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The born-again (very late thermal pulse) scenario revisited: the mass of the remnants and implications for V4334 Sgr

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ABSTRACT

We present 1D numerical simulations of the very late thermal pulse (VLTP) scenario for a wide range of remnant masses. We show that by taking into account the different possible remnant masses, the observed evolution of V4334 Sgr (a.k.a. Sakurai's object) can be reproduced within the standard 1D mixing length theory (MLT) stellar evolutionary models without the inclusion of any ad hoc reduced mixing efficiency. Our simulations hint at a consistent picture with present observations of V4334 Sgr. From energetics, and within the standard MLT approach, we show that low-mass remnants ($M \lesssim 0.6\,\mathrm{M_{\odot}}$) are expected to behave markedly differently from higher mass remnants ($M \gtrsim 0.6\,\mathrm{M_{\odot}}$) in the sense that the latter remnants are not expected to expand significantly as a result of the violent H-burning that takes place during the VLTP. We also assess the discrepancy in the born-again times obtained by different authors by comparing the energy that can be liberated by H-burning during the VLTP event.

Key words: stars: AGB and post-AGB – stars: evolution – stars: individual: V4334 Sgr.

1 INTRODUCTION

Hydrogen (H)-deficient post-asymptotic giant branch (post-AGB) stars display a wide variety of surface abundances, ranging from the almost pure helium atmospheres of O (He) stars to the helium (He)-, carbon (C)- and oxygen (O)-rich surfaces of WR-CSPN and PG1159 stars (see Werner & Herwig 2006, for a review). In particular the surface composition of the last group resembles the intershell region chemistry of AGB star models when some overshooting in the pulse-driven convection zone (PDCZ) is allowed during the thermal pulses (Herwig et al. 1997). For this reason, and due to the fact that the occurrence of late (i.e. post-AGB) helium flashes is statistically unavoidable in single stellar evolution modelling (Iben et al. 1983), a late helium flash is the most accepted mechanism for the formation of these stars (see, however, De Marco 2002). In particular during a very late helium flash (VLTP; Herwig 2001b), as the H-burning shell is almost extinguished, the PDCZ can reach the H-rich envelope. As a consequence H-rich material is carried into the hot C-rich interior, and violently burned (see Miller Bertolami et al. 2006, from now on Metal06, for a detailed description of the event). As was already noted by Iben et al. (1983), the time-scale in which H is burned is similar to that of convective motions and consequently the usually adopted instantaneous mixing approach is not valid. For this reason only few numerical simulations of the VLTP exist in the literature: Iben & MacDonald (1995), Herwig et al. (1999), Herwig (2001a), Lawlor & MacDonald (2002, 2003) and more recently Metal06.

The identification of V4334 Sgr (a.k.a. Sakurai's object) as a star undergoing a VLTP event (Duerbeck & Benetti 1996) has renewed the interest in this particular kind of late helium flash. V4334 Sgr has shown a very fast evolution in the Hertzsprung–Russell (HR) diagram of only a few years (Duerbeck et al. 1997; Asplund et al. 1999; Hajduk et al. 2005). In this context it is worth mentioning that the theoretical born-again times (i.e. the time it takes the star to cross the HR diagram from a white dwarf configuration to a giant star one) is a controversial issue: whilst Iben & MacDonald (1995), Lawlor & MacDonald (2002) and Metal06 obtain born-again times of the order of one or two decades, Herwig et al. (1999) obtain time-scales of the order of centuries.

The difference between the theoretical born-again times of Herwig et al. (1999) and the observed time-scale of V4334 Sgr (and also V605 Aql, Duerbeck et al. 2002) prompted Herwig (2001a) to propose a reduction in the mixing efficiency during the conditions of the violent proton burning (by about a factor of 100 for their $0.604\,\mathrm{M_{\odot}}$ sequence) in order to match the observed time-scale. However, in view of the lack of hydrodynamical simulations of the violent burning and mixing process during the VLTP, the idea has an important drawback: it introduces a free parameter (i.e. the mixing efficiency) that can only be calibrated with the situation that one wants to study, thus losing its predictive power. Aside from this philosophical aspect, current reduced mixing efficiency models (Herwig 2001a; Lawlor & MacDonald 2003; Hajduk et al. 2005) suffer from an internal inconsistency. Indeed, contrary to what is stated in Herwig (2001a) and Lawlor & MacDonald (2003),

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Table 1. Description of the sequences analysed in this work. The fourth column shows the total amount of H ($M_{\rm H}$) left at the moment of the VLTP. Note the strong dependence of $M_{\rm H}$ on the mass of the remnant. $t_{\rm BA}$ stands for the time elapsed from the maximum of proton burning to the moment when the sequence reaches $\log T_{\rm eff} = 3.8$. Mean values of the local convective turnover time-scales estimated with different prescriptions are shown for both He- and H-driven convective zones (He-DCZ and H-DCZ, respectively). The global estimation of the turnover time-scale for the H-DCZ is also shown. The last column shows the approximate mass location of the maximum of proton burning at the moment of its maximum.

