

# Evolution of low-mass close binary systems with a neutron star: its dependence with the initial neutron star mass

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## ABSTRACT

We construct a set of binary evolutionary sequences for systems composed by a normal, solar composition, donor star together with a neutron star. We consider a variety of masses for each star as well as for the initial orbital period corresponding to systems that evolve to ultra-compact or millisecond pulsar-helium white dwarf pairs. Specifically, we select a set of donor star masses of 0.50, 0.65, 0.80, 1.00, 1.25, 1.50, 1.75, 2.00, 2.25, 2.50, 3.00 and 3.50  $M_{\odot}$ , whereas for the accreting neutron star we consider initial mass values of 0.8, 1.0, 1.2 and 1.4  $M_{\odot}$ . Because the minimum mass for a proto-neutron star is approximately 0.9  $M_{\odot}$ , the value of 0.8  $M_{\odot}$  was selected in order to cover the whole range of possible initial neutron star masses. The considered initial orbital period interval ranges from 0.5 to 12 d.

It is found that the evolution of systems, with fixed initial values for the orbital period and the mass of the normal donor star, heavily depends upon the mass of the neutron star. In some cases, varying the initial value of the neutron star mass, we obtain evolved configurations ranging from ultra-compact to widely separated objects.

We also analyse the dependence of the final orbital period with the mass of the white dwarf. In agreement with previous expectations, our calculations show that the final orbital period–white dwarf mass relation is fairly insensitive to the initial neutron star mass value. A new period–mass relation based on our own calculations is proposed, which is in good agreement with period–mass relations available in the literature.

As a consequence of considering a set of values for the initial neutron star mass, these models allow finding different plausible initial configurations (donor and neutron star masses and orbital period interval) for some of the best observed binary systems of the kind we are interested in here. We apply our calculations to analyse the case of PSR J0437–4715, showing that there is more than one possible set of initial parameters (masses, period and the fraction  $\beta$  of matter accreted by the neutron star) for this particular system.

**Key words:** binaries: close – stars: evolution – white dwarfs.

## 1 INTRODUCTION

During past years, binary radio pulsars have been detected more and more often. Now, we are aware of the existence of 141 pulsars belonging to binary systems (ATNF Pulsar Catalogue: [www.atnf.csiro.au](http://www.atnf.csiro.au); Manchester et al. 2005). For these objects, both

estimations of the median mass of the companion (assuming an orbital inclination of  $60^{\circ}$ ) and orbital period of the binary system are available. If we restrict ourselves to low-mass companions ( $M < 0.35 M_{\odot}$ ), we find about 100 objects; approximately half of them located in globular clusters. Among this group of binary systems, we are interested in a subgroup composed by a neutron star (NS) and a low-mass white dwarf (WD). Presumably, these objects have a helium-rich interior and will be referred to as helium white dwarfs (HeWDs).

Remarkably, for some of the above-referred binary systems, it has been possible to make reliable determinations of the masses of both components. This has been possible taking advantage of the relativistic effect known as Shapiro delay (see Taylor & Weisberg 1989 and references therein). In Table 1, we list the main

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**Table 1.** The CBSs composed by a millisecond pulsar and a low-mass WD for which it has been possible to detect the Shapiro delay effect and measure the masses of both components. All these systems belong to the Galactic plane population. From left to right, the table presents the name of the pulsar, its spin period, the WD and pulsar masses, the orbital period and the relevant reference.

Name	$P_p$ (ms)	$M_{\text{WD}}$ ( $M_{\odot}$ )	$M_{\text{NS}}$ ( $M_{\odot}$ )	$P$ (d)	Reference
PSR J0437–4715	5.757	$0.236 \pm 0.017$	$1.58 \pm 0.18$	5.741	van Straten et al. (2001)
PSR J1713+0747	4.57	$0.28 \pm 0.03$	$1.3 \pm 0.2$	67.825	Splaver et al. (2005)
PSR B1855+09	5.362	$0.258^{+0.028}_{-0.016}$	$1.50^{+0.26}_{-0.14}$	12.327	Kaspi, Taylor & Ryba (1994)
PSR J1909–3744	2.947	$0.2038 \pm 0.0022$	$1.438 \pm 0.024$	1.533	Jacoby et al. (2005)

parameters of these binary systems. Apart from the data included there, Nice, Stairs & Kasian (2008) have reported further observations of the binary system containing PSR J0751+1807. They improved the values of the pulsar mass, finding it to be  $1.26 \pm 0.14 M_{\odot}$  (68 per cent confidence) or  $1.26 \pm 0.28 M_{\odot}$  (95 per cent confidence). These values are much lower than their previous claim (Nice et al. 2005), especially in connection with the inferred mass of the NS. We do not include them in Table 1 because the value of the WD mass is not yet available.

