Self-similarity and anti-self-similarity of the effective Landé $g_\perp$ factor in GaAs-(Ga,Al)As Fibonacci superlattices under in-plane magnetic fields

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A theoretical study of the effects of in-plane magnetic fields on the Landé $g_\perp$ factor associated to conduction electrons in GaAs-(Ga,Al)As Fibonacci superlattices is presented. We have used the Ogg-McCombe effective Hamiltonian, which includes nonparabolic and anisotropy effects, in order to describe the electron states in the Fibonacci heterostructure. We have expanded the corresponding electron envelope wave functions in terms of harmonic-oscillator wave functions, and obtained the Landé $g_\perp$ factor for magnetic fields related by even powers of the golden mean $\tau=(1+\sqrt{5})/2$. Theoretical results for GaAs-(Ga,Al)As Fibonacci superlattices, under magnetic-field values scaled by τ^{2n}, clearly exhibit a self-similar (for even n) or anti-self-similar (for odd n) behavior for the Landé $g_\perp$ factors, as appropriate.

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The physical properties of quasiperiodic semiconductor superlattices have been extensively studied in the past few decades, not only because they represent an accessible and intermediate case between periodic and disordered solids, but also for their potential applications in the fabrication of electronic and optoelectronic devices. In particular, since the pioneering work of Merlin and co-workers,[1] a considerable number of both experimental and theoretical work has been devoted to the understanding of the unique nature of the electronic properties associated with GaAs-(Ga,Al)As Fibonacci superlattices (FSLs).[2–5] Maan and co-workers[2–4] studied the effects of a magnetic field applied parallel to the layers on the electronic and optical properties of GaAs-(Ga,Al)As Fibonacci superlattices (FSLs) and, by measuring the magneto-optical spectra of this system, Toet *et al.*[3,4] suggested that the spectra exhibit self-similarity at field values scaled by τ^2, where $\tau=(1+\sqrt{5})/2$ is the golden mean.

More recently, the increasing potential applications of semiconductor nanostructures in a variety of semiconductor devices based on spin-electronic transport have attracted the attention of scientists, as the possibility of active manipulation of the spin degree of freedom in solid-state systems[6–9] is one of the important aspects in the development of quantum information processing and spintronics. In that respect, the dependence of the electron Landé g factor on carrier quantum confinement in semiconductor nanostructures has been the subject of a number of both experimental and theoretical[10–18] studies.

Following these studies, the aim of the present work is to investigate the effects of in-plane magnetic fields on the Landé $g_\perp$ factor associated to conduction electrons in GaAs-(Ga,Al)As FSLs. In the present theoretical calculations, these effects are taken into account by using the Ogg-McCombe effective Hamiltonian to describe the electron states in the Fibonacci heterostructure.

We have focused on the quasiperiodic FSL studied by Maan *et al.*,[2–5] i.e., a MBE-grown system consisting of alternating $Ga_{1-x}Al_xAs$ layers (elementary block a) and GaAs layers (elementary block b), which follows the Fibonacci sequence ω_n defined[2] as

$$\omega_n = \begin{cases} \omega_{n-2}|\omega_{n-1} & \text{if } n \text{ is odd,} \\ \omega_{n-1}|\omega_{n-2} & \text{if } n \text{ is even,} \end{cases} \qquad (1)$$

where $\omega_1=a$ and $\omega_2=b$. An important result, which is a consequence of the generation procedure described above, is the self-similarity of the resulting structure: the substitutions $(ab\to a, abb\to b)$ and $(b\to a, ab\to b)$ transform ω_n into ω_{n-2} and ω_n into the reverse of ω_{n-1}, respectively.[2–5] On the other hand, one may obtain a modified generation $\bar{\omega}_n$ by removing the first and last elementary blocks of a given generation ω_n. The resulting $\bar{\omega}_n$ contains an inversion center.[5] Also, for a FSL under an in-plane magnetic field and within the parabolic-band model, it is well known[2–5] that scaling of τ^{2n} in the magnetic field, and therefore of $\frac{1}{\tau^n}$ in the length (cyclotron radius), with integer n, leads to either self-similarity or anti-self-similarity in the energy-level structure, interband and intraband transition strengths and absorption coefficients, and to self-similarity in the density of states. The transformations $(ab\to a, abb\to b)$ and $(b\to a, ab\to b)$ correspond to scaling with $n=2$ and $n=1$, respectively. As discussed by Wang and Maan,[2] the relation $\frac{d_b}{d_a}=\tau$ (d_a and d_b are the thicknesses of barrier a and well b layers, respectively) is a necessary condition to study similarity properties in a FSL under in-plane magnetic fields. We restrict the present theoretical analysis to the cases of scaling of τ^{2n} in the magnetic field, with $n=1$ and $n=2$.