Remnant mass ($M_\odot$)	Initial mass ($M_\odot$)	Reference	Pre-flash $M_{\rm H}$ ($M_\odot$)	$t_{\rm BA}$ (yr)	He-DCZ $\tau^1_{\rm loc} - \tau^2_{\rm loc}$ (s)	H-DCZ $\tau^1_{\rm loc} - \tau^2_{\rm loc}$ (s)	H-DCZ $\tau_{\rm glo}$ (s)	$\frac{^{12}{\rm C}}{^{13}{\rm C}}$	$\frac{^{12}{\rm C}}{^{14}{\rm N}}$	Location H-burning peak ($M_\odot$)
0.515	1	d	2.24×10^{-4}	14	2183–2687	121–154	1931	6.6	13.4	0.486 112
0.530	1	b	2.30×10^{-4}	5	1399–1824	30–39	326	6.0	10.9	0.512 02
0.542	1	b	1.75×10^{-4}	5.1	1260–1637	33–42	333	6.3	12.1	0.527 29
0.561	1.8	e	1.00×10^{-4}	7.5	602–792	28–37	283	6.0	11.2	0.549 50
0.565	2.2	b	8.19×10^{-5}	10.8	486–642	28–35	329	6.1	11.1	0.553 97
0.584	2.5	a	8.70×10^{-5}	8.9	458–600	30–37	316	5.9	10.6	0.574 85
0.609	3.05	b	4.68×10^{-5}	157[a]	194–255	35–43	348	6.4	13.7	0.601 66
0.664	3.5	b	3.17×10^{-5}	106[a]	155–206	22–28	170	6.11	12.4	0.659 68
0.741	3.75	c	1.79×10^{-5}	65[a]	108–134	22–27	174	6.7	15.9	0.738 22
0.870	5.5	b	8.85×10^{-6}	–[b]	101–127	31–40	186	5.1[b]	7.8[b]	0.868 85

[a]In these sequences the return to the AGB is mainly powered by the He-shell flash. [b]The 0.870 $M_\odot$ sequence was stopped at high effective temperatures due to numerous convergence problems. References: (a) Metal06, (b) Miller Bertolami & Althaus (2006), (c) Córsico et al. (2006), (d) Althaus et al. (2007) and (e) unpublished.

changes in convective velocities are expected to affect convective energy transport. In fact, as shown in Appendix A, reducing convective velocities is completely equivalent to reducing the mixing length. But more importantly, the reproduction of the born-again time-scale does not necessarily make models completely consistent with observations. In fact, although Hajduk et al. (2005) claim that models with a reduction in the mixing efficiency reproduce 'the real-time stellar evolution of Sakurai's object', a closer inspection shows that such a claim should be taken with a pinch of salt. In particular the effective temperature (and also the cooling rate) of the model in its (first) return to the AGB contradicts the inferred effective temperature (with two different methods) during 1996–1998 (Duerbeck et al. 1997; Asplund et al. 1999). In the same line, the extremely high luminosity of their theoretical model in its first return to the AGB ($\sim 12\,500\,{\rm L}_\odot$) implies an extremely large distance of more than 8 kpc (by comparing with the values in Duerbeck et al. 1997). Surprisingly enough, this value is inconsistent with the 2 kpc adopted by Hajduk et al. (2005) as well as with independent distance estimations which place V4334 below 4.5 kpc and preferentially between 1.5 and 3 kpc (Kimeswenger 2002). Hence, even if the reduction in the mixing efficiency by a factor of 60 leads to born-again times (Hajduk et al. 2005) similar to those displayed by V4334 Sgr, that model fails to match other well established observed properties. On the other hand, whilst the reduced mixing efficiency models of Lawlor & MacDonald (2003) do not suffer from these inconsistencies, their models show high H-abundances which are not consistent with observations of both V4334 Sgr and PG1159 type stars.

In this context, it is worth noting that a strong reduction in the mixing efficiency does not seem necessary in the VLTP models of Iben & MacDonald (1995) and Metal06 to reproduce observations. Most of the existing computations of the VLTP have been performed for masses in a very narrow range around the canonical mass $\sim$0.6 $M_\odot$. In this context, we feel that an exploration of the (neglected) importance of the mass of the remnant for the born-again time-scale is needed. This is precisely the aim of this paper. Indeed some confusion seems to exist in the literature with respect to this issue: whilst it is usually stated that low-mass/low-luminosity models evolve more slowly after a VLTP (Pollaco 1999; Kimeswenger 2002), it is clear from Herwig's (2001a) 0.535 $M_\odot$ model that lower luminosities/masses lead to faster born-again evolutions.

2 DESCRIPTION OF THE WORK AND NUMERICAL/PHYSICAL DETAILS

In the present work we performed numerical simulations of the VLTP scenario for several different remnant masses. The simulations have been performed with the LPCODE (Althaus et al. 2005) by adopting the Sugimoto (1970) scheme for the structure equations as described in appendix A of Metal06. During the present work we have adopted the standard mixing length theory (MLT) with a mixing length parameter $\alpha = 1.75$. Convective mixing was considered as a diffusive process and solved simultaneously with nuclear burning as explained in Althaus et al. (2003). We adopt a diffusion coefficient for the convective mixing zones given by

$$D = \frac{1}{3}\, l\, v_{\rm MLT} = \frac{\alpha^{4/3}\, H_P}{3}\left[\frac{c\, g}{\kappa\, \rho}(1-\beta)\nabla_{\rm ad}(\nabla_{\rm rad}-\nabla)\right]^{1/3} \qquad (1)$$

as can be deduced from Cox & Giuli (1968).[1] As shown in Metal06, no difference would arise if the expression of Langer, El Eid & Fricke (1985) was to be adopted. Diffusive overshooting was allowed at every convective boundary with a value $f = 0.016$ (see Herwig et al. 1997, for a definition of f). By adopting the standard MLT we are ignoring the effect of chemical gradients during the violent proton burning that can certainly influence the results (Metal06). We choose not to include this effect for consistency, as our overshooting prescription does not include the effect of chemical gradients, and also for numerical simplicity. Also, as will be clear in the following sections, we do not intend to reproduce the exact evolution of born-again stars (something that would require a much more sophisticated treatment of convection) but instead to show the importance of the remnant mass for the subsequent evolution.

