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Reflection of magnetoelastic waves from ferromagnetic surfaces

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We present theoretical studies of the reflection of bulk magnetoelastic waves from the surface of a ferromagnet, with magnetization parallel to the surface. The calculations show a pronounced left-to-right asymmetry in the amplitude of the reflected wave. The origin of this asymmetry is in the boundary conditions which when applied to surface magnetoelastic waves theory lead to nonreciprocal dispersion relations. The frequency dependence of the reflectivity is studied here, for particular selected geometries which exhibit the left-to-right asymmetry.

I. INTRODUCTION

The propagation of surface magnetoelastic waves on ferromagnetic surfaces,[1] and on nonmagnetic substrates overlaid with ferromagnetic films[2] has been studied by a number of authors. One striking feature of all these analyses is that the propagation characteristics of the waves are nonreciprocal. That is, if we consider a surface wave of some particular frequency, then both the phase velocity (dispersion relation) and attenuation constant of the wave differ when the wave propagates from left to right across the magnetization (assumed parallel to the surface), than when the propagation is from right to left. As pointed out earlier by Scott and Mills,[1] the origin of the asymmetry lies in a breakdown of reflection symmetry produced by the presence of the surface. We repeat the argument below.

The purpose of this paper is to point out that the same breakdown of reflection symmetry leads to left-to-right asymmetries in the reflectivity of bulk magnetoelastic waves from the ferromagnetic surface. This is true even though the bulk dispersion relations do not display the nonreciprocity evident in the surface wave dispersion relations. We illustrate this point with calculations carried out for selected geometries which exhibit the asymmetry strongly. We also find that far from the ferromagnetic resonance frequency there can be a nearly complete transfer of energy from one elliptically polarized branch of the magnetoelastic wave spectrum to its nearly degenerate partner, as such that an incident wave reflects from the boundary.

The geometry we consider here is illustrated schematically in Fig. 1. We have a semi-infinite ferromagnet which occupies the half-space $y > 0$, and the magnetization $\vec{M}_s$ and external field $\vec{H}_0$ are parallel to each other, and also to the surface. A bulk magnetoelastic wave of wave vector $\vec{k}_0$ is incident on the surface, and a portion of the incident energy is carried away by the specularly reflected wave with wave vector $\vec{k}_R$. Our point is that the amplitude of the specular wave is in general different for the configuration in Figs. 1(a) and 1(b). In essence, the configuration in Fig. 1(b) is obtained from Fig. 1(a) by a time-reversal operation applied to the wave vectors of both the incident and specularly reflected wave (the magnetization is held fixed in direction).

One might believe that the breakdown of time reversal as a "good symmetry" operation of the crystal in the presence of magnetic order is responsible for the asymmetry in the reflection coefficient. Howev-

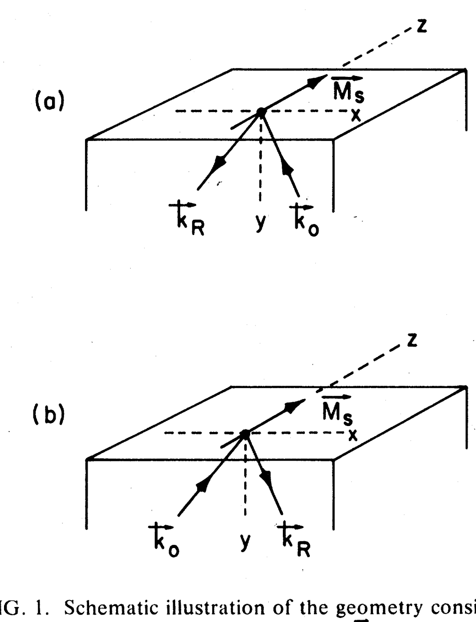

FIG. 1. Schematic illustration of the geometry considered in the present paper. The magnetization $\vec{M}_s$ lies parallel to the z direction and to the sample surface. In (a), the incident wave vector describes a wave that carries energy from right to left across the magnetization, and in (b) from left to right.

er, this is not the case. The asymmetry has its origin in the breakdown of reflection symmetry in the xz plane produced by the surface combined with the axial vector character of $\vec{M}_s$.

