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Slow solar magnetosonic waves and time variation in the solar neutrino data

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Abstract

We analyze the perturbations of the solar magnetic field generated by general solar plasma displacements and investigate their consequences in the left-right conversion probability of neutrinos produced in the Sun. We solve the Hain-Lüst equation and the evolution equations of neutrinos interacting with the solar magnetic field through a nonvanishing neutrino magnetic moment to conclude that the appearance of slow magnetosonic waves (with a period around 100 days) simulate the time behavior of solar neutrino data.

1. Whether the solar neutrino flux varies with time has been an interesting discussion in the context of the solar neutrino problem. In fact, time variation in the solar neutrino flux has been searched in the solar neutrino data [1–4]. Assuming a nonvanishing neutrino magnetic moment the interaction of neutrinos with the solar magnetic field [5] could generate an anticorrelation of the solar neutrino flux detected at the Earth with the solar activity (i.e., long period variations of around 11 years). Although Homestake data could suggest this anticorrelation, the solar neutrino data from other experiments (Kamiokande, Gallex and Sage) do not show any compelling evidence for this long period variation of the solar neutrino flux.

Day-night variations of the solar neutrino data (i.e., short period fluctuations of these data) have also been investigated as a possible indication of the MSW effect [6] in the Earth. Only real-time experiments, like the Kamiokande one, are able to detect such variations and no evidence in favor of this possibility was found in its data.

Nevertheless analyzing individually each experimental point obtained from solar neutrino detectors, we see that they are widely dispersed in Homestake [1] and ^{71}Ga experimental [3,4] results, presenting values that vary from approximately 0 to 1 times the standard solar model theoretical predictions [7]. Furthermore observing their experimental errors we can conclude that these experimental points are not compatible (at 1σ-level) with each other. Kamiokande data [2], on the other hand, do not present this dispersion since their experimental points are statistically compatible (at 1σ-level) with a certain mean value. This apparently different behavior of Homestake and ^{71}Ga experiments and the Kamiokande data could suggest that the former have observed a certain time variation in solar neutrino flux, while the latter show no compelling evidence for this variation.

In this paper we analyze the possibility of such data behavior being a consequence of the interaction of neutrinos produced in the Sun with a fluctuating so-

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lar magnetic field by means of a nonvanishing neutrino magnetic moment. Using a numerical procedure that calculates the fluctuations of the solar magnetic field as a consequence of the solar plasma motion [8], we show that the so-called slow magnetosonic waves appear in the solar magnetic field and have a typical period of order of one hundred days. Interestingly enough we show that these slow magnetosonic waves are sufficient to understand the mentioned apparently different behavior of the experimental data.

2. The magnetic waves are calculated assuming that they can be generated by small displacements of the solar plasma, $\boldsymbol{\xi}$, from an equilibrium configuration. The time evolution of these displacements is described by the linearized equation of motion [9]

$$\rho\frac{\partial^2\boldsymbol{\xi}}{\partial t^2} = \boldsymbol{F}(\boldsymbol{\xi}), \tag{1}$$

where

$$\boldsymbol{F}(\boldsymbol{\xi}) = \nabla(\gamma p\nabla.\boldsymbol{\xi}) - \boldsymbol{b}\times(\nabla\times\boldsymbol{B}_0) - \boldsymbol{B}_0\times(\nabla\times\boldsymbol{b}), \tag{2}$$

ρ is the density, p is the pressure, $\gamma = 1.0$, $\boldsymbol{B}_0$ is the equilibrium magnetic field and $\boldsymbol{b}$ is the magnetic perturbation generated by the displacement $\boldsymbol{\xi}$ given by

$$\boldsymbol{b} = \nabla\times(\boldsymbol{\xi}\times\boldsymbol{B}_0). \tag{3}$$

