

Design, analysis and reformulation of a didactic sequence for teaching the special theory of relativity in high school

(Proposta, análise e reformulação de uma sequência didática para o ensino da teoria da relatividade especial no Nível Médio)

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In this work, we design, implement and analyze a didactic sequence for the teaching of the basic topics of special relativity theory in high school. The sequence proposes a series of situations, specially designed to allow the emergence of the central aspects of special relativity. The conceptualization process is investigated from the point of view of the theory of conceptual fields of Vergnaud. By means of a careful analysis of classroom student productions we detect the key theorems-in-action they use, evidencing that most of conceptual errors are of pre-relativistic nature. This leads us to a reformulation of the sequence, which promotes the conceptualization of Galilean relativity and the principles of the special relativity. This previous step aims at bringing to students a firm basis to address the more complex aspects of the subject.

Keywords: physics didactics, special relativity, theory of conceptual fields, high school teaching.

Neste trabalho planejamos, implementamos e avaliamos uma sequência didática para ensinar aspectos básicos da teoria especial da relatividade (TER) no Ensino Médio. A sequência propõe um conjunto de situações especialmente planejadas para que os aspectos centrais da TER possam emergir. O processo de conceitualização é analisado segundo o ponto de vista da teoria dos campos conceituais de Vergnaud. Por meio de uma cuidadosa análise das produções dos estudantes durante as aulas, detectamos teoremas em ação e conceitos em ação, de natureza pré galileana. Isso nos conduziu à reformular a sequência para promover a conceitualização da relatividade galileana e os postulados da relatividade especial, antes de considerar aspectos mais complexos.

Palavras-chave: didática da física, relatividade especial, teoria dos campos conceituais, física no Ensino Médio.

1. Introduction

The curriculum of the high school in Argentina proposes the study of the basic concepts of relativistic physics. In particular, in Buenos Aires province, the topic special relativity (SR) is part of the discipline Classical and Modern Physics, of 6th year high school with natural-sciences orientation.

Contrary to the stipulations of the high school curriculum, the contents of modern and contemporary physics are (in general) not studied at this level. However, commonly students show interest and some level of knowledge in modern physics topics, due to the variety of media available [1]

It is clear that the study of SR is relevant due to

the deep revolution it caused on common sense aspects about space and time. But from a wider perspective it also brings sense to the study of Galilean-Newtonian relativity, as a previous step to its conceptualization within the SR framework.

The investigations on the teaching of the topic relativity, focusing on the conceptualization of the basic aspects, are scarce. Here we briefly review illustratively some of them. The works [2-5] analyze the conceptualization of relative motion in the Galilean context, especially at University level. Regarding proposals to teach SR, some results indicate that students do not use SR concepts, but keep their pre-SR ideas to interpret SR results [6] Finally other works [7-9] explore epistemological, historical and conceptual aspects of the SR with

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teachers and students of physics education. These investigations conclude that in general teachers introduce the concepts uncritically, with a weak knowledge of the basic theory, in detriment of an appropriate conceptualization by part of the students.

The aim of this work is contribute to the development of a Didactics of the special theory of relativity and the study of the conceptualization process of their fundamental notions in students of the last years of high school. The didactic component of our research requires the specification of the reference conceptual structure (RCS) [10] for the SR. This entails an epistemological and didactical analysis of the fundamentals of the SR, in order to propose a potentially viable sequence, adequate to high school.

The didactic performance and viability of the sequence are experienced in 6th year high school physics courses, together with the analysis of the conceptualization by part of the students. In this way, the investigation assumes the complementarity of the didactic and cognitive dimensions.

2. The theory of conceptual fields

The theory of conceptual fields (TCF) is a cognitive theory that brings a coherent and operative framework, organized around a set of basic principles to study the learning process and the development of complex concepts and competences. By providing a scenario for addressing learning aspects, the TCF is also relevant for Didactics [11]. From the point of view of the TCF, the conceptualization takes place in all areas of human experience: family, compulsory school, professional training, employment, etc.

However, there are contexts that can significantly stimulate the conceptualization process in some areas of knowledge. For instance, the learning of physics and mathematics topics requires a high level of conceptualization, which may emerge in situations that high school can recreate likely best than any other social institution [12].

The TCF proposes that in every field of knowledge, certain processes of conceptualization are needed. These processes emerge in some kind of situations and events, evoking the development of certain types of activity. Therefore, it is necessary to make explicit the knowledge of reference from which the teaching will be conceived, the knowledge to be taught and their transformations, as well as the one it is actually taught, taking into account the transpositive processes [13].

