A 2D honeycomb photonic crystal under applied magnetic fields

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ABSTRACT

The band-structure properties of a photonic two-dimensional honeycomb lattice formed by cylindrical semiconductor shell rods with dielectric permitivities ε_1 and ε_2, and embedded in a background with permitivity ε_3, is studied by using an standard plane-wave expansion. The properties of bandgaps and density of states, considering dispersive dielectric responses, are investigated together with the possibility of fabricating systems with tunable photonic bandgaps, due to the Voigt magneto-optical effect, under the influence of an external magnetic field.

Keywords: Photonics, honeycomb, magnetic field

1. INTRODUCTION

Most of the interesting electrodynamics effects reported by investigations on the band structure of photonics systems rely on the existence of complete or absolute band gaps, that is, those for which the propagation of electromagnetic waves is forbidden for all wave vectors. A great deal of the studies devoted to the understanding of bandgaps has been focused on non-dispersive and positive electric and magnetic responses. In the last decade, tunable bandgaps have been the object of many investigations, and other media with electrical and magnetic response functions that are dispersive and strongly dependent on external parameters have been widely used [1-8]. The basic idea is to fabricate tunable photonic bandgaps (PBGs) by externally controlling the response functions with the aim to optoelectronic applications such as sensor devices. Intrinsic semiconductors with dielectric functions strongly dependent on an external magnetic field, via magneto-optical effects such as the Voigt effect, have also been investigated [2,3]. Along these lines, the frequency range of the PBG of a square lattice composed by cylindrical air holes embedded in a background of GaAs, has been shown to be efficient for frequencies in the Terahertz domain. A study, from the symmetry point of view, has shown that symmetry reduction in honeycomb photonic lattices may increase the size of absolute PBGs [4]. Recently, a thorough study [5] was performed of a 2D hexagonal structure of circular rods composed by an internal region of dielectric permitivity ε_1 plus an anisotropic external shell of a material ε_2, embedded periodically in a background with permitivity ε_3, as depicted in Figure 1. A comparative study between the cases using solid Te rods and Te shells filled with another material has shown that the latter exhibits wider PBGs and also that these are shifted to higher frequency ranges [5] . This work is concerned with a theoretical investigation of non-dispersive dielectric permitivities of Te shell rods embedded in air (cf. Fig. 1), together with the modifications introduced on the photonic band structure (PBS) of a highly anisotropic honeycomb photonic crystal (PC) using a dispersive dielectric function of bulk GaAs in air, and considering the action of an externally applied magnetic field. The dielectric response of a n-doped GaAs semiconductor, used here to fill the cylindrical rods, depends on whether the optical field corresponds to E-polarization, or H-polarization, as defined in Figure 1. For the case of an externally applied magnetic field, we shall be particularly interested in the study of the H-polarization case and focus on the Voigt effect, i.e., we consider the electric field component of the optical field perpendicular to the external magnetic field. In Section 2 we present the basic equations which are numerically solved by a standard plane-wave expansion, and Section 3 is devoted to the present results and conclusions.

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2. THEORETICAL FRAMEWORK

The PBG in a periodic dielectric structure may be determined by studying the propagation of light within the framework of Maxwell's equations. Thus, beginning with the wave equation for the magnetic field in inhomogeneous dielectric materials, expanding the magnetic field in a sum of plane waves, and taking the Fourier transform of the inverse of $\varepsilon(\vec{r})$, denoted here by $\eta(\vec{G})$, one may obtain the following equation governing the dispersion of EM waves [6]:

$$H(\vec{r}) = \sum_{\vec{G}'} \left[\left|\vec{k}+\vec{G}\right| \left|\vec{k}+\vec{G}'\right| \eta(\vec{G}-\vec{G}') \begin{pmatrix} +\hat{e}_2 \cdot \hat{e}_2' & -\hat{e}_2 \cdot \hat{e}_1' \\ -\hat{e}_1 \cdot \hat{e}_2' & +\hat{e}_1 \cdot \hat{e}_1' \end{pmatrix} - \frac{\omega^2}{c^2} \delta_{\vec{G}',\vec{G}} \right] \begin{pmatrix} h_{\vec{G}',1} \\ h_{\vec{G}',2} \end{pmatrix} = 0 \qquad (1)$$

where c and ω are the velocity and the frequency of light, respectively, $\vec{G} = (G_x, G_y)$ represents a reciprocal lattice vector, $\vec{k}$ a wave vector in the first Brillouin zone, and $\hat{e}_\lambda$ (λ = 1, 2) orthogonal unit vectors perpendicular to $\vec{k}+\vec{G}$.

