Bosonization Approach for Bilayer Quantum Hall Systems at $\nu_T = 1$

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We develop a nonperturbative bosonization approach for bilayer quantum Hall systems at $\nu_T = 1$, which allows us to systematically study the existence of an exciton condensate in these systems. An effective boson model is derived and the excitation spectrum is calculated in both the Bogoliubov and the Popov approximations. In the latter case, we show that the ground state of the system is an exciton condensate only when the distance between the layers is very small compared to the magnetic length, indicating that the system possibly undergoes another phase transition before the incompressible-compressible one. The effect of a finite electron interlayer tunneling is included and a quantitative phase diagram is proposed.

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Introduction.—The recent conjecture of Bose-Einstein condensation (BEC) of excitons in a bilayer quantum Hall system (QHS) at a total filling factor one ($\nu_T = 1$) has attracted a great deal of attention [1,2]. This system consists of two quantum wells (layers) separated by a fixed distance d, under a perpendicular magnetic field B. Each layer has filling factor $\nu = n\phi_0/B = 1/2$, where n is the layer electron density and $\phi_0 = hc/e$ is the magnetic flux quantum. By increasing the ratio d/l, where $l = \sqrt{\hbar c/eB}$ is the magnetic length, the bilayer QHS at $\nu_T = 1$ undergoes a transition from an incompressible to a compressible phase [3]. Indeed, for small d/l, spontaneous interlayer coherence develops and the system behaves as a single-layer QHS with $\nu = 1$, whereas for large values of d/l, intralayer correlations become important and the system behaves as two independent QHSs with $\nu = 1/2$ [1,4].

The nature of the ground state of the bilayer QHS at $\nu_T = 1$ is an issue not completely settled yet. From the experimental point of view, tunneling conductance experiments [5] show a zero bias peak which does not seem to diverge with decreasing temperatures. In addition, a linear current-voltage characteristic, instead of a power law one [6], has been observed in magnetotransport measurements [7,8]. Both points raise doubts about the existence of a true exciton condensate, which might have superfluid properties [9]. Theoretically, diagrammatic calculations [10], which rely on the existence of an exciton condensate, indicate that the ground state of the system becomes unstable for $d/l \gtrsim 1.2$. Single-mode approximation [11] and generalized random phase approximation calculations [12] confirm the result. Recently, Fertig and Murthy suggested that this "imperfect" two-dimensional superfluid behavior of the bilayer may be understood in terms of a coherence network induced by disorder, which breaks up the system into large and small regions with, respectively, weak and strong interlayer coherence [13].

In this Letter, we develop a nonperturbative bosonization formalism for bilayer QHSs at $\nu_T = 1$, which allows us to properly explore the idea of exciton condensation in this system. An *interacting* boson model is derived and the excitation spectrum is calculated in both the Bogoliubov and the Popov approximations. In the latter case, we find that the ground state of the system is a true exciton condensate only when $d/l \leq (d/l)_c$. For the zero electron interlayer tunneling case, $(d/l)_c = 0.4$, but this parameter increases with the tunneling strength. This new phase transition (or crossover) takes place at a critical ratio $(d/l)_c$ *much smaller* than $(d/l)_c^{\text{I-C}}$, where the incompressible-compressible phase transition is experimentally observed. Our findings, which are based on a proper treatment of the Coulomb interaction, yield a quantitative phase diagram for the bilayers.

The model.—Let us consider a bilayer system with N spinless electrons moving in the $(x, y, z = 0)$ plane and N spinless electrons moving in the $(x, y, z = d)$ plane in an external magnetic field $\mathbf{B} = B\hat{z}$. For each layer, we restrict the Hilbert space to the lowest Landau level (LLL) and consider $N = N_\phi/2$, where N_ϕ is the Landau level degeneracy. We also introduce a pseudospin index ($\sigma = \uparrow, \downarrow$) in order to define to which layer each electron is associated.