The formation mechanism of such close binary systems (CBSs) is well established: a low-mass, normal star undergoes Roche lobe overflow (RLOF) and transfers mass to a NS companion. After a long, stable mass transfer episode the donor (normal) star has lost most of its mass. In the non-conservative case, only part of this mass is accreted by the NS which is spun up, allowing it to be detected as a millisecond pulsar (MSP) while its companion (initially a main-sequence star) cools down becoming a WD (see e.g. Bhattacharya & van den Heuvel 1991).

Let us make a brief discussion of the binary evolution results available in the literature related to the objects we are interested in. Sarna, Antipova & Muslimov (1998) investigated the evolution of CBSs to account for the binary system containing the MSP PSR J1012+53 and its low-mass companion. For the initial NS mass [ $(M_{\text{NS}})_i$ ], they assumed the ‘canonical’ value of  $1.4 M_{\odot}$ . Ergma, Sarna & Antipova (1998) made evolutionary calculations of low-mass CBSs in conservative and non-conservative cases, considering donor star masses in the range  $1.0 M_{\odot} \leq M \leq 1.5 M_{\odot}$ . Again, they set  $(M_{\text{NS}})_i = 1.4 M_{\odot}$ . Tauris & Savonije (1999) computed non-conservative evolution of CBSs with low-mass ( $1.0$ – $2.0 M_{\odot}$ ) donor stars and a  $(M_{\text{NS}})_i = 1.3 M_{\odot}$  accreting NS. The initial orbital periods range was between 2 and 800 d. Besides, they revisited the orbital period–WD mass relation ( $P - M_{\text{WD}}$ ) in wide binary WD-radio pulsar systems. Podsiadlowski, Rappaport & Pfahl (2002) performed a systematic study of the evolution of low- and intermediate-mass binary systems. In their calculations, they assumed  $(M_{\text{NS}})_i = 1.4 M_{\odot}$  for the NS, which accretes (at most) half of the transferred matter, while donor stars had initial masses between  $0.6$  and  $7 M_{\odot}$ . The initial orbital periods covered the interval from approximately 4 h to 100 d. Ergma & Sarna (2003) constructed binary evolution sequences to account for the observed binary parameters for PSR J1740–5340. Again, they considered  $(M_{\text{NS}})_i = 1.4 M_{\odot}$ . Nelson & Rappaport (2003) investigated possible scenarios for accretion-powered MSPs in ultra-compact binaries. They calculated a large set of evolutionary tracks corresponding to different donor masses and degrees of chemical evolution at the onset of mass transfer. The range of initial donor masses was between  $1.0$  and  $2.5 M_{\odot}$  and  $(M_{\text{NS}})_i = 1.4 M_{\odot}$ . They assumed a fully non-conservative mass transfer case. Benvenuto & De Vito (2005) computed the evolution of a set of binary systems leading to the

formation of HeWDs – MSP or ultra-compact systems considering diffusion. They also analysed possible progenitors for some of the best observed systems containing an MSP together with a low-mass WD. They set  $(M_{\text{NS}})_i = 1.4 M_{\odot}$  and  $\beta = 0.5$  ( $\beta$  is the fraction of transferred matter accreted by the NS), although in fitting the masses and orbital period of these systems, they allowed for lower values of  $\beta$ . Benvenuto, Rohrmann & De Vito (2006) found a possible original configuration that accounts for the observed parameters of PSR J1713+0747 binary system. They computed a set of binary evolution calculations in order to simultaneously account for the masses of both stars and the orbital period, again setting  $(M_{\text{NS}})_i = 1.4 M_{\odot}$ .

In spite of the fact that in most of theoretical studies aimed to explore the evolution of low-mass WD–NS binary systems the initial mass of the NS has been set to  $(M_{\text{NS}})_i = 1.4 M_{\odot}$ , observational evidence presented in Table 1 indicates that  $(M_{\text{NS}})_i$  may indeed be lower. At present, we do not know the value of the fraction  $\beta$ . The only physical limitation is the Eddington critical accretion rate  $\dot{M}_{\text{NS}} \leq \dot{M}_{\text{Edd}} = 2 \times 10^{-8} M_{\odot} \text{yr}^{-1}$  (where  $\dot{M}_{\text{NS}}$  is the accretion rate of the NS). Usually  $\beta$  is considered as a free parameter. Certainly, we may account for NS masses greater than  $1.4 M_{\odot}$  by setting an initial canonical value for  $(M_{\text{NS}})_i$  and adjusting  $\beta$ . However, this is *not* possible if observed NS masses are lower than  $1.4 M_{\odot}$  (e.g. the case of PSR J1713+0747; see Table 1). This fact induced us to perform a systematic exploration of the evolution of these CBSs varying the initial donor (normal) and accretor (neutron) stars masses (and also the initial orbital period). This is one of the main purposes of this paper.

