}}(\mathrm{hN})\big) = \sigma_{\mathrm{tot}}^{\mathrm{Gla}}\big(A,\sigma_{\mathrm{tot}}(\mathrm{hN})\big) - \Delta\sigma^{\mathrm{Kar}}. \tag{23}$$

These are a Glauber model cross section $\sigma_{\mathrm{tot}}^{\mathrm{Gla}}(A,\sigma_{\mathrm{tot}}(\mathrm{hN}))$ and an inelastic screening correction $\Delta\sigma^{\mathrm{Kar}}$.

The Glauber model cross section

According to [14], $\sigma_{\mathrm{tot}}^{\mathrm{Gla}}(A,\sigma_{\mathrm{tot}}(\mathrm{hN}))$ can be calculated by:

$$\sigma_{\mathrm{tot}}^{\mathrm{Gla}}(\mathrm{hA}) = 4\pi\,\Re e\left\{\int_0^\infty 1 - \left[1 - \frac{(1-\mathrm{i}\rho'_{\mathrm{hN}})}{2}\,\sigma_{\mathrm{tot}}(\mathrm{hN})T(b)\right]^{\mathrm{A}} b\,\mathrm{d}b\right\},$$

$$T(b) = \frac{1}{2\pi}\int_0^\infty J_0(qb)\mathrm{e}^{-B_{\mathrm{hN}}\frac{q^2}{2}}S(q)q\,\mathrm{d}q, \quad S(q) = \frac{4\pi}{q}\int_0^\infty r\sin(qr)\tilde{\rho}(r)\,\mathrm{d}r. \tag{24}$$

Here ρ'_{hN} is the real to the imaginary part of the elastic scattering amplitude in the forward direction observed in hadron–nucleon elastic scattering and b is the impact parameter. B_{hN} is the hadronic slope in hadron–nucleon elastic scattering and J_0 is a Bessel function of order zero. The nuclear density $\tilde{\rho}(r)$ is normalized as:

$$4\pi\int_0^\infty \tilde{\rho}(r)r^2\,\mathrm{d}r = 1. \tag{25}$$

The inelastic screening correction

The inelastic screening correction $\Delta\sigma^{\mathrm{Kar}}$, originally formulated in [15] for proton–nucleus reactions, is generalized by:

$$\Delta\sigma^{\mathrm{Kar}} = 4\pi\int_0^\infty \int_{(m_{\mathrm{p}}+m_\pi)^2}^{(\sqrt{s}-m_{\mathrm{p}})^2} \left(\frac{\mathrm{d}^2\sigma}{\mathrm{d}t\,\mathrm{d}M^2}\right)_{t=0} \mathrm{e}^{-\frac{1}{2}\sigma_{\mathrm{tot}}(\mathrm{hN})A\widetilde{\mathrm{T}}(b)}\big|F(q_{\mathrm{L}},\vec{b})\big|^2\mathrm{d}M^2\,\mathrm{d}^2b,$$

$$\widetilde{\mathrm{T}}(\vec{b}\,) = \int_{-\infty}^{+\infty} \tilde{\rho}(\vec{b}, z)\,\mathrm{d}z, \qquad F(q_{\mathrm{L}}, b) = A \int_{-\infty}^{+\infty} \tilde{\rho}(b, z)\mathrm{e}^{\mathrm{i}q_{\mathrm{L}}z}\,\mathrm{d}z,$$
$$q_{\mathrm{L}} = \left(M^2 - m_{\mathrm{p}}^2\right)\frac{m_{\mathrm{p}}}{s}. \tag{26}$$

Here m_{p} is the proton mass and m_π is the pion mass. The double differential cross section $\mathrm{d}^2\sigma/\mathrm{d}t\,\mathrm{d}M^2$ describes the inelastic reaction $\mathrm{h} + \mathrm{N} \to \mathrm{N} + \mathrm{X}$ of the incident hadron h with a nucleon N, where the resulting final state X has an invariant mass squared of M^2.

7.0.2. Input parameters for the total cross-section model

Model input parameters are $\sigma_{\mathrm{tot}}(\mathrm{hN})$, ρ'_{hN}, B_{hN}, $\tilde{\rho}(r)$ and $(\mathrm{d}^2\sigma/\mathrm{d}t\,\mathrm{d}M^2)|_{t=0}$. All of them are extracted from experimental data with $N = \mathrm{p}$.

Input parameter $\sigma_{\mathrm{tot}}(\mathrm{hN})$

Model calculations require values of $\sigma_{\mathrm{tot}}(\mathrm{hN})$ for a wide range of center of mass energies $\sqrt{s}$. We fit data on pp and π^-p total cross sections from [8] to a smooth function:

$$\sigma_{\mathrm{tot}}(\mathrm{hp}, s) = \frac{a_0}{s^{a_1}} + a_2 \log^2(s). \tag{27}$$

The fit-parameters a_i, their errors and the validity range of each parameterization, are shown in Table 4. The result of each parameterization is in mb, when using s in GeV^2.