To appreciate this, we return to the consideration of magnetoelastic surface waves, which have a nonreciprocal dispersion relation with $\omega_s(\vec{k}_\parallel) \neq \omega_s(-\vec{k}_\parallel)$, where $\vec{k}_\parallel$ is the wave vector of the surface wave. While there is nonreciprocal character in the surface wave dispersion relation,[3] the bulk magnetoelastic wave dispersion relations $\omega_B(\vec{k})$ are always *even* in the wave vector, i.e., $\omega_B(+\vec{k}) \equiv \omega_B(-\vec{k})$ for *each* branch. If nonreciprocity in the surface wave dispersion relation had its origin in the breakdown of time-reversal symmetry in the presence of the spontaneous magnetization $\vec{M}_s$, then both $\omega_s(\vec{k}_\parallel)$ and $\omega_B(\vec{k})$ should exhibit nonreciprocity.

For a crystal for which the xz, yz, and xy planes are reflection planes (in the absence of a surface), then with $\vec{M}_s \neq 0$, reflection symmetry requires $\omega_B(+\vec{k}) = \omega_B(-\vec{k})$. This is true if the magnetization is aligned along a principal axis of the crystal, and if all three principal axes are perpendicular to each other. To illustrate, consider a bulk magnetoelastic wave which propagates parallel to the x axis, so $\vec{k} = \hat{x}k$. A reflection operation R_{yz} in the yz plane takes $\vec{k}$ into $-\vec{k}$, but is not a good symmetry operation because $\vec{M}_s$ is an axial vector, so R_{yz} changes its sign. The reflection R_{xz} applied subsequently leaves $\vec{k}$ unchanged, but restores $\vec{M}_s$ to its original sign. Thus, for the ferromagnet, the combination $R_{xz}R_{yz}$ is a good symmetry operation when $\vec{M}_s \neq 0$, and this ensures $\omega_B(+\vec{k}) = \omega_B(-\vec{k})$ for bulk magnetoelastic waves that propagate parallel to $\hat{x}$. The argument is readily extended to show the bulk wave dispersion relation has inversion symmetry for any direction of $\vec{k}$.

If we consider a semi-infinite material, then R_{xz} takes the crystal from the half-space $y > 0$ and places it in the region $y < 0$. Thus, with surface present and $\vec{M}_s \neq 0$, the combination $R_{xz}R_{yz}$ no longer remains a good symmetry operation. This has the consequence that $\omega_s(+\vec{k}_\parallel) \neq \omega_s(-\vec{k}_\parallel)$, since there is no symmetry operation which changes the sign of $\vec{k}_\parallel$, and leaves both $\vec{M}_s$ and the configuration of the crystal unchanged. It is the breakdown of this reflection operation that renders the specular reflectivity in Fig. 1(a) different than that in Fig. 1(b).

From a theoretical point of view, the nonreciprocity of the surface wave dispersion curve and the behavior of the reflectivity are intimately linked. One finds the magnetoelastic dispersion curve by searching for the zeros of the determinant of a certain matrix $D(\vec{k}_\parallel, \omega)$. The reflectivity calculation involves inversion of the matrix, with the elements calculated through choice of a value for $\vec{k}_\parallel$ determined by the bulk magnetoelastic dispersion curves. In each analysis, the nonreciprocity has the same mathematical origin.

In Sec. II, we outline the method we have used to carry out the calculations, and in Sec. III we discuss our results. Section II is quite brief, since the analysis is straightforward in principle, though not in practical execution.

II. CALCULATION

The approach we have used is very similar to that described earlier by Scott and Mills,[1] in a study of magnetoelastic surface wave propagation on a semi-infinite ferromagnet with magnetization parallel to the surface. The Hamiltonian describes an isotropic elastic continuum with c_t and c_l the transverse and longitudinal sound velocity, respectively. The spin motion is described by a Bloch equation, with demagnetizing field included but exchange ignored. At the frequencies and wavelengths of interest in the present study, the neglect of exchange has little quantitative consequence; in the magnetoelastic surface wave problem, the influence of exchange has been studied by Camley and Scott,[1] and we refer the reader to their paper for a discussion of its role.

