Photoluminescence spectrum of an interacting two-dimensional electron gas at $\nu=1$

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We report on the theoretical photoluminescence spectrum of an interacting two-dimensional electron gas at filling factor one ($\nu=1$). We considered a model similar to the one adopted to study the x-ray spectra of metals and solved it analytically using the bosonization method previously developed for the two-dimensional electron gas at $\nu=1$. We calculated the emission spectra of the right and the left circularly polarized radiations for the situations where the distance between the two-dimensional electron gas and the valence band hole is smaller and greater than the magnetic length. For the former, we showed that the polarized photoluminescence spectra can be understood as the recombination of the so-called excitonic state with the valence band hole whereas, for the latter, the observed emission spectra can be related to the recombination of a state formed by a spin down electron bound to n spin waves. This state seems to be a good description for the quantum Hall skyrmion.

I. INTRODUCTION

Among all the strongly correlated electron systems, the two-dimensional electron gas (2DEG) at filling factor one ($\nu=1$) has received a great deal of attention lately.[1,2] The ground state of this system, the so-called quantum Hall ferromagnet, is formed by a spin polarized state, where all electrons completely fill the spin up lowest Landau level. The neutral elementary excitations are described as spin waves or *magnetic excitons*[3] while the low-lying charged one as a *charged spin texture* known as the quantum Hall skyrmion.[4]

The analysis of the polarized photoluminescence spectrum of the 2DEG around $\nu=1$ allows us to study the charged excitations of the system in the presence of a valence band hole. An interesting aspect is that the emission spectrum of the right circularly polarized (RCP) radiation is quite distinct from the left circularly polarized (LCP) one.[5] It has been observed that while the RCP spectrum is formed by only one sharp peak, the peak of the LCP spectrum is asymmetric with an additional spectral weight on the low-energy side. The existence of a low-energy tail in the LCP spectrum is related to the spin wave excitation which is left in the system after the recombination process.[6] In both cases, the energy of the peaks continuously decreases as the filling factor changes from $\nu<1$ to $\nu>1$. Here, the estimated distance d between the 2DEG and the valence band hole was $d<l$, where $l=\sqrt{\hbar c/(eB)}$ is the magnetic length. The distance d is an important parameter for the system as features have been observed in the photoluminescence spectrum when the distance d increases.

Osborne *et al.*[7] performed similar measurements in a single quantum well sample applying an electric field (perpendicular to the two-dimensional electron system), which polarizes the electrons and the holes to opposite sides of the quantum well, increasing the value of the distance d. For low intensity electric field, the spectrum has the same characteristics as the ones observed in Ref. 5. As the electric field increases, the energies of the peaks decrease and a discontinuity appears at $\nu=1$, namely, there is a blue shift of the energy of the peaks as the filling factor varies from $\nu<1$ to $\nu>1$. Furthermore, the width of the LCP spectral line on the low-energy side also increases. Those features are enhanced when the photoluminescence spectrum of double quantum well sample is analyzed. In this case, the distance between the centers of the two quantum wells is $d>l$. It has been observed that the spectra related to the electron-hole recombination in the same (direct) and between distinct (indirect) quantum wells are similar to the data recorded for the single quantum well sample in the limit of low and high electric fields, respectively. In particular, the increase of the width of the low-energy tail of the LCP spectrum is now more evident and the RCP spectrum also presents this behavior, differently from the $d<l$ case.

The discontinuity in the photoluminescence spectrum described above for a high applied electric field is understood as a change in the nature of the *photoexcited ground state* as the filling factor changes from $\nu<1$ to $\nu>1$.[6] This is defined as the state of the system after the photoexcitation and thermal relaxation processes which will recombine with the valence band hole.[7]

The nature of the photoexcited ground state is also strongly related to the distance d between the 2DEG and the valence band hole. For instance, at $\nu=1$, Cooper and Chklovskii[6] showed that, if $d<l$, the photoexcited ground state is formed by the completely filled spin-up lowest Landau level plus a spin-down electron bound to the valence band hole, the so-called *excitonic state*. In this case, the quantum Hall ferromagnet is inert after the photoexcitation as the Coulomb interaction between the spin down electron and the hole is stronger than the electron-electron interaction within the 2DEG. On the other hand, for larger values of the distance d, Osborne *et al.*[7] suggested that the photoexcited ground state should be described by a quantum Hall skyrmion (the charged excitation of the 2DEG at $\nu=1$) bound to the valence band hole, i.e., a *skyrmion-hole* state. Only the recombination of such a state can leave a spin wave excitation in the system and therefore we could understand the
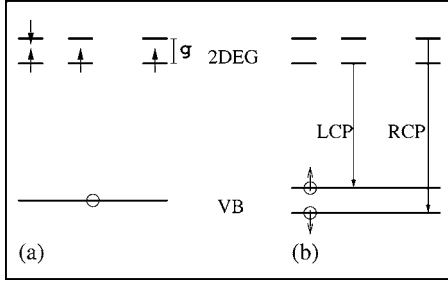
FIG. 1. Schematic representation of the photoluminescence experiment in the 2DEG at $\nu=1$, (a) the valence band hole created after the photoexcitation and the extra electron added to the lowest Landau level of the 2DEG; (b) the emission of the right (RCP) and left (LCP) circularly polarized radiations. VB is the valence band and g is the Zeeman energy.

existence of low-energy tails in both RCP and LCP spectra.