The Hamiltonian of the system is given by

$$\mathcal{H} = \mathcal{H}_T + \mathcal{V}, \tag{1}$$

where $\mathcal{H}_T$ describes the electron tunneling between the two layers,

$$\mathcal{H}_T = -\frac{1}{2}\Delta_{SAS}\sum_m c^\dagger_{m\uparrow}c_{m\downarrow} + \text{H.c.}, \tag{2}$$

and $\mathcal{V}$ is the Coulomb interaction term (unit area system)

$$\mathcal{V} = \frac{1}{2}\sum_{\mathbf{k}}\sum_{\sigma,\sigma'} v_{\sigma\sigma'}(k)\rho_\sigma(\mathbf{k})\rho_{\sigma'}(-\mathbf{k}), \tag{3}$$

with $k = |\mathbf{k}|$. Here Δ_{SAS} is the electron interlayer tunneling, the fermion operator $c^{\dagger}_{m\sigma}$ creates an electron with guiding center m in the LLL of the σ layer (see Fig. 1), $\rho_{\sigma}(\mathbf{k})$ is the Fourier transform of the projected density operator of pseudospin σ electrons, and $v_{\uparrow\uparrow}(k) = v_{\downarrow\downarrow}(k) = v_A(k) = (2\pi e^2/\epsilon k)\exp(-|kl|^2/2)$ and $v_{\uparrow\downarrow}(k) = v_{\downarrow\uparrow}(k) = v_E(k) = (2\pi e^2/\epsilon k)\exp(-|kl|^2/2)\exp(-kd)$ are, respectively, the Fourier transform of the intralayer and interlayer interaction potential.

First, we will concentrate on the limit $\Delta_{SAS} = 0$. The Hamiltonian (1) reduces to the Coulomb term (3), which can be rewritten in terms of the total electron density $\rho(\mathbf{k}) = \rho_{\uparrow}(\mathbf{k}) + \rho_{\downarrow}(\mathbf{k})$ and the z component of the pseudospin density $S_Z(\mathbf{k}) = [\rho_{\uparrow}(\mathbf{k}) - \rho_{\downarrow}(\mathbf{k})]/2$ operators as

$$\mathcal{V} = \frac{1}{2}\sum_{\mathbf{k}} v_0(k)\rho(\mathbf{k})\rho(-\mathbf{k}) + 2\sum_{\mathbf{k}} v_Z(k)S_Z(\mathbf{k})S_Z(-\mathbf{k}), \tag{4}$$

with

$$v_{0/Z}(k) = \frac{1}{2}[v_A(k) \pm v_E(k)] = \frac{\pi e^2}{\epsilon k} e^{-|kl|^2/2}(1 \pm e^{-kd}).$$

The restriction of the Hilbert space to the LLL together with the introduction of the pseudospin language renders the description of the bilayer QHS at $\nu_T = 1$ analogous to the one of the single-layer QHS at $\nu = 1$ when the electron spin degree of freedom is included. For the latter, it was shown that the particle-hole pair excitations (magnetic excitons) of the ground state (quantum Hall ferromagnet) can be approximately treated as bosons [14]. In this framework, the electron and the z component of the (pseudo)spin density operators are written as

$$\rho(\mathbf{k}) = \delta_{k,0}N_{\phi} + 2i\sum_{\mathbf{q}} \sin(\mathbf{k}\wedge\mathbf{q}/2)b^{\dagger}_{\mathbf{k}+\mathbf{q}}b_{\mathbf{q}},$$
$$S_Z(\mathbf{k}) = \frac{1}{2}\delta_{k,0}N_{\phi} - \sum_{\mathbf{q}} \cos(\mathbf{k}\wedge\mathbf{q}/2)b^{\dagger}_{\mathbf{k}+\mathbf{q}}b_{\mathbf{q}}, \tag{5}$$

with $\mathbf{k}\wedge\mathbf{q} \equiv l^2\hat{z}\cdot(\mathbf{k}\times\mathbf{q})$. Here, $b^{\dagger}_{\mathbf{q}}$ and $b_{\mathbf{q}}$ are boson operators which obey the canonical commutation relations $[b^{\dagger}_{\mathbf{q}}, b^{\dagger}_{\mathbf{q}'}] = [b_{\mathbf{q}}, b_{\mathbf{q}'}] = 0$ and $[b_{\mathbf{q}}, b^{\dagger}_{\mathbf{q}'}] = \delta_{\mathbf{q},\mathbf{q}'}$. When $b^{\dagger}_{\mathbf{q}}$ is applied to the quantum Hall ferromagnet, it creates a magnetic exciton whose momentum $\mathbf{q}$ is related to the vector $\mathbf{r}$ between the guiding centers of the excited electron and hole by $|\mathbf{r}| = l^2|\mathbf{q}|$ [Fig. 1(a)] [15]. In this formalism, the configuration of a bilayer QHS at $\nu_T = 1$ corresponds to a system with $N_{\phi}/2$ bosons.