In the effective-mass approximation and taking into account nonparabolicity effects for the conduction-band electrons, the Ogg-McCombe effective Hamiltonian[19–23] for an electron in a GaAs-$Ga_{1-x}Al_xAs$ FSL grown along the y axis and under an in-plane $\mathbf{B}=B\hat{z}$ magnetic field [we choose the gauge $\hat{\mathbf{A}}=(-yB,0,0)$ for the magnetic vector potential] is given by

$$\hat{H} = \frac{\hbar^2}{2}\hat{\mathbf{K}}\frac{1}{m(y)}\hat{\mathbf{K}} + \frac{1}{2}g(y)\mu_B B\hat{\sigma}_z + a_1\hat{\mathbf{K}}^4 + a_2\left(\frac{e}{\hbar c}\right)^2 B^2 + a_3[\{\hat{K}_x^2,\hat{K}_y^2\} + \{\hat{K}_y^2,\hat{K}_z^2\} + \{\hat{K}_z^2,\hat{K}_x^2\}] + a_4 B\hat{\mathbf{K}}^2\hat{\sigma}_z + a_5 B\{\hat{\sigma}\cdot\hat{\mathbf{K}},\hat{K}_z\} + a_6 B\hat{\sigma}_z\hat{K}_z^2 + \Gamma(\hat{\sigma}\cdot\hat{\tau}) + V(y), \qquad (2)$$

where $\hat{\mathbf{K}}=\hat{\mathbf{k}}+\frac{e}{\hbar c}\hat{\mathbf{A}}$, $\hat{\mathbf{k}}=\frac{1}{i}\frac{\partial}{\partial \mathbf{r}}$, the $\hat{\sigma}_i$ are the Pauli matrices' components of $\hat{\sigma}$, the operator $\hat{\tau}$ has components $\hat{\tau}_x=\hat{K}_y\hat{K}_x\hat{K}_y-\hat{K}_z\hat{K}_x\hat{K}_z$ and cyclic permutations, the $m(y)$ are the growth-direction position-dependent conduction-electron effective mass,[24] whereas the $g(y)$ are the Landé g factors[10] of the GaAs wells $[g(y)=g_w=-0.44]$ and $Ga_{1-x}Al_xAs$ barriers $[g(y)=g_b=0.54$, for $x=0.35]$. The values of a_i, $i=1,2,\ldots,6$, are obtained via a fitting with GaAs magnetospectroscopic measurements,[22] and Γ is a constant associated with the cubic Dresselhaus spin-orbit term[25] and associated to the GaAs lack of inversion symmetry, $\{\hat{a},\hat{b}\}$ is the anticommutator between the $\hat{a}$ and $\hat{b}$ operators, and $V(y)$ is the FSL confining potential (for electrons, we take the finite potential barrier as 60% of the $Ga_{1-x}Al_xAs$ and GaAs band-gap offset). Here we mention that the Hamiltonian (2) is reduced to the parabolic one[2–5] for $\Gamma=0$ and $a_i=0$, $i=1,2,\ldots,6$.

As $\hat{H}$ does not explicitly depend on x and z, k_x and k_z are good quantum numbers, and the eigenfunctions of $\hat{H}$ may be chosen as $\psi(\mathbf{r})=\varphi(y)e^{i(k_x x+k_z z)}/\sqrt{L_x L_z}$, where $\psi(\mathbf{r})$ and the $\varphi(y)$ are two-component wave functions, and L_x and L_z are the sample lengths along the x and z directions, respectively. One should notice that the presence of the FSL layer-confining potential together with the effects of the applied in-plane magnetic field lead to a dependence of the eigenvalues of (2) on the cyclotron orbit-center position $y_0=k_x l_B^2$, where $l_B=\sqrt{\frac{\hbar c}{eB}}$ is the cyclotron radius (or magnetic length). Moreover, at low temperatures, one may take $k_z=0$, as only the lowest energy levels are occupied. By neglecting the off-diagonal terms[23] in the Schrödinger equation, the ↑ spin-up and ↓ spin-down states become uncoupled. The Schrödinger equation may then be written as

$$\hat{H}_{m_s}\psi_{n,y_0,m_s}(y-y_0) = E_n(y_0,m_s)\psi_{n,y_0,m_s}(y-y_0), \qquad (3)$$

where the $\hat{H}_{m_s}$, with $m_s=\pm 1/2$, are the diagonal components of (2) for $k_z=0$, and n is the Landau magnetic-subband index. The above equation, for each m_s projection (↑ or ↓) of the electron spin along the magnetic-field direction, may be readily solved by expanding the wave function in terms of the harmonic oscillator wave functions, which are the natural solutions of Eq. (3) in the parabolic approximation and absence of the confining potential.