The aim of this paper is to calculate the form of the spectral line of the polarized photoluminescence spectra of the interacting two-dimensional electron gas at $\nu=1$, in the limit of small ($d<l$) and large ($d>l$) separations between the 2DEG and the valence band hole. *We will concentrate only on the filling factor equal to one.* Our model is similar to the ones considered by Schotte and Schotte to study the x-ray spectra of metals[8] and by Westfahl, Jr. *et al.* to study the x-ray edge problem for the 2DEG under a perpendicular magnetic field when $\nu \gg 1$.[9] We will show that the form of the spectral line can be analytically calculated using the bosonization method for the two-dimensional electron gas at $\nu=1$ presented in Ref. 10, and that the results are qualitatively in agreement with the experimental data of Plentz *et al.*[5] and Osborne *et al.*[7]

The paper is organized as follows. In the next section, we will present our model to describe the photoluminescence experiment. In Sec. III, we will calculate the form of the spectral line for both polarizations, for the case $d<l$, considering that the photoexcited ground state is given by the excitonic state, whereas in Sec. IV, we will do the same analysis for the $d>l$ case, assuming that the skyrmion-hole state is the photoexcited ground state. Finally, in Sec. V, we will present a summary of our results.

II. THE MODEL

The photoluminescence experiment for the 2DEG at $\nu=1$ is schematically described in Fig. 1. After the photoexcitation, a hole is created in the valence band and one electron is created in the spin-down lowest Landau level [Fig. 1(a)]. There are two possible recombination processes as illustrated in Fig. 1(b). If a spin-down electron recombines with a spin $-3/2$ valence band hole, the change of the z component of the total spin of the system is $\Delta S^Z=+1$ and therefore the emitted radiation is right circularly polarized. On the other hand, if a spin-up electron recombines with a spin $3/2$ valence band hole, $\Delta S^Z=-1$, and hence the emitted radiation is left circularly polarized.

We will consider that the distance between the planes of the 2DEG and the valence band hole is d. Following Refs. 8

and 9, we will analyze the system before and after the recombination process.

The Hamiltonian of the system prior to the recombination is given by

$$\mathcal{H}_i = \mathcal{H}^e + \mathcal{H}_0^h + \mathcal{H}_{\text{int}}^{e-h}. \tag{1}$$

Here, $\mathcal{H}^e$ is the Hamiltonian of the interacting two-dimensional electron gas at $\nu \approx 1$ with all the electrons restricted to the lowest Landau level. In the Landau level basis, it can be written as (see Ref. 10 for details)

$$\mathcal{H}^e = -\frac{1}{2}g\sum_\sigma \sum_m \sigma c_{m\sigma}^\dagger c_{m\sigma} + \frac{1}{2}\sum_{\sigma,\sigma'}\int \frac{d^2k}{4\pi^2}V(\mathbf{k})\rho_\sigma(\mathbf{k})\rho_{\sigma'}(-\mathbf{k}), \tag{2}$$

where $c_{m\sigma}^\dagger$ is a fermionic operator, which creates a spin σ electron in the lowest Landau level with guiding center m. The density operator of spin σ electrons is given by

$$\rho_\sigma(\mathbf{k}) = e^{-|lq|^2/2}\sum_{m,m'} G_{m,m'}(lk)c_{m\sigma}^\dagger c_{m'\sigma},$$

with the function $G_{m,m'}(x)$ defined in Appendix A. $g=g^*\mu_B B>0$ is the Zeeman energy, where g^* is the effective electron gyromagnetic factor in the host semiconductor, μ_B is the Bohr magneton, and B the external magnetic field. Finally, $V(k)=2\pi e^2/(\epsilon k)$ is the Fourier transform of the Coulomb potential in two dimensions and ϵ is the dielectric constant of the semiconductor.

The valence band hole is described by the Hamiltonian,

$$\mathcal{H}_0^h = \hbar w_0 h^\dagger h,$$

where the operator $h^\dagger$ creates a hole in the valence band with energy $-\hbar w_0$ (measured from the chemical potential). We assume that the hole wave function is localized at the origin of the system of coordinates and that it has s-wave symmetry.

The interaction between the 2DEG and the valence band hole is given by a contact potential at the origin

$$\mathcal{H}_{\text{int}}^{e-h} = V_0\sum_\sigma \Psi_\sigma^\dagger(0)\Psi(0)_\sigma h^\dagger h,$$

where $\Psi_\sigma^\dagger(\mathbf{r})$ is the fermionic field operator at the origin

$$\Psi_\sigma^\dagger(\mathbf{r}) = \sum_m \frac{1}{\sqrt{2\pi l^2}}e^{-|r|^2/4l^2}G_{0,m}(-ir^*/l)c_{m\sigma}^\dagger. \tag{3}$$

For $\mathbf{r}=0$, the function $G_{0,m}(-ir^*/l)$ vanishes except for $m=0$. This condition simplifies our calculations. Since the angular momentum is conserved during the recombination process (the electron-hole interaction potential is spherically symmetric), only electrons with guiding center $m=0$ will recombine with the valence band hole.

After the recombination process, we end up with only the two-dimensional electron system at $\nu=1$. Therefore the Hamiltonian of the system is given by Eq. (2),

$$\mathcal{H}_f = \mathcal{H}^e. \tag{4}$$

In order to describe the emission spectrum at $T=0$, we need to calculate the transition rate ($\hbar=1$),

$$W(w) \propto \sum_f |\langle f|\Psi_\sigma(0)|i\rangle|^2 \delta[w-(w_0+E_i-E_f)], \quad (5)$$

where $|f\rangle$ is the set of all final states of the system after the recombination process with energy E_f and $|i\rangle$ is the photoexcited ground state with energy E_i. Substituting Eq. (3) in the expression (5), the transition rate can be written as

$$W(w) \propto \sum_f |\langle f|c_{m=0\sigma}|i\rangle|^2 \delta[w-(w_0+E_i-E_f)]$$

$$\propto \sum_f \int_{-\infty}^{\infty} dt\, e^{-i(w-w_0-E_i+E_f)t} \langle i|c_{0\sigma}^\dagger|f\rangle \times \langle f|c_{0\sigma}|i\rangle$$

$$\propto Re \int_0^\infty dt\, e^{-i(w-w_0)t} \langle i|e^{i\mathcal{H}_i t}c_{0\sigma}^\dagger e^{-i\mathcal{H}_f t}c_{0\sigma}|i\rangle \quad (6)$$

$$\propto Re \int_0^\infty dt\, e^{-i(w-w_0-E_i)t} \langle i|c_{0\sigma}^\dagger e^{-i\mathcal{H}_f t}c_{0\sigma}|i\rangle. \quad (7)$$

Here, $\sigma=\uparrow$ and $\downarrow$ give the LCP and RCP transition rates, respectively.