The specificity of the acquisition processes in each conceptual field leads to Vergnaud linking cognitive development in a certain domain, with teaching, that is to say with Didactics [14]

3. Operational form and predicative form of knowledge

The operational form of knowledge is what allows the subject to act in a given situation, whereas the predicative form consists in stating the relations between objects. There is a huge complexity in doing and saying what is done [15]. But while teaching is irreplaceable in the process of conceptualization, it cannot be reduced to put into words the conceptual content of knowledge. The enunciation is essential in the process of conceptualization.

In particular, the difficulties students have in learning physics and mathematics show the complexity of the situations involved, and the thinking operations necessary to treat them.

4. Concept

Vergnaud proposes a pragmatic -useful and functional- definition of concept. A concept can be defined by the conjunction of three different sets, which are not independent of each other [14]

$$\text{Concept} = \text{def}(S, I, L),$$

where S is the set of situations that give sense to the concept, I is the set of operational invariants that integrate the schemes evoked in the situations and L is the set of linguistic and symbolic representations (algebraic, graphical, etc.) that allow representing the concepts and their interrelations

The operational invariants are of two types: *concepts in action*, defined as categories pertinent to the subject in the situation, and *theorems in action*, that are affirmations validated by the subject.

Therefore, the concept involves, on one hand, a component which is property of the subject but related to the situation, such as the operational invariants present in the schemes. On the other hand, a concept involves a link to "the real" as the types of situations that interact dialectically with the schemes. Finally, the concept comprises a semiotic component, which refers to the systems of signs or representations used to enunciate the concepts, their interrelations, and to refer to the objects [12]

5. Investigation methodology

The design and implementation of a didactic sequence involves three main phases. The first one, known as *priori analysis*, is the construction of a *reference conceptual structure* (RCS) which is the basis for the design of a number of situations, whose resolution requires the emergency of certain concepts. The second phase comprises the *design and development of the didactic sequence* itself, based in the priori analysis. Finally, the

phase of *testing and implementation* of the sequence in one or more pilot projects to generate a posteriori analysis, which in turn will allow an eventual sequence reformulation. This process generates a cycle that leads to a relative stabilization of the main parts of the sequence with the modification or addition of more tasks to enforce the conceptualization of the relevant topics if necessary, or conversely, reduce them.

This research comprises a first cycle, which leads to the testing and implementation of the original sequence in two courses of sixth year of secondary school (total number of students $N = 43$). This gives rise to a modification of the sequence, which is also described as part of the work. The research has exploratory, qualitative and ethnographic character. In each class a situation is proposed to students, who work in small groups. Class by class student protocols are collected and scanned, to be returned the next class. In addition, all classes are audio-recorded, and the teacher, who is also the researcher, carries out participant observation. Other researchers of the team perform non-participant observation. The protocols are analysed considering the situations, the theorems in action and representation systems used by students: verbal (oral and written) graphic, numeric and algebraic.

6. A reference conceptual structure for special relativity theory

Historically, the genesis of the theory of relativity is in Maxwell's equations. These equations predict the propagation of electromagnetic waves in the vacuum with a constant speed c . This linked the optics with the electric and magnetic phenomena, promoting the first big unification in the physics laws.

Nevertheless, beyond of the importance of the synthesis of Maxwell, the presence of a constant in the equations turned out enigmatic. Maxwell's equations are not invariant under the Galileo's transformation, which is contradictory with the principle of relativity. Facing this fact, different points of view arose. One possibility would be to consider the existence of a privileged system of reference in absolute rest, the "*ether*" respect of which Maxwell's laws were valid. Thereby, in other systems of reference the speed of the light would change in agreement to Galileo's transformation, that is to say, c would not be a constant.

Another possibility was to raise c to the range of universal constant, supporting the principle of relativity. But looking for the invariance of the Maxwell's equations implied to change Galileo's transformation, and therefore the law of speed addition together with the existence of an absolute and universal time, concepts strongly established in our daily experience. In

any case, the new transformation had to satisfy the requirement of leaving invariant to Maxwell's equations and tending mathematically to Galileo's transformation at human scale speeds. That is to say the Galileo's transformation might be considered an accurate law in the range of speeds much lower than c .