A detailed description of the sequences considered in the present work is listed in Table 1. HR diagram evolution of the sequences

[1] Note that there are two typos in the expression for D in footnote 6 of Metal06.

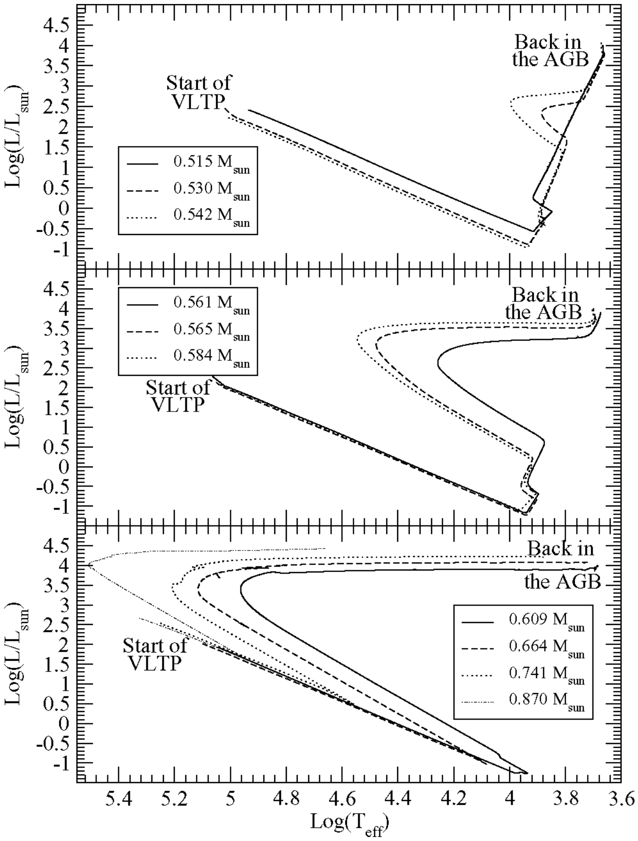

Figure 1. HR diagrams of the VLTP evolution for the sequences presented in this work during their first return to the AGB.

during their first return to the AGB during the VLTP is shown in Fig. 1. We mention that for more massive sequences this will be their only return to the AGB as they only experience the He-driven expansion. With exception of the sequence 0.561 M$_\odot$, the prior evolution of our VLTP sequences has been presented in our previous works (Metal06; Córsico, Althaus & Miller Bertolami 2006 and Althaus, Córsico & Miller Bertolami 2007). All of them are the result of full and consistent evolutionary calculations from the zero-age main-sequence to the post-AGB stage. In all the cases the initial metallicity was taken as $Z = 0.02$. One of the most remarkable features displayed by Table 1 is the strong dependence of the total amount of H at the moment of the VLTP (M_H) on the mass of the remnant. This is important in view of the discussion presented in Section 4. Note however that, aside from this strong dependence on the mass of the remnant, M_H will also depend on the previous evolution and on the exact moment at which the VLTP takes place. This is why the 0.584 (0.530) M$_\odot$ model has a higher M_H value than the 0.565 (0.515) M$_\odot$ model.

3 CONVECTION, TIME-SCALES AND TIME RESOLUTION

As was already mentioned in Metal06, an extremely high time resolution ($\sim 10^{-5}$ yr) during the violent proton burning is needed in order to avoid an underestimation of the energy liberated by proton burning.[2] Such small time-steps are close to the time-scale needed to reach the steady state described by the MLT (see Herwig 2001a). To be consistent with the treatment of convection the time-step should be kept above this value.

The convective turnover times can be estimated with different prescriptions which can lead to significantly different values. In order to have a feeling of what is really happening we have adopted three different estimations: τ^1_loc, τ^2_loc and τ_glo (two local and one global time-scale), which are defined as

$$\tau^1_\mathrm{loc} = \frac{1}{|N|} = \sqrt{\frac{kT}{\mu\, m_\mathrm{p}\, g^2\, |\nabla - \nabla_\mathrm{ad}|}}, \tag{2}$$

$$\tau^2_\mathrm{loc} = \frac{\alpha\, H_P}{2\, v_\mathrm{MLT}}, \tag{3}$$

$$\tau_\mathrm{glo} = \int_{R_\mathrm{base}}^{R_\mathrm{top}} \frac{\mathrm{d}r}{v_\mathrm{MLT}}. \tag{4}$$