In Refs. 8 and 9, the transition rate is calculated from the expression (6) using the respective bosonization methods. However, since we do not have an expression for the fermionic field operator in terms of the annihilation and creation bosonic operators and it is not possible to write down the bosonic form of the Hamiltonian $\mathcal{H}_i$, which describes the 2DEG at $\nu \approx 1$, Eq. (6) cannot be properly written in the bosonic form. Therefore, as our bosonization method for the 2DEG at $\nu=1$ presents some limitations, we will employ Eq. (7) in our calculations. All those points will become clearer in the next section.

Since we will not consider the Hamiltonian $\mathcal{H}_i$ explicitly, the energy E_i will be treated as a parameter of our model. Following this scheme, we are not able to determine the energy of the photoexcited ground state but once we have the form of the initial state $|i\rangle$ we can *analytically* calculate the emission spectrum.

III. THE EXCITONIC INITIAL STATE

First of all, we will analyze the case where the distance between the 2DEG and the valence band hole is small, namely, $d < l$. We will compare our results with the ones of Cooper and Chklovskii[6] and with the experimental data of Plentz et al.[5] in order to check that our scheme is quite reasonable to describe qualitatively the photoluminescence spectrum in this limit. We will consider *only* the case $\nu=1$.

As has been pointed out in Ref. 6, in this limit the photoexcited ground state can be described by the *excitonic state*, which is formed by the quantum Hall ferromagnet, $|FM\rangle$, plus one electron of spin-down in the lowest Landau level and one valence band hole. This state can be written as

$$|i\rangle = c_{m\downarrow}^\dagger |FM\rangle. \quad (8)$$

Since the electron-hole interacting potential is spherically symmetric and the valence band hole is assumed to be local-

ized at the origin, we can choose $m=0$ in the above expression.

Substituting Eq. (8) in the LCP transition rate expression [Eq. (7)], we have

$$W_{\mathrm{LCP}}^0(w) \propto Re \int_0^\infty dt\, e^{-i(w-w_0-E_i)t}$$

$$\times \underbrace{\langle FM|c_{0\downarrow}c_{0\uparrow}^\dagger e^{-i\mathcal{H}_f t}c_{0\uparrow}c_{0\downarrow}^\dagger|FM\rangle}_{\mathcal{F}_{\mathrm{LCP}}(t)}. \quad (9)$$

The function $\mathcal{F}_{\mathrm{LCP}}(t)$ can be easily calculated using the bosonization method for the two-dimensional electron gas at $\nu=1$ introduced in Ref. 10. It was shown that the Hamiltonian of the interacting two-dimensional electron gas at $\nu=1$, $\mathcal{H}^e$, can be mapped into an interacting two-dimensional bosonic model

$$\mathcal{H} = -\frac{1}{2}gN_\phi + \sum_q w_q b_q^\dagger b_q + \frac{2}{\mathcal{A}} \sum_{k,p,q} V(k)e^{-|lk|^2/2}\sin(k\wedge p/2)$$

$$\times \sin(k\wedge q/2)b_{k+q}^\dagger b_{p-k}^\dagger b_q b_p, \quad (10)$$

where the boson operators b are written in terms of the fermionic operators as

$$b_q = \frac{1}{\sqrt{N_\phi}}e^{-|lq|^2/4}\sum_{m,m'} G_{m,m'}(-lq)c_{m\uparrow}^\dagger c_{m'\downarrow}, \quad (11)$$

$$b_q^\dagger = \frac{1}{\sqrt{N_\phi}}e^{-|lq|^2/4}\sum_{m,m'} G_{m,m'}(lq)c_{m\downarrow}^\dagger c_{m'\uparrow}, \quad (12)$$

which obey the canonical commutation relations

$$[b_q^\dagger, b_{q'}^\dagger] = [b_q, b_{q'}] = 0,$$

$$[b_q, b_{q'}^\dagger] = \delta_{q,q'}. \quad (13)$$

The dispersion relation of the bosons b is given by

$$w_q = g + \frac{e^2}{\epsilon l}\sqrt{\frac{\pi}{2}}[1 - e^{-|lq|^2/4}I_0(|lq|^2/4)], \quad (14)$$

where $I_0(x)$ is the modified Bessel function of the first kind.[11] The state $b_q^\dagger|FM\rangle$ corresponds to a spin wave excitation with momentum q of the quantum Hall ferromagnet and to a very well separeted quasiparticle-quasihole pair, respectively, for $|lq| < 1$ and $|lq| \gg 1$. This result is exactly the one previously obtained by Kallin and Halperin.[3] Here, we will approximate $\mathcal{H}^e$ by the noninteracting bosonic model, i.e.,

$$\mathcal{H}_f = \mathcal{H}^e = -\frac{1}{2}gN_\phi + \sum_q w_q b_q^\dagger b_q. \quad (15)$$

As pointed out in the preceding section, our bosonization method can be applied only to the 2DEG at $\nu=1$ and therefore it cannot be used to bosonize the Hamiltonian $\mathcal{H}_i$, which describes the 2DEG at $\nu=1$ with an extra electron ($\nu \approx 1$). Therefore, we decide to keep the energy E_i as a parameter of our model. Despite the fact that we do not have an expression for the fermionic field operators in terms of the bosonic

operators b, we can bosonize products of creation and annihilation fermionic operators, such as $c_{0\downarrow}^\dagger c_{0\uparrow}$. The obtained expressions are presented in Appendix B. Based on the above-mentioned points, it is now clear why we need to choose Eq. (7) instead of Eq. (6) to calculate the transition rate and also why we need to write down the form of the photoexcited ground state.