The definition in favour of the second possibility came across several fronts. On the one hand, Lorentz and Fitzgerald found the laws of transformation that leave invariant Maxwell's equations, which rightly take the name of Lorentz's transformations. Nevertheless they were unaware about the physical consequences regarding the relativity of the space and the time.

On the other hand, from the experimental point of view, all the attempts to detect the hypothetical "*ether*" were fruitless, specially the famous interferometry experiments of Michelson and Morley. This was a very strong argument to break down the first option.

Finally, the most significant contribution in favour of the second possibility was carried out by Einstein, who independently analysing Maxwell's equations, decided to generalize the principle of relativity for all the physical systems, included the electromagnetism, and to assume the invariance of c for all inertial observers. He understood the deep consequences of these ideas working together, and synthesized this viewpoint in two postulates, to later derive their main physical consequences. For this reason Einstein is considered the father of the theory of the relativity

The SR describes the kinematic and dynamic behaviour of objects without taking into account gravitational effects. It is possible to develop the SR on the basis of the two postulates:

P1: The principle of relativity: The laws of physics are the same for all inertial observers.

P2: Invariance of the speed of light: The speed of light c is constant for all inertial observers in the vacuum and is the upper bound for any speed.

In summary: In the first postulate, Einstein generalized the principle of relativity to all physical systems. The second postulate raises the speed of light in vacuum to the *status* of universal constant. Therefore, the laws of physics are invariant (principle of relativity) under the Lorentz transformation, which for low speeds compared with c reduces mathematically to the Galilean transformation.

These two, seemingly harmless postulates, working together lead to a series of surprising predictions that challenge the ideas of space, time, mass and energy, deeply rooted in our everyday experience of low speeds (compared with c). Figure 1 shows schematically the interrelation between the different concepts involved in the RCS for SR.

