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Screening effect on the plasma heating by inverse bremsstrahlung

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The effect of Coulomb screening on the inverse-bremsstrahlung heating process in a plasma illuminated by a laser beam is discussed. It is shown that, although the screening effect actually lowers the electron-nuclei interaction, a situation may arise, namely, when the laser frequency approaches the plasma frequency, in which an enhancement of the collisional plasma heating can be expected.

I. INTRODUCTION

There has been recent interest in the study of the interaction of intense laser fields with plasmas[1-4] and semiconductors.[5-9] In particular, it has been suggested that the inverse-bremsstrahlung process may play an important role in the breakdown and the heating of a plasma by a laser beam.[10] The argument goes as follows:[2,10] in the simultaneous presence of the Coulomb nuclear field and of an electromagnetic wave of frequency ω and electric field intensity $\vec{E}_0$, an electron may absorb photons from the electromagnetic field. When the nucleus recoil energy is neglected, the gain in average kinetic energy of the electrons per process is $e^2\vec{E}_0^2/2m\omega^2$. Since, obviously, not all electron-nucleus collisions will be accompanied by inverse bremsstrahlung, let ν_{eff} be the appropriate effective collision rate. The rate of change of the kinetic energy of the electron will then be given by[2]

$$\frac{d}{dt}\langle\epsilon\rangle = \frac{e^2\vec{E}_0^2}{2m\omega^2}\nu_{\text{eff}}\ . \tag{1}$$

However, if this process is to contribute effectively towards the heating, to thermonuclear temperatures, when a plasma is illuminated by a laser beam, it must entail a rapid absorption of electromagnetic energy during the period of the laser pulse. The investigation of the rate of absorption by inverse bremsstrahlung was done by Seely and Harris,[2] who have shown the important result that, even though it is a comparatively slow process, it could eventually be made faster than collective instabilities and become the dominant heating mechanism. In their calculations,[2] however, the effect of the screening of the Coulomb interaction between the electron and the nuclei has been neglected. As is well known, the interaction of a charged particle with electrons is substantially lowered by Coulomb screening, thereby affecting the effective collision rate; it is consequently of interest to look for ways to reduce this effect. This leads us to the important question of how the estimates of Ref. 2 would be changed by Coulomb screening.

In the following we consider the effect of Coulomb screening on the inverse-bremsstrahlung heating process, and investigate the conditions under which the weakening effect of screening could be reduced. It is shown that, although the screening effect actually lowers the Coulomb interaction, one might accomplish a reduction of this weakening effect by illuminating the plasma with an electromagnetic wave with frequency ω near the plasma frequency ω_p. The plasma is assumed to be infinite and homogeneous, and we neglect the effects of external magnetic fields. The laser beam is treated as a classical plane electromagnetic wave in the dipole approximation. This is justifiable if the distance over which the amplitude of the electromagnetic wave changes is large in comparison to the size of the scattering centers, the initial Debye screening radius r_D, and the amplitude of the electron oscillations in the wave field. The electron states are described by the solution to the Schrödinger equation for an electron in the field of a classical electromagnetic wave. The inverse-bremstrahlung process is treated using first-order perturbation theory as done in Ref. 2.

We begin with the derivation of the screened potential of a static charge Ze placed in a plasma subjected to an electromagnetic wave. The transition probabilities for the electron collision with a nucleus (assumed to be fixed) are then used to write a kinetic equation for the electrons. After taking the classical limit, we calculate the rate of change of the kinetic energy of the electrons, and compare it with Eq. (1) to estimate the effective collision frequency.

II. SCATTERING POTENTIAL

The modification of Coulomb screening due to the presence of an electromagnetic wave has been discussed in previous papers.[11,12] Here we shall briefly outline the main results. We begin by writing the Hamiltonian of our system as

$$H(t)=\sum_{\vec{p}}\frac{1}{2m}\left(\hbar\vec{p}+\frac{e}{c}\vec{A}(t)\right)^2 c_p^\dagger c_p - e\sum_{\vec{p},\vec{k}}\varphi(\vec{k},t)c_{p+k}^\dagger c_p\ ,$$

where $\vec{A}(t)=(c/\omega)\vec{E}_0\cos\omega t$ describes the laser field, and the scalar potential φ describes the field of a static charge and the self-consistent field. The Fourier components of this scalar potential are given by the Poisson equation

