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Mobility and diffusion of a particle coupled to a Luttinger liquid

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We study the mobility of a particle coupled to a one-dimensional interacting fermionic system, namely, a Luttinger liquid. We bosonize the Luttinger liquid and find the effective interaction between the particle and the bosonic system. We show that the dynamics of this system is completely equivalent to the acoustic polaron problem where the interaction has purely electronic origin. This problem has a zero-mode excitation, or soliton, in the strong-coupling limit, which corresponds to the formation of a polarization cloud due to the fermion-fermion interaction around the particle. We find that, due to the scattering of the residual bosonic modes, the soliton has a finite mobility and diffusion coefficient at finite temperatures that depend on the fermion-fermion interaction. We show that at low temperatures the mobility and the diffusion coefficient are proportional to T^{-4} and T^{5}, respectively, and at high temperatures the mobility vanishes as T^{-1} while the diffusion increases as T.

Interest in the dynamics of particles coupled to one-dimensional systems has increased lately due to the unusual properties of these systems with respect to Fermi liquids[1] and their similarity to the strongly interacting electronic systems such as the cuprates.[2] These properties appear in the anomalous exponents in response and correlation functions[3] and in tunneling properties at low temperatures.[4] All these anomalies are related to the intrinsic nonlinear character of interactions in one dimension which are due essentially to the lack of phase space for scattering. The topological character of the excitations of one-dimensional systems has been a subject of discussion for many years and there is an enormous amount of literature on this subject.[5] More recently it has been found that the low-energy physics of interacting electronic models on the lattice, such as the Hubbard chain,[6] or interacting bosonic field theories in one dimension, such as the bosonic model with local interactions,[7] have solitonlike excitations. Indeed, the soliton formation is one of the main characteristics of nonlinear field theories in one dimension and we have shown recently that the soliton in quantum-field theory undergoes Brownian motion due to the scattering with environmental excitations.[8] The same type of Brownian motion can be found in the motion of ferromagnetic domain walls in higher dimensions.[9]

In this paper we discuss the problem of the formation of solitonlike excitations in one-dimensional systems from the point of view of an external particle which is added to the system. The only difference between the external particle and the particles of the one-dimensional system is distinguishability, that is, we can follow the motion of the external particle without confusing it with the environment. Then we can make predictions for its mobility and diffusion in this environment. Of course, a more profound problem would be to calculate the mobility of particles inside the system without making use of the artifact of distinguishability. Here we look at this problem from a semiclassical point of view, that is, from the point of view of a random-phase-approximation (RPA)-type scheme.[10] We want to understand the formation of a polarization cloud, and therefore the creation of the quasiparticle, by probing the system with an external particle. It is worth noticing that, although one-dimensional interacting electronic systems, such as Luttinger liquids, are not Fermi liquids because of the vanishing of the quasiparticle residue, their main properties can be described within the RPA scheme.[11] This result is true because the RPA fulfills all the sum rules at long wavelengths and low energies and then it is expected to describe the continuum limit of these models. Moreover, when indistinguishability is taken into account, it is natural to suppose that, instead of an isolated soliton excitation, or quasiparticle, we end up with a very complex nonlinear theory of interacting solitons which destroys the quasiparticle character of the excitations.

We limit ourselves to the problem of an external parti-

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cle of mass m moving in a lattice with intersite distance a and interacting via a local interaction potential with a system of interacting particles with the same mass and with same potential interaction. This problem can be thought of as the problem of the Hubbard chain with one spin down in a sea of spins up.[12] By diagonalizing exactly the fermionic system via bosonization we obtain the effective interaction between the particle and the bosonic system, the Luttinger liquid.[13] By transforming this problem back to real space we obtain a Hamiltonian which is equivalent to the problem of an electron interacting with acoustic bosons, that is, the acoustic polaron problem.[14] It means that the particle is dressed by the fermions of the environment in such a way to form a solitonlike excitation. This soliton, or quasiparticle, acquires a new mass, which now depends on the interaction within the system and a renormalized coupling constant with the bosonic environment.[8] Using the same technique employed for the strong-coupling limit of the polaron problem we calculate the mobility and diffusion coefficients of the soliton as a function of the fermion-fermion interaction and temperature. This calculation is a clear example of the nonlinear character of interacting one-dimensional fermionic systems and illuminates many aspects of the formation of such types of excitations in one dimension.

Our starting point is the Hamiltonian for a particle interacting with an interacting fermionic system (we consider spinless fermions for simplicity),

$$H = H_P + H_I + H_F, \tag{1}$$

where H_P is the Hamiltonian for the particle alone,

$$H_P = \frac{p^2}{2m}, \tag{2}$$

H_I is given by

$$H_I = \sum_{j=1}^{N} U(x - x_j), \tag{3}$$

and it couples the particle to a set of N other particles at positions x_j $(j = 1, 2, 3, ..., N)$. The fermionic system is described by

$$H_F = \sum_{j=1}^{N} \frac{p_j^2}{2m} + \frac{1}{2} \sum_{\substack{i,j=1 \\ j \neq i}}^{N} U(x_i - x_j). \tag{4}$$

In the second quantized form the interaction term in (3) and (4) can be written as $U \sum_i n_i n_{i+1}$, where i labels the sites on a chain.

