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Determination of the Δ^{++} magnetic dipole moment

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Abstract

We study the elastic and radiative π^+p scattering within a full dynamical model which incorporates the finite width effects of the Δ^{++} . The scattering amplitudes are invariant under contact transformations of the spin 3/2 field and gauge-invariance is fulfilled for the radiative case. The pole parameters of the Δ^{++} obtained from the elastic cross section are $m_\Delta = (1211.2 \pm 0.4)$ MeV and $\Gamma_\Delta = (88.2 \pm 0.4)$ MeV. From a fit to the most sensitive observables in radiative π^+p scattering, we obtain $\mu_\Delta = (6.14 \pm 0.51)e/2m_p$ for the magnetic dipole moment of the Δ^{++} .

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The description of resonances in particle physics has gotten a renewed interest with the advent of precise measurements of the Z^0 gauge boson properties at LEP [1]. The idea behind these recent works is to provide a consistent description of resonances based on general principles of quantum field theory such as gauge invariance and analyticity [2]. On another hand, it has long been recognized that mass and width are physical properties of resonances that can be determined in a model-independent and gauge-invariant way by identifying the pole position of the S-matrix amplitude [3].

However, the determination of the couplings of resonances to other particles necessarily involves the assumption of a dynamical model to describe how they enter the relevant amplitude. It is not idle to

mention that most of the values of masses, widths and branching ratios for hadronic resonances quoted in the Particle Data Book [1] correspond to parameters obtained from somewhat arbitrary parametrizations of the Breit–Wigner formula.

The aim of the present Letter is to determine the magnetic dipole moment (MDM) of the Δ^{++} resonance by using a *full* dynamical model which consistently describes elastic and radiative π^+p scattering data. The model, to be described below, reproduces very well the total and differential cross sections for elastic π^+p scattering close to the resonance region. This model also provides an amplitude for radiative π^+p scattering that satisfies electromagnetic gauge invariance when finite width effects of the Δ^{++} resonance are taken into account.

The spirit of our calculations is similar to the approaches developed in Ref. [4] to cure gauge invariance problems associated to a naive introduction of

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the finite width of the resonance. As it has been shown for the case of the W^\pm gauge boson, the use of propagators and electromagnetic vertices that include absorptive corrections coming from loops of fermions allows to introduce finite width effects in scattering amplitudes in a gauge-invariant way [4]. Similar conclusions has been reached for the ρ^\pm unstable meson by including loops of pions in absorptive corrections to its propagator and electromagnetic vertex [5]. The expressions for the propagator and electromagnetic vertex of unstable particles, when massless particles appears in absorptive loop corrections, are equivalent to the ones obtained by using a *complex mass scheme*. In the complex mass scheme, gauge invariance of the amplitudes is satisfied if the squared mass M^2 of unstable particles in Feynman rules is replaced by $M^2 - iM\Gamma$ with Γ being the decay width [6].

It is interesting to note that the mass and width parameters of the Δ^{++} resonance required to describe the π^+p elastic scattering within our model, are consistent with the ones obtained from a model-independent analysis of this process [7]. This feature cast confidence on the consistency of the dynamical model advocated in this Letter. Thus, the magnetic dipole moment of the Δ^{++} turns out to be the only adjustable parameter required to describe radiative π^+p scattering.

Some of the previous determinations of the Δ^{++} MDM have been summarized in Ref. [1]. Due to the large spread of central values, the Particle Data Group [1] prefers to quote a rough estimate for this multipole which lie in the range $\mu_\Delta \sim 3.7$ to 7.5 in units of $e/2m_p$. The most recent determinations of the Δ^{++} MDM are based on fits to the radiative π^+p scattering data of the SIN [8] and UCLA [9] experiments. Some of the models [10] used to extract the MDM rely on the soft photon theorem [11], and on a specific parametrization of the off-shell elastic amplitude to fix the terms of order ω_γ^0 (ω_γ is the photon energy in the radiative process) by requiring gauge-invariance. Furthermore, Ref. [10] ignores the effects of the finite width of the Δ^{++} and diagrams with vertices involving four particles (see Figs. 1(e–f) in Ref. [12]).