Table 4
Fit-parameters and validity range of the total cross-section parameterizations

Reaction	a_0	a_1	a_2	Momentum range
pp	49.51 ± 0.26	0.097 ± 0.002	0.314 ± 0.004	$10 \ldots 3000$ GeV/c
π^-p	55.2 ± 7.2	0.255 ± 0.032	0.346 ± 0.020	$80 \ldots 380$ GeV/c

Input parameter ρ'_{hp}

We parameterize $\rho'_{\mathrm{pp}}(p_{\mathrm{lab}})$ and $\rho'_{\pi^-\mathrm{p}}(p_{\mathrm{lab}})$, using data on ρ'_{pp} from [16–28] and data on $\rho'_{\pi^-\mathrm{p}}$ from [16,21,29,30], assuming that ρ' reaches a constant value when p_{lab} goes to infinity. Our fits are

$$\rho'_{\mathrm{pp}}(p_{\mathrm{lab}}) = +\frac{6.8}{p_{\mathrm{lab}}^{0.742}} - \frac{6.6}{p_{\mathrm{lab}}^{0.599}} + 0.124$$
$$\text{for } 0.8\ \mathrm{GeV}/c < p_{\mathrm{lab}} < 2100\ \mathrm{GeV}/c, \tag{28}$$
$$\rho'_{\pi^-\mathrm{p}}(p_{\mathrm{lab}}) = -\frac{0.92}{p_{\mathrm{lab}}^{0.54}} + 0.54$$
$$\text{for } 8.0\ \mathrm{GeV}/c < p_{\mathrm{lab}} < 345\ \mathrm{GeV}/c, \tag{29}$$

where p_{lab} is in GeV/c. Fig. 12 displays these fit-functions together with all data points included in the fit.

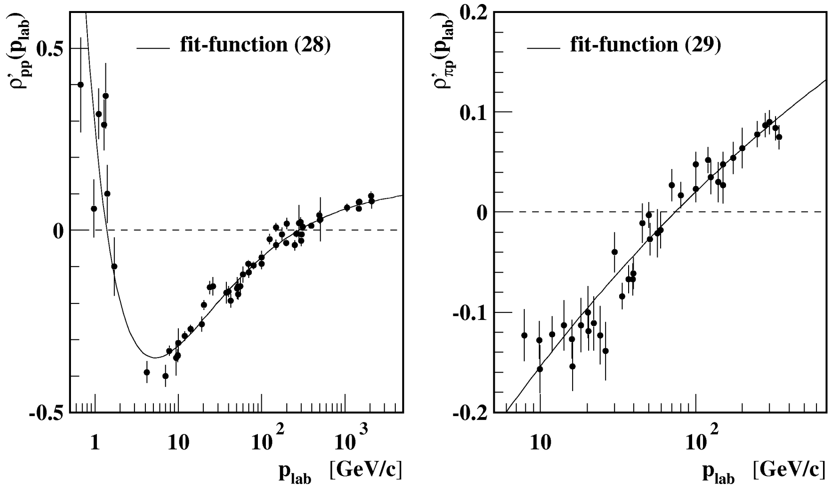

Fig. 12. Our parameterizations for $\rho'_{\mathrm{pp}}(p_{\mathrm{lab}})$ and $\rho'_{\pi^-\mathrm{p}}(p_{\mathrm{lab}})$ together with experimental data from [16–28].

Input parameter B_{hp}

For the hadronic slope parameters B_{pp} and $B_{\pi^-\mathrm{p}}$ we take the parameterizations presented in [31]:

$$B_{\mathrm{pp}}(p_{\mathrm{lab}}) = \begin{cases} B_{\mathrm{pp},1} = 11.13 - \frac{6.21}{\sqrt{p_{\mathrm{lab}}}} + 0.30\log\{p_{\mathrm{lab}}\}, & q^2 = 0.02, \\ B_{\mathrm{pp},2} = 9.26 - \frac{4.94}{\sqrt{p_{\mathrm{lab}}}} + 0.28\log\{p_{\mathrm{lab}}\}, & q^2 = 0.20, \\ B_{\mathrm{pp},3} = 9.67 - \frac{7.51}{\sqrt{p_{\mathrm{lab}}}} + 0.10\log\{p_{\mathrm{lab}}\}, & q^2 = 0.40, \end{cases} \tag{30}$$

$$B_{\pi\mathrm{p}}(p_{\mathrm{lab}}) = \begin{cases} B_{\pi\mathrm{p},1} = 9.11 + \frac{0.65}{\sqrt{p_{\mathrm{lab}}}} + 0.29\log\{p_{\mathrm{lab}}\}, & q^2 = 0.02, \\ B_{\pi\mathrm{p},2} = 6.95 + \frac{0.65}{\sqrt{p_{\mathrm{lab}}}} + 0.27\log\{p_{\mathrm{lab}}\}, & q^2 = 0.20, \\ B_{\pi\mathrm{p},3} = 6.13 + \frac{0.65}{\sqrt{p_{\mathrm{lab}}}} + 0.25\log\{p_{\mathrm{lab}}\}, & q^2 = 0.40. \end{cases} \tag{31}$$

Here, q^2 is in units of GeV^2/c^2. These parameterizations are linearly interpolated to account for the dependency of B_{hN} on both p_{lab} and q^2.

