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Response to ‘‘Comment on ‘Photoacoustic determination of thermal diffusivity of solids: Application to CdS’’’ [Appl. Phys. Lett. 4 7, 434 (1985)]

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Comment on "Photoacoustic determination of thermal diffusivity of solids: Application to CdS"

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A recent article[1] proposed a photoacoustic method for determining the thermal diffusivity of optically transparent solids, and gave experimental confirmation. However, the theoretical basis for the method was partially in error, was unnecessarily limited to a one-dimensional treatment of heat flow, and failed to incorporate the effect of finite beam size. As the method does appear to have the advantage of simplicity of experimental arrangement, a proper theoretical discussion is warranted.

The proposed method uses a periodically modulated laser beam passing laterally through a transparent sample, viz., at constant distance from the sample-gas interface of the photoacoustic cell (Figs. 1, 2 of Ref. 1). Optical absorption (coefficient β) leads to a periodic heat source distribution which is approximately independent of the lateral coordinate (z) if the lateral dimension of the sample (d) is smaller than the optical absorption length. A Gaussian beam of (e^{-2}) radius R then gives a source distribution

$$S(x,y) = \beta I_0 \exp\{-(2/R^2)[(x-x_0)^2 + y^2]\} \exp(j\omega t), \quad (1)$$

where β is the optical absorption coefficient, I_0 is the incident intensity at beam center, x_0 is the distance of beam axis from sample-gas interface, and $f(=\omega/2\pi)$ is the modulation frequency. The variation with x_0 of the photoacoustic signal from this source distribution will allow determination of the sample thermal diffusivity.

The distribution of Eq. (1) will clearly not give rise to one-dimensional heat flow, though in the configuration of Ref. 1 heat flow will be primarily transverse to the beam. The photoacoustic signal is then given[2] by

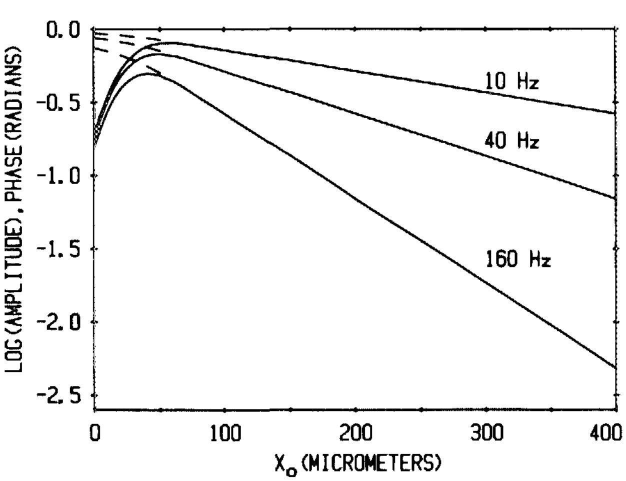

FIG. 1. Photoacoustic signal variation from Eq. (4), for three frequencies; $R = 50\,\mu$m. Dashed curves show phase variation.

$$P(t) = (\gamma P_0 \theta / T_0 l_g \sigma_g) \exp(j\omega t), \quad (2)$$

where γ is the gas specific heat ratio, P_0 and T_0 are the ambient pressure and temperature, l_g is the cell gas length, and $\sigma_g^2 = (j\omega/\alpha)$, with α the gas thermal diffusivity; θ is the sample surface temperature, *averaged over the gas-sample interface* (sample thermal expansion is neglected, as it is generally negligible for solids). This expression is technically valid only if there is negligible heat flow to the cell walls (in the gas region); however, the corrections[3] to Eq. (2) are independent of x_0, so will not affect the signal variation discussed here. A *unit point* source at depth x_0 gives an averaged surface temperature θ_p of[4]

$$\theta_p = (k_s\sigma_s)^{-1}\{(b+1)\exp(-\sigma_s x_0) - (b-1) \times \exp[-\sigma_s(2l - x_0)]\} \, [(g+1)(b+1) - (g-1)(b-1)\exp(-2\sigma_s l)], \quad (3)$$

where k_s is the sample thermal conductivity, $g = k_g\sigma_g/k_s\sigma_s$, $b = k_b\sigma_b/k_s\sigma_s$ (with subscript b indicating backing parameters), and l is the sample thickness. A beam of vanishing radius will thus give $\theta = \beta' I_0 \theta_p$, where $\beta' = \beta d'$, d' being the lateral dimension of the gas-sample interface ($d' < d$, see Fig. 2 of Ref. 1); this result replaces Eq. (5a) of Ref. 1. A beam of non-negligible radius requires that one integrate the product $S\theta_p$ over the effective source region:

$$\theta = \int_{-\infty}^{0} dx \int_{-\infty}^{\infty} dy \int_{0}^{d'} dz \, S(x,y)\theta_p = A \exp(-\sigma_s x_0) \tfrac{1}{2} \operatorname{erfc}\,[(\sigma_s R/2\sqrt{2}) - (\sqrt{2}x_0/R)], \quad (4)$$

where A is a complex constant, erfc is the complementary error function, and we have assumed a thermally thick sample, so $l \to \infty$. This result reduces to that of Ref. 1 ($l \to \infty$) in the limit of $R \to 0$.