$$k^2\varphi(\vec{k},t)=4\pi\rho(\vec{k})-4\pi e\sum_{\vec{p}}\langle c_{p-k}^\dagger c_p\rangle_t\ , \tag{2}$$

where $\rho(\vec{k})$ is the Fourier component of the static charge, and $\langle\ \rangle_t$ denotes averaging with the complete Hamiltonian. Constructing the equation of motion for $\langle c_{p-k}c_p\rangle_t$ within the random-phase approximation (RPA), solving it with the initial condition $\langle c_{p-k}^\dagger c_p\rangle_{t=-\infty}=0$, and substituting into Eq. (2), one gets[12]

$$\varphi(\vec{k},t)=\frac{4\pi\rho(\vec{k})}{k^2}-\frac{4\pi i e^2}{k^2}\int_{-\infty}^{t}dt'\,\varphi(\vec{k},t') \times\sum_{\vec{p}}(f_{p-k}-f_p)\exp\left(-\frac{(\epsilon_p-\epsilon_{p-k})(t-t')}{\hbar}\right) \times\exp[-i\vec{k}\cdot\vec{a}(\sin\omega t-\sin\omega t')]\ . \tag{3}$$

Here $\vec{a}=e\vec{E}_0/m\omega^2$ is the electron oscillation amplitude in the electromagnetic wave field, $\epsilon_p=\hbar^2p^2/2m$, and f_p is the electron occupation number. If we now define

$$\tilde{\varphi}(\vec{k},t)=\varphi(\vec{k},t)\exp(i\vec{k}\cdot\vec{a}\sin\omega t)\ ,$$
$$\tilde{\rho}(\vec{k},t)=\rho(\vec{k})\exp(i\vec{k}\cdot\vec{a}\sin\omega t)\ , \tag{4}$$

and combine Eqs. (4) with Eq. (3) we get

$$\tilde{\varphi}(\vec{k},\omega)=4\pi\tilde{\rho}(\vec{k},\omega)/k^2\epsilon(\vec{k},\omega)\ , \tag{5}$$

where $\epsilon(\vec{k},\omega)$ is the usual dielectric constant in the RPA.[13] It then follows from Eqs. (4) and (5) that $\varphi(\vec{k},t)$ can be written[12]

$$\varphi(\vec{k},t)=\sum_{n,\mu=-\infty}\frac{4\pi\rho(\vec{k})}{k^2\epsilon(\vec{k},n\omega)}J_{n+\mu}(\vec{k}\cdot\vec{a})J_n(\vec{k}\cdot\vec{a})\,e^{i\mu\omega t}\ , \tag{6}$$

where $J_n(z)$ is the Bessel function of order n. Equation (6) tell us that the presence of an electromagnetic wave affects the screening in such a way that the high-frequency components at the wave frequency and its harmonics contribute even to the low-frequency components of the potential.[1] In particular, the static component $\varphi_0(\vec{r})$ (i.e., where $\mu=0$) will be[12]

$$\varphi_0(\vec{r})=\frac{1}{(2\pi)^3}\int d^3k\,\frac{4\pi Ze}{k^2\epsilon_{\text{eff}}}\,e^{-i(\vec{k}\cdot\vec{r})}\ , \tag{7}$$

where

$$\frac{1}{\epsilon_{\text{eff}}}=\sum_{n=-\infty}^{+\infty}\frac{J_n^2(\vec{k}\cdot\vec{a})}{\epsilon(\vec{k},n\omega)}\ . \tag{8}$$

That is, the effect of a radiation field on the static potential of a point charge can be taken into account by introducing an effective dielectric constant dependent on both the frequency and the polarization of the electromagnetic field. In the zero-field limit (i.e., where $\vec{a}=0$) only the $n=0$ term in Eq. (8) survives, so that ϵ_{eff} reduces to the usual static dielectric constant $\epsilon(\vec{k},0)$.