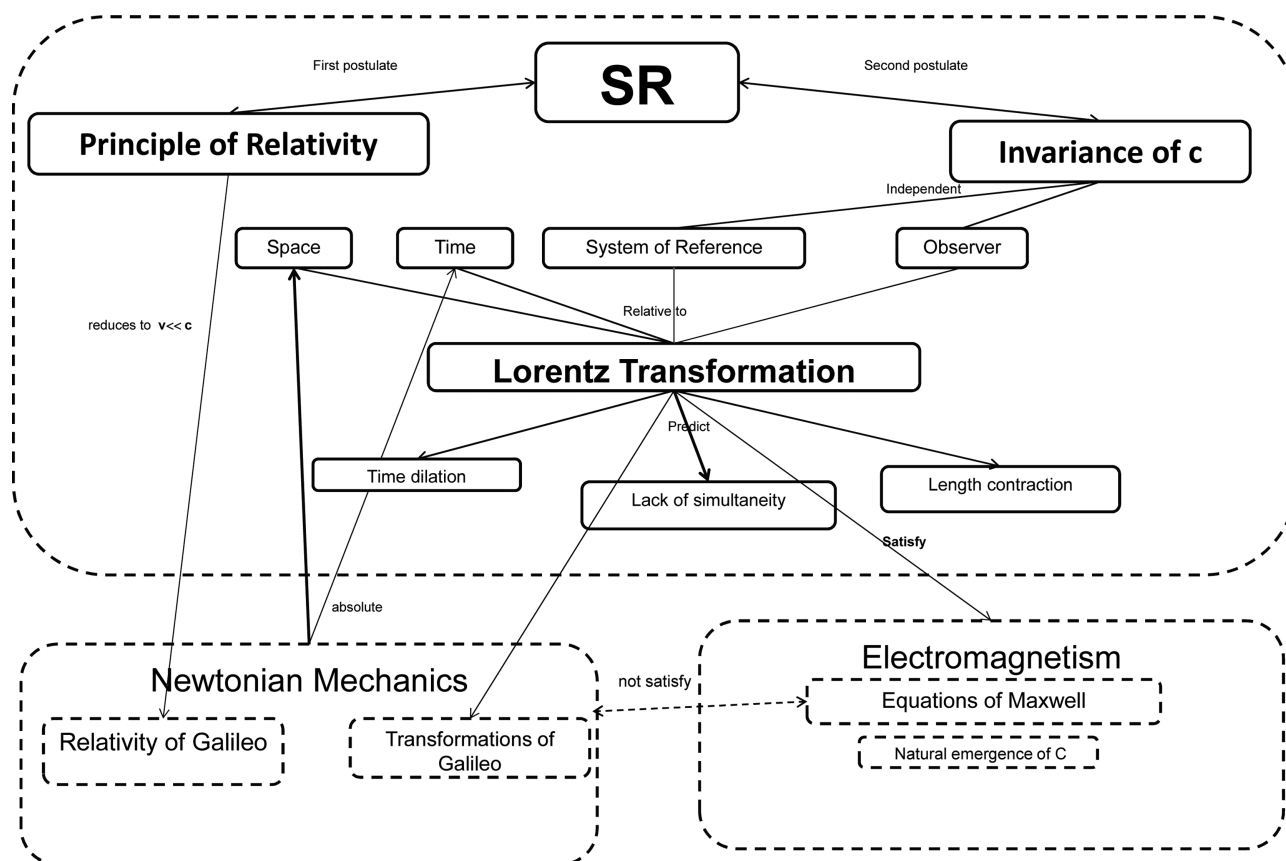


Figure 1 - Scheme of the reference conceptual structure (RCS) for the special theory of relativity.

In this proposal the effects of relativistic dynamics are not studied. Therefore some relevant topics, such as the relationship between mass and energy are beyond the scope of this research.

7. Didactic sequence

7.1. First part: Galileo relativity

In this part the concepts of reference system, observer, measuring of length and time are discussed. After that, the concept of relative motion, *i.e.*, respect to different reference systems, and the law of addition of velocities (much smaller than c) are discussed.

Finally, situations to evidence the impossibility of distinguishing between rest and uniform motion are proposed, and thus reconstructing the Galilean relativity principle in the RCS proposed.

7.2. Second part: Transition from Galileo to Einstein relativity

This part has the purpose of consolidate the concept of relativity and introduce the idea of invariance of light velocity, which are the two basic ingredients that allow constructing Einstein relativity.

At the beginning there is a situation presented to emphasize the Galilean relativity concept paving the

way for Einstein's relativity. Here it is expected that students apply Galileo relativity, so results are only correct for low velocities (compared to c).

At the end of this part we propose a situation designed to be analyzed by using both: relativity and c -invariance concepts together, to obtain a first counterintuitive result: relativity of simultaneity

7.3. Third part: Einstein relativity (kinematics)

After presenting the first counter-intuitive challenge to students in previous part, we propose situations where the phenomena of time dilation and length contraction are manifest. This gives rise to the analysis of relativistic law of velocities addition, Lorentz transformation and a discussion of the low speed limit (Galileo transformation).

It is important to emphasize the range in which relativistic aspects are relevant. The large value of c compared to the ordinary speeds which we are used, deprive us of experiencing phenomena such as time dilation and length contraction in everyday life.

To promote the conceptualization of these experimental results, we propose the didactical strategy of considering the hypothetical case of c being of same order of magnitude than ordinary speeds. In this case, relativistic effects would be observable and would not

represent a contradiction to intuition.

8. Situations proposed in the original sequence

8.1. First part: Galileo relativity

S1: How can anyone say that someone or something else is moving or not? Give examples, write your answer and if you want draw pictures

S2: I am traveling by car on a straight and long road. I see another car coming from the front. My travel partner says that this car is approaching us at 150 km/h. The speed limit on this road is 80 km / h. My partner says that this car is violating the maximum allowed speed. I say no, because we are traveling at the speed limit. Who is right? Could you mathematically represent this situation?

S3: I am moving with a speed v with respect to the street to benefit from the “green wave” on an avenue. A car traveling in the right lane is going twice as fast as I with respect to the road. What is its speed relative to me? Does the other car benefits from the “green wave”?

S4: Build a pendulum by tying a rubber ball at the end of a string and analyze what happens when you perform the following actions:

- You are walking in a straight line with your pendulum in one hand and you stop suddenly.*
- You are walking in a straight line with your pendulum in one hand without accelerating or braking.*
- You are walking in a straight line with your pendulum in one hand and you start to run.*
- You are standing with your pendulum in one hand.*
- You go by car or bike and take a curve.*

Draw pictures or diagrams of the different cases and explain them

S5: Suppose we were locked in a train wagon or in a car and can't see out, or take any external reference, but we have a pendulum. Is there anything we can do to find out if we are moving?

8.2. Second part: Transition from Galileo to Einstein relativity

S6: A person stands right in the middle of an empty truck trailer, which moves on a straight road with a

constant speed v (respect to the road). The observer has a device that can shoot rubber bullets or light beams (laser) forwards and backwards at the same time. If the person fires the rubber bullets, which one came first to the trailer edges?