Invariance under contact transformations ensures that physical amplitudes involving the Δ resonance are independent of any arbitrariness in the Feynman rules of a given theoretical model for this resonance [13,14].

Vertices and propagators depend on an arbitrary parameter A that changes as $A \rightarrow A' = (A - 2a)/(1 + 4a)$, when the transformation $\psi^\mu \rightarrow \psi^\mu + a\gamma^\mu\gamma_\alpha\psi^\alpha$ ($a \neq -1/4$) is done. Physical amplitudes, should however be independent of A . Other models (see, for example, [15]) make use of an amplitude that depends on A ; hence, the value of the MDM is quoted for an specific value of this arbitrary parameter. In Ref. [16] a determination of the MDM free of ambiguities related to contact transformations is provided. However, their method [16] requires to detach the decay process of the resonance $\Delta^{++} \rightarrow \pi^+p\gamma$ from the whole radiative π^+p process.

The main difference between previous works and ours, is that our model for the Δ^{++} resonance gives an amplitude for the radiative π^+p scattering that is gauge-invariant in the presence of finite width effects and independent upon the parameter associated to contact transformations. In addition, let us emphasize that the mass and width of the Δ^{++} required to fit the total cross section data of elastic π^+p scattering, are consistent with the model-independent analyses done in Ref. [7].

The dynamical model we use in this Letter includes the contributions of intermediate states with nucleons and Δ^{++}, ρ , and σ resonances. We will assume isospin symmetry for the masses, widths and strong couplings of the Δ 's and nucleons. The effective Lagrangian densities relevant for our calculations can be found in Ref. [17]. Some of the couplings entering those Lagrangians can be fixed from low energy phenomenology: $g_\rho^2/4\pi = 2.9$, $g_{\pi NN}^2/4\pi = 14.3$ [18] and the magnetic ρNN coupling $\kappa_\rho = 3.7$. The mass of the hypothetical σ meson was set to 650 MeV [18] (see Ref. [17] for other choices). The couplings $g_\sigma \equiv g_{\sigma\pi\pi}g_{\sigma NN}$ and $f_{\Delta N\pi}$ are left as free parameters to be determined from the π^+p total cross section data.

In order to see how the model works in the case of elastic π^+p scattering, let us focus on the Δ^{++} contribution to the $\pi^+(q)p(p) \rightarrow \pi^+(q')p(p')$ amplitude [12] (letters within brackets denote four-momenta):

$$\begin{aligned} \mathcal{M}(\pi^+p \rightarrow \pi^+p) \\ = i \left(\frac{f_{\Delta N\pi}}{m_\pi} \right)^2 \bar{u}(p') q'_\mu G^{\mu\nu}(p+q) q_\nu u(p), \end{aligned}$$

where $f_{\Delta N\pi}$ is the $\Delta N\pi$ coupling constant and the Δ^{++} propagator in momentum space $G^{\mu\nu}(p+q)$

Table 1
Fits to the cross section of π^+p scattering by including different intermediate states (Δ 's, N , ρ , σ)

Int. state	$f_{\Delta N\pi}^2/4\pi$	m_{Δ} (MeV)	Γ_{Δ} (MeV)	$g_{\sigma}/4\pi$	χ^2/dof
Δ 's	0.281 ± 0.001	1201.7 ± 0.2	69.8 ± 0.2	–	121.1
Δ 's, N	0.331 ± 0.003	1208.6 ± 0.2	87.5 ± 0.3	–	17.6
Δ 's, N , ρ	0.327 ± 0.001	1207.4 ± 0.2	85.6 ± 0.3	–	15.6
Δ 's, N , ρ , σ	0.317 ± 0.003	1211.2 ± 0.4	88.2 ± 0.4	1.50 ± 0.12	10.5