III. KINETIC EQUATION

Once we know the modification of the Coulomb potential of a fixed point charge in the plasma due to the presence of an electromagnetic wave, we can calculate the electron scattering by these static charges. In doing so, however, we shall consider only the static component of the potential, $\varphi_0(\vec{r})$, as given by Eq. (7). Treating the electron collision with a nucleus as a perturbation, we find the transition-probability amplitude for a transition from a state $1(\vec{p}_1)$ to a state $2(\vec{p}_2)$ to be

$$a(1\to2)=i/\hbar\int d^3r\int dt\,\psi_2^*(\vec{r},t)\,e\varphi_0(\vec{r})\psi_1(\vec{r},t)\ , \tag{9}$$

where $\psi(\vec{r},t)$ is the solution to the Schrödinger equation for an electron in the field of an electromagnetic wave[2]:

$$\psi(\vec{r},t)=V^{-1/2}\,e^{i\vec{p}\cdot\vec{r}}\,e^{i\vec{\delta}(t)\cdot\vec{p}}\,e^{-i\epsilon_p t/\hbar}\,e^{-i\eta(t)}\ , \tag{10}$$

with

$$\vec{\delta}(t)=(e\vec{E}_0/m\omega^2)\sin\omega t\ ,\quad \eta(t)=(e^2\vec{E}_0^2/4m\omega^2\hbar)t\ . \tag{11}$$

Substituting Eqs. (7) and (10) into Eq. (9), and performing the integrations as done in Ref. 2, we can write

$$a(1\to2)=\sum_{\nu=-\infty}^{+\infty}2\pi i\,\delta(\epsilon_2-\epsilon_1-\nu\hbar\omega)\frac{4\pi Ze^2J_\nu(z)}{V|\vec{p}_2-\vec{p}_1|^2\epsilon_{\text{eff}}(\vec{p}_2-\vec{p}_1)}\ , \tag{12}$$

with $z=(\vec{p}_2-\vec{p}_1)\cdot\vec{a}$. From the well-known relation between the scattering amplitude and the T matrix,[14] we can then use Eq. (12) to obtain the transition probability per unit time, $T_\nu(1\to2)$, for the transition from state 1 to state 2 with absorption ($\nu>0$) or emission ($\nu<0$) of $|\nu|$ photons. We get

$$T_\nu(1\to 2)=\frac{2\pi}{\hbar}J_\nu^2(z)\left|\frac{4\pi Ze^2}{V|\vec{p}_2-\vec{p}_1|^2\epsilon_{\text{eff}}(\vec{p}_2-\vec{p}_1)}\right|^2 \times\delta(\epsilon_2-\epsilon_1-\nu\hbar\omega). \quad (13)$$

The rate of change of the number of electrons $f(\vec{p}_2)$ with momentum $\hbar\vec{p}_2$ is then given in terms of the transition probability as[2]

$$\frac{\partial f(\vec{p}_2)}{\partial t}=\sum_{\nu=-\infty}^{+\infty}\sum_{\vec{p}_1}T_\nu(1\to 2)[f(\vec{p}_1)-f(\vec{p}_2)], \quad (14)$$

with the assumption that the electrons are far from degeneracy [i.e., $f(\vec{p})\ll 1$]. Letting the sum over $\vec{p}_1$ become an integral, assuming a Maxwellian distribution for the electrons, and using Eq. (13), we reduce Eq. (14) to

$$\frac{\partial f(\vec{p}_2)}{\partial t}=f(\vec{p}_2)\sum_{\nu=1}^{\infty}\frac{V}{(2\pi)^3}\int d^3k\,\frac{2\pi}{\hbar}\frac{J_\nu^2(\vec{k}\cdot\vec{a})(4\pi Ze^2)^2}{V^2|\vec{k}^2\epsilon_{\text{eff}}(\vec{k})|^2} \times\left\{\left[\exp\left(\frac{\nu\hbar\omega}{k_BT}\right)-1\right]\delta\left(\frac{\hbar^2\vec{k}\cdot\vec{p}_2}{m}-\nu\hbar\omega-\frac{\hbar^2k^2}{2m}\right)+\left[\exp\left(\frac{-\nu\hbar\omega}{k_BT}\right)-1\right] \times\delta\left(\frac{\hbar^2\vec{k}\cdot\vec{p}_2}{m}-\nu\hbar\omega+\frac{\hbar^2k^2}{2m}\right)\right\}, \quad (15)$$