Input parameter $(\mathrm{d}^2\sigma/\mathrm{d}t\,\mathrm{d}M^2)|_{t=0}$

To calculate the inelastic screening correction $\Delta\sigma^{\mathrm{Kar}}$, we use the parameterization of $(\mathrm{d}^2\sigma/\mathrm{d}t\,\mathrm{d}M^2)|_{t=0}$ for the process $\mathrm{p}+\mathrm{p}\to\mathrm{p}+\mathrm{X}$, given in [7]:

$$\left(\frac{\mathrm{d}^2\sigma}{\mathrm{d}t\,\mathrm{d}M^2}\right)_{t=0} = \begin{cases} 26.470\,(M^2-1.17) - 35.969\,(M^2-1.17)^2 \\ \quad + 18.470\,(M^2-1.17)^3 - 4.143\,(M^2-1.17)^4 \\ \quad + 0.341\,(M^2-1.17)^5 & \text{for } 1.17 < M^2 < 5\ \mathrm{GeV}^2/c^2, \\ 4.4/M^2 & \text{for } M^2 > 5\ \mathrm{GeV}^2/c^2. \end{cases} \tag{32}$$

In addition, we also use more recent parameterizations for $(\mathrm{d}^2\sigma/\mathrm{d}t\,\mathrm{d}M^2)|_{t=0}$ to describe the processes $\mathrm{p}+\mathrm{p}\to\mathrm{p}+\mathrm{X}$ and $\pi+\mathrm{p}\to\mathrm{p}+\mathrm{X}$, which are presented in [32] and are based on calculations of triple-Regge diagrams in [33]. For $M^2 \leqslant M_0^2$, these parameterizations

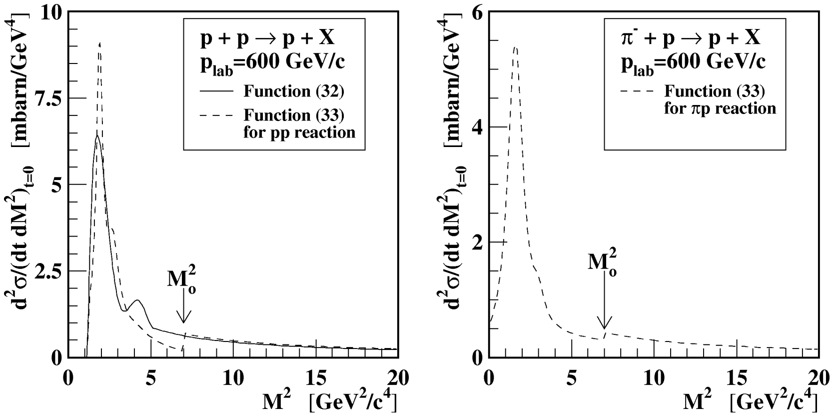

Fig. 13. The parameterizations (32) and (33) evaluated for $p_{\text{lab}} = 600\,\text{GeV}/c$.

consist of a background term and a sum of non-energy-dependent resonance terms. In case $M^2 > M_0^2$ the parameterizations consist of a sum over contributions from triple-Regge diagrams:

$$\left(\frac{\mathrm{d}^2\sigma}{\mathrm{d}t\,\mathrm{d}M^2}\right)_{t=0} = \begin{cases} \sum_i \frac{a_i}{(M^2-M_i^2)^2+\Gamma_i} + \frac{c_f(M^2-M_{\min}^2)}{(M^2-M_{\min}^2)^2+d_f}, & M^2 \leqslant M_0^2, \\ \sum_k V_k \left(\frac{M^2}{s}\right)^{\alpha_{k(0)}-\beta_{k(0)}-\beta'_{k(0)}} \frac{1}{s^{2-\alpha_{k(0)}}}, & M^2 > M_0^2\,. \end{cases} \tag{33}$$

Instead of displaying the large number of parameters for Eq. (33), which are taken from calculations in [33], we display the parameterizations (32) and (33) in Fig. 13.

Compared to (33), parameterization (32) has no s-dependence. Further, parameterization (33) is not continuous and the resonance sizes are quite different for $\mathrm{p} + \mathrm{p} \to \mathrm{p} + \mathrm{X}$.

Input parameter $\tilde{\rho}(r)$

In the calculations, we use density distributions $\tilde{\rho}(r)$ that are based on the harmonic-oscillator model:

$$\tilde{\rho}(r) = \rho_0\left[1 + \tilde{\alpha}\left(\frac{r}{a_{\text{rad}}}\right)^2\right]\mathrm{e}^{-\left(\frac{r}{a_{\text{rad}}}\right)^2}. \tag{34}$$

This offers the possibility to calculate some integrals in an analytic way and gives a better description of the (charge-) density distribution for light nuclei than a standard two-parameter Fermi parameterization. As reported in [7], we also find that the model does not provide a good description of neutron–nucleus total cross sections if one uses both $\tilde{\alpha}$ and a_{rad} from electron-scattering data [34]. Therefore, we used $\tilde{\alpha}$ values from [34] and adjusted the radius parameter a_{rad}, such that the model cross section $\sigma_{\text{tot}}^{\text{mod}}(\text{NA}, a_{\text{rad}})$ gives a best description of nA-cross section data in the momentum range $10-273\,\text{GeV}/c$. Adjusting of a_{rad} was done for each nucleus and for each of the parameterizations (32) and (33) separately. Table 5 gives a summary of the density parameters.