What if the person does the same with the light?

8.3. Third part: Einstein relativity (kinematics)

S7: An observer is sitting right in the middle of a closed truck trailer moving at a constant speed v , respect to the road. In the roof of the trailer there is a plane mirror. The observer has a device that can emit a beam of light perpendicular to the ceiling. The ray hits the mirror and is reflected back toward the viewer. How long does the ray of light take to go to the mirror and back to the observer?

a) For the observer in the truck

b) For another observer who is standing on the road.

S8: An observer is sitting right in the middle of a trailer that moves at a constant speed v , with respect to the road. The observer on the trailer says that the length of the trailer is L . What trailer length would measure another observer standing on the road? Consider different and coherent values for L and v .

Answer previous question by supposing (hypothetically) that $c = 300$ km/h.

9. Analysis of the results

In situations 1-3 the students managed the use of Galilean speeds addition in one dimension, and we could say that the situations functioned properly. In the situation 4 the students conducted experiments in the schoolyard with enthusiasm. Some brought a bike to experience what was happening with the pendulum while they were turning. The students expressed their ideas in more than one system of representation, which indicates an appropriate level of conceptualization It started implicit in action to become explicit in different representational formats. Most of the responses were correct and the drawings complete and coherent This would indicate that students correctly understand the Galilean principle of relativity and the indistinguishability between uniform motion and rest in an inertial frame, as suggest the analysis of protocols.

Table 1 - Frequency of the theorems-in-action identified in the situation 4.

Theorems in action related to the principle of relativity identified in S4		
The pendulum accompanies the motion	The pendulum moving with constant speed is the same that at rest	If the pendulum moves with constant speed, it does not move respect to me
38	41	23

As an example, in the Fig. 2, the protocol of A23 allows us to appreciate that this student understands what happens if he suddenly stops when walking in a straight line with constant speed, if she/he quickly starts moving from the rest, or moves in a straight line at constant speed (a)-(c) respectively, and when he is turning (e).

Something similar is observed in Fig. 3 about the protocol of A16, for the different possibilities proposed in Situation 4. Similarly, the Fig. 4 shows the protocol of B7, where the student has represented correct responses to the situations, reflecting an appropriate level of conceptualization.

In the Situation 5, the students had to use the ideas made explicit in the situation 4, assuming they were in an isolated system and had only a pendulum. Here they can predict what happens in the case of a speed variation but fail in the indistinguishability between uniform translation and rest. The difficulties are manifested by a drastic reduction of pictorial representations and the frequency of the theorems in action.

Regarding the few pictorial representations obtained in the Figure 5, protocols A23, B5, B6 and B7

show that students conceive the isolated system only seen from outside. The representation of the pendulum and arrows assume that the system is moving, which is undetectable from inside. This would indicate that they conceive movement as absolute, rather than relative.



Figure 2 - Protocol of A23 related to the Situation 4. Drawings suggest a correct understanding of Galilean relativity principle.

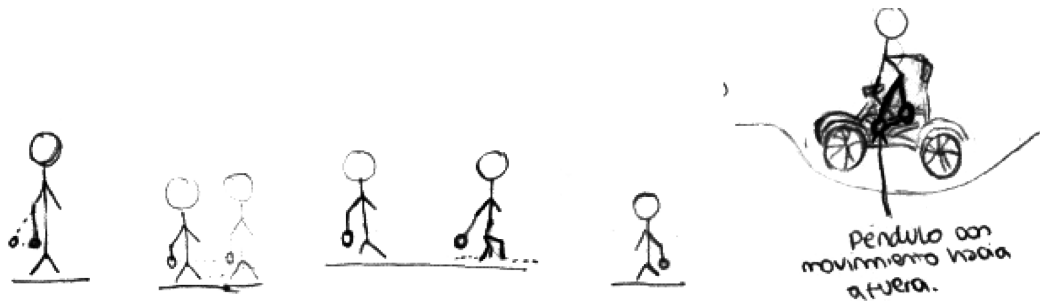


Figure 3 - Protocol of the student A16 related to the Situation 4, where she/he has drawn the pendulum motion when the bike turns.



Figure 4 - Protocol of the student B3 shows correct responses to every question of the situation 4.

Table 2 - Frequency of the theorems-in-action identified in the situation 5.

