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Twisting magnetic field and multiple resonance mechanism in the solar neutrino problem

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The magnetic moment solution to the solar neutrino problem is investigated in the context of a solar magnetic field with varying phase velocity on the transverse plane. This phase velocity is assumed to successively take values which are approximately proportional to the solar density, giving rise to the appearance of many resonances along the neutrino trajectory. We obtain a magnetic moment necessary for a neutrino reduction compatible with the experimental situation in the range $(6–7)\times 10^{-13}\mu_B$, an improvement by a factor of 4–6 relative to the one resonance case under the same field conditions.

The resonant spin-flavour conversion [1] of neutrinos weakly interacting into sterile neutrinos has proven to be an effective mechanism [2] for explaining the solar neutrino deficit relative to the solar standard model prediction [3] observed by the four existing solar neutrino experiments [4]. It requires the existence of a neutrino magnetic moment [5] in the range $\mu_\nu \geqslant 10^{-12}\mu_B$ which is at least six orders of magnitude above the electroweak theory prediction [6] and is limited by the firm laboratory bound $\mu_\nu \leqslant 4\times 10^{-10}\mu_B$ [7]. The idea of the mechanism [1] is based on the possible existence of one resonance inside the Sun in the neutrino trajectory arising from the interplay of the solar density and the quantities $\Delta^2 m_{21}/2E$ (flavour mass square difference divided by the neutrino energy) and $\mu_\nu B$ (the product of the neutrino magnetic moment by the intensity of the solar magnetic field). This operates in a way similar to the matter oscillation [8]. The resonance range is a privileged zone for neutrino conversion and in fact its existence can reduce by as much as two orders of magnitude the necessary magnetic moment for a significant reduction of active neutrinos. One question that most naturally arises may therefore be whether the existence of multiple resonances can allow for a further reduction.

The purpose of this letter is to investigate this possibility, by dividing conceptually the neutrino trajectory in intervals, each containing one resonance only. If the conversion probability P_i in each resonance is admittedly small in connection with a magnetic moment smaller than the usually required minimum value $O(10^{-12}\mu_B)$, the existence of a large number of resonances may lead to a sizeable overall probability satisfying $P\approx O(\sum P_i)$. The possibility of a neutrino crossing a large number of resonances along its trajectory can only be realized, as will be seen, for a solar field $\boldsymbol{B}$ with varying phase velocity. Our analysis will show that the multiple resonance mechanism can provide a large total probability with small values of the individual probabilities P_i. Furthermore, assuming multiple resonances, the minimum magnetic moment necessary for a suitable solution to the solar neutrino problem will be seen to lie almost one order of magnitude below the minimum for the conventional one resonance case.

As neutrinos travel through the Sun, they will most likely experience a magnetic field with varying phase

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velocity in the transverse plane. Such is the case of the twisting magnetic field introduced in the literature some time ago [9]. Although this is the only existing candidate for multiple resonances, such a possibility was never considered.

In order to evaluate the transition probability in a scenario with multiple resonances, let us take an electron neutrino ν_e produced in the core of the Sun which we will denote by (1, 0) on the basis (ν_e, ν_x) where ν_x refers to a sterile neutrino flavour. After travelling through a region containing one resonance, this becomes

$$\begin{pmatrix} 1-P_1 \\ P_1 \end{pmatrix}, \tag{1}$$

where P_1 is the probability for the $\nu_e \to \nu_x$ transition in this region. Writing this state as the combination

$$(1-P_1)\begin{pmatrix} 1 \\ 0 \end{pmatrix} + P_1 \begin{pmatrix} 0 \\ 1 \end{pmatrix}, \tag{2}$$

then, after a second resonance

$$\begin{pmatrix} 1 \\ 0 \end{pmatrix} \to \begin{pmatrix} 1-P_2 \\ P_2 \end{pmatrix}, \quad \begin{pmatrix} 0 \\ 1 \end{pmatrix} \to \begin{pmatrix} P_2 \\ 1-P_2 \end{pmatrix}, \tag{3}$$

where P_2 is the probability for $\nu_e \to \nu_x$ in the region containing the second resonance. Thus the transition probability from an active to a sterile neutrino after two resonances is

$$\begin{aligned} P_2^{\text{tot}} &= (1-P_1)P_2 + P_1(1-P_2) \\ &= P_1 + P_2 - 2P_1P_2 \,. \end{aligned} \tag{4}$$

Including a third resonance and repeating the argument, it is straightforward to derive the total probability for three resonances:

$$\begin{aligned} P_3^{\text{tot}} &= P_1 + P_2 + P_3 - 2P_1P_2 - 2P_1P_3 - 2P_2P_3 \\ &\quad + 4P_1P_2P_3 \,. \end{aligned} \tag{5}$$