Considering an exponential time dependence for the displacement $\boldsymbol{\xi}$, $\boldsymbol{\xi}(r,\theta,z,t) = e^{-iwt}\boldsymbol{\xi}(r,\theta,z)$, the frequency of the wave w and the displacement $\boldsymbol{\xi}$ can be found by solving the eigenvalue equations

$$-\rho w^2\boldsymbol{\xi} = \boldsymbol{F}(\boldsymbol{\xi}). \tag{4}$$

We are interested in the behavior of the magnetic field inside the Sun along the trajectory of the neutrinos that reach the Earth. This trajectory is localized around the plane of the solar equator. We can use therefore cylindrical coordinates to solve Eq. (4) to obtain the magnetic perturbations in this region. We consider the plane of the equator coincident with one of the planes of the cylinder perpendicular to the z axis. Employing cylindrical coordinates (r,θ,z) with periodicity in the coordinate z the displacement $\boldsymbol{\xi}$ can be written as

$$\boldsymbol{\xi}(r,\theta,z,t) = \sum_{m,k}\boldsymbol{\xi}^{m,k}(r)\exp\left[i(m\theta - kz - \omega t)\right]. \tag{5}$$

In cylindrical coordinates the linearized equations of motion present a very useful feature: for each Fourier component (m,k) they separate out into a second-order differential equation for ξ_r and two relations expressing ξ_θ and ξ_z in terms of ξ_r and $d\xi_r/dr$ [8]. This property permits us to obtain all the components of the displacement $\boldsymbol{\xi}$, and, consequently, all the components of the fluctuation of the magnetic field solving just the differential equation for ξ_r. This differential equation was first obtained by Hain and Lüst [10] and is given by

$$\frac{d}{dr}\left[f(r)\frac{d}{dr}(r\xi_r)\right] + g(r)(r\xi_r) = 0, \tag{6}$$

where

$$f(r) = \frac{\gamma p + B_0^2}{r}\frac{(w^2 - w_A^2)(w^2 - w_S^2)}{(w^2 - w_1^2)(w^2 - w_2^2)}, \tag{7}$$

$$rg(r) = (\rho\omega^2 - F^2) - \left(\frac{B_{0\theta}}{r}\right)^2\frac{(\alpha - \rho\omega^2\gamma p)4k^2}{\rho^2\omega^4 - H\alpha} + r\frac{d}{dr}\left[\frac{2kB_{0\theta}G\alpha}{r^2(\rho^2\omega^4 - H\alpha)}\right] - r\frac{d}{dr}\left(\frac{B_{0\theta}}{r}\right)^2, \tag{8}$$

$$w_A^2 = \frac{F^2}{\rho}, \quad w_S^2 = \frac{\gamma p}{\gamma p + B_0^2}\frac{F^2}{\rho}, \tag{9}$$

$$w_{1,2}^2 = \frac{H(\gamma p + B_0^2)}{2\rho} \times\left\{1 \pm \left[1 - 4\frac{\gamma p F^2}{(\gamma p + B_0^2)^2 H}\right]\right\}, \tag{10}$$

$$\alpha = \rho\omega^2(\gamma p + B_0^2) - \gamma p F^2,$$
$$F = \frac{m}{r}B_{0\theta} + kB_{0z}, \tag{11}$$

$$H = \frac{m^2}{r^2} + k^2, \quad G = \frac{m}{r}B_{0z} - kB_{0\theta}. \tag{12}$$

This equation is solved imposing appropriate boundary conditions. In this case we can use for the boundary condition at $r = 1$ the experimental data of the magnetic field in the solar surface indicating a magnitude of order 10–100 G (and can reach up to $O(10^3)$ in sunspots). At $r = 0$ the boundary condition is given by the expected behavior of ξ_r which is $\xi_r(r\to 0) = r^{|m|-1}$. The Hain-Lüst equation has singularities when

$f(r) = 0$, that is, when $w^2 = w_A^2$ or $w^2 = w_S^2$. In the interval $0 \leq r \leq 1$ the functions w_A^2 and w_S^2 take continuous values that define ranges of values of w that correspond to improper eigenvalues. So, the eigenvalues of the Hain-Lüst equation must be searched in the regions where $w^2 = w_A^2$ and $w^2 = w_S^2$, which are called Alfvén and slow continua, respectively.