Table 5
Parameters of the density distribution $\tilde{\rho}(r)$ from electron–nucleus elastic scattering [34] and the radius parameters resulting from a fit of $\sigma_{\mathrm{tot}}^{\mathrm{mod}}(a_{\mathrm{rad}}, p_{\mathrm{lab}})$ to nA cross-section data in the momentum range $10-273\,\mathrm{GeV}/c$

Nucleus	Data from [34] e–A scattering		Fit result using (32) in $\sigma_{\mathrm{tot}}^{\mathrm{mod}}(\mathrm{NA}, a_{\mathrm{rad}})$	Fit result using (33) in $\sigma_{\mathrm{tot}}^{\mathrm{mod}}(\mathrm{NA}, a_{\mathrm{rad}})$
	$\tilde{\alpha}$	a_{rad} [fm]	a_{rad} [fm]	a_{rad} [fm]
beryllium	0.611	1.791	1.89981	2.02914
carbon	1.067	1.687	1.79247	1.89277

7.0.3. Results of the model calculations

Results for nucleon–nucleus model cross sections

To show the quality of our model calculation after adjusting the nuclear density parameter a_{rad}, we evaluated the total cross sections $\sigma_{\mathrm{tot}}^{\mathrm{mod}}(\mathrm{Be}, \sigma_{\mathrm{tot}}(\mathrm{pp}))$ and $\sigma_{\mathrm{tot}}^{\mathrm{mod}}(\mathrm{C}, \sigma_{\mathrm{tot}}(\mathrm{pp}))$ using function (32). This was done for data on $\sigma_{\mathrm{tot}}(\mathrm{pp})$ taken from [8] and for values on $\sigma_{\mathrm{tot}}(\mathrm{pp})$ resulting from our fit (27). The calculations were done at many different values of p_{lab} to show the behavior over the entire high momentum region. Scatter in the model calculations (observed when experimental data on $\sigma_{\mathrm{tot}}(\mathrm{pp})$ are used) demonstrate the sensitivity of the model to small changes in $\sigma_{\mathrm{tot}}(\mathrm{pp})$.

Summaries of calculation and data are shown in Figs. 7 and 8. They show that the calculations reflect quite well the cross-section data for $p_{\mathrm{lab}} > 5\,\mathrm{GeV}/c$. The nBe data of [7] in the range $131-273\,\mathrm{GeV}/c$ suggest a rise of the nBe cross section with energy that is also indicated by the model calculation. Our data point does not show any rise for pBe. In the case of nC cross sections our measurements join both data at lower energy and calculation very nicely.

Results for π^-–nucleus model cross sections

We evaluated the cross sections $\sigma_{\mathrm{tot}}^{\mathrm{mod}}(\mathrm{Be}, \sigma_{\mathrm{tot}}(\pi^-\mathrm{p}))$ and $\sigma_{\mathrm{tot}}^{\mathrm{mod}}(\mathrm{C}, \sigma_{\mathrm{tot}}(\pi^-\mathrm{p}))$ using function (33) and the corresponding nuclear density parameter a_{rad}, which was determined by a fit of the model cross section to neutron–nucleus data. All further input parameters are specific for π^-p-reactions. The calculations were done for data on $\sigma_{\mathrm{tot}}(\pi^-\mathrm{p})$ taken from [8] and for values from function (27).

Results are shown in Figs. 9 and 10 together with data for π^-–nucleus total cross sections from [6] and the SELEX experiment. The figures show that the calculations match our measurements quite well and agree within errors with lower-energy data from [6].

8. Results for hadron–nucleon cross sections

The hadron–nucleon cross sections $\sigma_{\mathrm{tot}}(\Sigma^-\mathrm{N})$ and $\sigma_{\mathrm{tot}}(\pi^-\mathrm{N})$ were first determined by a CH_2–C method. As this method provides hadron–nucleon cross sections only with a precision on the order of 10%, we improved the precision using a method which takes advantage of the more precise hadron–nucleus cross-section ratios.

8.1. Hadron–nucleon cross sections using a CH_2–C difference method

The hadron–nucleon cross sections $\sigma_{\text{tot}}(\Sigma^-\text{N})$ and $\sigma_{\text{tot}}(\pi^-\text{N})$ can be deduced from corresponding cross sections measured on carbon and polyethylene by:

$$\sigma_{\text{tot}}(\text{hN}) = \frac{1}{2}\left[\sigma_{\text{tot}}(\text{hCH}_2) - \sigma_{\text{tot}}(\text{hC})\right], \tag{35}$$

where h denotes the incident hadron. Results obtained by this method are presented in Table 9. The quoted errors are calculated from the total errors in the hadron–nucleus cross sections given in Table 3.

8.2. Hadron–nucleon cross sections deduced from hadron–nucleus cross sections

In a second approach, we deduce hadron–nucleon cross sections from ratios of measured hadron–nucleus cross sections. To motivate the method, we first derive empirical relations between hadron–nucleon and hadron–nucleus cross section ratios, which we then refine using the model calculation described in Section 7.