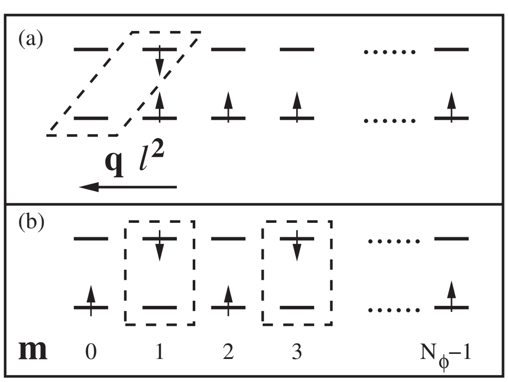

FIG. 1. Schematic representation of (a) one-boson state (magnetic exciton) of the quantum Hall ferromagnet with $|\mathbf{q}l^2| = 1$ and (b) the condensate of $N_{\phi}/2$ zero-momentum bosons. The guiding center quantum number is denoted by $\mathbf{m}$.

Substituting Eq. (5) into Eq. (4) and normal ordering the result, we obtain, apart from a constant related to the positive background, an interacting boson model,

$$\mathcal{H}_B = \sum_{\mathbf{q}} w_q b^{\dagger}_{\mathbf{q}}b_{\mathbf{q}} + \sum_{\mathbf{k},\mathbf{p},\mathbf{q}} 2v_{\mathbf{k}}(\mathbf{p},\mathbf{q})b^{\dagger}_{\mathbf{k}+\mathbf{p}}b^{\dagger}_{\mathbf{q}-\mathbf{k}}b_{\mathbf{q}}b_{\mathbf{p}}, \tag{6}$$

where the dispersion relation of the bosons is given by

$$w_q = \frac{e^2}{\epsilon l}\left(\sqrt{\frac{\pi}{2}} - \frac{d}{l} - l\int_0^{\infty} dk e^{-kd}e^{-(kl)^2/2}J_0(kql^2)\right),$$

with $J_0(x)$ denoting the Bessel function of first kind. The second term of w_q is similar to the capacitorlike one introduced phenomenologically by Girvin [4]. The boson interaction potential is given by

$$v_{\mathbf{k}}(\mathbf{p},\mathbf{q}) = v_0(k)\sin(\mathbf{k}\wedge\mathbf{p}/2)\sin(\mathbf{k}\wedge\mathbf{q}/2) + v_Z(k) \times \cos(\mathbf{k}\wedge\mathbf{p}/2)\cos(\mathbf{k}\wedge\mathbf{q}/2). \tag{7}$$

By taking the limit $d/l \rightarrow 0$ in Eq. (6), we recover the boson model for the single-layer QHS at $\nu = 1$ [14].

Bogoliubov approximation.—We start the analysis of the interacting boson model (6) within the Bogoliubov approximation [16]. We assume that the bosons condense in their lowest energy state, the $\mathbf{q} = 0$ mode, which means that the ground state of the system is roughly given by the state shown in Fig. 1(b). This assumption is in agreement with the scenario proposed in Ref. [2]. Therefore, the boson operators $b^{\dagger}_{\mathbf{q}=0}$ and $b_{\mathbf{q}=0}$ can be replaced by complex numbers, i.e., $b^{\dagger}_{\mathbf{0}}$, $b_{\mathbf{0}} \rightarrow \sqrt{N_0}$, where N_0 is the (macroscopic) number of bosons in the $\mathbf{q} = 0$ mode.