Our first step is to diagonalize the Hamiltonian (4) using the techniques of bosonization of one-dimensional systems.[3] We will skip the details here since this technique is already well known. It is possible to show that the Hamiltonian for the fermionic system can be written as (in our units $\hbar = K_B = 1$)

$$H_F = \sum_{q>0} E_q \beta_q^\dagger \beta_q, \tag{5}$$

which has exactly the same form as for a set of independent harmonic oscillators with energies given by

$$E_q = v_F |q| \sqrt{1 + \frac{U}{2\pi v_F}}. \tag{6}$$

Notice that the only effect of the interaction in the spectrum is a renormalization of the Fermi velocity v_F to a new value $\tilde{v}_F = v_F \sqrt{1 + \frac{U}{2\pi v_F}}$.

In this new representation the interaction term (3) is written as

$$H_I = -i \sum_q \left(\frac{|q|}{2\pi L} \right)^{1/2} U e^{iqx - \phi} \text{sgn}(q) \left(\beta_{-q} + \beta_q^\dagger \right), \tag{7}$$

where

$$\tanh(2\phi) = \frac{U}{4\pi v_F + U}. \tag{8}$$

Now we define new operators for the bosons in the coordinate and the momentum form

$$\beta_q = \sqrt{\frac{E_q m}{2}} \left(X_q + i \frac{P_{-q}}{E_q m} \right),$$
$$\beta_q^\dagger = \sqrt{\frac{E_q m}{2}} \left(X_{-q} - i \frac{P_q}{E_q m} \right), \tag{9}$$

which obey $[X_q, P_{q'}] = i\delta_{q,q'}$.

The Hamiltonian for the problem can also be rewritten in second quantized form as

$$H = \sum_k \epsilon_k d_k^\dagger d_k + \sum_q \left(\frac{P_q P_{-q}}{2m} + \frac{m E_q^2}{2} X_q X_{-q} \right)$$
$$- iU e^{-\phi} \left(\frac{\tilde{v}_F}{\pi L} \right)^{1/2} \sum_q q \rho(q) X_{-q}, \tag{10}$$

where d_k and $d_k^\dagger$ are the creation and annihilation operators for the particle in the state of momentum k, $\epsilon_k = \frac{k^2}{2m}$ is the dispersion relation for the particle (we have the constraint that there is just one particle in the system $\sum_k d_k^\dagger d_k = 1$ for all states of the Hilbert space), and $\rho(q) = \sum_k d_{k+q}^\dagger d_k$ is the density operator for the particle (and since the particle is a fermion, $\{d_k, d_{k'}^\dagger\} = \delta_{k,k'}$).

Now we go back to real space by defining the following field operators:

$$\eta(x) = \frac{1}{\sqrt{N}} \sum_q e^{iqx} X_q,$$
$$\Pi(x) = \frac{1}{\sqrt{La}} \sum_q e^{-iqx} P_q, \tag{11}$$
$$\psi(x) = \frac{1}{\sqrt{L}} \sum_k e^{ikx} d_k,$$

where $L = Na$ is the length of the system and N is the number of sites. It is easy to prove that the commutation relation between these above defined operators is $[\eta(x), \Pi(x')] = i\delta(x - x')$ and $\{\psi(x), \psi^\dagger(x')\} = \delta(x - x')$.

Finally we rewrite the Hamiltonian as

$$H = \int dx \left\{ \frac{\Pi^2}{2\nu} + \frac{\nu \tilde{v}_F^2}{2} \left(\frac{\partial \eta}{\partial x} \right)^2 + \frac{1}{2m} \frac{\partial \psi^\dagger}{\partial x} \frac{\partial \psi}{\partial x} + D \frac{\partial \eta}{\partial x} \psi^\dagger \psi \right\}, \tag{12}$$

where $\nu = m/a$ and

$$D = U e^{-\phi} \left(\frac{\nu \tilde{v}_F}{\pi} \right)^{1/2} = U \left(\frac{\nu v_F}{\pi} \right)^{1/2} \tag{13}$$

is the coupling between the fields. It is amazing to notice that the Hamiltonian (12) has the same form as the Hamiltonian for an electron coupled to acoustic phonons where D is the deformation potential coupling.[14] We could therefore apply the results for the strong-coupling limit obtained in Ref. 14 directly to the problem of the Luttinger liquid. The strong coupling is obtained when the deformation energy is much larger than the characteristic bosonic energy scale, that is,

$$\frac{Da}{\tilde{v}_F} = \frac{u}{\sqrt{1+u}} \sqrt{4\pi p_F a} \gg 1, \tag{14}$$

where $u = \frac{U}{2\pi v_F}$ is the dimensionless coupling constant and $p_F = m v_F$ is the Fermi momentum.