In our models, we consider masses for the donor stars in the range from  $0.50$  to  $3.50 M_{\odot}$  and accreting NSs with initial masses [ $(M_{\text{NS}})_i$ ] of  $0.8$ ,  $1.0$ ,  $1.2$  and  $1.4 M_{\odot}$ . The range of  $(M_{\text{NS}})_i$  we propose for our calculations needs some justification. It is well known that most of the accurately measured NS masses are near  $1.4 M_{\odot}$ . Also, it is well known (see e.g. Lattimer & Prakash 2004 for a recent tabulation; Lattimer & Prakash 2007) that the masses of some NSs are well below that value. In particular, these are the cases of the NSs in the X-ray binaries SMC X-1, Cen X-3 and 4U1538-52 that, following Lattimer & Prakash (2004), have masses of  $1.17^{+0.16}_{-0.10}$ ,  $1.09^{+0.20}_{-0.36}$  and  $0.96^{+0.19}_{-0.16} M_{\odot}$  respectively. More recently, van der Meer et al. (2007) have presented more accurate determinations for the masses of NSs in binary systems. Specifically, for the cases of SMC X-1 and Cen X-3, the authors find values of  $1.06^{+0.11}_{-0.10} M_{\odot}$  and  $1.34^{+0.10}_{-0.14} M_{\odot}$ , respectively. Note that in the case of SMC X-1, the NS is somewhat less massive, but for Cen X-3, the NS is notably more massive than the previous determination.

NSs have both minimum and maximum mass limits. The maximum mass is unknown, but lies in the range of  $1.44$  to  $3 M_{\odot}$ . The upper bound follows from causality arguments (Rhoades & Ruffini 1974), imposing that the speed of sound in dense matter must be

less than the speed of light, whereas the lower bound is set by the largest accurately measured pulsar mass,  $1.4408 \pm 0.0003 M_{\odot}$ , in the binary pulsar PSR 1913+16 (Weisberg & Taylor 2003).

Regarding the minimum NS mass value  $M_{\min}$ , it is important to remark that it is sensitive to the equation of state (EOS) of NS matter at sub-nuclear densities. Haensel, Zdzunik & Douchin (2002) calculate  $M_{\min}$  for cold NSs using two different EOSs. For non-rotating configurations they find  $M_{\min} = 0.094 M_{\odot}$  for the SLy EOS (Chabanat et al. 1998) and  $M_{\min} = 0.088 M_{\odot}$  for the FPS EOS (Lorenz, Ravenhall & Pethick 1993). However, we are interested in rotating NSs, i.e. the accreting companion of a donor star in CBSs. Haensel et al. (2002) performed accurate calculations of stationary, cold NSs configurations, rotating uniformly at  $\nu = 100$  Hz and  $\nu = 641$  Hz (which corresponds to the shortest observed pulsar period). The authors find for SLy EOS that minimum mass at  $\nu = 641$  is  $0.61 M_{\odot}$  and for FPS EOS, at the same rotation frequency,  $0.54 M_{\odot}$ . For the case of  $\nu = 100$  Hz and SLy EOS, the minimum mass finding is of  $0.13 M_{\odot}$ ,  $\approx 40$  per cent larger than that for static NSs.

If we consider newly born proto-NSs, both thermal (after core bounce the proto-NS has a temperature  $T \approx 10^{10}$  K) and neutrino-trapping effects are large, and are found to largely increase the  $M_{\min}$  value to  $0.9$ – $1.1 M_{\odot}$  (Goussard, Haensel & Zdzunik 1998; Strobel, Schaab & Weigel 1999). Thus, if NSs' formation corresponds to a gravitational collapse event, we should expect the existence of NSs with masses above the  $M_{\min}$  value corresponding to proto-NSs,  $0.9 M_{\odot}$ . Observational data support this lower mass limit.

There is a large gap between the values of  $M_{\min}$  for cold and proto-NS as estimated from the different models presented above. Still, an NS may reach mass values smaller than  $0.9$ – $1.1 M_{\odot}$  by mass loss after becoming a cold NS.<sup>1</sup> This possibility has been studied by Blinnikov et al. (1984), Colpi, Shapiro & Teukolsky (1991) and Sumiyoshi et al. (1998). However, analysing such possibility and its consequences is beyond the scope of this paper.

In view of the above discussion, the minimum NS mass value ( $0.8 M_{\odot}$ ) considered in our calculations may seem somewhat low. However, in any case, in performing our theoretical experiment, we select the minimum value of the accreting NS of  $0.8 M_{\odot}$ , somewhat less massive than that of the minimum, presented by the observations and for the theoretical calculations of proto-NS, simply to be sure we are exploring the whole meaningful NS mass interval.

It is well known that there exists a somewhat tight relation between the mass and the radius of the cores of low-mass giants (see e.g. Joss, Rappaport & Lewis 1987). Then, a  $P - M_{\text{WD}}$  relation can be derived. This will be valid if the star belongs to a CBS and undergoes RLOF as a giant. In the calculations to be presented below, some donor stars undergo RLOF as red giants; however, other experience RLOF when they are still much more compact. Thus, we explore the  $P - M_{\text{WD}}$  relation and test the claim (Rappaport et al. 1995) that it is nearly independent of  $(M_{\text{NS}})_i$  quantitatively and in more general conditions than those previously considered.

