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Local and global structure of a thick-domain-wall space-time

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The local and global properties of the Goetz thick plane domain-wall space-time are studied. It is found that when the surface energy of the wall is greater than a critical value σ_c, the space-time will be closed by intermediate singularities at a finite proper distance. A model is presented in which these singularities will give rise to scalar ones when interacting with null fluids. The maximum extension of the space-time of the wall whose surface energy is less than σ_c is presented. It is shown that for a certain choice of the free parameter the space-time has a black hole structure but plane symmetry.

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The study of topological domain walls is crucial for understanding the inflationary universe scenario. According to theory [1], the Universe undertakes an exponential expansion in the early stages of its evolution, triggered by some phase transitions associated with spontaneous symmetry breaking of the Higgs field of some grand unified theory. It was found that such a kind of expansion is needed in order to solve some long-standing puzzles in cosmology, such as the horizon, flatness, and monopole problems. The evolution of the Universe from the exponential expansion to its present Friedmann-Robertson-Walker form is completed by the spontaneous nucleation of bubbles of true vacuum. Guth's original idea has experienced several complementary modifications [2]. It is the general belief that inflation will finally solve the above-mentioned problems of the standard big-bang cosmology.

The work of Hill, Schramm, and Fry (HSF) [3], renewed interest in the cosmological significance of domain walls. In the HSF model, the phase transition happens after the time of recombination of matter and radiation. So, domain walls produced during this phase transition are very light and thick, and are assumed to provide the gravitational field necessary for the clustering of dark matter and baryons after the recombination. Those "soft" walls have promising implications for the Universe [4]. Following this line, an interesting solution of a thick plane domain wall to the Einstein field equations has been found recently by Goetz [5] and rederived by Mukherjee [6].

Motivated by the inflationary universe scenario and the interest of the walls on their own right, in this paper we shall present a detailed study of the Goetz solution and pay the main attention to its global properties. It is found that for some choice of the free parameter q appearing in the solution the space-time has a black hole structure with plane symmetry. It has two asymptotically flat regions separated from the catastrophic ones by event horizons. In each of the two asymptotically flat regions there is a wall. Because of the high symmetry of the Goetz solution, these walls also can be interpreted as bubbles, which are collapsing initially and expanding later with a constant acceleration. In this respect, we can see that the thick domain wall shares the same property as those with zero thickness [7,8]. Examples of single solutions with several different physical interpretations are not rare in general relativity [9]. It should be noted that domain walls with a causal lattice structure similar to the Reissner-Nordström and Kerr black holes have been found in supergravity [10], and some interesting features have been obtained [10,11].

The Goetz solution can be written as [5,6]

$$ds^2=e^{-\Omega}(dt^2-dz^2)-e^{-h}(dx^2+dy^2), \qquad (1)$$

with

$$\Omega=2q\ \ln[\cosh(pz)], \qquad h=2q\ \ln(\cosh(pz)]-2kt, \qquad (2)$$

where $p\equiv k/q$, and k and q are arbitrary constants subject to the conditions $k>0$, and $0<q<1$. The coordinates take the range $-\infty<t,z,x,y<+\infty$. From Eq. (1) we see that the Goetz wall is plane symmetric with the Killing vectors, ∂_x, ∂_y, and $y\partial_x-x\partial_y$. The corresponding energy-momentum tensor (EMT) is given by [12] $T_{\mu\nu}=\rho(g_{\mu\nu}+\xi_\mu\xi_\nu)+\nu\xi_\mu\xi_\nu$, where ρ denotes the energy density of the wall, and ν the pressure in the direction perpendicular to the wall; they are given by

$$\rho=-\left(\frac{q+2}{3q}\right)\nu=k^2\left(\frac{q+2}{q}\right)\{\cosh(pz)\}^{-2(1-q)}. \qquad (3)$$

The unity vector ξ_μ is the normal to the wall and given by $\xi_\mu=e^{-\Omega/2}\delta^z_\mu$. As shown in [5,6], the above solution corresponds to a Higgs scalar field with a kinklike shape $\phi(=\arctan[\sinh(pz)])$, self-interacting through the potential $V(\phi)=\{\cos^2\phi\}^{(1-q)}$. An interesting feature of this solution