After substituting $b^{\dagger}_{\mathbf{0}}$ and $b_{\mathbf{0}}$ by $\sqrt{N_0}$ in the Hamiltonian (6), including the chemical potential μ explicitly, i.e., $\mathcal{H}_B \rightarrow \hat{\mathrm{K}} = \mathcal{H}_B - \mu\hat{N}$, and keeping only the *quadratic* terms in the boson operators, we obtain

$$\hat{\mathrm{K}} \approx 2v_Z(0)N_0^2 + (w_0 - \mu)N_0 - (1/2)\sum_{\mathbf{q}\neq 0}\epsilon_q + \frac{1}{2}\sum_{\mathbf{q}\neq 0}[\epsilon_q(b^{\dagger}_{\mathbf{q}}b_{\mathbf{q}} + b_{-\mathbf{q}}b^{\dagger}_{-\mathbf{q}}) + \lambda_q(b^{\dagger}_{\mathbf{q}}b^{\dagger}_{-\mathbf{q}} + b_{-\mathbf{q}}b_{\mathbf{q}})], \tag{8}$$

where $\epsilon_q = w_q - \mu + 4N_0v_Z(0) + 4N_0v_Z(q)$ and $\lambda_q = 4N_0v_Z(q)$. The above Hamiltonian can be diagonalized by the canonical transformation $b_{\mathbf{q}} = \cosh(\gamma_q)a_{\mathbf{q}} - \sinh(\gamma_q)a^{\dagger}_{-\mathbf{q}}$ and $b^{\dagger}_{\mathbf{q}} = \cosh(\gamma_q)a^{\dagger}_{\mathbf{q}} - \sinh(\gamma_q)a_{-\mathbf{q}}$, with γ_q real, and therefore the dispersion relation of the quasiparticles is given by

$$\Omega_q = \sqrt{\epsilon_q^2 - \lambda_q^2}. \tag{9}$$

In the one-loop approximation [17], $\mu = w_0 + 4v_Z(0)N_0$, and thus, $\epsilon_q = w_q - w_0 + 4N_0 v_Z(q)$. Using that $\langle \hat{N} \rangle = N_0 + \sum_{\mathbf{q}\neq 0}\langle b_{\mathbf{q}}^\dagger b_{\mathbf{q}} \rangle = N_\phi/2$, we can calculate N_0 and determine Ω_q. The latter is illustrated in Fig. 2(a) for several values of d/l.

In the long wavelength limit, $\Omega_q \approx \hbar v q$, where the linear mode velocity is $v = \sqrt{2n_0\epsilon_B d/l}(l/\hbar)e^2/\epsilon l$. Here $\epsilon_B = -d/4l + \sqrt{\pi/32}(1 + d^2/l^2)\exp(d^2/2l^2)\mathrm{erfc}(d/\sqrt{2}l)$, and $\mathrm{erfc}(x)$ is the complementary error function. For $d/l =$ 0.2, 0.5, 0.8, 1.0, 1.5, and 2.0, we find $v =$ 4.47, 4.64, 3.92, 3.39, 2.27, and 1.52×10^4 m/s, respectively.

The dispersion relation $\Omega_{\mathbf{q}}$ of the quasiparticles increases with momentum for all values of d/l, in qualitative agreement with the calculations done in Refs. [10–12] in the region $d/l \leq 1.0$. However, these previous calculations found that a minimum (rotonlike excitation) appears around $|\mathbf{q}l| \sim 1.0$ for $d/l > 1.0$ and that this minimum becomes soft at $d/l > 1.2$, whereas, in our case, no minimum arises when d/l increases. The behavior of the condensate fraction $n_0 = N_0/(N_\phi/2)$ in terms of d/l shows that the Bogoliubov approximation for the boson model (6) holds only for small d/l. As the ratio d/l increases, the number of bosons in the condensate decreases continuously ($n_0 < 0.3$ for $d/l > 1.0$), indicating that the interaction between excited (out of the condensate) bosons starts to play an important role in the description of the system.

