Ion-Cyclotron Resonance Heating of Plasmas and Associated Longitudinal Cooling*

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We have investigated, via a 1-2/2 dimensional computer simulation, the possibility of forcing an initially isotropic, magnetized plasma into an anisotropic state by means of an external pump. Strong heating of the perpendicular ion temperature was observed together with a strong cooling of the longitudinal temperature. This mechanism could enhance particle trapping in tokamaks and increase confinement time in mirror machines. We use basic physical arguments to predict the maximum temperature ratio that can be obtained.

The propagation of ion-cyclotron waves in a plasma is a subject of considerable interest as these waves are very effective for heating ions in several types of devices, as demonstrated recently, for example, by Barter and Sprott and Edgley *et al.*[1] It has also been suggested that these waves play a role in the formation of auroras.[2] It is known that anisotropic velocity distributions can drive both ion-cyclotron waves and their electron counterpart, whistler waves, unstable. The linear analysis for unstable ion-cyclotron waves has been reported[3] and also the linear analysis for whistler waves in several regimes.[4] There are reports in the literature, including theory and computer simulation, on Weibel-type instabilities in unmagnetized plasmas,[5] and also on whistler instabilities.[6] Hasegawa and Birdsall[7] in an early paper used a computer-simulation model to study the growth and damping of ion-cyclotron waves for initially anisotropic, bi-Maxwellian ion-velocity-distribution functions. The electrons were considered as a hot neutralizing background and their motion was neglected. In the reports mentioned above,[2-7] the anisotropy in temperature was taken as an initial condition and its consequences were studied. In this paper we report on some recent studies of the absorption of ion-cyclotron waves which involve computer simulation and theoretical analysis. As opposed to previous reports, an initially isotropic plasma is forced into an anisotropic state by means of an applied external pump. These studies show that in addition to a strong heating of the ions in the perpendicular direction (perpendicular to the external magnetic field) there is a strong cooling of the parallel ion temperature. We use very basic thermodynamic agruments to predict the nonlinear saturation of the heating-cooling process, and the simulation results confirm the predictions. We also show that these arguments predict the known threshold for ion-cyclotron instabilities in an anisotropic plasma,[3,6] which is usually obtained from linear theory. It has been recognized for some time that absorption of ion-cyclotron waves would result in removal of energy from the parallel ion motion,[8] however, we know of no discussion of this effect in the literature.

An ion-cyclotron wave of frequency ω and the wave number k propagating along an external magnetic field resonates with particles of velocity V_R such that $kV_R = \omega - \omega_c$. Since the frequency of these waves is always less than the ion-cyclotron frequency the resonant particles and the wave propagate in opposite directions. The resonant particles either continually gain or lose energy to the wave. If the wave is absorbed, then on the average more particles take energy from the wave than give energy to it; on the other hand, if the wave is amplified more particles lose energy to the wave. Now, in addition to gaining (or losing) energy in the perpendicular direction, they also change their energy in the parallel direction. This is most simply seen from the following quantum argument although the effect is classical and can also be derived from conservation of energy and momentum using the Vlasov equation. Assume the particle absorbs n quanta from the wave. Its perpendicular energy will increase by $n\hbar\omega_c$. The wave supplies only $n\hbar\omega$ of this energy; thus the remainder $n\hbar(\omega_c - \omega)$ must come from the parallel motion of the particle. We thus find that in absorbing energy dW from the wave the perpendicular energy increases by $dQ_\perp = dW\omega_c/\omega$, and the parallel energy decreases by $dQ_\parallel = dW(\omega_c - \omega)/\omega$. From the thermodynamic point of view, we may regard the parallel and perpendicular motions as two independent systems with two temperatures, $T_\parallel$ and $T_\perp$. If, say, the perpendicular temperature is higher than the parallel temperature, then thermodynamically it is possible to remove an amount of heat dQ from the perpendicular system, extract an amount of work dW, and deposit the remaining energy $d\tilde{Q} = dQ - dW$ in the low-temperature system. In such a process the entropy must increase, or, at best, remain the same for a reversible process; thus