The remainder of this paper is organized as follows: In Section 2, we briefly describe the employed code. In Section 3, we present our calculations studying the dependence of the evolution of binary systems with  $(M_{\text{NS}})_i$  (Section 3.1) and discuss them in connection with the  $P - M_{\text{WD}}$  relation (Section 3.2). In Section 4, we discuss the possibility of finding different initial binary configurations to account for the observed characteristics of systems containing a

recycled pulsar and a low-mass WD and, as an example, we study the case of PSR J0437–4715 in detail. Finally, in Section 5 we present the main conclusions of this work.

## 2 THE COMPUTER CODE

The code employed here has been described elsewhere (Benvenuto & De Vito 2003). Briefly, we use a generalized Henyey technique that allows for the computation of the stellar structure and mass transfer episodes in a fully implicit way. The code has an updated description of opacities, EOS, nuclear reactions and diffusion, while we simultaneously compute orbital evolution considering the main processes of angular momentum loss: angular momentum carried away by the matter lost from the system, gravitational radiation and magnetic braking.

Regarding the inclusion of element diffusion, it has several effects on the chemical profile of these stars, especially in the WD and pre-WD stages (see e.g. Iben & MacDonald 1985; Althaus, Serenelli & Benvenuto 2001). For example, diffusion is responsible for the occurrence of almost pure hydrogen atmospheres in the case of cool enough DA WDs. Moreover, diffusion leads to the sink of hydrogen to layers hot enough for triggering the occurrence of nuclear burning. While in calculations neglecting diffusion, stellar models in the here-considered mass range suffer from the occurrence of (envelope) hydrogen thermonuclear flashes, it has been shown that diffusion forces the star to undergo supplementary flashes (Althaus et al. 2001).

In our treatment of the orbital evolution of the system, we consider that the NS is able to retain a fraction  $\beta$  of the material coming from the donor star:  $\dot{M}_{\text{NS}} = -\beta \dot{M}$  (where  $\dot{M}$  is the mass transfer rate from the donor star), as done in Benvenuto & De Vito (2005). We consider  $\beta$  as constant throughout all RLOF episodes; in particular, if not stated otherwise, we set  $\beta = 0.5$ , as done in Podsiadlowski et al. (2002). We assume that material lost from the binary system carries away the specific angular momentum of the compact object ( $\alpha = 1$ ; see e.g. Benvenuto & De Vito 2003).

In this work, we consider the Mixing Length Theory as described in Kippenhahn, Weigert & Hofmeister (1967), setting the Mixing Length parameter to  $l/H_p = 1.7432$  and including convective overshoot as in Demarque et al. (2004). Furthermore, we consider grey atmospheres and neglect the effects of the irradiation of the donor star by the pulsar.

## 3 NUMERICAL RESULTS

We select initial values for the system parameters (initial masses and orbital period) in order to obtain systems with HeWD companions, although some of them evolve to ultra-compact binaries avoiding the formation of WDs. The initial donor star masses are of  $0.50, 0.65, 0.80, 1.00, 1.25, 1.50, 1.75, 2.00, 2.25, 2.50, 3.00$  and  $3.50 M_{\odot}$  of solar composition. We combine these masses with accreting NSs with initial masses  $(M_{\text{NS}})_i$  of  $0.8, 1.0, 1.2$  and  $1.4 M_{\odot}$ .

The initial orbital period for the three smaller donor stars are of  $^2 0.175, 0.20$  and  $0.30$  d. For the other donor star masses, initial periods are of  $0.50, 0.75, 1.00, 1.50, 3.00, 6.00$  and  $12$  d. In all

<sup>1</sup> Note that the NS spends only several seconds in releasing most of its lepton and thermal content to become a cold NS.

<sup>2</sup> This choice is due to the fact that, if initial periods were shorter, the Roche lobe would be smaller than the star even for a Zero Age Main Sequence object; if they were longer, the star would not fill the Roche lobe on the Hubble time.

**Table 2.** Main results of our binary evolution calculations. First and second columns list the initial mass of the donor star and the initial orbital period of the systems, respectively. For each donor star and orbital period we compute the evolution of binary systems for different values of the initial NS mass:  $(M_{\text{NS}})_i = 0.8, 1.0, 1.2$  and  $1.4 M_{\odot}$ . For each system that evolves to ultra-compact or HeWD–MSP pair we list the final period, the donor remnant and NS masses. Numbers in italics denote systems that form a WD but we do not include in Fig. 5. For further details see the main text.