Theorems in action related to the principle of relativity identified in S5	
I only realize if the train stops or turns	I do not distinguish if the train is at rest or moves with constant speed
33	14

In the Situation 6, students have to analyze the motion of two small balls and then two light beams fired from the centre of a truck trailer, assuming two observer positions: inside outside the truck trailer.

The situation 6 with rubber bullets is presented to emphasize the Galilean relativity concept paving the way for Einstein's relativity. In this case, we "expect" students to predict simultaneity because the tools at their disposal at this stage are those of the Galilean relativity, even though it is clear that this is a low speed (respect to c) approximation.

On the other hand, the same situation 6 using light is designed to be the first real encounter with special relativity. We hope students use the concept of relativity and the invariance of light velocity for both observers, which has already been discussed by this stage. Therefore, we expect students to perform the first relativistic calculation ending up with loss of simultaneity, a counter-intuitive result.

The didactic strategy is revisiting the situation 6

with rubber bullets at the end of the sequence from a relativistic point of view, to obtain the same result for the case of light, illustrating the generality of the result. In the appendix we show the generality of the lack of simultaneity phenomenon in terms of Lorentz transformation

Contrary to our expectations, students conclude, mostly without surprise, that the balls will not come simultaneously to the walls of the trailer, neither when viewed from inside nor from outside the truck.

Even more unexpected is that they predict that the light rays come at the same time at the opposite sides of the truck, for both, the observer who is inside as well as the one is outside the truck trailer. In other words, they predict exactly the opposite than the expected.

A possible reason for these unexpected predictions could be that they seem to analyze the situation from outside the truck, *i.e.*, students always consider the truck in motion, which is undetectable from inside. Therefore, they do not apply the principle of relativity, regarding the motion as absolute.

On the other hand, for them, the light propagates at such a high speed, that in practical terms it propagates "instantaneously" thereby, the arrival time of light to the walls is always the same, for all observers. The following tables show the frequencies of the theorems in action for bullets and light for both observers.

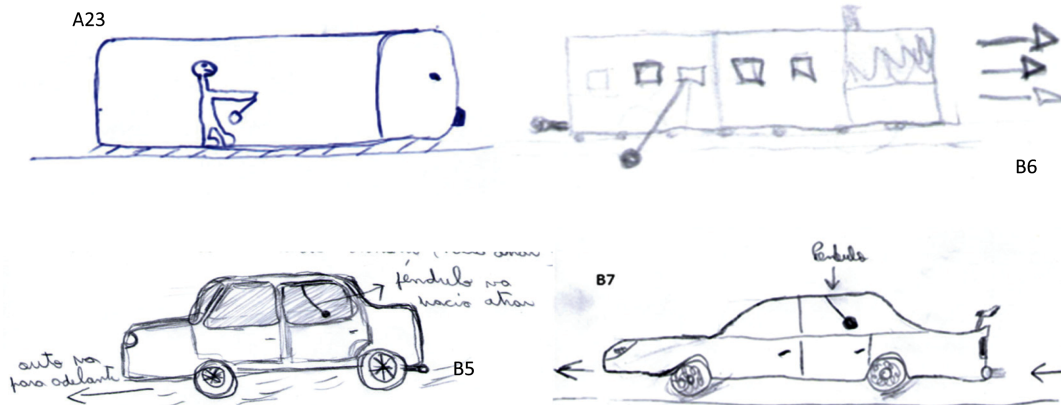


Figure 5 - Pictures of the students A23, B6, B5 and B7 show arrows indicating velocity, when the observer is inside, and the movement is undetectable.

Table 3 - Frequency of the theorems-in-action identified in the situation 6 when the observer is inside the trailer.

Observer inside the trailer			
Rubber bullets		Beams of Light	
They arrive together	It comes first the one that is going to behind	It comes first the beam of light that is going to behind	Both beams come together
15	27	2	37

Table 4 - Frequency of the theorems-in-action identified in the situation 6 when the observer is outside the trailer.

Observer outside the trailer		
Rubber bullets		Beams of Light
They arrive together	It comes first the one that is going to behind	Both beams come together
16	21	36

According to the results obtained, we conclude that the sequence as it was originally designed must be modified. It has to be re-designed in order to allow the students to correctly apply the principle of relativity, in particular disregarding the speed of the truck when they are inside it, and taking it into account when they are outside. Therefore, we have modified the situation 6 as follows:

New S₆ (non-relativistic)

An observer is sitting right in the middle of an empty truck trailer. Another observer standing at the side of the road determines that the truck moves with constant speed. The observer inside the truck has a device that can launch rubber bullets forward and backward at the same instant. Complete the following table for each observer, proposing different speeds for the truck and the projectiles.

a) Analyze for each observer, without doing calculations,

if the bullets arrive simultaneously or not at each edge of the trailer.

b) Calculate the meeting point (position and time) between the bullets and trailer walls, for each observer, considering different values of speeds.