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is the existence of coordinate singularities in each of the three spatial directions. The ones in the x and y directions are obvious, since the hypersurfaces $z=\text{const}$ are the (2+1)-dimensional de Sitter spaces. Thus, these coordinate singularities are the usual de Sitter horizons. The coordinate singularities in the z direction can be seen by computing the proper distance between $z=0$, the center of the wall, and $|z|=\infty$, which is found finite [5]. Consequently, the hypersurfaces $|z|=\infty$ represent coordinate singularities, too. Therefore, in general an extension in each of these three directions is needed. The extension in the x and y directions is simple and similar to its four-dimensional analogue of the de Sitter space-time given in [13]. Thus, in the following we shall restrict ourselves only to the extension in the z direction.

It will be useful to consider first the hypersurfaces $|z|=\infty$ in more details, specially the timelike geodesics perpendicular to the wall. From the first integral, it is found [5,6] that the timelike geodesic equations yield $dt/d\tau = E\cosh^{2q}(pz)$, $dz/d\tau = \pm\cosh^{q}(pz)\{E^2\cosh^{2q}(pz) - 1\}^{1/2}$, $dx/d\tau=0$, and $dy/d\tau=0$, where E is the energy of the test particle, and τ the proper time. Perpendicular to the timelike vector $\lambda^{\mu}_{(0)}(\equiv dx^{\mu}/d\tau)$, we have other three linearly independent spacelike vectors $\lambda^{\mu}_{(a)}(a=1,2,3)$ defined by $\lambda^{\mu}_{(1)}=(dz/d\tau)\delta^{\mu}_{t}+(dt/d\tau)\delta^{\mu}_{z}$, $\lambda^{\mu}_{(2)}=e^{h/2}\delta^{\mu}_{x}$, and $\lambda^{\mu}_{(3)}=e^{h/2}\delta^{\mu}_{y}$, where h is given by Eq. (2). One can show that such defined four unity vectors form an orthogonal tetrad and have the properties $\lambda^{\mu}_{(0);\nu}\lambda^{\nu}_{(0)}=0=\lambda^{\mu}_{(a);\nu}\lambda^{\nu}_{(0)}$. Thus, the three vectors $\lambda^{\mu}_{(a)}$ are parallel transported along the timelike geodesics, and together with $\lambda^{\mu}_{(0)}$ form a freely-falling frame. Computing the Riemann tensor in this frame, we find that it has only four independent components, one of which is given by

$$R_{\mu\nu\sigma\delta}\lambda^{\mu}_{(0)}\lambda^{\nu}_{(2)}\lambda^{\sigma}_{(0)}\lambda^{\delta}_{(2)} = \frac{k^2}{q}[\cosh(pz)]^{2(q-1)}\{E^2(1-q)\cosh^{2q}(pz)-1\}.$$

Clearly, as $|z|\to\infty$, this component becomes unbounded for $q>1/2$. From the geodesic equation we get that as $|z|\to\infty$, $e^{2pqz}\sim(\tau_{\infty}-\tau)^{-1}$. Thus the freely falling observer experiences a tidal force $\sim(\tau_{\infty}-\tau)^{-2(q-1/2)/q}$, where τ_{∞} is the observer proper time needed to reach $|z|=\infty$, which is finite. From the above expression we can see that the tidal forces experienced by the freely falling test particles are infinitely large as the hypersurfaces $|z|=\infty$ are approaching. This means that these surfaces are real space-time singularities for $q>1/2$, instead of horizons [5]. Since in the present case all the scalar invariants are finite, we conclude that these singularities are intermediate (or nonscalar) singularities [14,15]. On the other hand, the surface energy density of the wall per surface element is given by [5,6]

$$\sigma=\int_{-\infty}^{\infty}\rho(z)e^{-\Omega/2}dz=\sqrt{\pi}k(2+q)\frac{\Gamma(1-q/2)}{\Gamma(3/2-q/2)},$$

where $\Gamma(x)$ is the standard gamma function. From this expression we can see that σ is a monotonically increasing function of q. Note that the constant k has the meaning of the energy scale. So, we have that *when the surface energy density of the wall per surface element is greater than* σ_c $(\equiv\sigma|_{q=1/2})$, *the space-time will be closed at a finite proper distance by space-time singularities.* As King [16] suggested, the nonscalar singularities might not be stable against perturbations and give rise to scalar ones. The following analysis is in favor to King's conjecture, and will show that they are indeed turned into scalar ones when null fluids are present. In this vein, following [17] we make the substitution