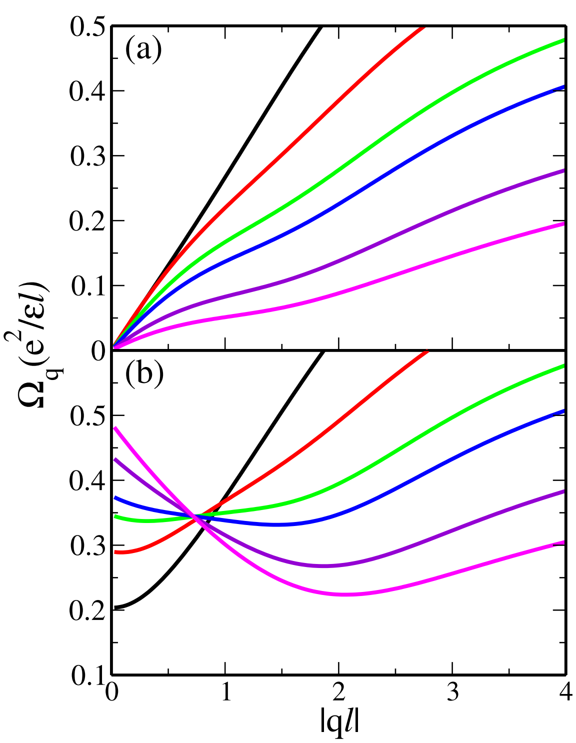

FIG. 2 (color online). Dispersion relation of the quasiparticles in the Bogoliubov approximation for different values of the ratio d/l: 0.2, 0.5, 0.8, 1.0, 1.5, and 2.0 (from top to bottom in the large momentum region). (a) $\Delta_{SAS} = 0$ and (b) $\Delta_{SAS} = 0.1e^2/\epsilon l$.

Popov approximation.—It is possible to go a step further in the analysis of the interacting boson Hamiltonian (6), by including the *interaction* between the excited bosons, which is neglected in the Bogoliubov approximation.

Let us consider the four-operator terms of the Hamiltonian (6) with $|\mathbf{q}l| \neq 0$ and treat them on the mean-field level, including only normal averages. This treatment resembles the so-called first-order Popov approximation for Bose gases at finite temperatures [18]. By applying this approximation to the boson model (6), we obtain a Hamiltonian similar to Eq. (8) with the replacement $\epsilon_q \to \bar{\epsilon}_q = w_q - \mu + 4N_0 v_Z(0) + 4N_0 v_Z(q) + 4\sum_{\mathbf{p}\neq\mathbf{0}}[v_Z(0) + v_{\mathbf{p}-\mathbf{q}}(\mathbf{q},\mathbf{q})]\langle b_{\mathbf{p}}^\dagger b_{\mathbf{p}}\rangle$. After diagonalizing the new Hamiltonian and calculating the chemical potential, we find the following set of self-consistent equations:

$$\lambda_q = 2N_\phi v_Z(q) - 4v_Z(q)\sum_{\mathbf{p}\neq 0}\mathcal{F}_p,$$
$$\bar{\epsilon}_q = w_q - w_0 + 2N_\phi v_Z(q) + 4\sum_{\mathbf{p}\neq 0}[v_{\mathbf{p}-\mathbf{q}}(\mathbf{q},\mathbf{q}) - v_Z(p) - v_Z(q)]\mathcal{F}_p,$$
$$\bar{\Omega}_q = \sqrt{\bar{\epsilon}_q^2 - \lambda_q^2}, \qquad \mathcal{F}_q = (\bar{\epsilon}_q/\bar{\Omega}_q - 1)/2, \tag{10}$$

where $\bar{\Omega}_q$ is the (new) quasiparticle dispersion relation.

Solutions for the above self-consistent problem can be obtained *only* for $d/l \leq (d/l)_c = 0.4$. In this case, the dispersion relation of the quasiparticles also vanishes linearly with momentum for all d/l, qualitatively confirming the results obtained within the Bogoliubov approximation. The velocities of the linear modes are renormalized, for instance, $v = 4.30 \times 10^4$ m/s for $d/l = 0.2$.

However, as the ratio d/l increases, it is not possible to find real solutions for Eqs. (10), implying that the ground state of the system is no longer a true condensate of bosons with zero momentum. This indicates that a new phase sets in above $(d/l)_c$ and below the well-known incompressible-compressible phase transition, which experimentally takes place at $(d/l)_c^{\text{I-C}}$. Notice that $(d/l)_c = 0.4$ is much smaller than the critical ratio $(d/l)_c^{\text{I-C}}$, which, in samples with negligible electron interlayer tunneling, is around 1.6–1.8 [7,8], indicating that the novel phase should be observable within a large region in the parameter space. The ground state of the bilayer QHS at $\nu_T = 1$ in the region $(d/l)_c < d/l < (d/l)_c^{\text{I-C}}$ should then be more complex than a pure exciton condensate, possibly a zero-momentum boson condensate coexisting with a charge-density wave state as suggested in Ref. [19]. We defer a detailed study of the properties of this novel phase, based on the interacting boson model (6), and the analysis of the nature of this new phase transition (or crossover) to later publications.