Substituting Eqs. (15), (B1), and (B2) in the expression (9) and using the fact that $|FM\rangle$ is the vacuum for the bosons b, the function $\mathcal{F}_{LCP}(t)$ reads

$$\mathcal{F}_{LCP}(t) = \sum_{\mathbf{q},\mathbf{q}'} \frac{e^{-|lq|^2/4 - |lq'|^2/4}}{N_\phi} \langle FM | b_\mathbf{q} e^{-i\mathcal{H}_0 t} b_{\mathbf{q}'}^\dagger | FM \rangle$$

$$= \sum_{\mathbf{q},\mathbf{q}'} \frac{e^{-|lq|^2/4 - |lq'|^2/4}}{N_\phi} \delta_{\mathbf{q},\mathbf{q}'} e^{-it(w_\mathbf{q} + E_f + E_{FM})},$$

where $E_{FM} = -gN_\Phi/2$ is the energy of the state $|FM\rangle$. In the second step above, we use the relation

$$e^{-it\Sigma_\mathbf{q} w_\mathbf{q} b_\mathbf{q}^\dagger b_\mathbf{q}} b_\mathbf{k}^\dagger = b_\mathbf{k}^\dagger \exp(-itw_\mathbf{k}) e^{-it\Sigma_\mathbf{q} w_\mathbf{q} b_\mathbf{q}^\dagger b_\mathbf{q}}, \quad (16)$$

which can be proved with the aid of the Baker-Hausdorff formula.[12] Note that the terms $e^{-|lq|^2/4}$ in the $\mathcal{F}_{LCP}(t)$ expression imply that only the long wavelength excitations are important and therefore we can write the energy of the bosons as

$$w_\mathbf{q} \approx g + \epsilon_B(lq)^2/4, \quad (17)$$

where $\epsilon_B = \sqrt{\pi/2} e^2/(\epsilon l)$ is a constant related to the Coulomb energy $e^2/(\epsilon l)$ and ϵ is the dielectric constant of the host semiconductor. Changing the sum over momenta into an integral,

$$\frac{1}{\mathcal{A}} \sum_\mathbf{q} \rightarrow \int \frac{d^2 q}{4\pi^2},$$

where the area of the system is related to the magnetic length and to the degeneracy of the lowest Landau level N_ϕ by $\mathcal{A} = 2\pi l^2 N_\phi$, we can calculate the function $\mathcal{F}_{LCP}(t)$ analytically, i.e.,

$$\mathcal{F}_{LCP}(t) = 2(4 + t^2\epsilon_B^2)^{-1/2} \exp[-it(g + E_{FM})]$$

$$\times \exp[-it \tan^{-1}(t\epsilon_B/2)]. \quad (18)$$

Substituting the above expression in Eq. (9), we can also solve the time integral analytically and obtain the emission rate for the left circularly polarized radiation,

$$W_{LCP}^0(w) \propto \frac{\pi}{\epsilon_B} \text{sign}(w_{LCP}^0 - w) \times \exp(2(w - w_{LCP}^0)/\epsilon_B), \quad (19)$$

where $w_{LCP}^0 = w_0 + E_i - E_{FM} - g$ and $\text{sign}(x)$ is the signal function.

As we can see in Fig. 2 (dashed line), the LCP spectrum is formed by a peak at $w = w_{LCP}^0$ with a low energy tail, which agrees with the results previously derived in Ref. 6 and with the experimental data of Ref. 5. The observed behavior is related to the spin wave excitation which remains after the
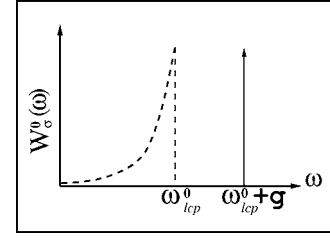

FIG. 2. Schematic representation of the emission spectra of the left (dashed line) and right (solid line) circularly polarized radiation when the photoexcited ground state is given by Eq. (8).

recombination process (see Fig. 1). Here, we should mention that the linewidth of the experimental spectrum is related to the disorder effects as showed in Ref. 6. Therefore, the low energy tail accounts for the asymmetry observed in the experimental spectrum.

Following an analogous procedure, we can calculate the transition rate for the RCP radiation. In this case, we have

$$W_{RCP}^0 \propto Re \int_0^\infty dt\, e^{-i(w - w_0 - E_i)t} \underbrace{\langle FM | c_{0\downarrow} c_{0\downarrow}^\dagger e^{-i\mathcal{H}_f t} c_{0\downarrow} c_{0\downarrow}^\dagger | FM \rangle}_{\mathcal{F}_{RCP}(t)}. \quad (20)$$

Using the expression of the operator $c_{0\downarrow}^\dagger c_{0\downarrow}$ in terms of the bosons [Eq. (B4)], it is possible to show that

$$\mathcal{F}_{RCP}(t) = e^{-iE_{FM}t}, \quad (21)$$

which implies that the emission rate of the RCP radiation is simply given by

$$W_{RCP}^0(w) \propto \delta[w - (w_{LCP}^0 + g)]. \quad (22)$$

The above expression is schematically illustrated in Fig. 2 (solid line). Based on this result, we can conclude that the RCP spectrum is formed only by a sharp peak, whose energy is greater than the energy of the LCP peak. Unlike the LCP spectrum, the form of the RCP spectral line is not asymmetric. Again, our results are in qualitative agreement with the experimental data.[5,7]

Despite the fact that our model is not able to determine the energy of the photoexcited ground state and therefore the energies of the peaks of the LCP and RCP spectra, it captures the main features of the experimental polarized photoluminescence spectra reported in Refs. 5 and 7, where the estimated value of the distance d was smaller than the magnetic length.

In the next section, we will extend our analysis to the $d > l$ case in order to check the suggestion of Osborne *et al.*,[7] which pointed the existence of skyrmions in the 2DEG prior to the recombination process. As far as we know, this is the first time that the form of the spectral line of the polarized photoluminescence spectrum of the 2DEG at $\nu = 1$ is calculated, including the presence of skyrmions in the system before the recombination.