The spin-lattice coupling enters the present calculation through the term

$$H_{ls} = \frac{b_2}{M_s}[M_x(e_{xz} + e_{zx}) + M_y(e_{yz} + e_{zy})] \quad , \tag{1}$$

and as in the work of Scott and Mills, we align the magnetization along the [100] direction so in the cubic crystal, only one magnetoelastic constant b_2 enters. We have set the relaxation time τ of the spins equal to infinity, so in the bulk of the material, the mean free path of the magnetoelastic wave is infinite. It is difficult to discuss the reflectivity calculation within a model that includes dissipation within the crystal.

Through use of the equations of motion displayed by Scott and Mills, one may find the dispersion relations of the bulk magnetoelastic waves, for any selected direction in the crystal. This must be done numerically, unfortunately. In general, we have a three-branch dispersion relation. For a particular geometry, we show dispersion curves in Fig. 2(b). These have been calculated for $\alpha = \beta = \frac{1}{4}\pi$, where α and β are defined by Fig. 2(a). We have chosen $b_2 = 2 \times 10^8$ ergs/cm^3, typical of a metal such as Ni, the external magnetic field is $H_0 = 2 \times 10^3$ Oe, the magnetization $4\pi M_s = 6 \times 10^3$ Oe, the transverse and longitudinal sound velocities have been taken to be 3.83×10^5 cm/sec and 5.4×10^5 cm/sec, respectively, and finally the density ρ has been chosen to be 8.9 g/cm^3. These material parameters have been employed in all the calculations reported in the present paper. Figure 2(b) shows two modes for each frequency that mix strongly with the spins. The third branch has a frequency which lies very close to, but

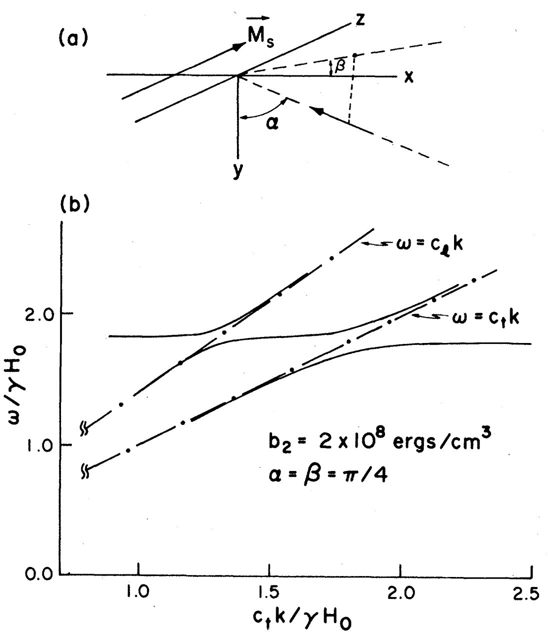

FIG. 2. (a) Orientation of the incident wave vector is described by the angles α and β illustrated here. The angle α is the angle between the wave vector and the y axis, normal to the surface, while β is the angle between the projection of the wave vector onto the xz plane and the x axis. (b) For parameters described in the text, we show the dispersion relation of the bulk magnetoelastic waves. One branch, not shown, couples weakly to the spins and has frequency close to $c_t k$.

not precisely equal to $c_t k$, with k the wave vector of the mode. The two branches which mix strongly with the spins show anomalous dispersion at the spin-wave frequency

$$[H_0(H_0+4\pi M_s \sin^2\theta_k)]^{1/2} \ ,$$

with θ_k the angle between the propagation direction and the magnetization. For the geometry illustrated in Fig. 2(a),

$$\sin^2\theta_k = \cos^2\alpha + \sin^2\alpha \cos^2\beta \ .$$

The basic calculation we explore here is to launch a bulk magnetoelastic wave from one of the strongly coupled branches of the dispersion curve up to the surface, and we study the amplitude of the mode specularly reflected from the surface. Of course, there are several reflected waves, only one of which is associated with the original branch. Each wave that comes off the surface has the same wave-vector components