To derive empirical relations between hadron–nucleon and hadron–nucleus cross section ratios we use data on hadron–nucleon cross sections around $137\,\text{GeV}/c$ from [4,8], and obtain the hadron–nucleon cross-section ratios:

$$\frac{\sigma_{\text{tot}}(\pi^-\text{p})}{\sigma_{\text{tot}}(\text{pp})} \approx 0.635 \pm 0.006, \qquad \frac{\sigma_{\text{tot}}(\Sigma^-\text{p})}{\sigma_{\text{tot}}(\text{pp})} \approx 0.901 \pm 0.012. \tag{36}$$

Next, we build nuclear cross-section ratios using our measurements for the Σ^-A, π^-A and pA cross sections from Table 3.

Our pA cross sections were measured at lower laboratory momentum than the corresponding Σ^-A or π^-A cross sections. To correct for this, we scale the pA cross sections by a factor k_{scale} before building the cross-section ratio. The scale factor takes into account the growth of the pA cross section from the laboratory momentum where it was measured to the larger laboratory momentum of the corresponding Σ^-A or π^-A cross section. Scaling factors are calculated using the model described in Section 7. They are displayed together with the nuclear cross-section ratios in Table 6.

The nuclear ratios show that the π^-A cross sections are about 0.7 times and the Σ^-A cross sections are about 0.92 times as large as the pA cross section.

To get a first relation between hadron–nucleon and hadron–nucleus cross sections, we ignore the weak energy dependence of the cross-section ratios. Calculating the ratios of hadron–nucleon to hadron–nucleus cross-section ratios using the above data gives the results presented in Table 7.

The double ratios show a small but significant deviation from one especially for ratios involving π^- cross sections. From this empirical observation it follows that a hadron–nucleon cross section $\sigma_{\text{tot}}(\text{hN})$ can be approximately derived from the pp cross section and a hadron–nucleus cross-section ratio using the relation:

Table 6
Nuclear cross-section ratios. The pA-cross section is scaled by k_{scale} to account for the discrepancy in laboratory momenta of the cross sections used in the ratio

Scaled cross-section ratio	p_{lab} [GeV/c]	k_{scale}
$\sigma_{\text{tot}}(\pi^-\text{Be})/\sigma_{\text{tot}}(\text{pBe}) = 0.698 \pm 0.006$	640	1.0058
$\sigma_{\text{tot}}(\pi^-\text{C})/\sigma_{\text{tot}}(\text{pC}) = 0.695 \pm 0.014$	590	1.0036
$\sigma_{\text{tot}}(\Sigma^-\text{Be})/\sigma_{\text{tot}}(\text{pBe}) = 0.922 \pm 0.008$	640	1.0058
$\sigma_{\text{tot}}(\Sigma^-\text{C})/\sigma_{\text{tot}}(\text{pC}) = 0.917 \pm 0.018$	590	1.0040

Table 7
Ratios of the hadronic cross-section ratios at 137 GeV/c and the nuclear cross-section ratios around 600 GeV/c

Double ratio	Result	Double ratio	Result
$\frac{\sigma_{\text{tot}}(\Sigma^-\text{p})/\sigma_{\text{tot}}(\text{pp})}{\sigma_{\text{tot}}(\Sigma^-\text{Be})/\sigma_{\text{tot}}(\text{pBe})}$	0.977 ± 0.016	$\frac{\sigma_{\text{tot}}(\pi^-\text{p})/\sigma_{\text{tot}}(\text{pp})}{\sigma_{\text{tot}}(\pi^-\text{Be})/\sigma_{\text{tot}}(\text{pBe})}$	0.910 ± 0.012
$\frac{\sigma_{\text{tot}}(\Sigma^-\text{p})/\sigma_{\text{tot}}(\text{pp})}{\sigma_{\text{tot}}(\Sigma^-\text{C})/\sigma_{\text{tot}}(\text{pC})}$	0.983 ± 0.023	$\frac{\sigma_{\text{tot}}(\pi^-\text{p})/\sigma_{\text{tot}}(\text{pp})}{\sigma_{\text{tot}}(\pi^-\text{C})/\sigma_{\text{tot}}(\text{pC})}$	0.915 ± 0.020
average (κ)	0.980 ± 0.014	average (κ)	0.913 ± 0.012

$$\sigma_{\text{tot}}(\text{hN}) \approx \kappa \times \sigma_{\text{tot}}(\text{pp}) \times \left(\frac{\sigma_{\text{tot}}(\text{hA})}{\sigma_{\text{tot}}(\text{pA})}\right), \tag{37}$$

where κ is a parameter specific for the cross section ratio (compare with Table 7). If we set $\kappa = 1$ for simplicity, we see that the precision of (37) is about 10%. The precision is improved by adequate adjusting of κ.

Unfortunately we cannot empirically derive κ from experimental cross sections for laboratory momenta around 600 GeV/c as necessary cross-section data is missing. Thus, as we want to deduce hadron–nucleon cross sections from nuclear cross-section ratios with best precision, we improve the relation between hadron–nucleon and hadron–nucleus cross sections using the total cross-section model that was introduced in Section 7.