The definition of the last two expressions is evident and the first is the inverse of the Brünt–Väisälä frequency and provides the time-scale for the growth of convective velocities in a convectively unstable region (see Hansen & Kawaler 1994, for a derivation). In Table 1 we list typical values of these time-scales. Values at the He-driven convection zone (He-DCZ) correspond to the moment just before the violent proton ingestion (second stage of proton burning, as described in Metal06) whilst values at the convective zone driven by the violent proton burning (H-DCZ) correspond to the moment of maximum energy release by proton burning (which is also the moment at which important amounts of H start to be burned). For the local estimations the averaged value over the whole convective zone is displayed. It is worth mentioning that whilst global and local turnover times coincide for the He-DCZ, the global estimation is, roughly, one order of magnitude larger in the H-DCZ. From the values in Table 1, one would be tempted to state that by adopting a minimum allowed time-step of $\sim 10^{-5}$ yr ($\sim$315 s) at the beginning of the violent stage of proton burning, we are allowing convective motions to develop and, thus, being consistent with the steady state assumption of the MLT. However, a point should be mentioned. Hydrodynamical simulations of 'standard' (i.e. without proton ingestion) helium shell flashes (Herwig et al. 2006) show that the steady state is not achieved in only one turnover time. In fact Herwig et al. (2006) find that, at a standard ($\epsilon_0 = 2 \times 10^{10}$ erg s^{-1} g^{-1}) heating rate, about 10 turnover times are necessary. They also find, however, that at an enhanced heating rate (30 times larger) steady state is achieved about two to three times faster. Then, as the heating rate at the base of the H-DCZ during the violent proton burning is about $\epsilon = 10^{13}$–10^{14} erg s^{-1} g^{-1} (1000–5000 times larger than the one at the base of the He-DCZ), one would expect that the steady state will probably be reached in no much more than one turnover time. Consequently it seems reasonable to choose a minimum allowed time-step of 10^{-5} yr, which according to our previous studies is needed to avoid an underestimation of the energy liberated by proton burning.[3] We have also checked that convective velocities remain subsonic during the stage of maximum proton burning, being in all the cases $v_\mathrm{MLT} < 0.12\, v_\mathrm{sound}$. This is important because during the maximum of proton burning convective velocities are high and the MLT is derived within the assumption of subsonic convective motions. In our view the main drawback of present simulations may come from the

[2] In fact this can be one of the reasons for the discrepancy in the born-again times of different authors.

[3] Time-steps of $\sim 10^{-5}$ yr are only needed during the (short) runaway burning of protons where most of the H is burned (see Metal06) and the energy liberated by proton burning is $L_\mathrm{H} \sim 10^7$–10^{11} L$_\odot$.

fact that convective mixing differs from a diffusion process. In fact, during a VLTP, H needs to be mixed with ^{12}C in order to be burned. It is probable that, initially, H may be transported downwards in the form of plumes and thus no important amounts of H would be (initially) mixed with ^{12}C. This situation would be very different from the picture of diffusive mixing with or without reduced mixing efficiency. How far our results are from reality will depend on how far the place at which most H is burned differs from the one predicted by diffusive mixing within the standard (without reduced mixing efficiency) MLT.

4 ENERGETICS

As mentioned in Section 1, born-again times from different authors can differ by almost two orders of magnitude. Also, as found in Metal06 for $\sim 0.59\,M_\odot$ models, numerical issues can change the born-again times completely and even suppress the H-driven expansion (if energy from H-burning is underestimated by a factor of ~ 2). Regarding the occurrence or absence of the H-driven expansion it is also interesting to note the sudden change in the born-again times that can be seen in Table 1 between the 0.584 and the $0.609\,M_\odot$ models. Whilst the $0.584\,M_\odot$ model shows an H-driven expansion and consequently short born-again times (8.9 yr), the $0.609\,M_\odot$ model does not, displaying born-again times of 157 yr. In this context we feel it interesting to analyse the energetics of the VLTP. To this end, we estimate the energy that can be liberated by proton burning if the whole H-content of the star is burned (E_H). This is shown in Fig. 2 as a shaded zone. This estimation can be done by noting that during the VLTP (almost) all the H is burned first through the chain $^{12}C + p \rightarrow {}^{13}N + \gamma \rightarrow {}^{13}C + e^+ + \nu_e$ (which liberates 3.4573 MeV per proton burned) and then through $^{12}C + p \rightarrow {}^{13}N + \gamma \rightarrow {}^{13}C + e^+ + \nu_e$ and $^{13}C + p \rightarrow {}^{14}N + \gamma$ working at the same rate (a process that liberates 5.504 MeV *per proton* burned, Metal06). These two values give rise to the lower and upper boundary in Fig. 2. We also calculate the gravitational binding energy E_g, just before the violent stage of proton burning, of the zone above the peak of proton burning. From numerical models we know this is the zone that expands due to the energy liberated by the H-burning (and will be denoted as *envelope* in the following):

$$E_g = -\int_{\rm envelope} \frac{Gm}{r}\,dm. \qquad (5)$$

We also estimate the internal energy of the envelope E_i as

$$E_i = \int_{\rm envelope} T c_v\,dm. \qquad (6)$$

From these values we compute the total energy of the envelope E_{tot} (which is negative for a gravitationally bound system) as

$$E_{tot} = E_g + E_i. \qquad (7)$$

$|E_{tot}|$ is then the energy needed to expand the envelope to infinity. Its value is lower than $|E_g|$ because (the virial theorem enforces) as the envelope expands it cools and internal energy is released, helping the expansion. By comparing E_H and $|E_{tot}|$ we can reasonably decide if the burning of H in a given remnant can drive an expansion of its envelope. The result is surprising, as inferred from Fig. 2. Note that for models above $0.6\,M_\odot$ the energy that can be liberated by proton burning *is not enough* to expand the envelope. This result is completely consistent with what is displayed by our detailed modelling of the process. Although this extreme consistency with such rough estimation can be just a coincidence, the different behaviour of E_{tot} and $|E_H|$ with the mass of the remnant is worth emphasizing. Whilst the former decreases only slightly with the stellar mass, the later shows a steeper behaviour. This is expected because the total H-amount of a remnant is a steep function of the stellar mass. This fact yields a transition at a certain stellar mass value below which the H-driven expansion is possible and above which the H-content of the remnant is not enough for H-burning to trigger an expansion.