$$\{\Omega,h\}\to\{\Omega+a(u)+b(v),h\},$$

in the metric coefficients of Eq. (1), where $a(u)$ and $b(v)$ are arbitrary functions of their indicated arguments, with $u\equiv(t+z)/\sqrt{2}$, and $v\equiv(t-z)/\sqrt{2}$. Then, corresponding to the new solution, the EMT is given by

$$T^{\mu}_{\nu}=\rho_1 l^{\mu}l_{\nu}+\rho_2 n^{\mu}n_{\nu}+\tilde{p}(\delta^{\mu}_{\nu}+\tilde{\xi}^{\mu}\tilde{\xi}_{\nu})+\tilde{\nu}\tilde{\xi}^{\mu}\tilde{\xi}_{\nu}, \tag{4}$$

where

$$\rho_1=-\sqrt{2}k[1-\tanh(pz)]a'(u),$$

$$\rho_2=-\sqrt{2}k[1+\tanh(pz)]b'(v),$$

$$\tilde{p}=e^{a(u)+b(v)}p, \qquad \tilde{\nu}=e^{a(u)+b(v)}\nu, \tag{5}$$

$\tilde{\xi}_{\mu}=e^{-(a+b)/2}\xi_{\mu}$, and l_{μ} and n_{μ} are two null vectors defined, respectively, by $l_{\mu}\equiv\partial u/\partial x^{\mu}=(\delta^{t}_{\mu}+\delta^{z}_{\mu})/\sqrt{2}$ and $n_{\mu}\equiv\partial v/\partial x^{\mu}=(\delta^{t}_{\mu}-\delta^{z}_{\mu})/\sqrt{2}$. The function ρ_1 represents the energy density of the null fluid moving along the $v=\text{const}$ hypersurfaces, and ρ_2 represents the energy density of the null fluid moving along the $u=\text{const}$ hypersurfaces. To have $\rho_{1,2}$ positive, in the following we shall assume that $a'(u),b'(v)<0$. Combining this assumption with Eq. (5) we can see that, because of the back reaction of the null fluids, the energy density and pressure of the wall become time dependent, and are always decreasing as the time develops. On the other hand, Eq. (5) also shows that ρ_1 is vanishing exponentially as one moves away from the center of the wall to the positive z direction, while ρ_2 is vanishing as one moves away from the center to the negative z direction. Thus, the new solution represents a domain wall emitting massless particles.

Corresponding to the new solution, the Kretschmann scalar can be written as

$$\mathscr{R}\equiv R^{\mu\nu\lambda\delta}R_{\mu\nu\lambda\delta} = e^{2(a+b)}\{\mathscr{R}_0+4e^{2\Omega}(\Phi^{(0)}_{00}\rho_1+\Phi^{(0)}_{22}\rho_2)+2e^{2\Omega}\rho_1\rho_2\}, \tag{6}$$

where $\mathscr{R}_0$ is the Kretschmann scalar of the background $\{\Omega,h\}$, and now is given by

$$\mathscr{R}_0=12k^4(1+q^2)q^{-2}[\cosh(pz)]^{4(q-1)}.$$

$\Phi^{(0)}_{00}$ and $\Phi^{(0)}_{22}$ are the components of the traceless Ricci tensor [18] and are now given by $\Phi^{(0)}_{00}=\Phi^{(0)}_{22}=k^2(1-q)/[2q\cosh^2(pz)]$. $\rho_{1,2}$ and Ω are given, respectively, by Eqs. (5) and (2). The first term on the right-hand side of Eq. (6)

represents the backreaction of the null fluids to the background. The second represents the interaction of the null fluids with the matter components $\Phi_{00}^{(0)}$ and $\Phi_{22}^{(0)}$, which now is proportional to $\cosh^{2(2q-1)}(pz)$. Thus, for $q>1/2$ this term will become unbounded as $|z|\rightarrow\infty$. That is, because of the interaction of the null fluids with the background, the intermediate singularities originally appearing at $|z|=\infty$ are now turned into scalar ones. The last term represents the interaction between the two null fluids, which now is also proportional to $\cosh^{2(2q-1)}(pz)$. Thus, the interaction between these two fluids also turns those intermediate singularities into scalar ones.