$$dS = -dQ/dT_\perp + d\tilde{Q}/dT_\parallel \geq 0. \tag{1}$$

It is also possible through the input of work dW to extract energy $d\tilde{Q}$ from the low-temperature parallel system and deposit $dQ = d\tilde{Q} + dW$ in the high-temperature system. For this process

$$-d\tilde{Q}/dT_\parallel + dQ/dT_\perp \geq 0. \tag{2}$$

We may regard cyclotron heating of the plasma as a process which pumps energy from the parallel motion into the perpendicular motion. There is then a maximum ratio of $T_\perp$ to $T_\parallel$ for which a cyclotron wave can cause such an energy transfer. Using $dQ_\parallel = d\tilde{Q}$ and $dQ_\perp = dQ$ in Eq. (2), we find

$$\omega_c/(\omega_c - \omega) \geq T_\perp/T_\parallel, \tag{3}$$

and this is an upper limit on the temperature anisotropy that can be achieved. This is a result that otherwise would be obtained only from nonlinear theory. Reversing the argument and regarding the unstable growth of an ion-cyclotron wave as a heat engine running on the transfer of heat from the perpendicular to the parallel motion, we find from Eq. (1) that the wave will only be unstable if

$$T_\perp/T_\parallel \geq \omega_c/(\omega_c - \omega). \tag{4}$$

This is a well-known condition,[3,6] usually obtained from linear theory, for instability to whistler waves of an anisotropic electron distribution and it applies equally well to ion-cyclotron waves.

We have carried out six computer experiments on the absorption of ion-cyclotron waves using a recently developed magnetostatic code.[9] We have observed strong parallel cooling accompanying the perpendicular heating. The highest temperature ratios $T_\perp/T_\parallel$ were obtained using two ion-cyclotron waves propagating in opposite directions so that particles moving in both directions lost energy. These waves were generated by pumping the plasma with resonant frequency and wave number, and the appropriate polarization; that is, ω and k lie on the ion-cyclotron branch of the dispersion relation for the isotropic plasma and the pump fields are circularly polarized and rotate in the same direction as the ions. The pump was turned on and off slowly to avoid strong transients and the maximum amplitude was $E_0 = m_i v_{Ti}/e\alpha$, where v_{Ti} is the ion thermal velocity and α is a numerical factor between 6 and 12. The perpendicular and parallel ion temperatures for one computer run are plotted against time in Fig. 1; the arrow indicates the time when the pump was turned off. Figure 2 shows the velocity distributions $f_i(v_x), f_i(v_y)$ at two instants of time (the wave motion has been subtracted out); the narrowing of $f_i(v_x)$ and the broadening of $f_i(v_y)$ demonstrate parallel cooling and perpendicular heating.

We also see from Fig. 1 that there is a clear saturation of the rise of $T_\perp$ and the drop of $T_\parallel$. Our thermodynamic argument suggests that this may result from the inability of the wave to transfer energy from the parallel to the perpendicular motion; it can also be viewed as saturation of ion-cyclotron damping due to the resulting anisotropy. We have compared the maximum $T_\perp/T_\parallel$