We have to choose an equilibrium profile for the magnetic field. Nevertheless it is very difficult to experimentally investigate the inner solar magnetic field because the solar material is opaque. Therefore large uncertainties are associated with models for the magnetic field in the sun. We take the model proposed by Akhmedov and Bychuk in Ref. [11]:

$$B_0(r) = \begin{cases} a_1 \left(\dfrac{0.2}{r+0.2}\right)^2 \mathrm{G} \\ \quad \text{for} \quad 0 \leq r \leq 0.7, \\ a_2 \left[1 - \left(\dfrac{r-0.7}{0.3}\right)^2\right] \mathrm{G} \\ \quad \text{for} \quad 0.7 < r \leq 1, \end{cases} \tag{13}$$

where $a_1 \approx 10^5$–10^7 and $a_2 \approx 10^4$–10^5 in such a way that the continuity of the magnetic field at the point $r = 0.7$ is satisfied (r is the radial distance from the center of the Sun normalized by the solar radius 6.96×10^5 km). We are interested in the behavior of the magnetic field in a transverse plane to the neutrino trajectory once that this is the field component felt by the neutrinos. Using cylindrical coordinates, the transverse plane to the neutrino trajectory is given by the z and θ components of the magnetic field, B_z and B_θ. The equilibrium magnetic field $\boldsymbol{B}_0$ is chosen in the z direction. We assume the matter density distribution, ρ, usually accepted to fit the predictions of the standard solar model [7], i.e., an approximately monotonically decreasing exponential function in the radial direction from the center to the surface of the Sun [7]. We consider the pressure tó be proportional to the density.

The results of our calculations indicate that a known kind of stable fluctuations are found in the region of squared frequencies of order $w^2 \approx 10^{-15}$ s^{-2}, corresponding to a period of around 100 days. These fluctuations, localized out of the Alfvén and slow continua and characterized y $\nabla \cdot \boldsymbol{\xi} \neq 0$, are usually referred to as slow magnetosonic waves. A set of these results are shown in Fig. 1, where the displacement are normalized such that $|\boldsymbol{\xi}| \leq 1$. Using $\boldsymbol{\xi}$ in Eq. (3) one builds the perturbations to be summed to the equilibrium magnetic field, Eq. (13).

Since we are assuming a nonvanishing neutrino magnetic moment, the interaction of neutrinos with a magnetic field will be given by the evolution equations [12]

$$\frac{d}{dr}\begin{pmatrix} \nu_R(r) \\ \nu_L(r) \end{pmatrix} = \begin{pmatrix} \frac{\sqrt{2}}{2} G_F N_e(r) & -\frac{\Delta m}{4E} \mu_\nu |\boldsymbol{B}_\perp(r)| \\ \mu_\nu |\boldsymbol{B}_\perp(r)| & -\frac{\sqrt{2}}{2} G_F N_e(r) + \frac{\Delta m}{4E} \end{pmatrix} \times \begin{pmatrix} \nu_R(r) \\ \nu_L(r) \end{pmatrix}, \tag{14}$$

where ν_L (ν_R) is the left (right) handed component of the neutrino field, $\Delta m = |m_L^2 - m_R^2|$ is their squared mass difference, E is the neutrino energy, G_F is the Fermi constant, $N_e(r)$ is the electron number density distribution and $|\boldsymbol{B}_\perp(r)|$ is the transverse component of magnetic field given by $\boldsymbol{B}_\perp^2 = (B_0 + b_z)^2 + b_\theta^2$.

Solving Eqs. (14) we can calculate the survival probability of left-handed neutrinos $P(\nu_L \to \nu_L)$ produced in the Sun to arrive at the Earth after interacting with the solar magnetic field $|\boldsymbol{B}_\perp(r)|$ perturbated by the fluctuations calculated in this paper. We can therefore compare this survival probability with experimental data. We will analyze data coming from the two oldest solar neutrino experiments, namely, Homestake and Kamiokande, from 1987 up to now, when both collected data contemporary. Our analysis relies on the fact that while the Homestake experiment takes one experimental point each 70 days approximately, Kamiokande puts the results of its observations over a period of around 260 days together.