Note that at the level of the Higgs scalar field, the potential corresponding to the new solution becomes $\tilde{V}=e^{a+b}V$. At the field level the new solution is obtained by formally introducing a nontrivial dependence of a coupling "constant" on the coordinates u and v, while the scalar field is kept unchanged. When the functions $a(u)\approx 0$ and $b(v)\approx 0$ the coupling "constant" is effectively unity. Note that the above conclusions regarding the singular behavior of the space-time at the spacelike infinity $|z|=\infty$ hold for any $a(u)$ and $b(v)$.

On the other hand, considering the covariant derivatives of the Riemann tensor in the freely falling frame, we find

$$R_{(i)(j)(k)(l);(k_1)\cdots(k_n)}\rightarrow\exp\{[2(2q-1)+2nq]p|z|\}$$

$$\rightarrow(\tau_\infty-\tau)^{-[n+2(q-1/2)/q]} \quad \text{as } |z|\rightarrow\infty,$$

where indices inside parentheses denote tetrad components. Clearly, for any given q the derivatives up to a certain order will become singular on the hypersurfaces $|z|=\infty$. In particular, for $q>\frac{1}{3}$ the first-order derivatives will become unbounded. This indicates the existence of mild singularities on these surfaces even for the solution with $0<q\leqslant\frac{1}{2}$. For example, for $q>\frac{1}{3}$ we have that the "difference" of tidal forces become infinitely large but the integral on the surface is still finite. Therefore, one cannot exclude the possibility of an extension for $0<q\leqslant\frac{1}{2}$. The above argument can be further justified by the following consideration. As we shall show below, the Riemann tensor for the extended solution is C^r across $|z|=\infty$ with $r>0$. According to the classifications given in [14], these hypersurfaces are C^r regular surfaces.

Following [19] (see also [10]), we first make the coordinate transformation

$$u=\alpha^{-1}e^{-k(t+|z|)}, \qquad v=-\alpha^{-1}e^{k(t-|z|)}, \tag{7}$$

where $\alpha^2=2k^2 4^{-q}$. Then, in terms of u and v the solution of Eqs. (1) and (2) reads

$$ds^2=[1+(-\alpha^2uv)^{1/q}]^{-2q}\{2du dv-2k^2v^2(dx^2+dy^2)\}, \tag{8}$$

where $0<q\leqslant 1/2$. From Eq. (7) we can see that the coordinate transformations are restricted to the regions $uv<0$. To extend the solution into the whole (u,v)-plane, one just simply forgets the way how to get Eq. (8) and lets u and v be any values. Regarding to such an extension, there are two different points of view. The first is due to Cvetič and coworkers [10], namely, considering it as two coordinate transformations, each of which is independently performed in the regions $z\leqslant 0$ and $z\geqslant 0$. The resulting space-time of the wall is the gluing of these two extended spaces along the wall. The second is to consider Eq. (7) as one, and take Eq. (8) as the complete extension of the space-time of the wall. Physically, the latter is equivalent to identify the two extended spaces of the former at the same values of the coordinates. In the following, we shall adopt the second point of view. From Eq. (8) we conclude that in order to extend the solution to all the values of u and v, we must distinguish different cases depending on the solutions of the algebraic equations $(-1)^{1/q}=-1,1,i$. We shall consider

$$\text{(a)}\ q=\frac{2n+1}{2m+1}, \quad \text{(b)}\ q=\frac{2n+1}{2m},$$

$$\text{(c) the rest} \qquad (n,m=0,1,2,\ldots).$$

We shall study the last case first.

Case (c). The metric coefficients of Eq. (8) in this case become complex in the regions $uv>0$, which indicates that Eq. (8) cannot be considered as the proper extension beyond the surfaces $uv=0$, and other possibilities must be considered. One way is to set the conformal factor in Eq. (8) as $[1+(\alpha^2uv)^{1/q}]^{-2q}$ in the regions $uv>0$. One can show that such an extension is C^1 in the sense of [13] and the hypersurfaces $u=0$ and $v=0$ are free of any kind of matter. In the regions $uv>0$, introducing the coordinates t and z via the relations [cf. Eq. (7)]