This energetic point of view not only holds the clue to understand the distinct behaviour of our sequences but also helps to understand the differences in previous works. Note that models with masses close to the $\sim 0.6\,M_\odot$ transition value will be very sensitive to numerical issues that can eventually alter the release of H-burning energy. This could explain why an underestimation by a factor of 2 in the H-burning energy reported in Metal06 strongly affects the born-again times. It also helps to explain why Althaus et al. (2005), in which no extreme care of the time-step during the violent proton burning was taken, reported longer born-again times (20–40 yr) than Metal06 for the same initial model.[4] In this view, the short born-again times found by Lawlor & MacDonald (2002) are probably due to the low mass of their models ($0.56-0.61\,M_\odot$). Also the difference in born-again times between Iben & MacDonald (1995) and Herwig et al. (1999) simulations can probably be understood in this context. In fact, whilst Iben & MacDonald (1995) and Herwig et al. (1999) sequences have similar remnant masses (0.6 and $0.604\,M_\odot$, respectively) the total amount of H is markedly different in both cases, being $\sim 2.37\times 10^{-4}$ and $\sim 5\times 10^{-5}\,M_\odot$, respectively. This leads to energies (E_H) of $3.29-5.23\times 10^{47}$ erg for Herwig et al. (1999) (similar to the values of our own $0.609\,M_\odot$ sequence, see Fig. 2) and $1.56-2.48\times 10^{48}$ erg for Iben & MacDonald (1995). This difference of almost a factor of 5 in the energy released by proton burning is very probably the reason why Iben & MacDonald (1995) find an H-driven expansion whilst Herwig et al. (1999) do not.

On the basis of these arguments, the finding of Herwig (2001a) that a reduction in the mixing efficiency can lead to shorter born-again times can be understood as follows. By reducing the mixing

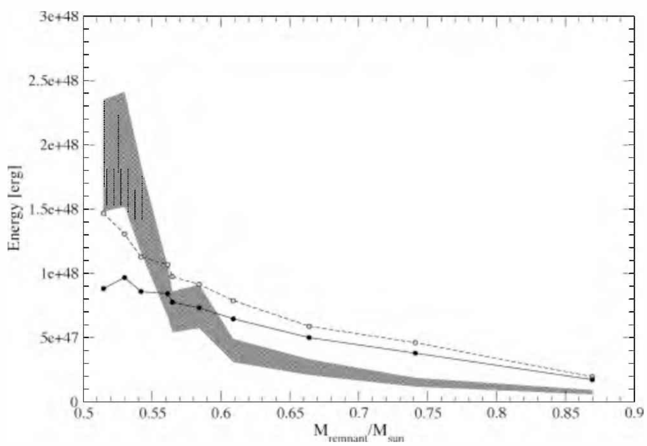

Figure 2. Comparison of the total amount of energy that can be liberated during the violent proton burning (E_H; shaded region) with the energy necessary to expand the envelope above the point of maximum proton burning ($|E_{tot}|$; solid line). Broken line shows the gravitational binding energy of that envelope ($|E_g|$).

[4] This was also due to a difference in the definition of the diffusion coefficient by a factor of 3 (see Metal06).

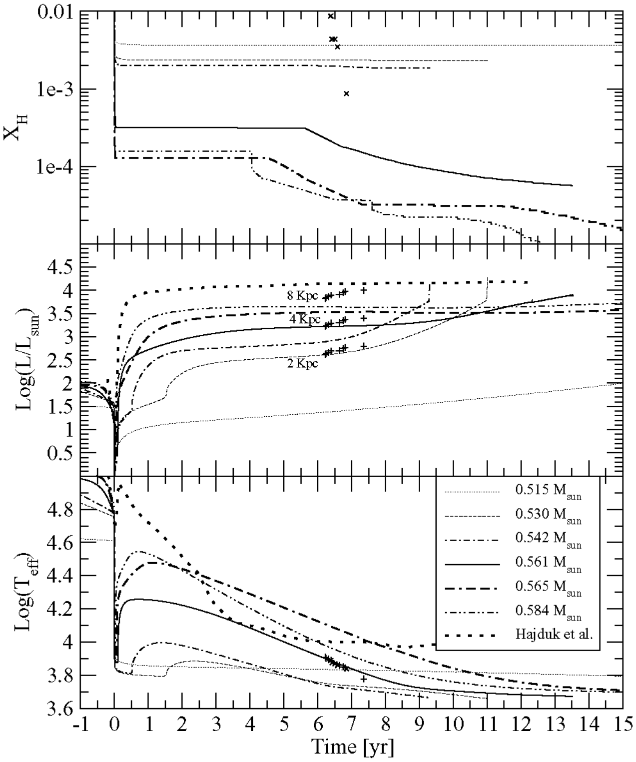

Figure 3. Bottom and middle panels show the evolution of luminosity and effective temperature of the models after the VLTP (set at 0 yr) compared with the observations of V4334 Sgr (Duerbeck et al. 1997, Asplund et al. 1999; + and × signs, respectively). The evolution of luminosity and effective temperature of Hajduk et al. (2005), extracted from fig. 2 of that work, is shown for comparison. The zero-point in the x-axis of the observations was arbitrarily set to allow comparison with the models. Upper panel shows the evolution of the H-abundance at the outermost layer of the models (which should be close to the surface abundance) compared with the observed abundances at V4334 Sgr (Asplund et al. 1999).