The expression one uses to define the $g_\perp^{(n)}$ effective Landé factor in the in-plane direction (perpendicular to the y-growth axis) associated to the $E_n(y_0,m_s,B)$ Landau levels reads

$$g_\perp^{(n)}(y_0,B) = \frac{E_n(y_0,\uparrow,B) - E_n(y_0,\downarrow,B)}{\mu_B B}. \qquad (4)$$

Notice that Eq. (4) is an adequate way of defining the $g_\perp^{(n)}$ effective Landé factor due to the fact that the ↑ and ↓ spin states, in the present calculations, are decoupled. Moreover, as the GaAs-$Ga_{1-x}Al_xAs$ FSL is grown along the y axis, it is clear that the effective $g_\perp^{(n)}$ factor will, in principle, depend on the y_0 orbit-center position and on the applied magnetic field. In the parabolic approximation ($\Gamma=0$ and $a_i=0$, $i=1,2,\ldots,6$), the matrix elements of the Zeeman contribution [second term in (2)] depend on y_0 due to the spatial dependence of g (which is different for wells and barriers), and are different for electron states with different spin projections along the applied magnetic field. Therefore, one may also expect an orbit-center position dependence on the $g_\perp^{(n)}$ effective Landé factor even in the parabolic case.

Here we comment that the well width d_b and barrier width d_a were chosen[2] as $d_b=1.69$ nm and $d_a=1.12$ nm. Note that, with this choice for d_a and d_b, one has $\frac{d_b}{d_a}\approx\tau$, and anti-self-similarity or self-similarity in the length scale for two different magnetic fields related by τ^2 or τ^4, respectively, is expected to be guaranteed, according to previous works.[2–5] In this sense, we have chosen $B''=20$ T, $B'=B''/\tau^2=7.64$ T, and $B=B''/\tau^4=2.92$ T. The corresponding magnetic lengths are related as $l_B=\tau l_{B'}=\tau^2 l_{B''}$ and the transformations ($ab\to a, abb\to b$) and ($b\to a, ab\to b$) correspond to scaling of τ^2 and τ in the length, respectively.

In Fig. 1 we display, for the ground-state Landau magnetic levels, the $g_\perp$ effective Landé factors as functions of the orbit-center position, for the three values of the scaled magnetic fields. Theoretical calculations were performed in the parabolic approximation [see Fig. 1(a)], and by taking into account the effects of nonparabolicity [cf. Fig. 1(b)]. The magnetic field dependence of the $g_\perp$ factor is due both to field effects on the ↑ and ↓ energy band structure as well as to magnetic field-confining effects on the spin wave functions. On the other hand, the orbit-center position dependence on the $g_\perp$ factor is more pronounced as the magnetic field is increased. Of course, the localization region of the ground-electron state is proportional to the magnetic length l_B, which is large for small values of the in-plane magnetic field. Therefore, no thin details of the confining potential are seen by the conduction electron in this case, and a weak orbit-center position dependence of the Landé $g_\perp$ factor is obtained. Notice [cf. Figs. 1(a) and 1(b)] the importance of nonparabolicity effects, a result already demonstrated in the case of GaAs-(Ga,Al)As semiconductor quantum wells.[23] Moreover, one clearly sees from Fig. 1(b) that, by increasing the strength of the applied magnetic field, one may change the sign of the Landé $g_\perp$ factor, a property that may prove useful in future applications in spintronic devices. The Landé $g_\perp$ factor, as a function of the orbit-center position, also manifests a self-similar or anti-self-similar behavior for magnetic-field values scaled by even powers of τ. Theoretical results are summarized in Fig. 2, where we have plotted the numerical results obtained considering the nonparabolicity effects in the Hamiltonian (2) and displayed in Fig. 1(b),