$$\vec{k}_\parallel = \hat{x}k \sin\alpha \cos\beta + \hat{z}k \sin\beta \ ,$$

as the incoming wave, but their wave-vector components $k_\perp$ normal to the surface are different. In fact, in certain regimes of frequency, some of the normal wave-vector components are purely imaginary, so we have a disturbance localized near the surface with displacement and magnetization that falls to zero exponentially as one moves into the crystal interior. We require a program that calculates the perpendicular components $k_\perp$ associated with a wave of frequency ω, and wave vector $\vec{k}_\parallel$ parallel to the surface. This program is similar to that used in the magnetoelastic surface wave investigations.

For a given incident wave direction, once the various allowed values of $k_\perp$ are found, the modes are superimposed and subjected to boundary conditions. The boundary conditions are that the stress tensor components σ_{yi} must vanish at the surface, and the normal component of the $\vec{b} = \vec{h} + 4\pi\vec{M}_T$ is continuous along with the tangential components of $\vec{h}$. Here $\vec{h}$ is the demagnetizing field generated by the transverse magnetization $\vec{M}_T = \hat{x}M_x + \hat{y}M_y$ associated with the spin motion. Outside the crystal ($y < 0$) we have a magnetic field $\vec{h}^< = -\nabla\phi_M$ in the vacuum, where

$$\phi_M = \phi^< \exp(i\vec{k}_\parallel \cdot \vec{x}_\parallel + k_\parallel y) \ .$$

The key feature of the calculation is inclusion of the magnetoelastic contributions to the elements σ_{yz} which appears in the boundary condition. As earlier,[1] we have

$$\sigma_{yz} = \sigma_{yz}^{(0)} + \frac{b_2}{M_s} M_y \ , \qquad (2)$$

while the other elements of the stress tensor involved in the boundary condition have a form identical to the limit $b_2 = 0$. The second term in Eq. (2) introduces nonreciprocal behavior into the reflection coefficient, since the first and second terms behave differently with respect to reflection in the yz plane. The first term is invariant under such a reflection, and the second changes sign.

The calculation of the reflection coefficient has been carried out as described above, and we now turn to our results.

III. DISCUSSION OF THE RESULTS

In Fig. 3, we present calculations of the frequency variation of the reflection coefficient for two choices of b_2, and for $\alpha = \beta = \frac{1}{4}\pi$. The incident wave thus propagates in the same direction chosen for the dispersion curves displayed in Fig. 2. The incident wave has been taken to lie on the lowest branch of the magnetoelastic wave dispersion curve in Fig. 2.

For both choices of b_2, we see a clear asymmetry in the reflection coefficient, for frequencies which lie somewhat below the asymptotic frequency of the branch. The asymmetry is largest in the vicinity of the "knee" of the dispersion curve, where the mode

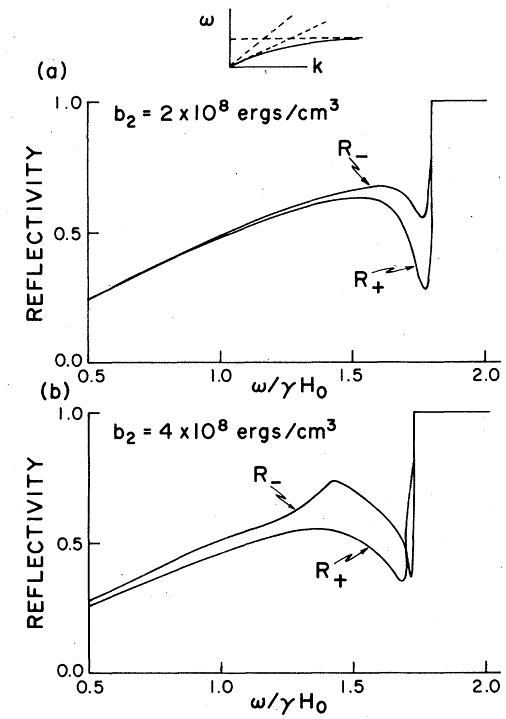

FIG. 3. Calculations of the reflection coefficient for $\alpha=\beta=\frac{1}{4}\pi$, for two choices of the magnetoelastic coupling constant b_2. Both (a) and (b) are calculated for an incident wave on the lowest branch of the coupled-mode dispersion curve, as illustrated in the inset. The curve labeled R_- gives the reflectivity for propagation across the magnetization from right to left, and that labeled R_+ is for propagation from left to right.

is a strong admixture of both spin and lattice motion. For the smaller value of b_2, the two curves are qualitatively similar in shape, but as b_2 is increased, the frequency variations in the two directions become rather different.