In Fig. 2a we put the experimental panorama of both experiments which is to be compared with our calculations (shown in Fig. 2b). In this figure, solid circles correspond to Homestake measurements while solid squares are related to Kamiokande. Note that no experimental errors are shown in Fig. 2a, but they are around 0.2 (in units of theoretical solar neutrino flux) for Homestake and around 0.1 for Kamiokande. To solve Eq. (14) we use the values 10^{-7} eV^2 and $3 \times 10^{-12} \mu_B$ for the mass squared difference Δm and the neutrino magnetic moment μ_ν. Note however that

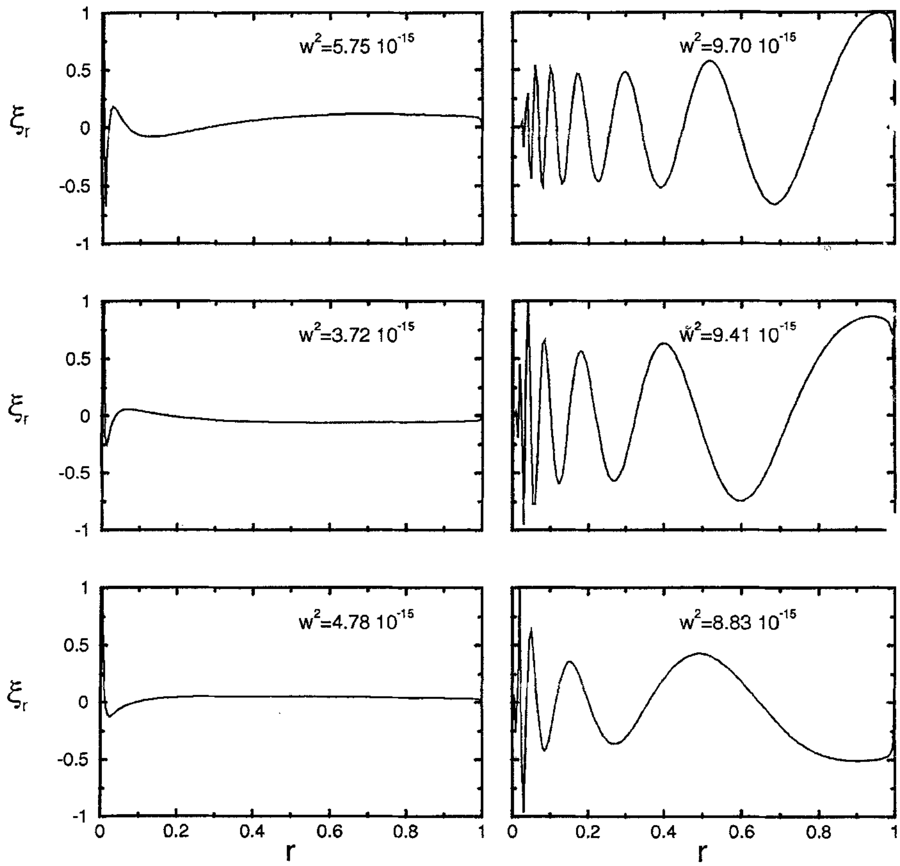

Fig. 1. Displacements $\boldsymbol{\xi}$, characterized by the wave numbers $m = 2$, $k = 10^{-7}$ and frequencies shown in the figure, obtained by solving the Hain-Lüst Eq. (6).

the relevant quantity to build Fig. 2b is the product $\mu_\nu|\boldsymbol{B}_\perp(r)|$ which appears in the evolution equations (14). The mentioned value of the neutrino magnetic moment μ_ν requires $a_1 = 5 \times 10^6$ and $a_2 = 2.5 \times 10^5$ in Eq. (13). We can diminish these values for a_1 and a_2 if we increase the neutrino magnetic moment μ_ν by the same factor. In our calculations the perturbation of the magnetic field is obtained through Eq. (3), using the displacement $\boldsymbol{\xi}$, characterized by $m = 2$, $k = 10^{-7}$, and $w^2 = 9.41 \times 10^{-15}$, presented in Fig. 1. The results present in Fig. 2b are the survival probability integrated in time over periods of 70 (solid circles) and 260 (solid squares) days.