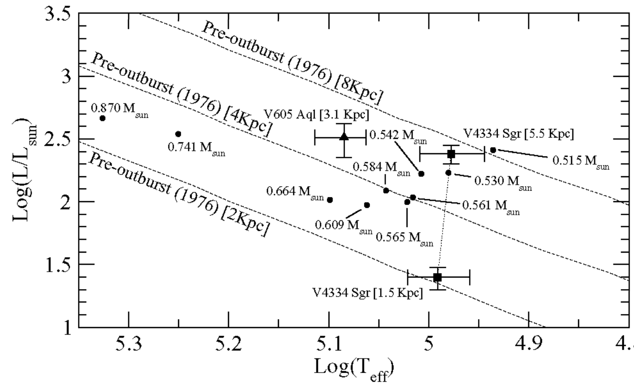

Figure 4. Location of the models in the HR diagram at the moment of the VLTP. The lines show the possible 1976 detection of V4334 (taken from Herwig 2001a, and rederived for different assumed distances). Also inferred pre-flash location of V4334 Sgr and V605 Aql (Kerber et al. 1999; Lechner & Kimeswenger 2004) are shown for comparison. Note the strong dependence of the $T_{\rm eff}$ on the mass of the remnant.

efficiency the point at which the energy is liberated by proton burning is moved outwards. Then the value of $|E_{\rm tot}|$ is lowered whilst the total amount of H remains the same and thus no change in $E_{\rm H}$ will happen. Then, changing the mixing efficiency moves the solid line in Fig. 2 up and down, which alters the transition mass value at which H-driven expansion begins to be possible.[5]

5 COMPARISON WITH THE OBSERVED BEHAVIOUR OF V4334 SGR

In Fig. 3 we compare the evolution of luminosity and effective temperature of our sequences with that observed at V4334 Sgr (Duerbeck et al. 1997; Asplund et al. 1999). Also the pre-outburst location of V4334 Sgr and V605 Aql (from Kerber et al. 1999; Herwig 2001a) is compared with the location of the sequences at the VLTP in Fig. 4. Luminosities have been rederived for different distances to allow comparison. This was done by making use of the fact that interstellar extinction is supposed to be constant in that direction of the sky for distances above 2 kpc. For remnants that display an H-driven expansion ($\lesssim$0.584 $M_\odot$) the main feature can be described as follows. Whilst the pre-outburst location of least massive remnants ($\lesssim$0.542 $M_\odot$) is compatible with higher distances (>4 kpc) their luminosity curves are compatible with lower distances (<4 kpc). However, for the 0.561-, 0.565- and 0.584-$M_\odot$ models, pre-outburst locations are compatible with a distance of ~4 kpc in agreement with the distance inferred from their luminosity curves (4–6 kpc). It is worth noting that all models between 0.530 and 0.584 $M_\odot$ display cooling rates which are compatible with that observed in V4334 Sgr. Also, their born-again times range from 5 to 11 yr not far from that observed at V4334 Sgr (or V605 Aql, Duerbeck et al. 2002).

It is particularly interesting to note the behaviour of our 0.561-$M_\odot$ model. It reaches the effective temperature at which V4334 Sgr was discovered in about ~6 yr after the VLTP, showing at that point a very similar cooling rate as that measured at V4334 Sgr with two different methods (Duerbeck et al. 1997; Asplund et al. 1999). On the other hand, its luminosity at the moment of discovery would imply a distance of about 3–4 kpc which is completely consistent with pre-outburst determinations when that distance is adopted.

As shown in Table 1 all of our models predict $^{12}C/^{13}C \sim 6$ (by mass fractions) not far from the inferred $^{12}C/^{13}C$ at V4334 Sgr (Asplund et al. 1999). Our models also predict high ^{14}N abundances at their first return to the AGB (see Table 1), at variance with Herwig (2001b) but in agreement with observations of V4334 Sgr (Asplund et al. 1999).

In Fig. 3 the evolution of the reduced mixing efficiency model of Hajduk et al. (2005) is shown for comparison. Although this model evolves initially faster than our sequences the behaviour of its luminosity and temperature are not consistent with those observed at V4334 Sgr. In particular all the effective temperature determinations of V4334 Sgr in its first return to the AGB lie beyond the minimum effective temperature attained by that sequence. Also its luminosity at low effective temperatures implies a distance of d > 8 kpc, in contradiction with all independent distance determinations (Kimeswenger 2002).

As is shown in the top panel of Fig. 3, the 0.561-, 0.565- and 0.584-$M_\odot$ sequences – those which are both compatible with pre- and post-outburst inferences for the temperature and luminosity of

[5] There is also a secondary effect which comes from the fact that the closer to the surface the energy is released, the less time it takes the liberated energy to reach the surface of the star.

V4334 Sgr – also reproduce qualitatively the drop at low effective temperature in the H-abundance of V4334 Sgr observed by Asplund et al. (1999). Note however that there is a quantitative disagreement by more than one order of magnitude between observations and model prediction. This may be either due to an intrinsic failure of the models or because the quantities plotted are not exactly the surface abundances of the models but, instead, the H-abundance at the outermost shell of the models (which can differ from the actual surface value).