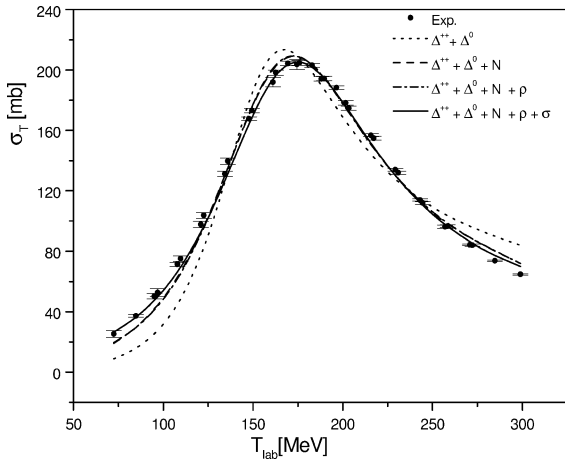


Fig. 1. Elastic π^+p cross section as a function of incident π^+ kinetic energy.

is given in Eq. (10) of Ref. [12]. According to the complex mass scheme, we must replace $m_{\Delta}^2 \rightarrow m_{\Delta}^2 - im_{\Delta}\Gamma_{\Delta}$ in $G^{\mu\nu}(p+q)$, where m_{Δ} and Γ_{Δ} are the mass and width of the Δ^{++} . As shown in [12], the above amplitude is explicitly independent of the parameter associated to contact transformations.

Since the Δ^{++} largely dominates the elastic scattering amplitude in the resonance region, we expect the contributions from other mesonic resonances and of crossed channels with intermediate nucleon and Δ^0 states to play the role of background terms to the Δ^{++} resonance. If we add coherently these contributions to $\mathcal{M}(\pi^+p \rightarrow \pi^+p)$, we can fit the experimental results data for the total cross section [19] of the elastic π^+p scattering with four free parameters (m_{Δ} , Γ_{Δ} , $f_{\Delta N\pi}$ and g_{σ}) in the range $75 \leq T_{\text{lab}} \leq 300$ MeV for energies of incident pions. In order to compare the size of the different contributions, we have chosen to fit the data by adding a new contribution in each fit. The results are shown in Table 1 and Fig. 1. Let us note that

since non-resonant contributions are included at the tree-level (they are real), and the decay width in the Δ^{++} propagator is taken as a constant within the complex mass scheme, our amplitude for elastic scattering will not be unitary. Note, however (see the appendix of Ref. [17]) that the terms neglected in our approximation would be reflected in a slight increase of the Δ decay width.

Using the results of Table 1, we can *predict* the angular distribution of pions in elastic π^+p scattering. In Fig. 2 we compare our prediction for the differential cross section for $T_{\text{lab}} = 263.7$ and 291.4 MeV with the corresponding experimental data from Refs. [20–22]. It is interesting to observe that data in Fig. 2 are well described despite the fact that kinetic energies of incident pions in Fig. 2 lie in the upper tail of the Δ^{++} resonance shape (see Fig. 1). This is very important because kinetic energies of incident pions in radiative π^+p scattering to be considered below, correspond to those particular values.

Some interesting features are worth to be pointed out from this analysis. First, the agreement with data improves when the contributions from all intermediate states (Δ^{++} , Δ^0 , N , ρ and σ) are included, both for the total and the differential cross sections (the last χ^2/dof in Table 1, actually drops to 4.5 when the last three points in the cross section are excluded). Second, the values obtained for the mass and width of the Δ 's, namely $m_{\Delta} = (1211.2 \pm 0.4)$ MeV and $\Gamma_{\Delta} = (88.2 \pm 0.4)$ MeV, are similar to the pole parameters obtained from a model-independent analysis of the same data, namely [7]: $M = (1212.20 \pm 0.23)$ MeV and $\Gamma = (97.06 \pm 0.35)$ MeV.¹ This is a non-trivial

¹ Resonance parameters defined from the pole position are smaller than the ones obtained from a Breit-Wigner with energy-dependent width [1].

feature given the different nature of both approaches: in Ref. [7] the amplitude for elastic π^+p scattering was written as a sum of a pole plus a background term as dictated by the analytic S-matrix theory [3]. The above information indicates that the contributions to the elastic scattering other than the Δ^{++} , indeed represent well the background.