where we have written $\vec{p}_1$ as $\vec{p}_1=\vec{p}_2-\vec{k}$. Upon taking the classical limit by letting $\hbar\to 0$ such that[13]

$$\frac{\hbar\vec{p}}{m}\to\vec{v},\quad \sum_{\vec{p}}(\cdots)f(\vec{p})\to V\int d^3v\,(\cdots)f(\vec{v}), \quad (16)$$

we finally get for the kinetic equation for the electrons

$$\frac{\partial f(\vec{v}_2)}{\partial t}=f(\vec{v}_2)\sum_{\nu=1}^{\infty}\frac{V}{(2\pi)^3}\int d^3k\,2\pi\frac{\nu^2\omega^2}{(k_BT)^2} \times\frac{(4\pi Ze^2)^2J_\nu^2(\vec{k}\cdot\vec{a})}{V^2|k^2\epsilon_{\text{eff}}(\vec{k})|^2}\delta(\vec{k}\cdot\vec{v}-\nu\omega). \quad (17)$$

Here ϵ_{eff} is given by Eq. (8).

IV. EFFECTIVE COLLISION FREQUENCY

As mentioned in the Introduction, the rate of change of the average kinetic energy of the electrons should now be evaluated and compared with Eq. (1) to get the effective collision frequency ν_{eff}. This is done using Eq. (17) for the kinetic equation of the electrons. The result for the rate of change $d\langle\epsilon\rangle/dt$ is then

$$\frac{d\langle\epsilon\rangle}{dt}=V\int d^3v_2\frac{m\vec{v}_2^2}{2}\frac{\partial f(\vec{v}_2)}{\partial t} =\sum_{\nu=1}^{\infty}\int d^3v_2\int d^3k\frac{m\vec{v}_2^2f(\vec{v}_2)}{2(2\pi)^2}\frac{\nu^2\omega^2}{(k_BT)^2} \times\frac{(4\pi Ze^2)^2J_\nu^2(\vec{k}\cdot\vec{a})}{|k^2\epsilon_{\text{eff}}(\vec{k})|^2}\delta(\vec{k}\cdot\vec{v}_2-\nu\omega), \quad (18)$$

which after comparison with Eq. (1) gives us

$$\nu_{\text{eff}}=\sum_{\nu=1}^{\infty}\int d^3v_2\int d^3k\frac{v_2^2f(\vec{v}_2)}{(2\pi)^2\vec{a}^2}\frac{\nu^2J_\nu^2(\vec{k}\cdot\vec{a})}{(k_BT)^2} \times\frac{(4\pi Ze^2)^2}{|k^2\epsilon_{\text{eff}}|^2}\delta(\vec{k}\cdot\vec{v}_2-\nu\omega). \quad (19)$$

Equation (19) is the general expression for the effective frequency, determining the rate at which the energy is retained by the electrons owing to the absorption of an arbitrary number of photons.

In the following we shall restrict ourselves to the case of weak field ($\vec{k}\cdot\vec{a}\ll 1$). In this case, the Bessel functions can be approximated by

$$J_\nu^2(\vec{k}\cdot\vec{a})=[1/(\nu!)^2](\tfrac{1}{2}\vec{k}\cdot\vec{a})^{2\nu}, \quad (20)$$

and consequently only the $\nu=1$ term should be retained; i.e., in the weak-field limit only single-photon processes are significant. Using Eq. (20) and retaining only the $\nu=1$ term, we get the following expression for the effective collision frequency:

$$\nu_{\rm eff} = \int d^3k \int d^3v_2 \frac{v_2^2 f(\vec{v}_2)}{4(2\pi)^2 \vec{a}^2} \frac{(\vec{k}\cdot\vec{a})^2}{(k_B T)^2} \times \frac{(4\pi Z e^2)^2}{|k^2 \epsilon_{\rm eff}|^2} \delta(\vec{k}\cdot\vec{v}_2 - \omega). \quad (21)$$

Replacing $f(\vec{v}_2)$ by $n_0(\pi v_T^2)^{-3/2} e^{-v^2/v_T^2}$, where $v_T^2 = 2k_B T/m$, and performing the integration over $\vec{v}_2$ using the δ function, we have

$$\nu_{\rm eff} = \frac{4(Ze^2)^2 n_0}{\pi^{1/2} \vec{a}^2 m^2 v_T^3} \int d^3k \frac{(\vec{k}\cdot\vec{a})^2}{k^5 |\epsilon_{\rm eff}(\vec{k})|^2} \times \left(\frac{k_D^2}{k^2} - 1\right) e^{-k_D^2/k^2}, \quad (22)$$

with $k_D = \omega_p / v_T$.