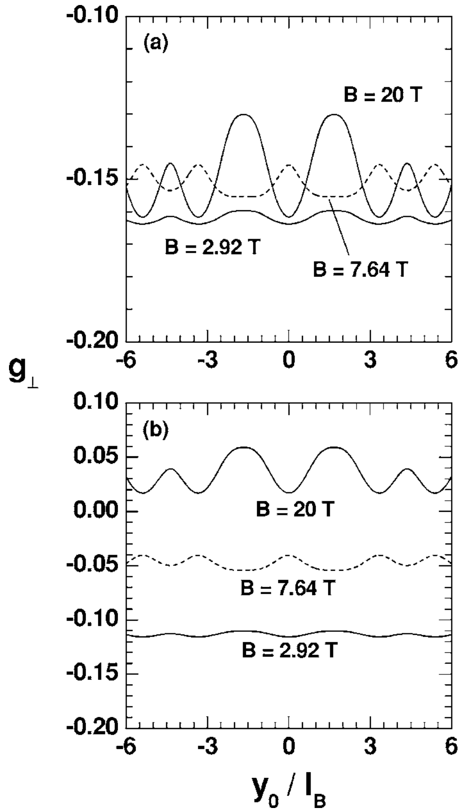

FIG. 1. Orbit-center position dependence of the Landé $g_\perp$ factor in the ω_{13} GaAs-$Ga_{0.65}Al_{0.35}$A FSL under in-plane magnetic fields of $B''=20$ T, $B'=B''/\tau^2=7.64$ T, and $B=B''/\tau^4=2.92$ T. Theoretical results shown in (b) take into account the effects of nonparabolicity, whereas in (a) these effects are disregarded, i.e., $a_1=a_2=\cdots=a_6=0$.

but appropriately scaled and shifted.[26] Notice that, as expected, the self-similar and anti-self-similar properties for $g_\perp$ are beautifully displayed (and that the scaled $g_\perp$ results for 20 T and 2.92 T are essentially identical in the scale used in Fig. 2).

Summing up, we have performed a theoretical study of the effects of an in-plane magnetic field on the effective Landé $g_\perp$ factor in GaAs-(Ga,Al)As FSLs. Theoretical results were presented for magnetic fields related by even powers of the golden mean $\tau=\frac{1+\sqrt{5}}{2}$. For magnetic-field values

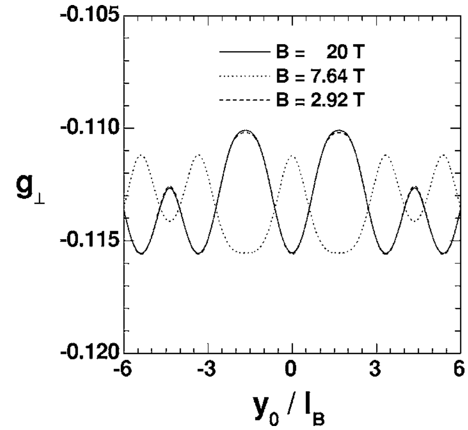

FIG. 2. Orbit-center position dependence of the Landé $g_\perp$ factor in the ω_{13} GaAs-$Ga_{0.65}Al_{0.35}$As FSL under in-plane magnetic fields of $B''=20$ T, $B'=B''/\tau^2=7.64$ T, and $B=B''/\tau^4=2.92$ T. Theoretical results are appropriately scaled and shifted in order to emphasize the self-similarity and the anti-self-similarity behaviors of the Landé $g_\perp$ factor.

scaled by τ^4 or by τ^2, one finds that the effective Landé $g_\perp$ factor as a function of the orbit-center position, properly scaled and shifted, manifests a self-similar or anti-self-similar behavior, respectively. To our knowledge, up to now there have been no experimental results on the effective Landé $g_\perp$ factor in GaAs-(Ga,Al)As quasiperiodic FSL. We do hope, however, that the theoretical results presented here may motivate such experiments and contribute to their understanding. Moreover, the present work clearly indicates that, by increasing the strength of the applied magnetic field, one may change the sign of the FSL Landé $g_\perp$ factor, which raises the possibility of manipulating spin-polarized currents in semiconductor systems, a property that may have possible applications in the area of spintronics devices.