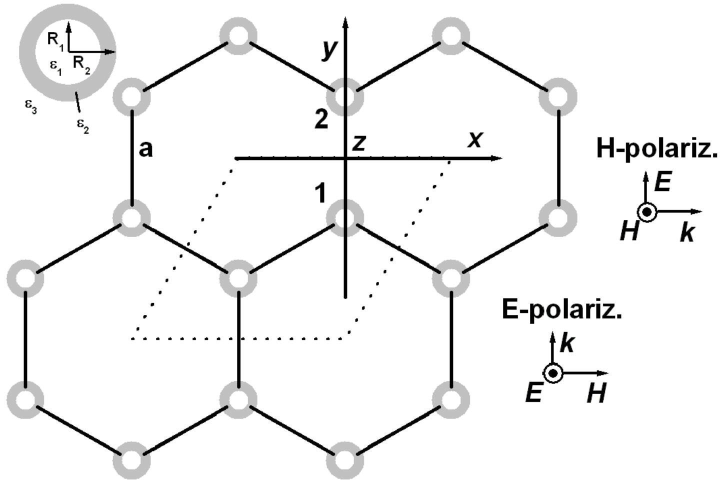

Fig. 1. Pictorial view of the 2D honeycomb PC studied in this work. The unit cell is depicted with dotted lines and the basis of two shell rods are denoted by numbers 1 and 2. The first-neighbors distance is denoted by a . The two radii for the shell rod and the three different dielectric functions are also shown.

Let us consider the periodic array of dielectric rods with axes parallel to the z -axis so that the intersection of these rods with the $x-y$ plane form a 2D periodic dielectric pattern according to the scheme presented in Fig. 1: an hexagonal structure of rods whose dielectric functions are $\varepsilon_1 = n_1^2$ for the internal rod and $\varepsilon_2 = n_2^2$ for the anisotropic external shell, embedded periodically in a background of dielectric function $\varepsilon_3 = n_3^2$. The anisotropic external shell [5] has the ordinary-refractive index $n_2 = n_o$ and extraordinary-refractive index $n_2 = n_e$. The Fourier coefficients for such a system may be written as

$$\eta(\vec{G}) = \frac{1}{\Omega} \int_{cell} \varepsilon^{-1}(\vec{r})\, e^{-i\vec{G}\cdot r} \qquad (2)$$

where the integral is performed over the area Ω of a unit cell of the lattice. The problem defined in Eq. (1) is reduced to solving two eigenvalue equations according to the particular state of polarization of light. So, for E-polarization, i.e., for the electric field parallel to the z -axis and thus perpendicular to the plane of propagation, the wave equation becomes:

$$\sum_{\vec{G}'} \left|\vec{k}+\vec{G}\right| \left|\vec{k}+\vec{G}'\right| \eta(\vec{G}-\vec{G}')\, h_{\vec{G}',2} = \frac{\omega^2}{c^2} h_{\vec{G},2} \qquad (3)$$

whereas in the case of H-polarization., i.e., for the magnetic field parallel to the z -axis, the wave equation is written as

$$\sum_{\vec{G}'} \left(\vec{k}+\vec{G}\right)\cdot\left(\vec{k}+\vec{G}'\right)\eta\left(\vec{G}-\vec{G}'\right)h_{\vec{G}',1} = \frac{\omega^2}{c^2}h_{\vec{G},1}\,. \tag{4}$$

If one considers the extraordinary axis parallel to the z -axis, the eigenequations for E and H polarizations are the same as those for isotropic PCs, except for the fact that the dielectric indices of the anisotropic outer shell are $n_2 = n_e$ and $n_2 = n_o$ for E and H polarizations, respectively [5]. To express the dielectric function of the system in this configuration, we introduce the step functions

$$f_{\text{rod}}(\vec{r}) = \begin{cases} 1, & 0 < r < R_1 \\ 0, & \text{otherwise} \end{cases} \quad \text{and} \quad f_{\text{shell}}(\vec{r}) = \begin{cases} 1, & R_1 < r < R_2 \\ 0, & \text{otherwise.} \end{cases} \tag{5}$$

The dielectric response for the system in this configuration may, therefore, be written as

$$\frac{1}{\varepsilon(\vec{r})} = \frac{1}{\varepsilon_3} + \left(\frac{1}{\varepsilon_1}-\frac{1}{\varepsilon_3}\right)F_{\text{rod}} + \left(\frac{1}{\varepsilon_2}-\frac{1}{\varepsilon_3}\right)F_{\text{shell}} \quad \text{where} \quad F_{\text{rod(shell)}} = \sum_i\sum_{\vec{R}} f_{\text{rod(shell)}}\left(\vec{r}-\vec{u}_i-\vec{R}\right). \tag{6}$$