$M_i$ ( $M_{\odot}$ )	$P_i$ (d)	$(M_{\text{NS}})_i = 0.80 M_{\odot}$			$(M_{\text{NS}})_i = 1.00 M_{\odot}$			$(M_{\text{NS}})_i = 1.20 M_{\odot}$			$(M_{\text{NS}})_i = 1.40 M_{\odot}$		
		$P_f$ (d)	$M_{\text{WD}}$ ( $M_{\odot}$ )	$M_{\text{NS}}$ ( $M_{\odot}$ )	$P_f$ (d)	$M_{\text{WD}}$ ( $M_{\odot}$ )	$M_{\text{NS}}$ ( $M_{\odot}$ )	$P_f$ (d)	$M_{\text{WD}}$ ( $M_{\odot}$ )	$M_{\text{NS}}$ ( $M_{\odot}$ )	$P_f$ (d)	$M_{\text{WD}}$ ( $M_{\odot}$ )	$M_{\text{NS}}$ ( $M_{\odot}$ )
0.50	0.175	0.0585	0.0366	1.0317	0.0595	0.0362	1.231	0.0603	0.0355	1.4322	0.0611	0.0351	1.6325
0.65	0.20	0.0589	0.0368	1.1066	0.0602	0.0362	1.306	0.0609	0.0356	1.5072	0.0616	0.0351	1.7075
0.80	0.30	0.0537	0.0323	1.1839	0.0568	0.0325	1.383	0.0571	0.0315	1.5842	0.0485	0.0447	1.7777
	0.50	0.0274	0.0305	1.2848	0.0361	0.0154	1.492	0.0384	0.0154	1.6923	0.0365	0.0148	1.8926
	0.75	1.8603	0.1878	1.2053	2.2487	0.1918	1.403	2.5233	0.1966	1.6010	2.9950	0.2011	1.7983
	1.00	4.7508	0.2180	1.1755	6.0133	0.2258	1.386	6.9490	0.2287	1.5829	8.1373	0.2317	1.7831
1.00	1.50	9.5741	0.2360	1.1251	12.6974	0.2420	1.371	15.5468	0.2471	1.5673	17.7987	0.2505	1.7706
	3.00	21.6577	0.2593	1.0832	29.3771	0.2667	1.349	35.0054	0.2719	1.5630	39.8687	0.2759	1.7611
	6.00	40.9639	0.2778	1.0471	55.4442	0.2871	1.323	66.4038	0.2929	1.5486	75.4193	0.2972	1.7506
	12.00	73.3650	0.2971	1.0098	98.2015	0.3079	1.283	117.7680	0.3141	1.5229	133.8719	0.3187	1.7270
	0.50	0.0445	0.0408	1.4046	0.0526	0.0286	1.610	0.0532	0.0277	1.8112	0.0546	0.0281	2.0109
	0.75	<i>0.0407</i>	<i>0.1414</i>	<i>1.3543</i>	<i>0.0516</i>	<i>0.1531</i>	<i>1.548</i>	<i>0.0336</i>	<i>0.1609</i>	<i>1.7445</i>	0.4501	0.1672	1.9403
	1.00	4.4800	0.2172	1.2943	5.4725	0.2229	1.512	6.0909	0.2253	1.7115	6.6807	0.2273	1.9106
1.25	1.50		$\dot{M}$ divergent		15.8528	0.2472	1.413	18.9614	0.2546	1.6786	22.2105	0.2587	1.8931
	3.00							44.2353	0.2789	1.6365	52.2558	0.2841	1.8746
	6.00							81.7321	0.3003	1.5641	97.9597	0.3062	1.8424
	12.00							144.7212	0.3223	1.5527	169.8661	0.3277	1.7152
	0.50	0.0590	0.0335	1.5332	0.0475	0.0434	1.728	0.0465	0.0389	1.9305	0.0545	0.0278	2.1361
	0.75	0.0483	0.0245	1.5377	0.0466	0.0220	1.739	0.0475	0.0199	1.9401	0.0496	0.0197	2.1401
	1.00	<i>0.0403</i>	<i>0.1601</i>	<i>1.4489</i>	0.5110	0.1741	1.660	1.6702	0.1943	1.8521	4.0510	0.2047	2.0468
1.50	1.50		$\dot{M}$ divergent			$\dot{M}$ divergent		28.9438	0.2693	1.7149	31.7147	0.2708	2.0139