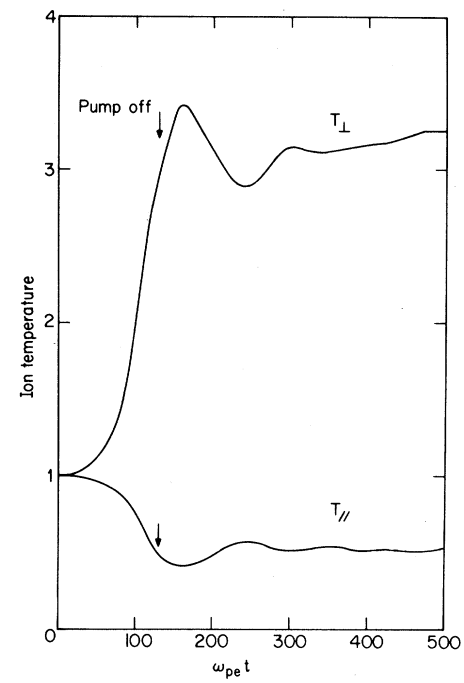

FIG. 1. The transverse ion temperature $T_\perp$ and the longitudinal ion temperature $T_\parallel$ plotted versus normalized time for a typical computer experiment. The pump frequency is $\omega = 0.17\omega_{pe}$ and wave number is $0.086\lambda_D^{-1}$. All runs were made with 5120 electrons and 5120 ions; system length of 512 cells, each cell one Debye length long. Normalization is $\omega_{pe} = 1$, $\lambda_D = 1$, $c = 12$. Mass ratio $m_i/m_e = 5$; $\omega_{ce} = 1$, $\omega_{ci} = 1/\sqrt{5}$, thermal velocities $v_{Te} = 1$, $v_{Ti} = 1/\sqrt{5}$ $(T_e = T_i)$; time step $\Delta t = 0.2\omega_{pe}^{-1}$. Total energy conservation is better than 0.5% after pump turnoff.

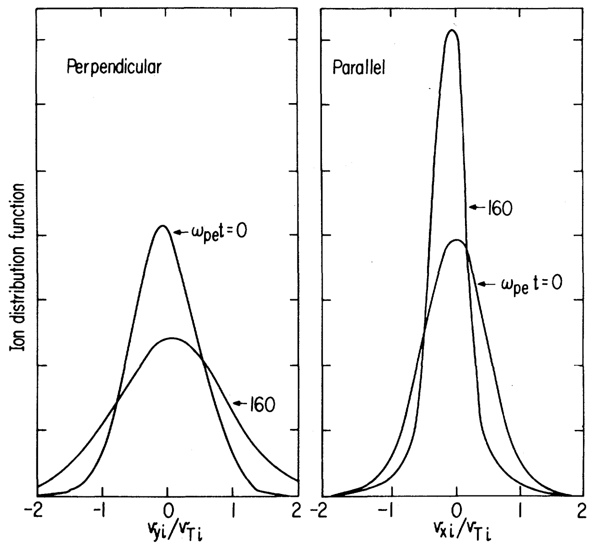

FIG. 2. The ion distribution function in the parallel direction (x) and in one of the perpendicular directions (y) plotted in arbitrary units versus the normalized velocities at two instants in time. v_{Ti} is the initial ion thermal velocity, corresponding to an isotropic Maxwellian distribution. Similar plots are obtained for the other perpendicualr direction (z) and are not shown.

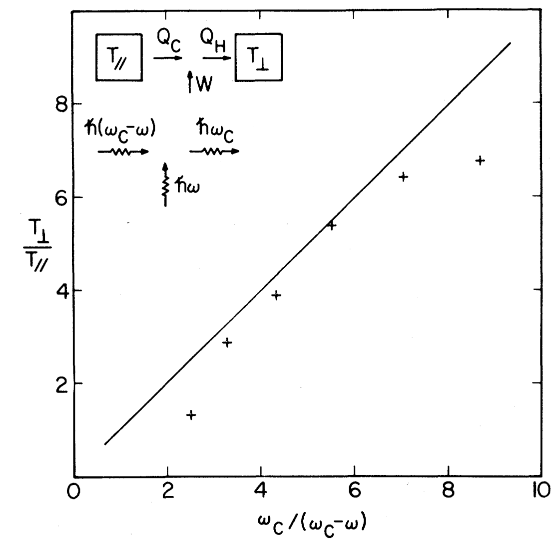

FIG. 3. The thermodynamic limit of the temperature ratio $T_\perp/T_\parallel$ is the straight line. The points are the results of six computer experiments with the same parameters as in Fig. 1 but with different pump modes. The inset shows the Carnot cycle when external work is done on the system and the corresponding exchange of quanta.

achieved for six different runs and these are plotted versus $\omega_c/(\omega_c-\omega)$ in Fig. 3. All parameters were kept the same, except the frequency ω (and the wave number) of the pump. The straight line corresponds to the thermodynamic maximum given by Eq. (3). We see that indeed all the experimental values lie below the maximum predicted by Eq. (4). The two points on the right correspond to pump modes that have $V_R = 0.52 v_{Ti}$ and $V_R = 0.74 v_{Ti}$. Particles with high parallel velocities are probably little affected by these waves and hence not cooled very much, which results in a tail on the parallel distribution which holds $T_\parallel$ up. Further, these modes are heavily damped so that the wave never reaches large amplitudes, and it is possible that with stronger pumping the points would move closer to the line. For the first point on the left $V_R = 4.8 v_{Ti}$ and there are, in fact, no resonant particles to be cooled.