The unit cell of an hexagonal lattice contains two rods located at $(0,+a/2)$ and $(0,-a/2)$, where a represents the first-neighbors distance. The Fourier coefficients of the hexagonal structure of shelled circular rods, are then given by

$$\eta(\vec{G}) = \begin{cases} \dfrac{1}{\varepsilon_3} + \left(\dfrac{2\pi R_1^2}{\Omega}\right)\left(\dfrac{1}{\varepsilon_1}-\dfrac{1}{\varepsilon_3}\right) + \dfrac{2\pi}{\Omega}\left(R_2^2-R_1^2\right)\left(\dfrac{1}{\varepsilon_2}-\dfrac{1}{\varepsilon_3}\right), & \vec{G}=0 \\ \dfrac{4\pi}{\Omega G}\cos\left(\dfrac{a\,G_y}{4}\right)\left[\left(\dfrac{1}{\varepsilon_1}-\dfrac{1}{\varepsilon_3}\right)R_1 J_1(R_1 G) + \left(\dfrac{1}{\varepsilon_2}-\dfrac{1}{\varepsilon_3}\right)\left[R_2 J_1(R_2 G) - R_1 J_1(R_1 G)\right]\right], & \vec{G}\neq 0 \end{cases} \tag{7}$$

where $J_1(x)$ is the Bessel function of the first kind. In this paper, we shall be interested in establishing tunable properties of the PBS based on the consequences of the Voigt effect, which is occurs when the applied magnetic field is perpendicular to the propagation direction. Let us then consider the external magnetic field pointing in the direction parallel to the rods axes. In this case, the dielectric response for **E**-polarization is constant and thus, independent of the external magnetic field, and defined just like for an ordinary semiconductor, that is [2],

$$\varepsilon_{\parallel}(\omega) = \varepsilon_0\left(1-\frac{\omega_p^2}{\omega^2}\right), \tag{8}$$

where ε_0 is the rods-dielectric constant and ω_p the semiconductor plasma frequency, a function of the carriers density and effective mass. On the other hand, for H-polarization [2] the electrical field is perpendicular to the external field so that the electric response is modified by the applied field according to

$$\varepsilon_{\perp}(\omega) = \varepsilon_0\left(1-\frac{\omega_p^2}{(\omega^2-\omega_c^2)} - \frac{\omega_p^4\,\omega_c^2}{\omega^2(\omega^2-\omega_c^2)(\omega^2-\omega_c^2-\omega_p^2)}\right), \tag{9}$$

where $\omega_c = eB/(m^*c)$ is the cyclotron frequency and B the externally applied magnetic field.

3. DISCUSSION AND CONCLUSIONS

Fig. 2 illustrates the PBS and corresponding density of states of the hexagonal lattice composed by Te solid rods in air. The PBS is obtained numerically by solving equations (3) and (4) using 243 plane waves. Compared with the case of the solid Te rods [5], for which the frequency range is situated between 0.278 (2πc/a) and 0.3068 (2πc/a), the absolute bandgap is wider and shifted to a higher frequency range [cf. Fig. 2 (a)] with its correspondent null density of states, whereas the flat band situated at about 0.45 (2πc/a) exhibits a large density of states [cf. Fig. 2(b)]. Figure 3 shows the photonic gap map for for E-polarization ($\varepsilon_2 = 38.44$) and H-polarization ($\varepsilon_2 = 23.04$) for a 2D honeycomb lattice of shell Te rods in air ($R_1 = R_2/2$; $\varepsilon_1 = \varepsilon_3 = 1.00$), and shows a dashed-grey region where the gaps of both polarizations, E and H, overlap, a situation which may be helpful in the design of optoelectronic devices.