	3.00							52.5323	0.2884	1.5327	65.1049	0.2941	1.8746
	6.00								$\dot{M}$ divergent		111.8036	0.3134	1.8673
	12.00										198.2880	0.3367	1.7854
	0.50	0.0588	0.0337	1.4818	0.0590	0.0321	1.782	0.0468	0.0389	2.0357	0.0530	0.0262	2.2619
	0.75	0.0501	0.0269	1.4823	0.0494	0.0246	1.785	0.0473	0.0211	2.0426	0.0493	0.0197	2.2652
	1.00	0.0460	0.0219	1.5051	0.0465	0.0205	1.820	0.0491	0.0186	2.0657	<i>0.0490</i>	<i>0.1648</i>	<i>2.1913</i>
1.75	1.50	10.5640	0.2459	1.1158	22.5014	0.2537	1.500	34.1985	0.2630	1.8120	40.5855	0.2814	2.0920
	3.00		$\dot{M}$ divergent			$\dot{M}$ divergent			$\dot{M}$ divergent		68.7946	0.3104	1.6614
	6.00											$\dot{M}$ divergent	
	0.50	0.0583	0.0345	1.2849	0.0603	0.0332	1.732	0.0564	0.0288	2.0303	0.0523	0.0258	2.3501
	0.75	0.0504	0.0279	1.3204	0.0518	0.0273	1.714	0.0481	0.0228	2.0155	0.0487	0.0196	2.3257
2.00	1.00	0.0464	0.0233	1.3209	0.0472	0.0227	1.717	0.0482	0.0196	2.0160	0.0324	0.0268	2.2870
	1.50	0.0301	0.0266	1.3108	1.1441	0.2047	1.670	3.6701	0.2317	1.9679	17.7005	0.2621	2.2028
	3.00		$\dot{M}$ divergent			$\dot{M}$ divergent			$\dot{M}$ divergent			$\dot{M}$ divergent	
	0.50		$\dot{M}$ divergent		0.0602	0.0343	1.644	0.0594	0.0315	1.9949	0.0535	0.0264	2.3217
	0.75				0.0523	0.0282	1.637	0.0500	0.0248	1.9593	0.0472	0.0201	2.2686
2.25	1.00		in all		0.0476	0.0236	1.610	0.0477	0.0219	1.9453	0.0517	0.0196	2.2417
	1.50				0.0265	0.0224	1.589	2.6496	0.2176	1.8582	6.6082	0.2427	2.1434
	3.00		cases			He burning			He burning			He burning	
	0.50		$\dot{M}$ divergent		0.0604	0.0348	1.488	0.0614	0.0338	1.9351	0.0468	0.0378	2.2212
	0.75				0.0526	0.0289	1.524	0.0416	0.0412	1.8920	0.0479	0.0209	2.2090
2.50	1.00		in all		0.0477	0.0246	1.505	0.0486	0.0234	1.8692	0.0487	0.0196	2.1790
	1.50				2.7871	0.2170	1.355	3.6504	0.2217	1.7351	6.1671	0.2367	2.0551
	3.00		cases			He burning			He burning			He burning	
	0.50		$\dot{M}$ divergent			$\dot{M}$ divergent			$\dot{M}$ divergent		0.0611	0.0327	2.1107
	0.75							0.0491	0.0242	1.7349	0.0490	0.0229	2.1056
3.00	1.00		in all			in all		0.0468	0.0184	1.7217	4.5711	0.2053	1.9641
	1.50							12.6423	0.2572	1.5379	15.4657	0.2612	1.8645
	3.00		cases			cases			He burning			He burning	
	0.50		$\dot{M}$ divergent			$\dot{M}$ divergent			$\dot{M}$ divergent			$\dot{M}$ divergent	
	0.75										0.0479	0.0197	1.9640
3.50	1.00		in all			in all			in all		15.9734	0.2378	1.8293
	1.50										21.0553	0.2755	1.7556
	3.00		cases			cases			cases			He burning	