In the case of rubber bullets, students could complete a table, parametrizing with different speeds and formulate the equations of motion with the established parameters.

It is important to stress the non-relativistic character of the calculations involved in this part. After that, students could calculate the meeting point and the corresponding time, verifying that it is the same, for both within and outside the truck. The aim here is that students would be able to write the equations of motion (at least numerically) as we can see below in the Table 6.

Table 5 - Complete the speed of bullets and the trailer considering the observer position (non-relativistic calculation: Galilean addition of velocities).

Observer inside the trailer			Observer outside the trailer		
V_t (m/s)	v_{br} (m/s)	v_{bl} (m/s)	v_{br} (m/s)	v_{bl} (m/s)	V_t (m/s)
	V_b	$-V_b$	$V_t + V_b$	$V_t - V_b$	V_t

Table 6 - Equations of motion to find the meeting point in the case of the observer on the trailer or on the road (non-relativistic calculation: Galilean addition of velocities used).

Observer on the trailer		Observer on the road	
Left	Right	Left	Right
$x_{wl} = -L$	$x_{wr} = L$	$x_{wl} = -L + v_t t_l$	$x_{wr} = L + v_t t_r$
$x_{bl} = -v_b t_l$	$x_{br} = v_b t_r$	$x_{bl} = (v_t - v_b) t_l$	$x_{br} = (v_t + v_b) t_r$
$x_{wl} = x_{bl}$	$x_{wr} = x_{br}$	$x_{wl} = x_{bl}$	$x_{wr} = x_{br}$
		$-L + v_t t_l = (v_t - v_b) t_l$	$L + v_t t_r = (v_t + v_b) t_r$
		$-L + v_t t_l = v_t t_l - v_b t_l$	$L + v_t t_r = v_t t_r + v_b t_r$
		$-L = -v_b t_l$	$L = v_b t_r$
$t_l = \frac{-L}{-v_b} = \frac{L}{v_b}$	$t_r = \frac{L}{v_b}$	$t_l = \frac{-L}{-v_b} = \frac{L}{v_b}$	$t_r = \frac{L}{v_b}$

Having analyzed what happens with the rubber bullets in a non-relativistic context, we propose considering the case of the rays of light in the new situation 7.

New S₇

An observer is sitting right in the middle of an empty truck trailer. Another observer standing at the

side of the road determines that the truck moves with constant speed. The observer inside the truck has a device that can shoot laser light beams forward and backward at the same instant. Complete the following table for each observer, proposing different speeds for the truck.

Table 7 - Complete the speed of light beams and the trailer considering the observer position.

Observer inside the trailer			Observer outside the trailer		
V_t	V_{Ll}	V_{Lr}	V_t	V_{Ll}	V_{Lr}

a) Analyze for each observer, without doing calculations, if laser light arrives simultaneously or not at each edge of the trailer.

b) Calculate the meeting point (position and time) between the light beams and the trailer walls, consider-

ing different values of truck speed.

Here students should apply both principles of SR together. Although in this case the numerical solutions would not be appropriate to assess the difference in time, due to the large value of c , it is expected that the students would be able to write the analytical equations of motion as is showed in the Table 8. In particular, from outside the trailer, where the lack of simultaneity is explicit. Note that at this stage it is not expected that students distinguish between proper L_0 length and L (measured from outside the trailer). However it is not necessary to determine simultaneity inside the trailer and non-simultaneity for the observer on the road.

Table 8 - Equations of motion to find the meeting point in the case of the observer on the trailer or on the road (relativistic calculation c-invariance explicit).