Next we focus on the determination of the Δ^{++} MDM from $\pi^+(q)p(p) \rightarrow \pi^+(q')p(p')\gamma(\epsilon, k)$ (letters within brackets denote four-momenta and ϵ the photon polarization). Using the Lagrangians given in Ref. [17] and the complex mass scheme to include the finite width of the Δ^{++} we obtain the following amplitude for the resonance contribution:

$$\begin{aligned} \mathcal{M}(\pi^+p \rightarrow \pi^+p\gamma) &= -e \left(\frac{f_{\Delta N \pi}}{m_\pi} \right)^2 q'_\mu q_\nu \bar{u}(p') \\ &\times \left[G^{\mu\nu}(P') \left(\frac{q \cdot \epsilon}{q \cdot k} + \frac{p \cdot \epsilon - R(p) \cdot \epsilon}{p \cdot k} \right) \right. \\ &- \left(\frac{q' \cdot \epsilon}{q' \cdot k} + \frac{p' \cdot \epsilon - R(p') \cdot \epsilon}{p' \cdot k} \right) G^{\mu\nu}(P) \\ &+ 2i G^{\mu\alpha}(P') \Gamma_{\alpha\beta\rho} \epsilon^\rho G^{\beta\nu}(P) \\ &+ \frac{1}{q \cdot k} G^{\mu\rho}(P') F_\rho^\nu \\ &\left. - \frac{1}{q' \cdot k} F_\rho^\mu G^{\rho\nu}(P) \right] u(p), \end{aligned} \quad (1)$$

where $F^{\rho\sigma} \equiv \epsilon^{\rho k \sigma} - \epsilon^{\sigma k \rho}$, and $R_\mu(x) \equiv \frac{1}{4}[\not{k}, \gamma_\mu] + \frac{\kappa_p}{8m_N} \{\not{k}, \gamma_\mu, \not{x}\}$; e is the proton charge, κ_p denotes the anomalous magnetic moment of the proton, and $P = p + q$, $P' = p' + q'$, such that $P = P' + k$. This amplitude is explicitly gauge-invariant and does not depend on the parameter associated to contact transformations [12]. It can also be verified that this amplitude satisfies Low's soft photon theorem [11] as required. The electromagnetic vertex of the Δ^{++} appearing in Eq. (1) is given by:

$$\begin{aligned} \Gamma_{\alpha\beta\rho} &= \left(\gamma_\rho - \frac{i\kappa_\Delta}{2m_\Delta} \sigma_{\rho\sigma} k^\sigma \right) g_{\alpha\beta} - \frac{1}{3} \gamma_\rho \gamma_\alpha \gamma_\beta \\ &- \frac{1}{3} \gamma_\alpha g_{\beta\rho} + \frac{1}{3} \gamma_\beta g_{\alpha\rho}, \end{aligned}$$

where κ_Δ is related to the total magnetic moment of the Δ^{++} by $\mu_\Delta = 2(1 + \kappa_\Delta)(e/2m_\Delta)$.