As was already noted by Metal06, models without reduced mixing efficiency fail to reproduce the fast reheating of V4334 Sgr (as reported by Hajduk et al. 2005). One may argue that this can be due to the fact that mass-loss was ignored in the models, whilst V4334 Sgr displayed strong mass-loss episodes once its temperature dropped below ~6000 K. In fact if we impose a mass-loss rate similar to that inferred in V4334 Sgr ($2 \times 10^{-4}\,\mathrm{M_\odot\,yr^{-1}}$, Hajduk et al. 2005) we find that the 0.561-$\mathrm{M_\odot}$ model reheats (reaches temperatures greater than 10 000 K) in only 27 yr and the 0.584 $\mathrm{M_\odot}$ model does it in 24 yr. This is certainly faster than in the absence of mass-loss but still a factor of ~3–4 greater than what was observed at V4334 Sgr. However, this failure to reproduce the late photospheric evolution of V4334 Sgr should not be a surprise if we consider the fact that the hydrostatic equilibrium condition is completely broken in the outer layers once the born-again star reaches the low-temperature/high-luminosity region of the HR diagram, and thus there is no reason to expect that present hydrostatic sequences will reproduce reality accurately at that point of evolution.

In any case, the present simulations show that it is possible to attain short born-again times without imposing an ad hoc reduction of the mixing efficiency if different remnant masses are allowed. Even more, this approach (aside from not introducing a free parameter) leads to models which are more consistent with the observations of V4334 Sgr than models with a reduced mixing efficiency (Herwig 2001a; Hajduk et al. 2005).

6 DISCUSSION AND FINAL REMARKS

In the present work we have presented 1D hydrostatic evolutionary sequences of the VLTP scenario for different remnant masses. In Section 4 we have made an analysis of the energetics of the VLTP that shows the importance of the mass of the remnant for the born-again time-scale. In particular, that argument shows that, within the standard MLT with no reduction of the mixing efficiency, it is expected that the H-driven expansion that leads to short born-again times will only be present in VLTPs of low-mass remnants. The transition remnant mass value below which the H-driven expansion (i.e. a short born-again time-scale) takes place is close to the canonical mass value ~0.6 $\mathrm{M_\odot}$. The precise value of this transition mass depends on the exact location at which most H is burned, and thus on a detailed description of the mixing and burning process. A more accurate value of the transition mass will have to wait until hydrodynamical simulations of the violent H-ingestion became available. We have also shown that the energetic point of view discussed in Section 4 can help to understand the differences in the calculated born-again times by different authors.

We have also compared our predictions with the observations of real VLTP objects (mainly V4334 Sgr). As was noted early in this work we do not expect from such simplified models and treatment of convection (as those presented in this and all previous works) to reproduce the exact evolution observed at real VLTP stars (like V4334 Sgr or V605 Aql). In fact this is one of the reasons why inferences such as the need of a reduction in the mixing efficiency coming from the fitting of the born-again times of 1D hydrostatic sequences for a single remnant mass should be taken with care. Also the born-again times given in Table 1 should be taken with care, as, for example, a reduction by a factor of 3 in the mixing efficiencies leads to a reduction of a factor of 2 in the born-again times of one of the sequences presented by Metal06. However, as argued in Section 2, we have some reasons to believe that present models may not be that far from reality. In this context the comparison with observations in Section 5 shows that it is possible to roughly reproduce the observed behaviour of V4334 if a mass of ~0.56 $\mathrm{M_\odot}$ for the remnant is assumed and a distance of ~3–4 kpc is adopted. Interestingly enough, these distances are similar to the ones derived by most distance determination methods (Kimeswenger 2002). What makes this model very interesting is not only that it avoids the inclusion of a completely free parameter but also that it solves the inconsistency problems of reduced efficiency models (Herwig 2001a; Hajduk et al. 2005) mentioned in Section 1 (and that can be appreciated in Fig. 3). Interestingly enough, ~0.56–0.59 $\mathrm{M_\odot}$ sequences also reproduce qualitatively the drop in the H-abundance observed in V4334 Sgr at low effective temperatures. As this late drop in H-abundance is partially due to dilution of the outer layers of the envelope into deeper layers of the star (and also due to a deepening of the shell at which $\tau_{2/3}$ is located), it is expectable that this drop in H would be accompanied by a raise in s-process elements abundances, just as observed in V4334 Sgr.

The main drawback of our models is that they fail to reproduce the fast reheating of V4334 Sgr (Hajduk et al. 2005) by about a factor of 4. However, as discussed in Section 5 this is not a surprise as one of the main hypotheses of the modelling (that of hydrostatic equilibrium) is explicitly broken in the outer layers of the models once the VLTP star is back to the AGB at very low temperatures. Thus, there is no reason to expect that the sequences of this work will accurately reproduce reality at that course of evolution.

We conclude then that V4334 Sgr is not 'incomprehensible' (Herwig 2001a) within the standard MLT approach and that there is 'a priori' no need for a reduction of the mixing efficiency. Needless to say, this statement does not imply that mixing efficiency is not reduced during the proton ingestion in the VLTP nor that the MLT approach is correct during the VLTP – something that will only be known once hydrodynamical simulations of the H-ingestion and burning become available – but only shows that it is possible to roughly reproduce the observations within the standard MLT approach.

In any case, the main conclusion of the present work is that different remnant masses *have to* be considered when comparing theoretical expectations with real VLTP objects.