The idea of the model-based ratio method is the following: Rewriting (37) yields the following relation between the experimental hadron–nucleus and the model based hadron–nucleus cross-section ratios.

$$\underbrace{\frac{\overline{\sigma}_{\text{tot}}(\text{hA})}{\overline{\sigma}_{\text{tot}}(\text{pA})}}_{\text{experimental}} = \underbrace{\frac{\sigma^{\text{mod}}_{\text{tot}}(\text{A}, \sigma_{\text{tot}}(\text{hN}))}{\sigma^{\text{mod}}_{\text{tot}}(\text{A}, \sigma_{\text{tot}}(\text{pN}))}}_{\text{theory } + \sigma_{\text{tot}}\text{-data}}. \tag{38}$$

Taking the ratio of model based quantities reduces the effect of uncertainties in the cross-section model. Because precise data for $\sigma_{\text{tot}}(\text{pp})$ is available over a large energy range, it is convenient to use proton–nucleus cross sections in the denominator. The energy dependence of the pp cross section is known at SELEX energies. The model is adjusted to describe NA cross sections for $p_{\mathrm{lab}} > 10\ \mathrm{GeV}/c$. Therefore the energy dependence of $\sigma_{\mathrm{tot}}(\mathrm{hp})$, which we want to determine, is determined by the known energy dependence of the pp cross section.

To deduce the cross section $\sigma_{\mathrm{tot}}(\mathrm{hN})$ from the measured nuclear cross-section ratio, we fix $\sigma_{\mathrm{tot}}(\mathrm{pN})(=\sigma_{\mathrm{tot}}(\mathrm{pp}))$ first and calculate the denominator $\sigma_{\mathrm{tot}}^{\mathrm{mod}}(A, \sigma_{\mathrm{tot}}(\mathrm{pp}))$ by taking $\sigma_{\mathrm{tot}}(\mathrm{pp})$ from parameterization (27) evaluated at the laboratory momentum of the nuclear cross-section ratio as given in Table 6. We adjust the model input parameter $\sigma_{\mathrm{tot}}(\mathrm{hN})$ until the model based total cross-section ratio in (38) equals the experimental one. At SELEX energy we interpret the value of the parameter $\sigma_{\mathrm{tot}}(\mathrm{hN})$ to be equal to $\sigma_{\mathrm{tot}}(\mathrm{hp})$.

8.2.1. *Results for* $\sigma_{\mathrm{tot}}(\Sigma^-\mathrm{N})$ *and* $\sigma_{\mathrm{tot}}(\pi^-\mathrm{N})$ *using the ratio method*

Results of the ratio method are presented in Table 9 together with the results from the $\mathrm{CH_2}$–C method. The errors of hadron–nucleon cross sections resulting from the ratio method include both the error in the measured nuclear cross-section ratio and model uncertainties. Model uncertainties are taken into account by adding the error of a model cross-section ratio in quadrature to the error of the corresponding experimental cross-section ratio given in Table 6. The error in the model cross-section ratio is derived from the discrepancy between model and measured cross sections observed for pA and π^-A total cross sections. Typical sizes of these discrepancies are shown in Table 8.

Further, as two different parameterizations for $(\mathrm{d}^2\sigma/\mathrm{d}t\,\mathrm{d}M^2)|_{t=0}$ are available, we evaluate the ratio method for both, average the results and include their difference in the error of the mean.

Finally, we want to mention that as little data exists for Σ^- scattering, we insert in the computation of $\sigma_{\mathrm{tot}}^{\mathrm{mod}}(\Sigma^-\mathrm{A})$ for $B_{\Sigma^-\mathrm{N}}$, $\rho'_{\Sigma^-\mathrm{N}}$ and $(\mathrm{d}^2\sigma/\mathrm{d}t\,\mathrm{d}M^2)|_{t=0}$, the parameterizations from pp-reactions.

Comparing the hadron–nucleon cross sections of the ratio and the difference method, we find that the results agree well within their errors. As final result, we average the hadron–nucleon cross-section values from all methods. These total averages are presented in the last row of Table 9 together with a corresponding averaged laboratory momentum.

Table 8
Discrepancy between model and measured total cross sections. The measured pA cross sections are scaled by k_{scale}

Reaction	Measured cross section $\times k_{\mathrm{scale}}$ [mb]	Calculated cross section [mb]	Cross-section difference [mb]	Nominal p_{lab} [GeV/c]
$\sigma_{\mathrm{tot}}(\pi^-\mathrm{Be})$	188.7	188.8	0.1	640
$\sigma_{\mathrm{tot}}(\pi^-\mathrm{C})$	234.1	231.4	2.7	590
$\sigma_{\mathrm{tot}}(\mathrm{pBe})$	270.2	277.0	6.8	640
$\sigma_{\mathrm{tot}}(\mathrm{pC})$	336.8	335.9	0.9	590

Table 9
The total cross sections $\sigma_{\rm tot}(\Sigma^- N)$ and $\sigma_{\rm tot}(\pi^- N)$ resulting from all methods and their average