The low value obtained for the reflection coefficient for frequencies small compared to the spin-wave frequencies is an effect of magnetoelastic coupling, even though only a small fraction of the energy in the wave is stored in the spin motion. Suppose b_2 vanishes identically. Then the normal shearlike modes of the bulk may be chosen to be linearly polarized and are degenerate with velocity $c_t k$. If we send a shear wave up to the surface, and its displacement is parallel to the surface, the reflection coefficient is unity for the angle of incidence we have used. The same is true for a shear wave with displacement parallel to the plane of incidence, as one sees from the analysis presented by Landau and Lifshitz.[4] Symmetry forbids mode mixing upon reflection from the surface, i.e., with $b_2 \equiv 0$ one cannot convert energy stored in an incident wave of s polarization into p-polarized shear wave energy by this means.

Even far from the spin-wave frequencies, where the influence of magnetoelastic coupling is small in quantitative terms, the two degenerate linearly polarized eigenvectors are mixed strongly to form normal modes with elliptical polarization when $b_2 \neq 0$. If one such mode is launched toward the surface, conversion of energy from a wave with one sense of elliptical polarization to the other is now possible, and the amplitude of the reflected wave with polarization the same as the incident wave can become small compared to unity if this conversion is very efficient. Our calculations show that there is nearly complete conversion at low frequency. This seems to be the case for each angle of incidence we have examined. We have checked the amplitude of both low-frequency elliptical waves with $\omega \cong c_t k$, to verify that 100% of the incident energy is carried away from the surface by the combination of the two, in the low-frequency limit. It is striking to see this influence of the magnetoelastic coupling far from the spin-wave frequencies. The origin of the effect is, as just described, that nearly degenerate waves are mixed strongly by even a weak perturbation.

For the incident wave on the lower branch, at wave vectors sufficiently far out on the dispersion curve, well beyond the "knee" and on the flat portion of the curve, the values of $k_\perp$ for all reflected waves, save for the specularly reflected wave, become pure imaginary. Here the reflectivity is necessarily unity. The region of the dispersion curve is not of great experimental interest, since the group velocity of the wave is very close to zero, and its mean free path is short.

In Fig. 4, we show calculations of the frequency variation of the reflection coefficient for the case where the incident magnetoelastic wave is directed along the [111] direction. We see a larger contrast between the reflectivity for the two directions of propagation, when these calculations are compared to these given in Fig. 3. Thus, the magnitude of the

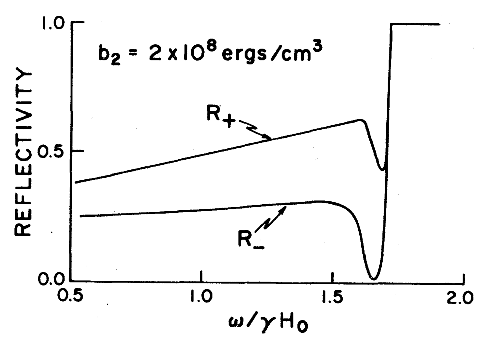

FIG. 4. Reflection coefficient for the case where the incident magnetoelastic wave is directed along the [111] direction. R_+ and R_- have the same meaning as in Fig. 3.

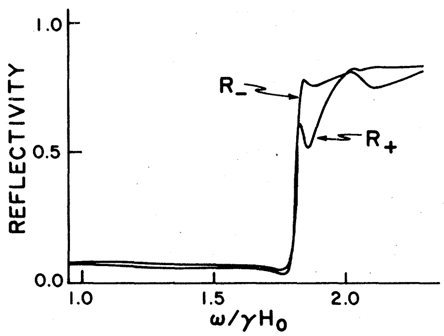

FIG. 5. Frequency variation of the reflection coefficient, for an incident wave on the branch of the magnetoelastic wave spectrum that approaches $c_l k$, as $k \to 0$.