Our studies open the possibility of the existence of a two-component phase in bilayers, based solely in an appropriate treatment of the Coulomb interaction. Disorder

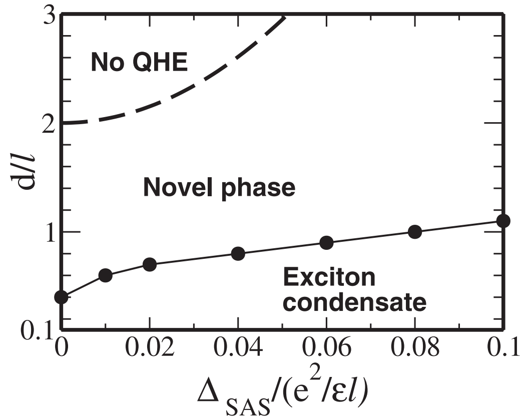

FIG. 3. Phase diagram ($d/l \times \Delta_{SAS}$) of the bilayer QHS at $\nu_T = 1$: the solid circles are the calculated critical ratio $(d/l)_c$ (the solid line is a guide to the eyes) and the dashed line is the experimental estimate for $(d/l)_c^{\text{I-C}}$ (from Ref. [3]), setting the transition to a phase where no quantum Hall effect (QHE) can be observed.

[13], though probably relevant, does not necessarily need to be invoked in order to generate a richer phase diagram.

Finally, we should mention that our findings are quite different from previous theoretical calculations [10–12] which point out that an exciton condensate is stable for $d/l \leq 1.2$. Within our approach, this instability appears at even smaller values of d/l. Moreover, in Ref. [11], the softness of the rotonlike excitation at $d/l \sim 1.2$ is associated with the incompressible-compressible phase transition, which is not the case here.

Finite electron interlayer tunneling.—The effect of a finite electron interlayer tunneling ($\Delta_{SAS} \neq 0$) can be easily included in our formalism. Now the complete Hamiltonian (1) should be considered. It is easy to show that the bosonic representation of $\mathcal{H}_T$ [Eq. (2)] is quadratic in the boson operators. Adding this extra term to the boson model (6) and following the same steps as before, we find, in the Bogoliubov approximation, that the dispersion relation of the quasiparticles is also given by Eq. (9), but now $\epsilon_q = \Delta_{SAS}(1/4 + 1/n_0)\sqrt{n_0/2} + w_{\mathbf{q}} - w_0 + 4N_0 v_Z(q)$. Notice that the spectrum is no longer gapless. As Δ_{SAS} increases, a minimum (rotonlike excitation) develops in the excitation spectrum around $|\mathbf{q}l| = 2$ for $d/l \geq 1.0$. The energy of the minimum reduces as the ratio d/l increases. The case $\Delta_{SAS} = 0.1e^2/\epsilon l$ is illustrated in Fig. 2(b) for several values of d/l.

In the Popov approximation, the final set of self-consistent equations are similar to Eqs. (10) with the replacement $\bar{\epsilon}_q \longrightarrow \bar{\bar{\epsilon}}_q = \bar{\epsilon}_q + \Delta_{SAS}(3n_0 + 2)/\sqrt{32n_0}$. The inclusion of a finite electron interlayer tunneling increases the critical ratio $(d/l)_c$ (see Fig. 3). However, the novel phase should remain robust in a sizable region because $(d/l)_c^{\text{I-C}}$ also increases with the tunneling strength as verified experimentally [3]. For $d/l \leq (d/l)_c$, the dispersion relation of the quasiparticles is quite similar to the results found in the Bogoliubov approximation apart from small renormalizations.

Conclusions.—By developing a nonperturbative bosonization approach for bilayer QHSs at $\nu_T = 1$ and by considering the *fully* interacting boson model, we are able to properly explore the conjecture of BEC of excitons in the bilayer QHS at $\nu_T = 1$. We show that an exciton condensate is stable only at very small values of the ratio d/l, indicating that the bilayer undergoes a phase transition (or crossover) at $(d/l)_c$. A new candidate for the ground state of the system in the region $(d/l)_c < d/l < (d/l)_c^{\text{I-C}}$ is requested because its thorough understanding will be important for the correct description of the incompressible-compressible phase transition. Details of the above calculations will be presented elsewhere.

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