Method description	$\sigma_{\rm tot}(\Sigma^- N)$ [mb]	$\sigma_{\rm tot}(\pi^- N)$ [mb]	$p_{\rm lab}$ [GeV/c]
difference method	33.7 ± 3.1	26.0 ± 2.1	585
ratio method, Be data	37.4 ± 1.3	27.1 ± 1.5	640
ratio method, C data	37.0 ± 0.8	26.4 ± 1.3	595
total average	37.0 ± 0.7	26.6 ± 0.9	610

8.3. Comparison to models

8.3.1. Comparisons for $\sigma_{\rm tot}(\pi^- p)$

Most of the models and parameterizations for hadron–nucleon cross sections exploit the interplay of two contributions: the Pomeron contribution, which dominates asymptotics at high energies; and the Regge contribution, which is important at low and medium energies. Many models (e.g., [35,36]) describe the energy dependence of total cross sections quite well. We display in Fig. 14 experimental data from [8] and SELEX along with the parameterization for $\sigma_{\rm tot}(\pi^- p, s)$:

$$\sigma_{\rm tot}(\pi^- p, s) = 35.9\, s^{-0.45} + 13.7\, s^{+0.079}$$
$$\text{for } p_{\rm lab} > 10\,\text{GeV}/c, \quad \sigma_{\rm tot} \text{ in mb}, s \text{ in GeV}^2, \tag{39}$$

from the 1996 Particle Data Group formulation [37].

We point out that so far the total cross section $\sigma_{\rm tot}(\pi^- p)$ has been measured only up to $p_{\rm lab} = 370\,\text{GeV}/c$ [38]. Thus, the SELEX total average for $\sigma_{\rm tot}(\pi^- N)$ at $610\,\text{GeV}/c$ is the first new measurement at higher laboratory momentum.

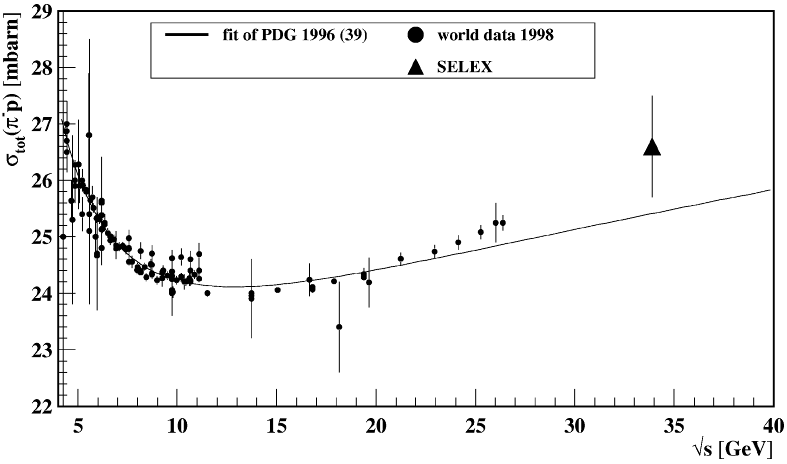

Fig. 14. Existing data for $\sigma_{\rm tot}(\pi^- p)$ in comparison with our results and parameterization (39) of the particle data group 1996.

In Fig. 14 the parameterization (39) of the Particle Data Group, which uses a Pomeron intercept of 0.079, is overlaid to the data. Qualitative inspection of (39) suggests that it is strongly weighted by the huge number of low energy data points and does not sufficiently well take into account the very accurate data of [38] at high energy. Our result seems to strengthen the trend observed in data of [38], implying a faster rise of the π^-p cross section with increasing energy than represented by (39). We just want to point out this observation, which may turn out to be in conflict with the belief that the energy increase of hadronic cross sections is universal. We do not give any quantitative estimate of the Pomeron intercept for the π^-p cross section. Its value is correlated to the assumed Regge contribution at low energy and its determination requires a careful analysis of all the data.

8.3.2. Comparisons for $\sigma_{\text{tot}}(\Sigma^-\text{p})$

Data on the total cross section $\sigma_{\text{tot}}(\Sigma^-\text{p})$ are scarce. In the past, there have been only two hyperon-beam experiments [4,39] giving information about the behavior of $\sigma_{\text{tot}}(\Sigma^-\text{p})$ in the momentum range 19–136.9 GeV/c. The SELEX result for $\sigma_{\text{tot}}(\Sigma^-\text{N})$ provides the first high energy data. Fig. 15 shows a compilation of data from previous experiments together with the SELEX result. Our measurement is 2.9 mb larger than the data point at 136.9 GeV/c from [4], indicating a rise of $\sigma_{\text{tot}}(\Sigma^-\text{p})$ with increasing beam energy.

Overlaid on the experimental data is the prediction for $\sigma_{\text{tot}}(\Sigma^-\text{p}, p_{\text{lab}})$ from H. Lipkin (see [36]):

$$\sigma_{\text{tot}}(\Sigma^-\text{p}, p_{\text{lab}}) = 19.5\left(\frac{p_{\text{lab}}}{20}\right)^{0.13} + 13.2\left(\frac{p_{\text{lab}}}{20}\right)^{-0.2}$$
$$\text{for } p_{\text{lab}} > 10\,\text{GeV}/c, \quad \sigma_{\text{tot}} \text{ in mb}, \ p_{\text{lab}} \text{ in GeV}/c. \tag{40}$$

The corresponding curve in Fig. 15 shows good agreement between our measurement and this prediction.