The variation of θ with x_0 is shown in Fig. 1, for three frequencies, and $R = 50\,\mu$m. For $x_0 > 100\,\mu$m straight lines are obtained (for *both* log amplitude and phase), consistent with the experimental data of Ref. 1; the slope is $a_s = (\pi f/\alpha_s)^{1/2}$. The present result is derived without assuming one-dimensional heat flow, and incorporates the effect of finite beam size.

[1]C. L. Cesar, H. Vargas, J. M. Filho, and L. C. M. Miranda, Appl. Phys. Lett. **43**, 555 (1983).
[2]F. A. McDonald, J. Photoacoustics **1**, 171 (1982).
[3]F. A. McDonald, J. Phys. Coll. **44**, C6, 21 (1983); F. A. McDonald, J. Appl. Phys. **52**, 381 (1981).
[4]F. A. McDonald, Appl. Phys. Lett. **36**, 123 (1979).

Response: In the preceding Comment[1] McDonald has criticized the theoretical basis of the photoacoustic (PA) method described by the present authors[2] for measuring the thermal diffusivity of solids. In his criticism it is claimed that the model of Ref. 2 "was unnecessarily limited to a one-dimensional treatment of the heat flow and failed to incorporate the effect of finite beam size" and that a proper treatment of the effects of three-dimensional heat flow is presented in his Comment.

In this reply we respond to these criticisms by firstly commenting on his statement on the first paragraph of his paper that the model of Ref. 2 "was unnecessarily limited to a one-dimensional treatment of the heat flow and failed to incorporate the effect of finite beam size." Reading carefully the description of our experiment the reader will find that we were aware of the need for taking into account the finite radius of the heating beam and this was indeed experimentally taken into account as described in page 556 of Ref. 2 "...Here we note that, because of the finite waist of the laser beam, the illumination position is defined relative to the beam waist. That is suppose that we are illuminating the sample from the gas side. As soon as part of the beam reaches the sample, the PA signal begins to grow; on further moving the sample, the PA signal eventually saturates when the entire beam is illuminating the sample. Up to this point we have moved the sample a distance of roughly 100 μm, which corresponds to the beam waist. From this point on, the PA signal decreases when moving the sample. Thus, when plotting our data we have subtracted the beam waist in the illumination position scale." This means that care has been taken, through our experimental procedure, to account for the effect of a finite beam radius. The resulting experimental data were then verified to decay exponentially as predicted by a one-dimensional model. In other words, the restriction to a one-dimensional model was not a question of choice but rather dictated by the experimental results. In this sense we have not "failed" to incorporate the effects of the finite beam size. Rather on the contrary, by taking into account the finite beam size as outlined above was that we have found that the simple one-dimensional model was perfectly adequate to describe our data. The effect of the finite beam radius, as discussed in McDonald's Comments, is only relevant up to one beam diameter. This is clearly shown in Fig. 1 of the preceding Comment in which, after an initial increasing up to a beam waist distance (100 μm), the PA signal decays exponentially as $\exp(-a_s s_0)$, where $a_s = (\pi f/\alpha_s)^{1/2}$ and x_0 is the illumination position. Since the finite beam radius effect can always be taken into account by adopting the experimental procedure outlined in Ref. 2, the more elaborate calculation of McDonald adds nothing.

The second point which is worth commenting regards the McDonald's claim in the abstract of his Comment that a proper treatment of the effects of three-dimensional heat flow is presented in his paper. This is certainly not what is done as is evident by the one-dimensional Green's function used in his paper [cf. Eq. (3) of the preceding Comment]; θ_p is a function only of x_0. This means that all that is mathematically done in his Comment is essentially to include the effect of a Gaussian heat source distribution on the one-dimensional model for the PA signal production. The result of the taking into account of this Gaussian beam of radius R, as shown in Fig. 1 of his Comment, is that for distances greater than one beam diameter the PA signal should decay exponentially as $\exp(-a_s x_0)$ in agreement with the one-dimensional model of Ref. 2.

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[2]C. L. Cesar, H. Vargas, J. Mendes Filho, and L. C. M. Miranda, Appl. Phys. Lett. **43**, 555 (1983).