These results may have some significance for ion-cyclotron heating of plasmas. They are of interest to experimentalists for they demonstrate the existence of an upper bound for the temperature anisotropy. And even more, this limit is optimistic, since the computer experiments are performed under rather idealized conditions as compared to a real experiment. If ion-cyclotron heating is done in a tokamak, both the increase in the perpendicular temperature and the reduction in the parallel temperature will enhance the trapping of particles on the outside of the torus. It may be possible to make use of this effect to enhance trapping of particles in a mirror machine or in a long, straight multimirror machine. The hot ions in a mirror machine lose energy to the colder electrons. If energy can be supplied to the ions at the rate they are losing energy to the electrons, while at the same time keeping them out of the loss cone, a type of "wetwood burner" operation might be maintained. For a long straight system with multiple mirrors to inhibit end losses, maintaining the perpendicular temperature above the parallel one would result in a significant improvement. We are only beginning to investigate these possibilities.

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Parametric Instabilities in Strongly Relativistic, Plane-Polarized Electromagnetic Waves*

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We investigate the stability of long-wavelength, plane-polarized electromagnetic waves when the oscillatory energy of the electrons exceeds their rest energy. The strong electron-mass variation destabilizes electron modes polarized along the electric field of the pump $\vec{E}_0$. The electron and ion modes decouple in the presence of such an intense pump and hence the ions do not strongly influence the instability.

The relativistic-mass oscillation in the presence of a large-amplitude, long-wavelength electromagnetic wave can strongly enhance the anomalous absorption of the wave in plasma. Tsintsadze[1] has shown that this mass variation can parametrically excite plasma waves. More recently, Drake *et al.*[2] have shown that, because of the electron-mass changes, electrostatic perturbations of long-wavelength electromagnetic waves "slow down" in regions of high intensity, steepen, and then break.

In this paper we investigate the stability of long-wavelength, plane-polarized electromagnetic waves when the oscillatory energy of the electrons exceeds their rest energy. The electron velocity oscillates periodically between $\pm c$. We show (1) that the wave is unstable to pure electron perturbations polarized along $\vec{E}_0$, and (2) that the decay into coupled electron and ion modes, which occurs in the nonrelativistic limit,[3] does *not* take place. This result is in sharp contrast with the calculations of Max and Perkins[4] for a circularly polarized pump where the ions play an important role.

The novelty of the present calculation is associated with the treatment of the highly anharmonic mass variation of the electrons as their velocity oscillates between $\pm c$. Rather than expand the time dependence of the mass in an infinite Fourier series, we assume that the electrons are so massive that they only respond to the perturbed fields as their velocity passes through zero in crossing between $\pm c$. This approximation is valid to lowest order in an expansion in the small parameter $(1+P_m^2)^{-1/2}$, P_m being the amplitude of the electron momentum (normalized to $m_e c$) in response to $\vec{E}_0$.

The electron motion follows from the relativistic kinetic equation in one dimension,

$$\{\partial/\partial t+[p/(1+p^2)^{1/2}]\,\partial/\partial x-[\tilde{E}(x,t)+E_0(t)]\,\partial/\partial p\}f(x,p,t)=0, \tag{1}$$

where we have normalized the electric field, momentum, distance, and time to $m_e\omega_p c/|e|$, $m_e c$, c/ω_p, and ω_p^{-1}, respectively, ω_p and m_e being the electron plasma frequency and rest mass. With neglect of the perturbed field $\tilde{E}$, f is a function of $q=p-p_0(t)$, where $p_0(t)=\int_0^t d\tau E_0(\tau)$ is the oscillatory momentum of the electron in E_0. The self-consistent time dependence of $p_0(t)$ was investigated by Akhiezer and Polovin[5] (see also Ref. 2 for a simpler discussion), who found that $p_0(t)$ is periodic over the time interval $\tau_0=2\pi/\omega_0=4(2P_m)^{1/2}$ when $P_m\gg 1$. We will investigate the stability of small perturbations around $p_0(t)$. It is convenient to change variables from p to q in (1). We then linearize the resultant