As we have shown in Ref. 14, the physics of the problem can be understood in terms of the formation of a self-consistent potential around the particle due to the formation of the polarization cloud. In the strong-coupling limit the particle stays in the ground state of this potential, with ground-state energy $E_0 = \frac{g^2}{8m}$, and can only undergo virtual transitions to excited states. Moreover, the deformation of the bosonic system has the form $\eta_0(x-vt) = -\frac{g}{2mD} \tanh\left(\frac{g(x-vt)}{2}\right)$, where v is the soliton velocity and g is the renormalized coupling constant of the theory which is given by

$$g = m\nu \left(\frac{D}{\tilde{v}_F \nu} \right)^2 = 4\pi \frac{u^2}{1+u} p_F. \tag{15}$$

This result shows that the displacement of the bosonic field has topological character, that is, it interpolates between two different vacuums in the asymptotic limit $[\lim_{x\to\pm\infty} \eta_0(x) = \mp \frac{g}{2mD}]$. Thus we see that the soliton rides on the wave produced by the distortions of the bosonic field.

The reader can find the details of the calculation in Ref. 14. Here we just sketch the main steps leading to our results. We use the saddle-point approximation for the static configuration in order to generate an expansion of the form $\eta(x,t) = \eta_0(x) + \rho(x,t)$. Then we expand the field configuration up to second order in $\rho(x,t)$ and end up with a theory for the renormalized bosons which has a zero mode with the form

$$\rho_0(x) = \sqrt{\frac{3ag}{8}} \text{sech}^2\left(\frac{gx}{2}\right). \tag{16}$$

We can also show that the soliton mass is given by[14]

$$M_s = \frac{32}{3} m \left(\frac{E_0}{\tilde{v}_F g} \right)^2 = \frac{8\pi^2}{3} \frac{u^4}{(1+u)^3} m. \tag{17}$$

These results give the complete picture of the quasiparticle formation: the interaction between the particle leads to the creation of a polarization cloud, which renormalizes the mass of the particle and the interaction with the environment.

The existence of the zero mode in the problem leads naturally to the quantization of its motion via the canonical coordinate formalism.[5] It is possible to show that the soliton is scattered by the residual (long-wavelength) bosonic excitations in the system and it leads to finite mobility and diffusion.[8,15] The mobility of the soliton μ as a function of the temperature T is given by[14]

$$\mu(T) = \frac{32\pi M_s}{g^2 I\left(\frac{T_c}{T}\right)}, \tag{18}$$

where ($E_F = v_F p_F$ is the Fermi energy),

$$T_c = \frac{g \tilde{v}_F}{2} = 2\pi \frac{u^2}{\sqrt{1+u}} E_F \tag{19}$$

is the characteristic temperature for the bosonic excitations, and

$$I(S) = S \int_0^\infty d\kappa\, \kappa^2\, R(\kappa)\, \frac{e^{S\kappa}}{(e^{S\kappa}-1)^2} \tag{20}$$

depends on the reflection coefficient R of residual bosons, which are scattered by the soliton. At low temperatures $T \ll T_c$, we can use the long-wavelength result for the reflection coefficient[16] $R(\kappa) \approx \frac{9\kappa^2}{4}$. Thus

$$\mu(T) = \frac{16\pi M_s}{27 g^2} \left(\frac{T_c}{T} \right)^4 \approx 483 \frac{u^8}{(1+u)^3} \frac{E_F^3}{T^4}. \tag{21}$$

This result shows that the soliton is almost free at low temperatures due to the absence of bosonic degrees of freedom. Remarkably enough in the strong-coupling limit the mobility increases with the interaction strength. This unusual behavior is typical of the nonlinear character of this system and it has many interesting properties in the study of tunneling in one dimension.[4]

Using the same methods we can also show that the diffusion coefficient in momentum space is given by

$$\overline{D}(T) \approx \frac{27 g^2 \hbar T^5}{16\pi\, T_c^4} \approx 0.054 \frac{1}{u^4} \frac{T^5}{v_F^4 p_F^2}. \tag{22}$$

Again, observe that at low temperatures the soliton moves ballistically. Observe that, contrary to the mobility, the diffusion decreases with the increasing interaction.

At high temperatures $T_c \ll T$, we use the short-wavelength limit of the reflection coefficient[16] $R(\kappa) \approx 4\pi^4 \kappa^6 e^{-2\pi\kappa}$. One finds

$$\mu(T) = \frac{64\pi^4 M_s}{315 g^2}\frac{T_c}{T} \approx 41.45\frac{u^2}{(1+u)^{3/2}}\frac{1}{T} \tag{23}$$

and

$$D(T) = \frac{315 g^2 T}{16\pi^5} \approx 10.16\frac{u^4}{(1+u)^2}p_F^2 T. \tag{24}$$

Observe that both the mobility and the diffusion coefficient increase with the interaction at high temperatures.

In summary, in this paper we calculate the mobility and diffusion coefficients of a particle coupled to a Luttinger liquid as functions of temperature and the coupling constant by mapping this problem in the strong-coupling regime of the problem of the acoustic polaron. We draw a picture for the formation of a solitonlike excitation in this problem which resembles the creation of a quasiparticle. We calculate explicitly the renormalized mass and coupling constant as a function of the fermion-fermion interaction. This problem has relevance for the understanding of the structure of excitations of one-dimensional interacting systems.

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