IV. THE SKYRMION-HOLE INITIAL STATE

Now, we will consider that the distance between the 2DEG and the valence band hole is $d > l$. In this case, the

Coulomb interaction between the spin-down electron and the spin-up electrons is greater that the interaction between the former and the valence band hole (remember that, at $\nu=1$, the radius of the cyclotron orbit of each electron is equal to l). As a result, the photoexcited ground state changes from the excitonic state [Eq. (8)] to a quantum Hall skyrmion bound to the valence band hole (skyrmion-hole state). This scenario is also corroborated by the numerical calculations of Portengen et al.,[13] who showed that the skyrmion-hole state is more stable than the excitonic state when $d>l$. The presence of skyrmions in the system prior to the recombination is also discussed in Ref. 14 and the references therein.

The above modification in the nature of the photoexcited ground state, in relation to the one considered in the last section, can be easily accommodated in our model. It can be done considering that the skyrmion (a charged excitation of the quantum Hall ferromagnet with spin $S^z>1/2$) can be described by a state formed by a spin-down electron bound to a determined number of spin wave excitations. In fact, this idea was suggested by Palacios and Fertig,[15] but only Oaknin et al. studied this model in details.[16] It is also considered in our previous work.[10]

If we assume that the skyrmion is formed by one spin-down electron bound to n spin waves, we can write down the photoexcited ground state as

$$|i\rangle \sim \prod_{i=1}^{n} b_{\mathbf{q}_i}^{\dagger} c_{m\downarrow}^{\dagger} |\text{FM}\rangle, \qquad (23)$$

where $|lq_i|<1$. In the above expression, we considered the fact that, within our bosonization method, the spin wave excitations of the interacting two-dimensional electron gas at $\nu=1$ can be described approximately as bosons.

Substituting Eq. (23) in (7), the transition rate for the left circularly polarized radiation reads

$$W_{\text{LCP}}(w) \propto Re \int_0^{\infty} dt\, e^{-i(w-w_0-E_i)t}$$

$$\times \underbrace{\langle \text{FM} | c_{m\uparrow} \prod_{i=1}^{n} b_{\mathbf{q}_i} c_{0\uparrow}^{\dagger} e^{-i\mathcal{H}_f t} c_{0\uparrow} \prod_{i=1}^{n} b_{\mathbf{q}_i}^{\dagger} c_{m\downarrow}^{\dagger} | \text{FM} \rangle}_{\mathcal{F}_{\text{LCP}}(t)}. \qquad (24)$$

Here, we will follow the same procedure of the preceding section, namely, the energy of the initial state E_i will be treated as a parameter and the Hamiltonian $\mathcal{H}_f$ will be given by Eq. (15). As $[b_{\mathbf{q}}^{\dagger}, c_{m\uparrow}^{\dagger}]=0$, it is possible to reorder the fermionic operators in Eq. (24) and, using the bosonic representation of the product of fermionic operators $c_{m\sigma}^{\dagger} c_{m\sigma'}$ [see Eqs. (B1) and (B2)], we have

$$\mathcal{F}_{\text{LCP}}(t) = \frac{1}{N_\phi} \sum_{\mathbf{k}} e^{-|lk|^2/2} G_{m,0}(lk) G_{0,m}(-lk)$$

$$\times \exp[-it(\epsilon + w_{\mathbf{k}} + E_{\text{FM}})]$$

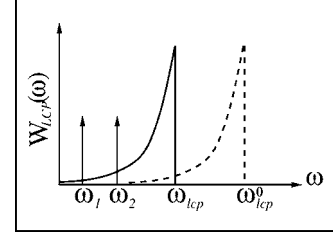$$+ \frac{1}{N_\phi} \sum_{i=1}^{n} e^{-|lq_i|^2/2} G_{m,0}(lq_i) G_{0,m}(-lq_i)$$

FIG. 3. Schematic representation of the emission spectra of the left circularly polarized radiation when the photoexcited ground state is given by Eq. (23) with $n=2$ (solid line) and by Eq. (8) (dashed line).

$$\times \exp[-it(\epsilon + w_{\mathbf{q}_i} + E_{\text{FM}})], \qquad (25)$$

where $\epsilon = \Sigma_{i=1}^{n} w_{\mathbf{q}_i}$ is the energy of the spin waves present in (23). Again, we will change the sum over momenta into an integral and expand $w_{\mathbf{k}}$ as in Eq. (17). Assuming that $m=0$, the function $G_{0,0}(x)=1$. As a result, the integral over momenta of the above expression can be calculated analytically,

$$\mathcal{F}_{\text{LCP}}(t) = 2(4 + t^2 \epsilon_B^2)^{-1/2}$$

$$\times \exp[-it(\epsilon + g + E_{\text{FM}})] \exp[-i \tan^{-1}(t\epsilon_B/2)]$$

$$+ \frac{1}{N_\phi} \exp[-it(\epsilon + E_{\text{FM}})] \sum_{i=1}^{n} \exp(-it w_{\mathbf{q}_i} - |lq_i|^2/2). \qquad (26)$$

Now, substituting Eq. (26) in (24), we can analytically calculate the temporal integral and show that the emission rate for the LCP radiation is simply given by

$$W_{\text{LCP}}(w) \propto \frac{2\pi}{\epsilon_B} \text{sign}(w_{\text{LCP}} - w) \exp[2(w - w_{\text{LCP}})/\epsilon_B]$$

$$+ \frac{1}{N_\phi} \sum_{i=1}^{n} \exp(-|lq_i|^2/2) \delta[w - (w_{\text{LCP}} + g - w_{\mathbf{q}_i})], \qquad (27)$$

where $w_{\text{LCP}} = w_0 + E_i - E_{\text{FM}} - g - \epsilon$.