cases, the initial periods refer to its value at the onset of the first RLOF. Calculations start from the Zero Age Main Sequence (we set zero age there) and are followed up to the formation of a HeWD or an ultra-compact system. We computed the evolution of the donor star up to an age far in excess of Hubble time of 20 Gyr, or when the donor has a luminosity lower than  $1 \times 10^{-5} L_{\odot}$ . However, in some cases we stop the computations earlier. We do so if helium is ignited at the stellar core or if mass transfer becomes very intense ( $\dot{M} \geq 10^{-4} M_{\odot} \text{ yr}^{-1}$ ). In Table 2, we present the main results of our calculations.

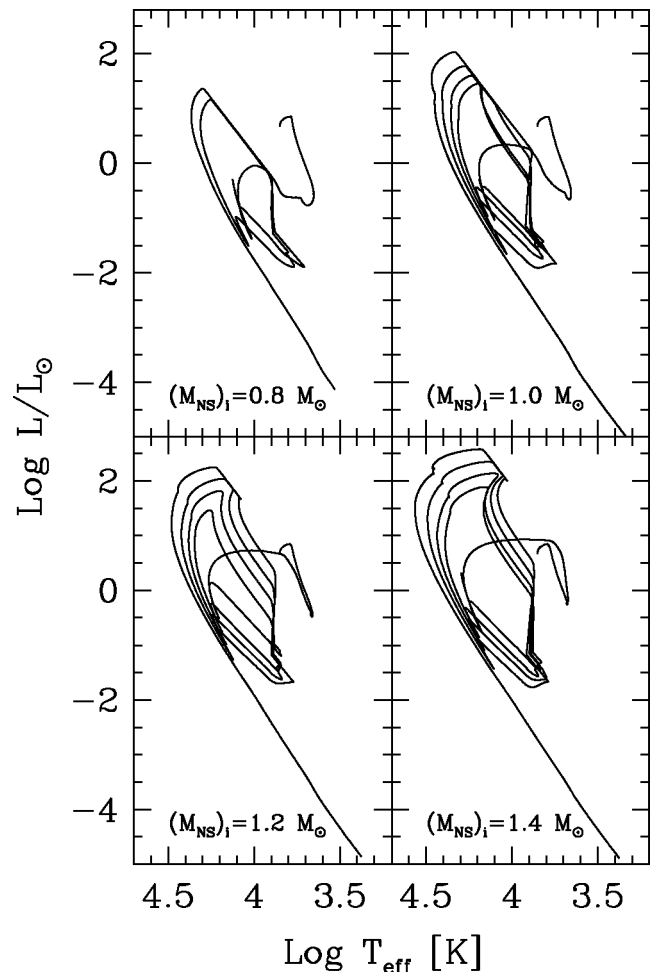
If a system suffers from a very large ( $\dot{M} \geq 10^{-4} M_{\odot} \text{ yr}^{-1}$ ), and still growing, mass transfer rate, we indicate this in Table 2 as ‘ $\dot{M}$  divergent.’ This behaviour can be explained in terms of the occurrence of a common envelope (CE) phase. A CE episode can be the consequence of a dynamical mass transfer event. Dynamical mass transfer is associated typically with mass being transferred from the more massive component, in a stage in which it possesses a deep convective envelope [e.g. if the onset of a RLOF occurs when the donor star is on the red giant branch (RGB) phase] or if the mass ratio of the system is large. In such conditions, the star is unable to contract as rapidly as its Roche lobe (in fact it expands), thus an unstable mass transfer process ensues (Paczynski & Sienkiewicz 1972). As a consequence of the high accretion rate, the accretor star, driven out thermal equilibrium, starts expanding (especially if the accretion rate exceeds the Eddington limit) and fills its own Roche lobe. The resulting mass flow leads to the formation of the CE configuration (see e.g. Yungelson 1973; Webbink 1977; Livio 1989; Han & Webbink 1999). This is the case we find in our calculations. The donor star fills its Roche lobe when it is in the RGB phase, with a deep convective envelope, being donor star the more massive component, and with super-Eddington values for  $\dot{M}$ . Then, we consider that this leads to a CE situation. Also, divergent  $\dot{M}$  episodes are found for the case of donor status with very short orbital periods and masses  $M_i \geq 3.0 M_{\odot}$ . For these systems, the onset of the RLOF occurs during core hydrogen burning and should be associated with a ‘delayed dynamical’ unstable mass transfer as found by Podsiadlowski et al. (2002). Note that ‘ $\dot{M}$  divergent’ systems are found at shorter initial orbital periods the lighter is the NS. This again indicates that the evolution of the systems heavily depends on the initial mass of the NS.

### 3.1 The dependence of the evolution of close binary systems upon the initial mass of the neutron star

Among the results presented in Table 2, we may select a subset of evolutionary calculations, for a given initial donor star mass and orbital period to study the behaviour of CBSs when we change the initial NS mass. In Table 3, we present supplementary data for the case of a donor star of  $1.5 M_{\odot}$ , initial period of 1 d and different values for the initial NS mass. In Fig. 1, we show the evolutionary tracks corresponding to the binary systems included in Table 3. In all of the selected cases, the donor star undergoes several thermonuclear hydrogen flashes. These flashes are responsible for the quasi-cyclic behaviour in the Hertzsprung–Russell diagram. Let us briefly quote that these events are due to the heating of the bottom of the hydrogen envelope of the (then) pre-WD object. At that place matter is degenerate, forcing the onset of unstable nuclear burning. For further details on the evolution of a pre-WD object undergoing thermonuclear flashes in presence of diffusion, see Althaus et al. (2001).

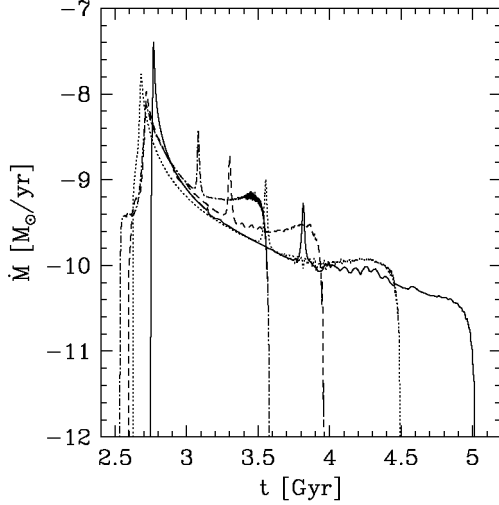
**Table 3.** Main characteristics of the evolution of systems that initially have a donor star mass of  $1.5 M_{\odot}$ , an orbital period of 1 d and different values for the initial NS mass. From left to right we list the initial mass of the accreting NS, the time for the onset of the first RLOF, the time spent during this RLOF, the final values of the WD and the NS masses, and the final orbital period of the system.