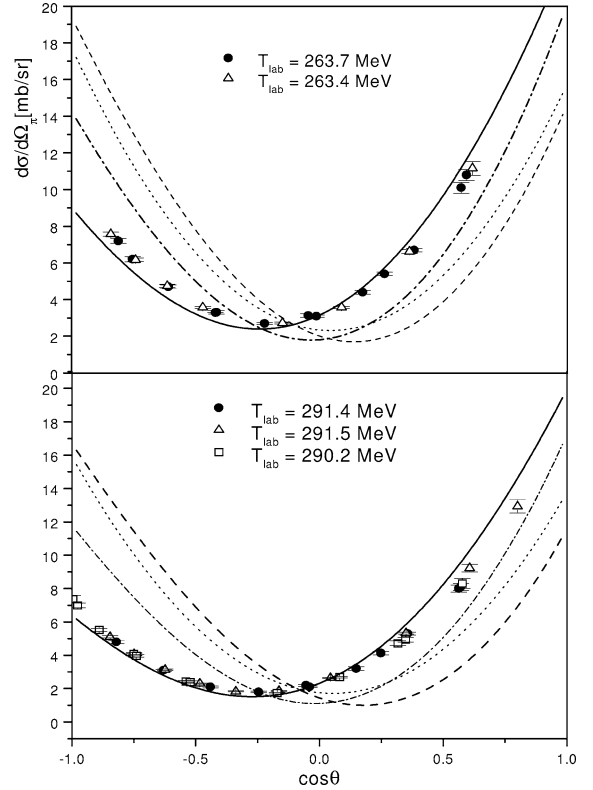


Fig. 2. Differential cross section for elastic π^+p scattering. The curves (with same convention as in Fig. 1) denote our prediction for $T_{\text{lab}} = 263.7$ (upper box) and 291.4 MeV (lower box).

The only adjustable parameter in radiative π^+p scattering is the Δ^{++} MDM. The contributions to this process coming from other intermediate states (Δ^0 , ρ , N and σ) can be added to Eq. (1) in a gauge-invariant way (see, for example, [17]). We are interested in the description of the differential cross section $d\sigma/d\omega_\gamma d\Omega_\pi d\Omega_\gamma$, as a function of the photon energy for fixed energies of incident pions and photon angle emission. We have chosen to fit a subset of data of Ref. [9] where photons are detected in angular configurations as shown in Table 2. According to Ref. [23] one expects the differential cross section to be more sensitive to the effects of μ_Δ in this case [17]. Furthermore, we chose the range of photon energies $20 \leq \omega_\gamma \leq 100$ MeV where we expect Low's soft photon approximation to be more reliable. Details of the fits for other angular configurations and its sensitivity with μ_Δ are given elsewhere [17].

Table 2
Fits to the differential cross section of radiative π^+p scattering (see text for labels)

T_{lab} (MeV)	Geometry	θ_γ	ϕ_γ	κ_Δ	χ^2/dof
269	G7	120°	0°	3.27 ± 0.76	1.99
	G4	140°	0°	3.01 ± 0.67	2.48
	G1	160°	0°	2.74 ± 0.87	1.73
298	G7	120°	0°	3.10 ± 0.87	2.68
	G4	140°	0°	2.90 ± 0.75	4.75
	G1	160°	0°	2.61 ± 1.00	1.47

The results of the fits for κ_Δ are shown in Table 2 for two energies of incident pions [9]. Fig. 3 displays the differential cross section as a function of the photon energy for three different geometries of photon emission (G1, G4, G7 as indicated in Table 2) and incident pions of energy $T_{\text{lab}} = 269$ MeV [9]. The solid line in Fig. 3 corresponds to the best fit, and the prediction when $\kappa_\Delta = 1$ (dashed line) is shown for comparison. Although the experimental data are rather scarce, Fig. 3 clearly indicates that the photon spectrum for the chosen geometries is indeed sensitive to the effect of the Δ^{++} MDM.

The determinations of κ_Δ shown in Table 2 are consistent among themselves, making meaningful to quote a weighted average over the six different fits. If we express the weighted average in units of nuclear magnetons we obtain:

$$\begin{aligned} \mu_\Delta &= 2(1 + \kappa_\Delta) \frac{m_p}{m_\Delta} \left(\frac{e}{2m_p} \right) \\ &= (6.14 \pm 0.51) \frac{e}{2m_p}. \end{aligned} \quad (2)$$

The effects associated to the non-unitarity of the elastic π^+p scattering amplitude have been estimated to decrease the central value in Eq. (2) by 2% (see appendix of Ref. [17]).

This result is compatible with the prediction $\mu_\Delta = 5.58(e/2m_p)$ obtained in the SU(6) quark model [24] and with the result $\mu_\Delta = 6.17(e/2m_p)$ from a recent quark model calculation that includes non-static effects associated to pion exchange and orbital excitations [26], and it is somewhat larger than the prediction obtained from bag-model corrections to the quark model, $\mu_\Delta = (4.41 \sim 4.89)(e/2m_p)$ [25].