Now, to perform the integration over $\vec{k}$ in Eq. (22) one should specify $\epsilon_{\rm eff}(\vec{k})$. Going back to Eq. (8), we see that $\epsilon_{\rm eff}$ is simplified noticeably when ω is near ω_p. In this case we recall that the high-frequency dielectric constant is $\epsilon = 1 - \omega_p^2/\omega^2$, and since the dominant term in Eq. (8) is the resonant one ($n = \pm 1$), we can approximate

$$\frac{1}{\epsilon_{\rm eff}} \simeq \frac{2J_1^2(\vec{k}\cdot\vec{a})}{1-\omega_p^2/\omega^2} = \frac{1}{2} \frac{(\vec{k}\cdot\vec{a})^2}{1-\omega_p^2/\omega^2}. \quad (23)$$

Substituting this expression for $\epsilon_{\rm eff}$ into Eq. (22), we get

$$\nu_{\rm eff} = \frac{Z^2 e^4 n_0}{\pi^{1/2} \vec{a}^2 m^2 v_T^2} \frac{1}{(1-\omega_p^2/\omega^2)^2} \times \int d^3k \frac{(\vec{k}\cdot\vec{a})^6}{k^5} \left(\frac{k_D^2}{k^2} - 1\right) e^{-k_D^2/k^2}, \quad (24)$$

which, after integration, reduces to

$$\nu_{\rm eff} = \frac{4\lambda}{7} \frac{\pi^{1/2} Z^2 e^4 n_0}{m^2 v_T^3} \frac{(k_D a)^4}{(1-\omega_p^2/\omega^2)^2} \quad (25)$$

where

$$\lambda = \int_0^1 dx\, x(1-x^2)\, e^{-1/x^2}.$$

Equation (25) is the expression for $\nu_{\rm eff}$ that we want to discuss. Comparing it with the equivalent expression obtained by Seely and Harris [see Eq. (16) of Ref. 2], one notices that they differ essentially by the last factor of Eq. (25), namely, $(k_D a)^4/(1-\omega_p^2/\omega^2)^2$. In the first place, when screening effects in the electron-nucleus interaction are taken into account, the effective collision frequency becomes field dependent; it then varies with the square of the laser intensity. Secondly, it is proportional to the electron density cubed, rather than being linearly proportional, as found in Ref. 2. Thirdly, as ω gets closer to the plasma frequency, $\nu_{\rm eff}$ becomes increasingly large, whereas as one gets away from resonance, Eq. (25) tells us that collisional absorption becomes a negligible heating mechanism. Physically, all these features may be understood as follows. The introduction of screening effects should, of course, weaken collisional absorption. This is because the first consequence of screening is a reduction in the strength of the Coulomb interaction, and thereby a reduction in the effective number of electron-nuclei collisions. However, if the plasma is illuminated by a radiation field of a frequency close to the natural frequency of oscillation of the screening cloud (ω_p), a resonant condition is reached, with the result that the screening cloud is destroyed. This screening breakdown in turn ensures that the electron-nuclei interaction will then regain strength, and therefore an enhancement of the plasma heating collisional absorption should be expected. Finally, we believe the effects discussed in this paper get adequate support from recent experimental observations.[15,16] These suggest an enhancement of the Brillouin backscattering of the ruby-laser beam from a dense (helium or hydrogen) plasma illuminated with an additional high-power CO_2 laser. This was attributed to the laser heating of the plasma. This heating effect can be construed as supporting our claim that inverse bremstrahlung is the dominant heating mechanism under the conditions stated in this paper. However, to get an accurate conception of the actual electron heating mechanism, similar experiments would have to be performed at different CO_2 power settings in order to see how the electron temperature varies with the pumping power. The results of these experiments could then be compared to our prediction that electron temperature rises as a cubic power of the CO_2-laser intensity.

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