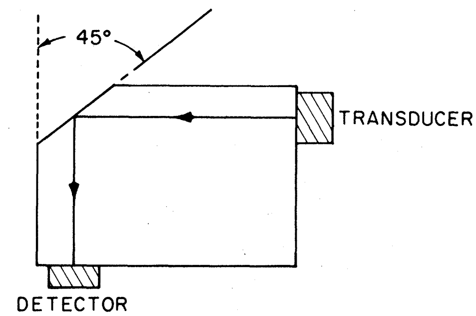

FIG. 6. Possible geometry for which asymmetries similar to those calculated here may be observed. The crystal within which the magnetoelastic waves propagate is a ferromagnet, with magnetization normal to the plane of the figure. Reversal of the direction of the magnetization will alter the reflection coefficient.

asymmetry is quite sensitive to the angle of incidence of the wave. Note particularly that the wave incident from the left side now has the largest reflectivity, while the converse is true for the geometry in Fig. 4.

Figure 5 gives reflectivities calculated for that branch of the magnetoelastic wave spectrum for which the frequency approaches $c_l k$ as $k \to 0$, with c_l the longitudinal sound velocity. The reflectivity is very low for frequencies below the spin-wave frequencies. What happens here is that almost all the energy which comes off of the surface is carried by the (nearly) p-polarized transverse waves. The reflection coefficient here agrees well with that calculated from elasticity theory[4] with b_2 set to zero. Above the spin-wave frequencies, this branch shows asymmetry in the reflection coefficient once again. When the results in Figs. 3, 4, and 5 are combined, we find a small asymmetry for the reflection coefficient on those portions of the magnetoelastic dispersion curve that lies near the "bare" longitudinal phonon frequency $c_l k$.

We conclude in Fig. 6 with an illustration of a geometry where asymmetries such as those calculated here may be observed, at least in principle. Imagine a transducer and detector arranged so magnetoelastic waves can be detected after reflection from a corner cut as shown. If the magnetization is perpendicular to the plane of the figure, its direction can be reversed through 180° through use of a suitable magnetic field. Thus, reversal of the magnetic field with transducer and detector fixed in location should lead to a change in the reflection coefficient of the magnetoelastic wave from the surface. We would find such measurements most intriguing.

ACKNOWLEDGMENTS

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[1]A number of authors have discussed the propagation of magnetoelastic surface waves on ferromagnets of semi-infinite extent. Representative studies which, as in the present work, ignore the influence of exchange on the spin motion are J. P. Parekh and H. L. Bertoni, J. Appl. Phys. 45, 434 (1974); P. R. Emtage, Phys. Rev. B 13, 3063 (1976); and R. Q. Scott and D. L. Mills, *ibid.* 15, 3545 (1977). The influence of exchange has been explored by R. E. Camley and R. Q. Scott, *ibid.* 17, 4327 (1978).

[2]A discussion of surface magnetoelastic wave propagation for a semi-infinite, nonmagnetic substrate covered with a ferromagnetic film has been given by R. E. Camley, J. Appl. Phys. 50, 5272 (1979).

[3]There are other instances where nonreciprocal behavior has been discussed for surface excitations, in the presence of a magnetic field. For example, see the discussion of surface polaritons on a doped semiconductor with magnetic field applied parallel to the surface. A theoretical discussion of these modes has been presented by J. J. Brion, R. F. Wallis, A. Hartstein, and E. Burstein, Phys. Rev. Lett. 28, 1455 (1972). The nonreciprocal propagation characteristics of these modes has been explored experimentally by A. Hartstein and E. Burstein, Solid State Commun. 14, 1223 (1974).

[4]See L. D. Landau and E. M. Lifshitz, in *Theory of Elasticity* (Addison-Wesley, London, 1959), pp. 101–103.