For an arbitrary number of resonances, one therefore obtains the general formula for the classical transition probability

$$P_n^{\text{tot}} = \sum_{m=1}^{n} \sum_{\substack{i_1, i_2, \ldots, i_m = 1 \\ i_1 < i_2 < \ldots < i_m}}^{n} (-2)^{m-1} P_{i_1} P_{i_2} \ldots P_{i_m} \,. \tag{6}$$

The simplifying assumption of taking approximately equal probabilities at each resonance is particularly useful to provide an insight on the behaviour of this expression. It may also be not too distant from a realistic realization of the multiple resonant scenario as will be seen. Instead of eq. (6) we can write for equal probabilities $P_1 = P_2 = \ldots = P_i = \ldots = P_n = P$

$$P_n^{\text{tot}} = \sum_{m=1}^{n} (-2)^{m-1} \binom{n}{m} P^m \,, \tag{7}$$

which, for instance, taking $n=100$, $P=0.01$ gives $P_{100}^{\text{tot}} = 0.43$. The first term in eq. (7) is the sum of all probabilities $n \cdot P$ while the following ones provide alternatively negative and positive contributions which converge to zero. The net result is thus a reduction to a number within the same order of magnitude of the quantity $n \cdot P$.

The implementation of the multiple resonance scenario is most naturally done, as referred to above, in the case of the so called twisting magnetic field or, more generally, a field with varying phase velocity on the transverse plane. The neutrino evolution equation is in such cases [10–11]

$$\mathrm{i}\frac{\mathrm{d}}{\mathrm{d}t}\begin{pmatrix} \nu_e \\ \nu_x \end{pmatrix} = \begin{pmatrix} B_0 & \mu_\nu B_1 \mathrm{e}^{-\mathrm{i}\phi} \\ \mu_\nu B_1 \mathrm{e}^{\mathrm{i}\phi} & -B_0 \end{pmatrix} \begin{pmatrix} \nu_e \\ \nu_x \end{pmatrix}. \tag{8}$$

Here ϕ is the field phase, $\boldsymbol{B}(t) = (B_1 \cos\phi, B_1 \sin\phi, 0)$, B_1 its modulus, $B_0 = G_F(N_e - N_n)/\sqrt{2}$ and, along the neutrino trajectory, the neutron density $N_n = \frac{1}{6}N_e$ with the electron density N_e decreasing exponentially [3]. Applying the unitary transformation

$$\begin{pmatrix} \nu_e \\ \nu_x \end{pmatrix}' = \begin{pmatrix} \mathrm{e}^{\mathrm{i}\phi/2} & 0 \\ 0 & \mathrm{e}^{-\mathrm{i}\phi/2} \end{pmatrix} \begin{pmatrix} \nu_e \\ \nu_x \end{pmatrix}, \tag{9}$$

one can eliminate the imaginary part of the evolution Hamiltonian, so eq. (8) becomes

$$\mathrm{i}\frac{\mathrm{d}}{\mathrm{d}t}\begin{pmatrix} \nu_e \\ \nu_x \end{pmatrix}' = \begin{pmatrix} B_0 - \frac{1}{2}\dot{\phi} & \mu_\nu B_1 \\ \mu_\nu B_1 & -B_0 + \frac{1}{2}\dot{\phi} \end{pmatrix} \begin{pmatrix} \nu_e \\ \nu_x \end{pmatrix}'. \tag{10}$$

It is therefore seen that a non-vanishing phase velocity $\dot{\phi}$ opens the possibility for one or more resonances to occur through the condition of equal diagonal elements:

$$\dot{\phi} = 2B_0 \,. \tag{11}$$

For a field whose direction changes permanently and irregularly on a plane perpendicular to the direction of motion, as is possibly the case, this condition may be verified a large number of times along the neutrino trajectory. We note that the effect of the phase velocity would be a direct increase in the transition probability if instead it appeared in the off-diagonal elements of eq. (10). It therefore plays a similar role to the mass square difference in the MSW mechanism [8] or the spin flip with only one resonance [1]. Thus it may only indirectly provide an increase in the transition probability through the multiple use of the resonance condition (11).

Using the system of equations (10), it is straightforward to get the second order differential equation for the evolution of A_x, the amplitude for $\nu_e \to \nu_x$. It is well known that the quantity B_0 introduced in eq. (8) decreases exponentially along the neutrino trajectory [3]. Since however we divide this trajectory in relatively short intervals we will assume, within limits, a constant B_0 in each interval as well as a constant phase velocity $\dot{\phi}$ and field magnitude B_1. In this way we obtain a simple Laplace equation for A_{xi}

$$\frac{d^2 A_{xi}}{dt^2} + [(B_{0i} - \tfrac{1}{2}\dot{\phi}_i)^2 + \mu_\nu^2 B_{1i}^2] A_{xi} = 0 . \qquad (12)$$