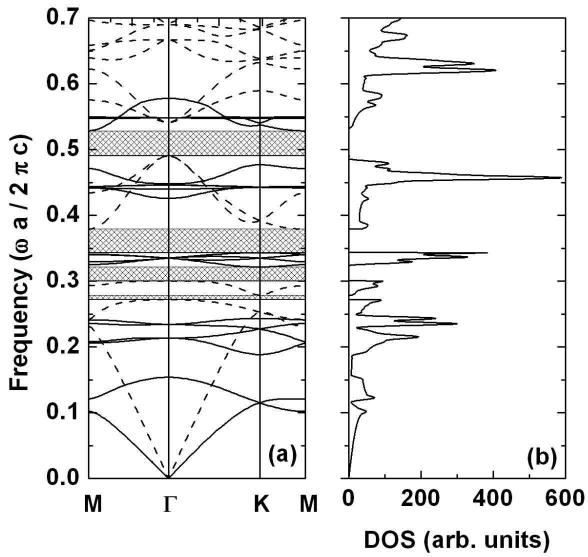

Fig. 2. (a) The photonic band structure of the 2D Honeycomb lattice of shell Te rods in air ($R_1 = R_2/2$; $R_2 = 0.3\,a$; $\varepsilon_1 = \varepsilon_3 = 1.00$) for E-polarization (solid lines; $\varepsilon_2 = 38.44$) and H-polarization (dashed lines; $\varepsilon_2 = 23.04$) modes. In (b) the corresponding density of states in arbitrary units. The absolute band gaps at the low-frequency range are shown by shaded region.

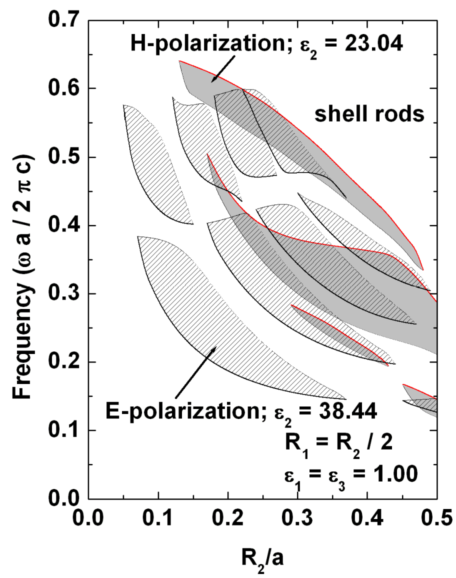

Fig. 3. (a) The photonic gap map (normalized frequency $\omega a/(2\pi c)$ versus relative radius R_2/a) for the 2D honeycomb lattice of shell Te rods in air ($R_1 = R_2/2$; $\varepsilon_1 = \varepsilon_3 = 1.00$) for E-polarization (dashed regions; $\varepsilon_2 = 38.44$) and H-polarization (grey regions; $\varepsilon_2 = 23.04$) modes. The absolute band gaps are obtained by the intersection between the dashed and grey regions.

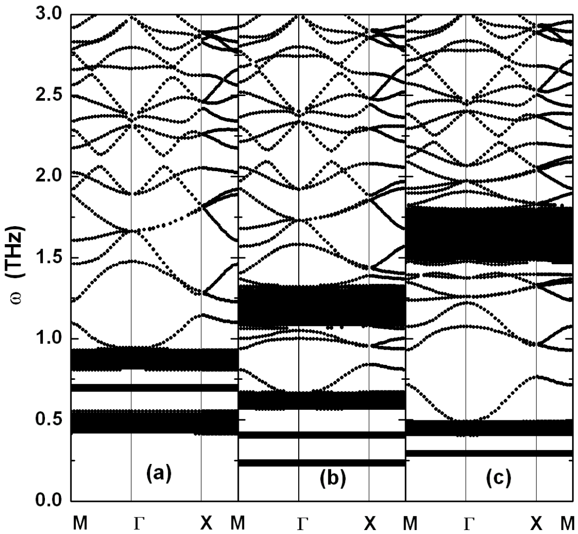

Fig. 4. Photonic band structure for the H-polarization in a honeycomb structure made of circular rods of n-doped [6] GaAs (a is the first-neighbor distance and R the radius of the rods) embedded in air, and for different values of an external magnetic field applied parallel to the rods. Different values of the applied magnetic field have been considered: $B = 0$ (a), 0.25 T (b), and 0.5 T (c). Here $R = a/2$, with a = 0.06 cm, ε_0 = 12.9 for the static dielectric constant of the GaAs, and ω_p = 0.942 THz.

In Figure 4, the PBS for H-polarization in a honeycomb structure made of circular rods of n-doped GaAs embedded in air, for different values of an external magnetic field applied parallel to the rods, is illustrated. The plasma frequency of the n-doped semiconductor [2] is $\omega_p = 0.942$ THz, and the dielectric response is taken as in Eq. (9). Above the plasma frequency, a relative wide bandgap between dispersive modes evolves under the action of the externally applied magnetic field [cf. Fig. 4 (b)]. In conclusion, present results indicate that semiconductor-based PCs may be highly sensitive to the action of an externally applied magnetic field. The anisotropic geometry breaks up the symmetry and unfolds degeneracies which may be an efficient strategy in the fabrication of systems with tunable bandgaps.

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