$$u=\alpha^{-1}e^{-k(t+|z|)}, \qquad v=\alpha^{-1}e^{k(t-|z|)},$$

we find that the metric in these regions can be written as

$$ds^2=\cosh^{-2q}(pz)\{dz^2-dt^2-e^{2kt}(dx^2+dy^2)\} \quad (uv>0),$$

which is the continuation of the metric (1) across the hypersurfaces $|z|=\infty$, and clearly shows that the coordinate t becomes spacelike and z timelike. Therefore, the hypersurfaces $|z|=\infty$ are acting as Rindler horizons [20]. It should be noted that the extension in this case is not an analytic extension, but it is maximal, in the sense that the extended space-time is geodesically complete [13]. Other possibilities exist, for example, in the regions $uv>0$, one can replace the conformal factor in Eq. (8) by 1. This extension is C^1 across $|z|=\infty$ and not analytic. The space-time in these regions are flat and the scalar field becomes constant.

Case (b). The metric coefficients of Eq. (8) in this case are well defined for all the values of u and v. It can be shown that when $1/q$ is an integer, the extension is the maximal analytic extension, and the extended space-time is geodesically complete. When $1/q$ is not an integer, the extension is only a maximal extension but not analytic. The space-time in the extended regions [cf. Fig. 1] is asymptotically flat.

Case (a). This is the most interesting case, as after the extension it yields a black hole space-time structure, which is quite similar to that of the Schwarzschild space. The extension in this case is the maximal analytic extension when $1/q$ is an integer, and only a maximal extension otherwise. To show that the space-time indeed has a black hole structure, let us first note that the metric coefficients become singular on the hypersurfaces $uv=\alpha^{-2}$ in this case. On the surface $uv=\alpha^{-2}$ the Kretschmann scalar

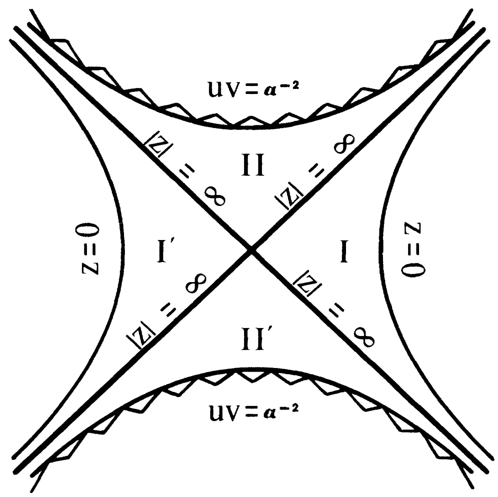

FIG. 1. The projection of the space-time onto the uv-plane. The hypersurfaces $|z| = +\infty$ ($uv = 0$) are event horizons. The center of the walls is at $z = 0$ ($uv = -\alpha^{-2}$). For Case (a) where $q = (2n+1)/(2m+1)$, the space-time is singular on $uv = \alpha^{-2}$, and consequently regions II and II′ represent two catastrophic regions. Regions II, II′, III, and III′ are the four extended regions that are missing in the (t,z) coordinates, where III $\equiv \{uv < -\alpha^{-2}, u > 0\}$, and III′ $\equiv \{uv < -\alpha^{-2}, v > 0\}$.

$$\mathscr{R} \equiv R^{\mu\nu\lambda\delta} R_{\mu\nu\lambda\delta} = \frac{48\alpha^4(1+q^2)}{q^2} \frac{(\alpha^2 uv)^{2(1-q)/q}}{[1-(\alpha^2 uv)^{1/q}]^{4(1-q)}}$$

becomes unbounded. In other words, a space-time singularity appears in the extended regions, II and II′ [cf. Fig. 1]. From Fig. 1 we can see that this singularity is spacelike. Note that in the space-time essentially we have two walls, each of which is located on one of the two branches of the hyperbola $uv = -\alpha^{-2}$. These walls are causally disconnected one from the other and behave like the Rindler particles [20]. The horizons at $|z| = \infty$ (or equivalently $uv = 0$) are event horizons. Thus, it is concluded that the solution given by Eq. (8) in this case represents a black hole but with plane symmetry. The plane is defined by the three Killing vectors ∂_x, ∂_y, and $x\partial_y - y\partial_x$.