This is the equation of motion describing the propagation of neutrinos within the ith region, each being characterized by a fixed value of B_{0i}, B_{1i} and $\dot{\phi}_i$. It is also assumed, as we have seen, that the resonance condition (11) is satisfied once and only once in each of these sections of the trajectory. Hence, on the basis of these assumptions, the transition probability for $\nu_e \to \nu_x$ in one interval calculated from the solution of eq. (12) is the more accurate, the closer one stays to the resonance condition within the interval. In other words the expression

$$\begin{aligned} P_i(\nu_e \to \nu_x) &= |A_{xi}|^2 \\ &= \frac{\mu_\nu^2 B_{1i}^2}{(B_{0i} - \frac{1}{2}\dot{\phi}_i)^2 + \mu_\nu^2 B_{1i}^2} \\ &\quad \times \sin^2([(B_{0i} - \tfrac{1}{2}\dot{\phi}_i)^2 + \mu_\nu^2 B_{1i}^2]^{1/2} \Delta t_i) \end{aligned} \qquad (13)$$

(where Δt_i is the length of the interval) gives a better approximation of the actual probability, the stronger the inequality

$$(B_{0i} - \tfrac{1}{2}\dot{\phi}_i)^2 \ll \mu_\nu^2 B_{1i}^2 \qquad (14)$$

is fulfilled throughout the whole interval. In this way a "quasi-resonance" condition remains valid within this length. For short intervals this condition is of course likelier to be ensured than for comparatively larger ones. Let us therefore assume that eq. (14) holds in each section of the trajectory and thus the probability P_i is accurately given by its maximum value

$$P_i = \sin^2(\mu_\nu B_{1i} \Delta t_i) . \qquad (15)$$

In order to get a rough but safe estimate of the maximum allowed value of Δt_i for the approximation to be accurate, let us consider a linearly decreasing density extending to both sides within the vicinity of the resonance. Thus

$$B_0 = B_{0i} \exp(-\beta t) \approx B_{0i}(1 - \beta t) . \qquad (16)$$

Here $\beta = 1/0.09 R_S$ with R_S being the solar radius ($R_S = 696\,000$ km) and $t=0$ at the ith resonance point. Applying the resonance condition (11), we obtain from the inequality (14)

$$\max \Delta t_i \approx 1.28 \times 10^{15} \mathrm{eV}^{-1} \frac{\mu_\nu B_{1i}}{\dot{\phi}_i} . \qquad (17)$$

The phase velocity $\dot{\phi}_i$ entering eq. (17) is therefore fixed by the density at resonance:

$$\dot{\phi}_i = 2B_{0i} = \sqrt{2} G_F \cdot \tfrac{5}{6} N_{ei} \qquad (18)$$

which, using [1] $N_{ei} = 2.4 \times 10^{26} \exp(-x_i/0.09)$ cm^{-3}, becomes

$$\dot{\phi}_i = 2.5 \times 10^{-11} \exp\left(\frac{-x_i}{0.09}\right) \mathrm{eV} , \qquad (19)$$

where x_i is the fraction of the solar radius at the resonance point, $x_i = r_i / R_S$. In other words, we are assuming the phase velocity of the field successively to reach along the neutrino trajectory a value given by eq. (19), so that at each such point the ith resonance occurs.

We now investigate under what conditions the total transition probability can be maximized for a given field and magnetic moment. A maximum probability must obviously result from a balance between the maximization of the intervals size and the number of intervals: the large one is, the smaller the other. If the regions are too long we approach the well known one resonance situation [2], whereas if they become too small, the probabilities P_i, eq. (15) become negligible. For the solar field we take one of the Akhmedov model distributions [12]

$$B_1(x)=10^7\times\left(\frac{0.2}{x+0.2}\right)^2 \mathrm{G} \quad \text{for } 0\leqslant x\leqslant 0.65\,,$$
$$=10^5\times\left[1-\left(\frac{x-0.7}{0.3}\right)^2\right]\mathrm{G} \quad \text{for } 0.65<x\leqslant 1\,. \tag{20}$$

Using eqs. (15) and (17)-(20) we first calculated $P_i(\max\Delta t_i)$ at each given interval. The minimum of these quantities was chosen as a common value for the probabilities so that the simplified formula, eq. (7), for the total probability could be applied. In this way the corresponding interval length Δt_i is often much smaller than $\max\Delta t_i$. In table 1 we list the values of $\max\Delta t_i$, eq. (17), the value Δt_i obtained in this way at several points of the trajectory, the corresponding values $\dot{\phi}(x_i)$, eq. (19) and the magnetic field, eq. (20). In this way we are in the most favourable case for applying eq. (7). For $\mu_\nu=7\times10^{-13}\mu_B$, we have $P_1\approx P_2\approx...\approx P_n\approx P\approx2\times10^{-3}$, while for $\mu_\nu=6\times10^{-13}\mu_B$, $P\approx10^{-3}$. The results of eq. (7) for these two values of μ_ν are listed in table 2, together with the approximate number of resonances encountered by the neutrinos, for these being produced at 0.05, 0.06 and 0.1 of the solar radius. This approximate number of resonances refers of course to the whole neutrino trajectory, from their production point up to the surface of the Sun.