It would be certainly desirable to find the Pomeron intercept for the Σ^-p cross section. The lack of low energy data does not allow any reasonable estimate of the intercept.

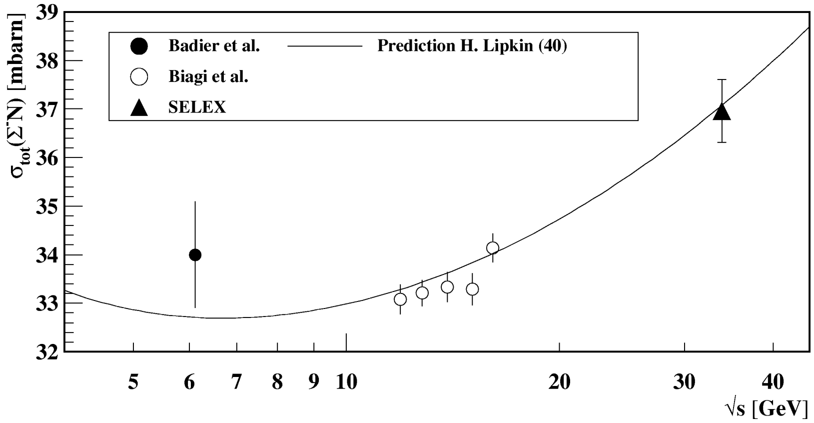

Fig. 15. Existing data for $\sigma_{\text{tot}}(\Sigma^-\text{p})$ in comparison with our results and prediction (40).

9. Conclusions

The SELEX collaboration has measured the total cross sections $\sigma_{tot}(\pi^-\mathrm{Be})$, $\sigma_{tot}(\pi^-\mathrm{C})$, $\sigma_{tot}(\pi^-\mathrm{Cu})$, $\sigma_{tot}(\pi^-\mathrm{CH_2})$, $\sigma_{tot}(\Sigma^-\mathrm{Be})$, $\sigma_{tot}(\Sigma^-\mathrm{C})$, $\sigma_{tot}(\Sigma^-\mathrm{Cu})$, $\sigma_{tot}(\mathrm{pBe})$ and $\sigma_{tot}(\mathrm{pC})$ in a broad momentum range around 600 GeV/c using a transmission method that was adapted to the specifics of the SELEX spectrometer. The accuracy of the results is within 0.6–1.5% for Be, C and CH_2 and about 17.5% for Cu.

The ratios of hadron–nucleus cross sections for Be and C show that π^-–nucleus cross sections are a about factor of 0.7 lower than corresponding proton–nucleus cross sections. Furthermore, we find that the Σ^-–nucleus cross sections are about a factor of 0.92 smaller than corresponding proton–nucleus cross sections.

We observe that the results for $\sigma_{tot}(\mathrm{pBe})$, $\sigma_{tot}(\mathrm{pC})$, $\sigma_{tot}(\pi^-\mathrm{Be})$, $\sigma_{tot}(\pi^-\mathrm{C})$ and $\sigma_{tot}(\pi^-\mathrm{Cu})$ join smoothly onto corresponding cross-section data at lower energy. The good agreement of the proton–nucleus and the π^-–nucleus cross sections to Glauber model calculations with an inelastic screening correction and one adjustable parameter in the density distribution justifies the deduction of $\sigma_{tot}(\Sigma^-\mathrm{p})$ and $\sigma_{tot}(\pi^-\mathrm{p})$ from the nuclear cross sections.

We deduced the hadron–nucleon cross sections $\sigma_{tot}(\pi^-\mathrm{N})$ and $\sigma_{tot}(\Sigma^-\mathrm{N})$, which we regard as $\sigma_{tot}(\pi^-\mathrm{p})$ and $\sigma_{tot}(\Sigma^-\mathrm{p})$, from our nuclear data using a CH_2–C difference and a model based ratio method. Results from the difference method have an accuracy of 8.1–9.2%, while results from the ratio method have an accuracy of 2.2–5.5%.

The total averages of all methods represent first measurements for $\sigma_{tot}(\pi^-\mathrm{p})$ and $\sigma_{tot}(\Sigma^-\mathrm{p})$ near 600 GeV/c. Our result for $\sigma_{tot}(\Sigma^-\mathrm{p})$ shows clearly a rise of this cross section with increasing beam energy, which agrees with the prediction of [36].

Our result for $\sigma_{tot}(\pi^-\mathrm{p})$ joins nicely onto the trend of the high energy data of [38]. As mentioned in Section 8.3.1, the data of [38] and our result may indicate a faster increase of the π^-p cross section than predicted by the parameterization given by the Particle Data Group in 1996.

This indication of a faster increase of the π^-p cross section compared to the pp (and $\overline{\mathrm{p}}$p) one can be verified only by a high statistics measurement using a π^- beam and a hydrogen target to avoid some systematic errors inherent to the method used in this experiment. In our opinion a measurement of the π^-p cross section at 600 GeV/c or higher is the only experimentally accessible opportunity to test if the energy variation of a hadronic cross section might be different from that for pp and (and $\overline{\mathrm{p}}$p) cross sections.

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