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APPENDIX A: CONSISTENT TREATMENT OF REDUCED CONVECTIVE VELOCITIES

It has been proposed in some papers (Herwig 2001a; Schlattl et al. 2001) that convective mixing efficiency can be reduced during violent H-flashes. Even more, Herwig (2001a) mentions that convective material transport can be changed in main-sequence stellar models by orders of magnitude without any change in stellar parameters. Going even further Lawlor & MacDonald (2003) conclude that, as reduced mixing efficiency does not produce significant changes in usual stellar evolutionary stages, reduced mixing velocities could be usual in stellar evolution. In what follows it is shown that, if a reduction of mixing velocities is considered in the full treatment of the MLT, the effect of changing the convective velocities is indistinguishable from changing the value of α (the mixing length to pressure scaleheight ratio). Then the previous conclusions come from an inconsistent treatment of the reduction in mixing velocities.

A1 Reduced convective velocities and convective energy flux

Following Kippenhahn & Weigert (1990) we can derive the mean work done by buoyancy on the convective elements to be

$$\bar{W}(r) = g\delta(\nabla - \nabla_{\rm e})\frac{{l_m}^2}{8H_P}, \tag{A1}$$

where the symbols have their usual meanings. Then to propose a reduced mixing velocity within the MLT, we propose that only a fraction of the work done by buoyancy goes into the kinetic energy of the convective elements. We can write this as

$$\bar{v} = f_v\left[\frac{g\delta(\nabla - \nabla_{\rm e})}{8H_P}\right]^{1/2} l_m. \tag{A2}$$

Here f_v is a new (and, a priori, 'free') parameter giving the factor by which the standard value of $\bar{v}$ is reduced. Following Kippenhahn & Weigert (1990) and inserting this relation in the equation for convective energy transport,

$$F_{\rm con} = \rho v c_P DT, \tag{A3}$$

and replacing DT in terms of $(\nabla - \nabla_{\rm e})$ we get

$$F_{\rm con} = \frac{C_P \rho T\sqrt{g\delta}}{4\sqrt{2}}{l_m}^2\left(\frac{\nabla - \nabla_{\rm e}}{H_P}\right)^{3/2} f_v. \tag{A4}$$

The convective flux is thus accordingly reduced by the factor f_v.

A2 Modified dimensionless equations

From the relation

$$\frac{\nabla_{\rm e} - \nabla_{\rm ad}}{\nabla - \nabla_{\rm e}} = \frac{6acT^3}{\kappa\rho^2 C_P l_m \bar{v}} \tag{A5}$$

and defining the usual dimensionless quantities U and W:

$$U = \frac{3acT^3}{c_P\rho^2\kappa {l_m}^2}\sqrt{\frac{8H_P}{g\delta}}, \tag{A6}$$

$$W = \nabla_{\rm rad} - \nabla_{\rm ad}, \tag{A7}$$

we get new equations (equivalent to equations 7.14 and 7.15 in Kippenhahn & Weigert 1990) for U:

$$\nabla_{\rm e} - \nabla_{\rm ad} = 2\frac{U}{f_v}\sqrt{\nabla - \nabla_{\rm e}}, \tag{A8}$$

$$(\nabla - \nabla_{\rm e})^{3/2} = \frac{8}{9}\frac{U}{f_v}(\nabla_{\rm rad} - \nabla). \tag{A9}$$

It seems natural then to define the quantity $U' = U/f_v$:

$$U\prime = \frac{3acT^3}{c_P\rho^2\kappa {l_m}^2 f_v}\sqrt{\frac{8H_P}{g\delta}}. \tag{A10}$$

Then defining the quantity $\zeta = +\sqrt{\nabla - \nabla_{\rm ad} + U'^2}$ we get that the new dimensionless cubic equation for the MLT is

$$(\zeta - U')^3 + \frac{8U'}{9}\left(\zeta^2 - U'^2 - W\right) = 0. \tag{A11}$$

This becomes the new equation to be solved in order to obtain the real value of ∇. This is the same equation as 7.18 given in Kippenhahn & Weigert (1990), the only difference being the definition of U'.

A3 The diffusion coefficient

In the context of diffusive convective mixing, material transport and mixing is ruled by the diffusion coefficient, which is usually defined as $D = (1/3) l_m v$. Taking equation (A2), replacing $(\nabla - \nabla_e)$ with equation (A9) and using the definition of U' we get for the mean velocity of convective motions the expression

$$\bar{v} = \left(\frac{{f_v}^2 l_m}{H_P}\right)^{1/3} \left[\frac{cg\nabla_{\rm ad}(1-\beta)}{\rho\kappa}(\nabla_{\rm rad} - \nabla)\right]^{1/3}. \qquad \text{(A12)}$$

Introducing this into the definition of D we get (using $l_m = H_P \alpha$)

$$D = \frac{1}{3}\left(\alpha^2 f_v\right)^{2/3} H_P \left[\frac{cg(1-\beta)}{\rho\kappa}\nabla_{\rm ad}(\nabla_{\rm rad} - \nabla)\right]^{1/3}. \qquad \text{(A13)}$$

A4 Conclusion

As U' (and thus ∇) and D depend only on the product $\alpha^2 f_v$ (and not on both quantities independently) then changes in the mixing length and changes in the mixing velocities are indistinguishable, making it unnecessary to consider a new free parameter within the MLT. A reduction in the mixing efficiency should be regarded as equivalent to reducing the mixing length. Then, as a consequence, changing the mean velocity of convective motions would produce an appreciable difference in any case in which non-adiabatic convection takes place (lower main-sequence, red giant branch, AGB stellar models).

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