As illustrated in Fig. 3 for the case $n=2$ (solid line), W_{LCP} is formed by a main peak localized at w_{LCP} with a low-energy tail, features which were observed in W_{LCP}^0. It is possible to estimate the energy E_i of the photoexcited ground state $|i\rangle$ and show that $w_{\text{LCP}} < w_{\text{LCP}}^0$. In fact, writing the energy of the state (8) as E_i^0 and the spin waves spin-down electron binding energy as E_b, we have $E_i \approx E_i^0 + \epsilon - E_b$ and therefore $w_{\text{LCP}}^0 - w_{\text{LCP}} = E_b$. From this analysis, we can conclude that there is a redshift in the energy of the main peak of W_{LCP} in relation to the one of W_{LCP}^0 and that this redshift is given by the binding energy of the spin waves spin-down electron.

Moreover, the LCP spectrum also has a set of secondary sharp peaks at $w_i = w_{\text{LCP}} + g - w_{\mathbf{q}_i}$ whose intensities are smaller than the intensity of the main peak. Notice that the redshift of the energy of each secondary peak w_i, in relation to the main one of W_{LCP}, is mainly given by the energy of the spin wave with momentum $\mathbf{q}_i$, which is present in the state
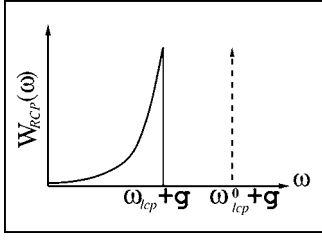
FIG. 4. Schematic representation of the emission spectra of the right circularly polarized radiation when the photoexcited ground state is given by Eq. (23) (solid line) and by Eq. (8) (dashed line).

(23). The presence of those small peaks will increase the spectral weight of $W_{\text{LCP}}(w)$ on the low energy side.

Finally, using the same approach, we can calculate the emission rate for the right circularly polarized radiation. In this case, Eq. (7) reads

$$W_{\text{RCP}}(w) \propto Re \int_0^\infty dt\, e^{-i(w - w_0 - E_i)t}$$

$$\times \underbrace{\langle \text{FM}| c_{m\downarrow} \prod_{i=1}^{n} b_{\mathbf{q}_i} c_{0\uparrow}^\dagger e^{-i\mathcal{H}_f t} c_{0\uparrow} \prod_{i=1}^{n} b_{\mathbf{q}_i}^\dagger c_{m\downarrow}^\dagger |\text{FM}\rangle}_{\mathcal{F}_{\text{RCP}}(t)}. \tag{28}$$

After a lengthly calculation (see Appendix C), it is possible to show that $\mathcal{F}_{\text{RCP}}(t)$ can be written as

$$\mathcal{F}_{\text{RCP}}(t) \approx \delta_{m,0} \left(1 - \frac{2n}{N_\phi} + \frac{n(n-1)}{N_\phi^2} \right) \exp[-it(E_{\text{FM}} + \epsilon)]$$

$$- 2i \frac{n}{N_\phi} \exp[-it(E_{\text{FM}} + \epsilon)] \frac{1}{t\epsilon_B - 2i}$$

$$+ \frac{1}{N_\phi^2} \sum_{j,l \neq j} \exp(-|l(\mathbf{q_j} - \mathbf{q_l})|^2/2)$$

$$\times \exp[-it(\epsilon + E_{\text{FM}} - w_{\mathbf{q_j}} + w_{\mathbf{q_i}})]. \tag{29}$$

Substituting $\mathcal{F}_{\text{RCP}}(t)$ in (28), neglecting the terms of order $1/N_\phi^2$, and after the temporal integration, we have

$$W_{\text{RCP}}(w) \propto \delta(w - w_{\text{LCP}} - g)$$

$$+ \frac{4\pi n}{\epsilon_B N_\phi} \text{sign}(w_{\text{LCP}} + g - w)$$

$$\times \exp[2(w - w_{\text{LCP}} - g)/\epsilon_B] + \mathcal{O}(1/N_\phi^2). \tag{30}$$

Equation (30) is schematically illustrated in Fig. 4 (solid line). Unlike $W_{\text{RCP}}^0(w)$, $W_{\text{RCP}}(w)$ has a sharp peak at $w_{\text{LCP}} + g$ in addition to a low-energy tail [second term of Eq. (30)], which implies that the RCP spectral line is also asymmetric. Here, the recombination of a spin-down electron with the valence band hole also leaves spin waves in the 2DEG as the photoexcited ground state is formed by a spin down electron bound to n-spin waves. We can also observe a redshift in the energy of this peak in relation to the one of $W_{\text{RCP}}^0(w)$, which

is related to the spin waves spin-down electron binding energy.

The expressions (27) and (30) are in good qualitative agreement with the experimental photoluminescence spectra reported by Osborne et al.[7] for a single quantum well sample under high applied electric field and for the indirect process in double quantum well samples. As pointed out in Sec. I, a redshift of the energies of the main peaks of the LCP and RCP spectra was observed in relation to the results obtained for the single quantum well samples under a low electric field. Our analysis showed that this redshift is related to the change in the nature of the photoexcited ground state from the excitonic state to the skyrmion-hole one. In particular, this redshift is proportional to the binding energy of the spin waves to the spin-down electron, which form the skyrmion-hole state. Moreover, as the distance d increases, the width of the low energy tail of the LCP spectrum also increases and now the RCP spectrum is also asymmetric. The expression (27) has a low energy tail in addition to a set of secondary peaks which account for the increase of the width of the low energy tail of the LCP spectral line. Note that the spectral line (30) is also asymmetric.

Therefore, the polarized photoluminescence spectrum of the two-dimensional electron gas at $\nu=1$, in the limit where $d > l$, can be described by the recombination of a state formed by a spin-down electron bound to n-spin wave excitations with the valence band hole as suggested by Osborne et al.[7]

Furthermore, we can also conclude that this state is a good candidate to describe the elementary charged excitation of the quantum Hall ferromagnet. Remember that, in the limit of larger d, the effect of the hole over the two-dimensional electron gas is smaller than in the case $d < l$. As a consequence, after the addition of the spin-down electron, the system probably relaxes to this charged excited state before the recombination.