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[1] R. Merlin, K. Bajema, R. Clarke, F. Y. Juang, and P. K. Bhattacharya, Phys. Rev. Lett. **55**, 1768 (1985).
[2] Y. Y. Wang and J. C. Maan, Phys. Rev. B **40**, 1955 (1989).
[3] D. Toet, M. Potemski, Y. Y. Wang, J. C. Maan, L. Tapfer, and K. Ploog, Phys. Rev. Lett. **66**, 2128 (1991).
[4] J. C. Maan, V. Chitta, D. Toet, M. Potemski, and K. Ploog, in *Springer Series in Solid-State Sciences*, edited by G. Landwehr (Springer, Berlin, 1992), Vol. 101, p. 549.
[5] A. Bruno-Alfonso, L. E. Oliveira, and M. de Dios-Leyva, Appl. Phys. Lett. **67**, 536 (1995); A. Bruno-Alfonso, E. Reyes-Gómez, L. E. Oliveira, and M. de Dios-Leyva, J. Appl. Phys. **78**, 15 (1995); M. de Dios-Leyva, A. Bruno-Alfonso, E. Reyes-Gómez, and L. E. Oliveira, J. Phys.: Condens. Matter **7**, 9799 (1995).
[6] Michael A. Nielsen and Isaac L. Chuang, *Quantum Computation and Quantum Information* (Cambridge University Press, Cambridge, 2000).
[7] G. Salis, Y. K. Kato, K. Ensslin, D. C. Driscol, A. C. Gossard, and D. D. Awschalom, Nature (London) **414**, 619 (2001).
[8] I. Zutic, J. Fabian, and S. Das Sarma, Rev. Mod. Phys. **76**, 323 (2004).
[9] H-A. Engel and D. Loss, Science **309**, 586 (2005).
[10] C. Hermann and C. Weisbuch, Phys. Rev. B **15**, 823 (1977); P. Le

Jeune, D. Robart, X. Marie, T. Amand, M. Brosseau, J. Barrau, V. Kalevich, and D. Rodichev, Semicond. Sci. Technol. **12**, 380 (1997).

[11] A. Malinowski and R. T. Harley, Phys. Rev. B **62**, 2051 (2000).

[12] V. F. Sapega, T. Ruf, M. Cardona, K. Ploog, E. L. Ivchenko, and D. N. Mirlin, Phys. Rev. B **50**, 2510 (1994).

[13] G. Medeiros-Ribeiro, M. V. B. Pinheiro, V. L. Pimentel, and E. Marega, Appl. Phys. Lett. **80**, 4229 (2002).

[14] R. Hanson, B. Witkamp, L. M. K. Vandersypen, L. H. Willems van Beveren, J. M. Elzerman, and L. P. Kouwenhoven, Phys. Rev. Lett. **91**, 196802 (2003).

[15] E. I. Rashba and A. L. Efros, Phys. Rev. Lett. **91**, 126405 (2003).

[16] R. de Sousa and S. Das Sarma, Phys. Rev. B **68**, 155330 (2003).

[17] S. J. Prado, C. Trallero-Giner, A. M. Alcalde, V. Lopez-Richard, and G. E. Marques, Phys. Rev. B **69**, 201310(R) (2004).

[18] C. F. Destefani and S. E. Ulloa, Phys. Rev. B **71**, 161303(R) (2005).

[19] N. R. Ogg, Proc. Phys. Soc. London **89**, 431 (1966).

[20] B. O. McCombe, Phys. Rev. **181**, 1206 (1969).

[21] M. Braun and U. Rössler, J. Phys. C **18**, 3365 (1985).

[22] V. G. Golubev, V. I. Ivanov-Omskii, I. G. Minervin, A. V. Osutin, and D. G. Polyakov, Sov. Phys. JETP **61**, 1214 (1985).

[23] M. de Dios-Leyva, E. Reyes-Gómez, C. A. Perdomo-Leiva, and L. E. Oliveira, Phys. Rev. B **73**, 085316 (2006).

[24] E. H. Li, Physica E (Amsterdam) **5**, 215 (2000).

[25] G. Dresselhaus, Phys. Rev. **100**, 580 (1955).

[26] It is easy to see in a simple way that self-similarity (or anti-self-similarity) occurs for magnetic-field values scaled by τ^4 (or τ^2): according to the Fibonacci generation, the shape of the function $g(y)$ appearing in the Zeeman term of (2) is a simple array of wells and barriers, and in the parabolic approximation (i.e., by neglecting nonparabolicity effects), one may use a similar reasoning as Wang and Maan (Ref. 2), and show that, apart from a rigid shift, $g_\perp(B,y_0/l_B)\simeq[g_\perp(B''=B\tau^4 y_0/l_{B''})]/\tau^4$, and, analogously, that $g_\perp(B,y_0/l_B)\simeq-[g_\perp(B'=B\tau^2,y_0/l_{B'})]/\tau^2$. By including the effects of nonparabolicity, one may numerically show that similar expressions are valid with scaling factors $\tau^{4.3}$ and $\tau^{2.4}$, respectively.