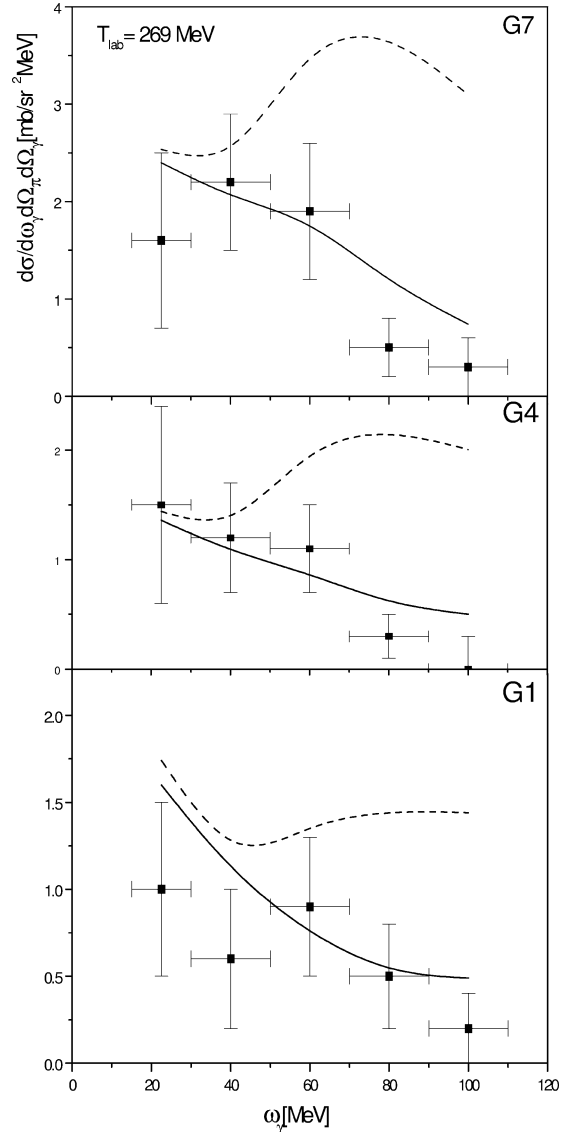


Fig. 3. Differential cross section in radiative π^+p scattering for $T_{\text{lab}} = 269$ MeV. The G1, G4 and G7 geometries are defined in Table 2. The solid line corresponds to the best fit and the dashed line to $\kappa_\Delta = 1$.

Our results are in agreement with previous determinations from experimental data of Refs. [15,16]: $5.58 \sim 7.53(e/2m_p)$ and $(5.6 \pm 2.1)(e/2m_p)$, respectively. Our result in Eq. (2) is larger than the one obtained from a variant of the soft-photon approximation [10]: $\mu_\Delta = (3.7 \sim 4.9)(e/2m_p)$.

In conclusion, we have analyzed the elastic and radiative π^+p scattering within a full dynamical model which gives amplitudes that are gauge-invariant when finite width effects of the Δ^{++} are introduced. These amplitudes are free of ambiguities related to contact transformations on the spin 3/2 fields. The relevant parameters of the Δ^{++} are fixed from the total cross section of the elastic scattering and prediction for the differential cross section is in satisfactory agreement with data. From a fit to the differential cross section of the radiative process we have obtained a determination of the Δ^{++} MDM, Eq. (2), that is in agreement with recent predictions based on the quark model [26]. For completeness, let us mention that our model describes a wider set of radiative π^+p data [17]. This is to our knowledge, the first determination of the Δ^{++} MDM from a full dynamical model that consistently incorporates its finite width and that is free of ambiguities related to contact transformations.