V. SUMMARY

We studied the polarized photoluminescence spectrum of the two-dimensional electron gas at $\nu=1$ in the limit of small and large separation between the 2DEG and the valence band hole.

In order to do that, we considered a model analogous to the one adopted to study the x-ray spectra of metals. The RCP and LCP spectral lines were calculated using a previously developed bosonization method for the 2DEG at $\nu=1$.

For small distances d, we use the fact that the photoexcited ground state can be described by an excitonic state and showed that the LCP spectrum is formed by a peak with a low-energy tail, which is related to the spin wave excitation left over after the recombination process takes place, and that the RCP spectral line is formed by a sharp peak.

For large distances d, we assume that the photoexcited ground state is given by the skyrmion-hole state, which was described by a spin-down electron bound to n-spin waves. We showed that, in addition to the main peak, the LCP spectrum also presents a set of secondary peaks which accounts for the increase of the width of the experimental spectral line

on the low-energy side. Furthermore, there is a redshift in the energy of the main peak, which is closely related to the binding energy between the spin-down electron and the spin wave, in relation to the energy of the peak of the LCP spectrum obtained in the limit of small d. In addition, we also showed that the recombination of the skyrmion-hole state might be responsible for the low-energy tail experimentally observed in the RCP spectrum.

Despite the fact that our model is not able to determine the energy of the photoexcited ground states, we can analytically calculate the spectral lines and also study a model for the quantum Hall skyrmion.

From our analysis, we can conclude that the photoluminescence experiment also corroborates the existence of a composite excitation in the 2DEG at $\nu=1$ as the quantum Hall skyrmion.

APPENDIX A: THE $G_{m,m'}(x)$ FUNCTION

The function $G_{m,m'}(x)$ is related to the matrix element of the operator $\exp(-i\mathbf{q}\cdot\mathbf{r})$ in the lowest Landau level. It is defined as[10]

$$G_{m,m'}(lq) = \theta(m'-m)\sqrt{\frac{m!}{m'!}}\left(\frac{-ilq^*}{\sqrt{2}}\right)^{m'-m}L_m^{m'-m}\left(\frac{|lq|^2}{2}\right)$$
$$+ \theta(m-m')\sqrt{\frac{m'!}{m!}}\left(\frac{-ilq}{\sqrt{2}}\right)^{m-m'}L_m^{m-m'}\left(\frac{|lq|^2}{2}\right),$$

where $L_{m'}^{m-m'}(x)$ is the generalized Laguerre polynomial.

APPENDIX B: BOSONIC FORM OF THE PRODUCT OF FERMIONIC OPERATORS

Using the bosonization method for the two-dimensional electron gas at $\nu=1$ introduced in Ref. 10, it is possible to show that the product of fermionic operators can be written in a bosonic language as

$$c_{m\downarrow}^\dagger c_{m'\uparrow} \equiv \frac{1}{\sqrt{N_\phi}}\sum_{\mathbf{q}} e^{-|lq|^2/4}G_{m',m}(-lq)b_{\mathbf{q}}^\dagger, \tag{B1}$$

$$c_{m\uparrow}^\dagger c_{m'\downarrow} = \frac{1}{\sqrt{N_\phi}}\sum_{\mathbf{k}} e^{-|lk|^2/4}G_{m',m}(lk)b_{\mathbf{k}}$$
$$- \sum_{\mathbf{p},\mathbf{q},\mathbf{k}} \frac{e^{-|lk|^2/4}}{N_\phi^{3/2}}\cos\left(\frac{(\mathbf{k}+\mathbf{p})\wedge(\mathbf{p}-\mathbf{q})}{2}\right)$$
$$\times G_{m',m}(lk)b_{\mathbf{k}+\mathbf{p}+\mathbf{q}}^\dagger b_{\mathbf{p}}b_{\mathbf{q}}, \tag{B2}$$

$$c_{m\uparrow}^\dagger c_{m'\uparrow} = \delta_{m,m'} - \frac{1}{N_\phi}\sum_{\mathbf{k},\mathbf{q}} e^{-|l(k-q)|^2/4}e^{-i\mathbf{k}\wedge\mathbf{q}}G_{m',m}(lq-lk)b_{\mathbf{k}}^\dagger b_{\mathbf{q}}, \tag{B3}$$

$$c_{m\downarrow}^\dagger c_{m'\downarrow} = \frac{1}{N_\phi}\sum_{\mathbf{k},\mathbf{q}} e^{-|l(k-q)|^2/4}e^{i\mathbf{k}\wedge\mathbf{q}}G_{m',m}(lq-lk)b_{\mathbf{k}}^\dagger b_{\mathbf{q}}, \tag{B4}$$

where $\mathbf{k}\wedge\mathbf{q}=l^2\hat{z}\cdot(\mathbf{k}\times\mathbf{q})$. The above expressions are quite similar to the ones derived in Ref. 10 for the spin density operators.

APPENDIX C: THE $\mathcal{F}_{\mathrm{RCP}}(t)$ FUNCTION FOR THE SKYRMION-HOLE INITIAL STATE

As $[b_{\mathbf{q}},c_{m\downarrow}]=0$, the function $\mathcal{F}_{\mathrm{RCP}}(t)$, defined in Eq. (28), can be written as

$$\mathcal{F}_{\mathrm{RCP}}(t) = \langle\mathrm{FM}|\prod_{i=1}^{n} b_{\mathbf{q_i}}(\delta_{m,0} - c_{0\downarrow}^\dagger c_{m\downarrow})$$
$$\times e^{-i\mathcal{H}_f t}(\delta_{m,0} - c_{m\downarrow}^\dagger c_{0\downarrow})\prod_{i=1}^{n} b_{\mathbf{q_i}}^\dagger|\mathrm{FM}\rangle = \sum_{m=0}^{4} \mathcal{F}_{\mathrm{RCP}}^{(m)}(t). \tag{C1}$$

Let us analyze each of the four terms separately.