However, in [7] (see also [8]) Ipser and Sikivie found that all the plane domain walls with zero thickness can be also interpreted as bubbles. The following considerations show explicitly that this is also the case for a thick domain wall. Let us first note that the metric inside the curly brackets of Eq. (8) is flat. As a matter of fact, by performing the coordinate transformations

$$T = \{(u+v) + k^2 v(x^2+y^2)\}/\sqrt{2},$$

$$Z = \{(u-v) + k^2 v(x^2+y^2)\}/\sqrt{2},$$

$$X = -\sqrt{2}kvx, \qquad Y = -\sqrt{2}kvy, \tag{9}$$

one can bring this part to the standard Minkowski form. If we further introduce the spherical coordinates $\{R, \theta, \varphi\}$, which are related to the coordinates $\{T, Z, X, Y\}$ in the usual way, we find that Eq. (8) takes the form

$$ds^2 = \left\{1 + \left[\frac{\alpha^2}{2}(R^2 - T^2)\right]^{1/q}\right\}^{-2q} \times \{dT^2 - dR^2 - R^2(d\theta^2 + \sin^2\theta d\varphi^2)\}, \tag{10}$$

where $R^2 = X^2 + Y^2 + Z^2$. From the above expression we can see that the space-time of the wall has spherical symmetry. On the other hand, from Eq. (10) we have

$$R^2 - T^2 = -2uv. \tag{11}$$

Thus, the wall (the center of which is at $uv = -\alpha^{-2}$) is an accelerated bubble with the constant acceleration given by $\alpha/\sqrt{2}$ in the above Minkowski coordinates. It starts to collapse at the moment $T = -\infty$ until the moment $T = 0$, where the radius of it is $R_{\min} = \sqrt{2}/\alpha$. Since the acceleration is outward, the wall will start to expand afterwards. Note that the physical radius of the wall is given by $r_{ph} = 2^{-q}R = 2^{-q}(2\alpha^{-2} + T^2)^{1/2}$. In [9], Bonnor provided some examples in which one solution of the Einstein field equations could have several physical interpretations, when it is considered in different coordinate systems. The ambiguity actually comes from the high symmetry that the space-time possesses. As shown above, the Goetz solution is conformally flat. Then, according to *Theorem 32.2* and *Table 32.1* in [21], it has at least six linearly independent Killing vectors, which generate a group, say, G_r, where $r \geqslant 6$. In the present case, the groups $G_3 IX$ and $G_3 VII_0$ defined in [21], which are generated by the three Killing vectors, respectively, of the spherically symmetric space-times and of plane symmetric space-times, are the subgroups of G_r. When different coordinate systems are used, different symmetries of G_r will be manifested. However, in the present case we argue that the interpretation of the above solution as representing a plane domain wall is more favorable than that as representing a bubble. This is due to the following considerations. First, for any given moment of time, say, $T = T_0$, according to Eq. (11) the center of the bubble is located on the hypersurface $R = (T_0^2 + 2\alpha^{-2})^{1/2}$, and the hypersurfaces $|z| = \infty$ is located on $R = |T_0|$, which is always inside the wall. That is, the space-time consists of a bubble that connects two compact spherically symmetric shells, and each of them is inside the bubble. Second, from Eq. (11) we can see that the coordinate transformations (9) map region I (or I′) in Fig. 1 to the region where $R \in (|T|, (T_0^2 + 2\alpha^{-2})^{1/2}]$, and the part $D \equiv \{x^\mu : T^2 \geqslant uv > 0, u > 0\}$, of region II (or $D' \equiv \{x^\mu : T^2 \geqslant uv > 0, u < 0\}$ of region II′) to the region where $R \in [0, |T|)$, while the part $E \equiv$ II $-D$ (or $E' \equiv$ II′ $-D'$) to a region where the coordinate R takes complex values. Therefore, in order to have a geodesically complete spacetime, one is forced to include a region where R is complex, which is clearly physically meaningless.

In summary, we have studied the local and global properties of the Goetz thick plane domain wall solution. It has been found that for $1/2 < q < 1$, intermediate singularities appear on the hypersurfaces $|z|=\infty$. When null fluids are present, these singularities become scalar ones. The solution with $0<q\leqslant 1/2$ has been extended beyond the horizons $|z|=\infty$. The extended space-time for some choice of the free parameter q has a black hole structure with plane symmetry.

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