A comparison of the results for the total suppression probability in the multiple resonance case with the one resonance case [2] shows that, under the same field conditions, an analogous probability is obtained for a neutrino magnetic moment 4–6 times smaller. In fact a magnetic moment of order $(6\text{–}7)\times10^{-13}\mu_B$ can give the necessary suppression of active neutrinos compatible with the experimental situation, to be compared with $\mu_\nu\approx(2\text{–}4)\times10^{-12}\mu_B$ in the one resonance case. A different feature of the multiple resonance framework with respect to both MSW and one resonance mechanisms is that all neutrinos are equally suppressed, regardless of their energy, and the location of the resonances is also energy independent. This is a consequence of the fact that the present scenario works independently of mass differences between

Table 1
The values of the magnetic field, eq. (20), at a fraction x of the solar radius, the phase velocity (19), the maximum interval length (17) and the value of Δt_i used in the calculation of the probability P_i in the corresponding interval.

$x=r/R_S$	$B_1(x)\,(\times10^6\mathrm{G})$	$\dot{\phi}(x)$ (eV)	$\max\Delta t_i\,(\mathrm{eV}^{-1})$	$\Delta t_i\,(\mathrm{eV}^{-1})$
0.04	6.9	1.61×10^{-11}	2.22×10^{12}	1.70×10^{12}
0.05	6.4	1.44×10^{-11}	2.29×10^{12}	1.84×10^{12}
0.06	5.9	1.28×10^{-11}	2.37×10^{12}	1.99×10^{12}
0.1	4.4	8.26×10^{-12}	2.75×10^{12}	2.75×10^{12}
0.2	2.5	2.72×10^{-12}	4.74×10^{12}	4.74×10^{12}
0.3	1.6	8.95×10^{-13}	9.21×10^{12}	7.37×10^{12}
0.4	1.1	2.93×10^{-13}	1.97×10^{13}	1.05×10^{13}
0.5	0.82	9.69×10^{-14}	4.34×10^{13}	1.44×10^{13}
0.6	0.62	3.19×10^{-14}	1.01×10^{14}	1.90×10^{13}

Table 2
The number of resonances encountered by neutrinos produced at 0.05, 0.06 and 0.1 of the solar radius and their total expected transition probability into sterile ones for two values of the magnetic moment. The solar magnetic field is taken from eq. (20).

$x=r/R_S$	Approximate number of resonances	P^{tot}	
		$\mu_\nu=7\times10^{-13}\mu_B$	$\mu_\nu=6\times10^{-13}\mu_B$
0.05	400	0.41	0.28
0.06	380	0.39	0.27
0.1	320	0.36	0.24

neutrino flavours and does not require neutrino mass.

From eq. (15) it is clear that there is the possibility of an anti-correlation of the neutrino flux with the sunspot activity in the convective zone of the Sun. This time dependence has been claimed by the Homestake experiment [4] but the Kamiokande data do not seem to confirm it. In the one resonance case, such a strong effect in one experiment and its absence in the other can be explained by the assumption of a resonance located in the convective zone for the beryllium neutrino sector only. This would lead us to expect an even stronger time dependence in gallium experiments [4,13], as the Be fraction is more important there. In the multiple resonance case, however, where the location of the resonances is determined by the field phase velocity and not by the mass difference, a possible reliable explanation lies in the comparison of the neutrino scattering effects in Homestake and Kamiokande. While Homestake observes a purely weak process, the neutrino–electron scattering observed by Kamiokande has an electromagnetic contribution from the magnetic moment which does not flutuate in time. So the Kamiokande effect is expected to be smoother. However, this smoothening is only apparent if $\mu_\nu \geqslant 10^{-10}\mu_B$ [14]. Although this range possibly leads in the present scenario to probabilities that are far too large, the large experimental uncertanties involved allow us to invoke this explanation.

The phase of solar magnetic field probably rotates in space and in time along the neutrino trajectory but in an unknown fashion. It is therefore possible that it may successively reach values on the way of the neutrinos which are proportional to the local solar density. Such a coincidence is not unlikely to be realized in practice but it requires some fine tuning to ensure that the phase velocity, whose range of variation is unknown, goes through the required values at the appropriate locations. Thus this mechanism, although possible in practice, is not necessarily implied by the existence of a variable magnetic field phase. This scenario thus offers a viable alternative for a solution to the solar neutrino problem within the assumption of a non-zero neutrino magnetic moment.

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