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References

- [1] C. Caso et al., Particle Data Group, *Eur. Phys. J. C* 3 (1998) 1.
- [2] R.G. Stuart, *Phys. Lett. B* 262 (1991) 113;
R.G. Stuart, *Phys. Lett. B* 272 (1991) 353;
R.G. Stuart, *Phys. Rev. Lett.* 70 (1993) 3193;
A. Sirlin, *Phys. Rev. Lett.* 67 (1991) 2127;
H. Veltman, *Z. Phys. C* 62 (1994) 35.
- [3] R.E. Peierls, in: E.H. Bellamy, R.G. Moorhouse (Eds.), *Proceedings of the 1954 Glasgow Conference on Nuclear and Meson Physics*, Pergamon Press, 1955, p. 296 ;
M. Lévy, *Nuovo Cimento* 13 (1959) 115;
R. Eden, P. Landshoff, D. Olive, J. Polkinghorne, *The Analytic S-matrix*, Cambridge Univ. Press, Cambridge, 1966.
- [4] U. Baur, D. Zeppenfeld, *Phys. Rev. Lett.* 75 (1995) 1002;
E. Argyres et al., *Phys. Lett. B* 358 (1995) 339;
M. Beuthe et al., *Nucl. Phys. B* 498 (1997) 55;
W. Beenakker et al., *Nucl. Phys. B* 500 (1997) 255.
- [5] G. López Castro, G. Toledo Sánchez, *Phys. Rev. D* 61 (2000) 033007.
- [6] G. López Castro, J.L. Lucio M., J. Pestieau, *Mod. Phys. Lett. A* (1991);
G. López Castro, J.L. Lucio M., J. Pestieau, *Int. J. Mod. Phys. A* 10 (1996);
A. Pilaftsis, M. Nowakowski, *Z. Phys. C* 60 (1993) 121.
- [7] A. Bernicha, G. López Castro, J. Pestieau, *Nucl. Phys. A* 597 (1996) 623.
- [8] A. Bosshard et al., *Phys. Rev. D* 44 (1991) 1962;
C.A. Meyer et al., *Phys. Rev. D* 38 (1988) 754.
- [9] B.M.K. Nefkens et al., *Phys. Rev. D* 18 (1978) 3911.
- [10] D. Lin, M.K. Liou, Z.M. Ding, *Phys. Rev. C* 44 (1991) 1819.
- [11] F.E. Low, *Phys. Rev.* 110 (1958) 974.
- [12] M. El-Amiri, G. López Castro, J. Pestieau, *Nucl. Phys. A* 543 (1992) 673.
- [13] K. Johnson, E.C.G. Sudarshan, *Ann. Phys.* 13 (1961) 126.
- [14] L.M. Nath, B. Etemadi, J.D. Kimel, *Phys. Rev. D* 3 (1971) 2153;
R.E. Behrends, C. Fronsdal, *Phys. Rev.* 106 (1958) 277;
J. Urias, Ph.D. Thesis, Université catholique de Louvain, Belgium, 1976.
- [15] R. Wittman, *Phys. Rev. C* 37 (1988) 2075.
- [16] P. Pascual, R. Tarrach, *Nucl. Phys. B* 134 (1978) 133.
- [17] G. López Castro, A. Mariano, *nucl-th/0010045*.
- [18] See for example:
B.C. Pearce, B.K. Jennings, *Nucl. Phys. A* 528 (1991) 655;
C. Schütz, J.W. Durso, K. Holinde, J. Speth, *Phys. Rev. C* 49 (1994) 2671.
- [19] E. Pedroni et al., *Nucl. Phys. A* 300 (1978) 321.
- [20] P.J. Bussey et al., *Nucl. Phys. B* 58 (1973) 363.
- [21] M.E. Sadler et al., *Phys. Rev. D* 35 (1987) 2718.
- [22] V.A. Gordeev et al., *Nucl. Phys. A* 364 (1981) 408.
- [23] V.I. Zakharov, L.A. Kondratyuk, L.A. Ponomarev, *Yad. Fiz.* 8 (1968) 783, *Sov. J. Nucl. Phys.* 8 (1968) 456 (in English).
- [24] M.A.B. Beg, B.W. Lee, A. Pais, *Phys. Rev. Lett.* 13 (1964) 514.
- [25] G.E. Brown, M. Rho, V. Vento, *Phys. Lett. B* 97 (1980) 423.
- [26] J. Franklin, *hep-ph/0103139*.