The first term is simply given by

$$\mathcal{F}_{\mathrm{RCP}}^{(1)}(t) = \delta_{m,0}\langle\mathrm{FM}|\prod_{i=1}^{n} b_{\mathbf{q_i}}e^{-i\mathcal{H}_f t}\prod_{i=1}^{n} b_{\mathbf{q_i}}^\dagger|\mathrm{FM}\rangle.$$

Writing $\mathcal{H}_f$ as in Eq. (15) and considering the long wavelength limit of the boson dispersion relation $w_{\mathbf{q}}$ [Eq. (17)], with the aid of the expression (16), the first term reduces to

$$\mathcal{F}_{\mathrm{RCP}}^{(1)}(t) = \delta_{m,0}\exp[-it(\epsilon + E_{\mathrm{FM}})]. \tag{C2}$$

For the second one, we have

$$\mathcal{F}_{\mathrm{RCP}}^{(2)}(t) = -\delta_{m,0}\langle\mathrm{FM}|\prod_{i=1}^{n} b_{\mathbf{q_i}}e^{-i\mathcal{H}_f t}c_{0\downarrow}^\dagger c_{0\downarrow}\prod_{i=1}^{n} b_{\mathbf{q_i}}^\dagger|\mathrm{FM}\rangle$$
$$= -\frac{1}{N_\phi}\delta_{m,0}\sum_{\mathbf{k},\mathbf{k}'} e^{-|l(k-k')|^2/4}e^{i\mathbf{k}\wedge\mathbf{k}'}$$
$$\times\langle\mathrm{FM}|\prod_{i=1}^{n} b_{\mathbf{q_i}}e^{-i\mathcal{H}_f t}b_{\mathbf{k}}^\dagger b_{\mathbf{k}'}\prod_{i=1}^{n} b_{\mathbf{q_i}}^\dagger|\mathrm{FM}\rangle.$$

Here, in the second step, we use the fact that the bosonic form of the operator $c_{0\downarrow}^\dagger c_{0\downarrow}$ is a linear combination of the product $b_{\mathbf{k}}^\dagger b_{\mathbf{k}'}$ [see Eq. (B4)]. Reordering the bosonic operators, we end up with

$$\mathcal{F}_{\mathrm{RCP}}^{(2)}(t) = -\frac{n}{N_\phi}\delta_{m,0}\exp[-it(\epsilon + E_{\mathrm{FM}})], \tag{C3}$$

where n is the number of spin waves in the state (23). The third term is similar to the second one and therefore the former is equal to (C3).

Finally, the last term is given by

$$\mathcal{F}_{\mathrm{RCP}}^{(4)}(t) = \langle \mathrm{FM}| \prod_{i=1}^{n} b_{\mathbf{q_i}} c_{0\downarrow}^{\dagger} c_{m\downarrow} e^{-i\mathcal{H}_f t} c_{m\downarrow}^{\dagger} c_{0\downarrow} \prod_{i=1}^{n} b_{\mathbf{q_i}}^{\dagger} |\mathrm{FM}\rangle.$$

Again, substituting Eqs. (B1) and (B2) in the above expression, choosing $m=0$ and reordering the bosonic operators, we have

$$\mathcal{F}_{\mathrm{RCP}}^{(4)}(t) = \frac{1}{N_\phi^2} \sum_{j=1}^{n} \sum_{\mathbf{p}} e^{-|l(\mathbf{p} - \mathbf{q_j})|^2/2}$$

$$\times \exp[-it(\epsilon + E_{\mathrm{FM}} - w_{\mathbf{q_j}} + w_{\mathbf{p}})]$$

$$+ \frac{1}{N_\phi^2} \sum_{j=1}^{n} \sum_{l \neq j} \{ e^{-|l(\mathbf{q_j} - \mathbf{q_l})|^2/2}$$

$$\times \exp[-it(\epsilon + E_{\mathrm{FM}} - w_{\mathbf{q_j}} + w_{\mathbf{q_l}})]$$

$$+ \exp[-it(\epsilon + E_{\mathrm{FM}})] \}. \quad (C4)$$

Substituting $\mathbf{p} \to \mathbf{p} + \mathbf{q_j}$ in the first term of the above expression, changing the sum over momenta into an integral, and rescaling the momentum $p \to lp$, we find that

$$\mathcal{F}_{\mathrm{RCP}}^{(4)}(t) = -\frac{2i}{N_\phi} \sum_{j=1}^{n} \exp[-it(\epsilon + E_{\mathrm{FM}})]$$

$$\times \frac{1}{t\epsilon_B - 2i} \exp\left(\frac{-i(lq_j t\epsilon_B)^2}{8i - 4t\epsilon_B} \right)$$

$$+ \frac{1}{N_\phi^2} \sum_{j=1}^{n} \sum_{l \neq j} \{ e^{-|l(\mathbf{q_j} - \mathbf{q_l})|^2/2}$$

$$\times \exp[-it(\epsilon + E_{\mathrm{FM}} - w_{\mathbf{q_j}} + w_{\mathbf{q_l}})] \}$$

$$+ \frac{1}{N_\phi^2} n(n-1) \exp[-it(\epsilon + E_{\mathrm{FM}})]. \quad (C5)$$

As we assume that the bosons momenta are $|lq_j| \ll 1$, we can expand the first term of Eq. (C5), i.e.,

$$\exp\left(\frac{-i(lq_j t\epsilon_B)^2}{8i - 4t\epsilon_B} \right) \approx 1 + \mathcal{O}(|lq_j|^2). \quad (C6)$$

Therefore, from the expressions (C2), (C3), and (C5), we find that the function $\mathcal{F}_{\mathrm{RCP}}(t)$ is given by Eq. (29).

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