Comparing Figs. 2a and 2b we see that some general features of the referred experimental data can be found also in the calculated survival probability. In particular we observe that this survival probability varies from 0.15 to 0.65 for Homestake while its amplitude is much smaller for Kamiokande.

3. Neutrinos have been indicated as a possible source of experimental information on the inner part of stars and supernovae, which is in general very difficult to obtain through other observational methods. The solar magnetic field behavior in the very interior of the Sun has no direct experimental evidence other than the solar neutrino flux observations (if we assume a nonvanishing neutrino magnetic moment). In this paper we analyze the possibility of interpreting these observations as evidence of the existence of slow magnetosonic waves in the Sun. We calculate the consequences of general solar plasma displacement

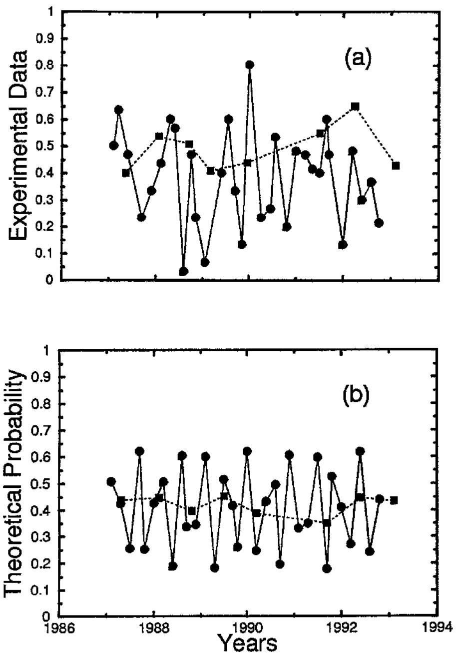

Fig. 2. (a) Neutrino flux at the Earth, normalized by the solar standard model predictions, measured in Homestake (circles) and in Kamiokande (squares) experiments. (b) Theoretical survival probability obtained solving Eq. (11), calculated for Homestake (circles) and Kamiokande (squares) experimental conditions.

on the solar magnetic field and how they influence the detectable solar neutrino flux. Note that the crucial point of our results is the fact that such slow magnetosonic waves have an approximate 100 days period, which is the right period to leads us to the time behavior of solar neutrino data presented in Fig. 2a. Interesting enough, this period is not an imposition in our calculation but derives naturally from the solution of the Hain-Lüst equation, Eq. (6).

Two remarks are in order. Although statistical fluctuations cannot, in the light of the present data, be discarded to explain the behavior of the solar neutrino data, it is not possible also to discard the time behavior of these experimental observations as a consequence of a physical effect in the Sun.

Finally we say that an analysis of the short period (around 10–100 days) time variations in the Kamiokande data could be of interest to obtain new information on the possible existence of solar slow magnetic waves. Nevertheless a conclusive analysis of the consequences of slow magnetic waves will have to wait for more precise experimental data to be obtained in future solar neutrino experiments (see [13] and references therein). Experiments like Superkamiokande [14] and SNO [15] will measure the spectrum of high energy neutrinos in a quite accurate way. Time variations of the neutrino flux having a period of approximately 100 days could be interpreted as the effect of slow solar magnetosonic waves. This effect can be distinguished from flux modulations due to seasonal variation in Sun–Earth distance and vacuum oscillations. This is the reason why it will be possible to investigate flux variations of neutrinos with a fixed energy. Such time variations will be different if neutrinos have evolution equations like (14) or evolve as in MSW [6] or vacuum oscillation phenomenon. Note, for instance, that the two free parameters μ_ν and Δm appear in different elements of the evolution matrix (14). Therefore, differently from what happens in the pure oscillation phenomenon, where the corresponding free parameters, vacuum mixing angle and squared mass difference appear, in the same elements in the evolution matrix, fitting future experimental data using the expected theoretical probability equations will allow precise determination of Δm and the value of the average of the product $\mu_\nu |B_\perp|$. Together with other possible model independent analyses we will be able to use solar neutrino data as a source of information about the physics of neutrinos as well as the inner part of the Sun.

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