Observer inside the trailer		Observer outside the trailer	
Left	Right	Left	Right
$x_{wl} = -L_0$	$x_{wr} = L_0$	$x_{wl} = -L + v_t t_l$	$x_{wr} = L + v_t t_r$
$x_{Ll} = -c t_l$	$x_{Lr} = c t_r$	$x_{Ll} = -c t_l$	$x_{Lr} = c t_r$
$x_{wl} = x_{Ll}$	$x_{wr} = x_{Lr}$	$x_{wl} = x_{Ll}$	$x_{wr} = x_{Lr}$
		$-L + v_t t_l = -c t_l$	$L + v_t t_r = c t_r$
			$L = (c - v_t) t_r$
$t_l = \frac{-L_0}{-c} = \frac{L_0}{c}$	$t_r = \frac{L_0}{c}$	$t_l = \frac{-L}{-(c+v_t)} = \frac{L}{(c+v_t)}$	$t_r = \frac{L}{(c-v_t)}$

After these situations, the sequence continues with situations 7 and 8 that are now renumbered into 8 and 9, respectively. Finally, at the end we reconsider situation 6 with rubber bullets from a fully relativistic point of view, to conceptualize the generality of the lack of simultaneity phenomenon.

10. Conclusions

We have designed, implemented and analyzed a didactic sequence for the teaching of basic aspects of special relativity theory in high school level. The didactic strategy proposed in this work promotes building Einstein’s concept of relativity strongly based on Galilean relativity. Our expectations have been solely based on the students’ current knowledge and their ability to build (in-action) on the subject at each stage of the sequence. A central part of the study consisted in determine to what extent these expectations are fulfilled, analyzing the underlying theorems in-action that students use, from the point of view of the theory of conceptual fields of Vergnaud.

A careful analysis of the results based on 43 student protocols from a first cycle-implementation let us conclude that the most complex aspects of the SR for students are related on the one hand with the principle of relativity itself. During the first five situations they tried to conceptualize this principle without success. In this case, we identify the main obstacle in the use of the underlying theorem-in-action: “motion is ab-

solute” which is not correct. Note that this conceptual problem is not specifically related with SR, moreover it is a pre-Galilean misconception.

On the other hand, regarding the second postulate, students accept the invariance of the light speed. However its very large value, compared with low speed everyday experience, translates into the theorem-in-action: “the light is instantaneous”. For this reason, students incorrectly predict simultaneity in the case of light for all observers.

To face these obstacles, new situations were designed, aiming the emergence of appropriate operational invariants. To reach this higher level of conceptualization, the teaching of classical pre-relativistic kinematics is fundamental. Hence, it is necessary to conceptualize first, Galilean relativity as a previous step to its generalization in the framework of SR. This can only be accomplished by designing study programs revaluating classical physics, with a view towards relativistic physics.

Regarding the idea that light propagates instantaneously and therefore arrives simultaneously everywhere, the use of equations of motion to solve meeting point problems brings the possibility of direct application of the invariance of light and thus the prediction of lack of simultaneity. Although this only provides a mathematical root to the correct results, it may be considered as a first step to the conceptualization process of the relativity of the simultaneity.

Once the concept of absolute time is refused in fa-

vor of the relativity of simultaneity, the student is better prepared to deal with the concepts of time dilation and length contraction, but there is long way up to this point. Unfortunately, traditional textbook approaches usually comprise a brief analysis of the postulates to quickly go to the “spectacular” parts of the theory. It does not have any sense moving towards the main topics without a prior process of conceptualization of the basic postulates. In this sense, our contribution promotes a firm conceptual basis of postulates, paving the way to address significantly the core issues of the special relativity.

Appendix

Lack of simultaneity from Lorentz transformation

In this appendix we show for completeness the general formula for lack of simultaneity as a direct consequence of Lorentz transformation.

Let us recall that Lorentz transformation relates spatial and temporal coordinates for a given event as measured from two inertial reference systems R and R_0 whose relative velocity is u . Considering here one spatial dimension x for simplicity, they are given by

$$x_0 = \gamma(x - ut), \quad t_0 = \gamma\left(t - \frac{ux}{c^2}\right), \quad (\text{A-1})$$

or equivalently

$$x = \gamma(x_0 + ut_0), \quad t = \gamma\left(t_0 + \frac{ux_0}{c^2}\right), \quad (\text{A-2})$$

where $\gamma = \frac{1}{\sqrt{1 - \frac{u^2}{c^2}}}$. In particular the second equation in

Eq. (A2) predicts that two events that are simultaneous in R , *i.e.* $\Delta t_0 = 0$ and separated by a distance Δx_0 are not simultaneous in R , being the interval of time given by

$$\Delta t = \frac{\gamma u \Delta x_0}{c^2}. \quad (\text{A-3})$$

Note that this expression is valid in general, independently of the type of event. Therefore the lack of

simultaneity in situation 6 is the same both for rubber bullets than for laser beams in a fully relativistic context.

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