$(M_{\text{NS}})_i$ ( $M_{\odot}$ )	$t_{i-\dot{M}}$ (Gyr)	$\Delta t_{\dot{M}}$ (Gyr)	$M_{\text{WD}}$ ( $M_{\odot}$ )	$M_{\text{NS}}$ ( $M_{\odot}$ )	$P$ (d)
0.80	2.740	2.295	0.1601	1.4489	0.0403
1.00	2.624	1.886	0.1741	1.6608	0.5110
1.20	2.595	1.371	0.1943	1.8521	1.6702
1.40	2.529	1.055	0.2047	2.0468	4.0510

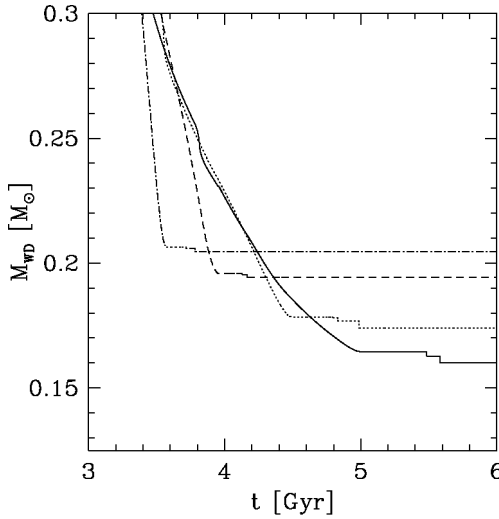


**Figure 1.** The evolutionary tracks for a normal donor star with initial mass of  $1.5 M_{\odot}$ , evolving in binary systems with different initial NS masses. The initial orbital period is of 1 d. The loops are due to hydrogen thermonuclear flashes (see the main text for further details).

In Fig. 2, we show the mass-loss rate for the same set of systems. The lower is the initial mass for the accreting NS, the longer is the time spent by the system in the RLOF episode. Note that the onset of the RLOF occurs later, the less massive is the NS, because the Roche lobe of the donor star is bigger (see e.g. Eggleton 1983). We find less massive WD remnants for less massive accreting NS, as we can see in Fig. 3. Fig. 4 shows the evolution of the orbital period



**Figure 2.** The temporal evolution of the mass transfer rate for the systems considered in Fig. 1. Solid, dot, short-dash and dot–short-dash lines show the mass transfer rates of the objects corresponding to initial NS masses of  $0.8$ ,  $1.0$ ,  $1.2$  and  $1.4 M_{\odot}$ , respectively. Here, we show only the mass-loss episodes not induced by thermonuclear flashes in the envelope of the star.



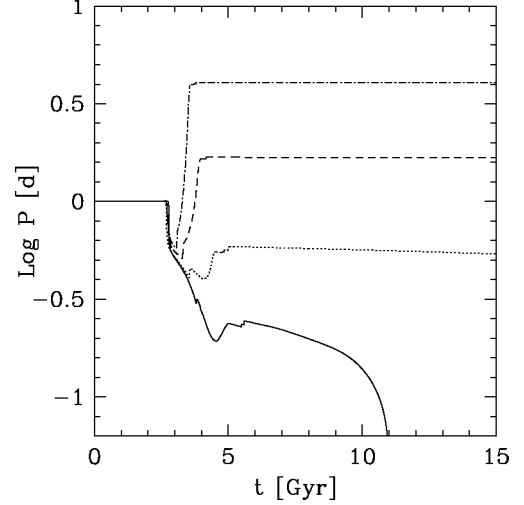
**Figure 3.** Evolution of the mass of the donor star with time, for systems depicted in Fig. 1. The meaning of lines are the same as in Fig. 2.

as a function of time for the same subset of systems. We see that systems having less massive NSs evolve to tighter configurations.

From the results presented above, we find that the evolution of the donor star *heavily* depends on the value of the mass of the NS. This is one of the main findings of the present paper.

### 3.2 The orbital period–WD mass relation

As stated above, one of the aims of the present paper is to explore the dependence of the  $P - M_{\text{WD}}$  relation upon the initial NS mass. Rappaport et al. (1995) claim that this relation should be fairly insensitive to changes in the initial NS mass. Their argument is based on the well-known fact that there exist a somewhat tight relation between the mass and the radius of the cores of low-mass giants (see e.g. Joss et al. 1987). Clearly, this is applicable only for the case of donor stars that undergo the onset of the RLOF as giants. Consequently, our calculations provide the opportunity to



**Figure 4.** The evolution of the orbital period for the systems depicted in Fig. 1. The meaning of lines are the same as in Fig. 2.

test the validity of the conclusions of Rappaport et al. (1995) in a quantitative way, at least for the case of HeWDs.

Let us repeat, for the sake of completeness, the argument of Rappaport et al. (1995) in detail. For the kind of binary systems studied in this work, the orbit is considered circular because of tidal dissipation since the onset of the first RLOF and should remain nearly circular thereafter. During later phases of mass transfer (once the mass of the donor star has become smaller than that of the NS), an approximate expression for the radius of the Roche lobe,  $R_L$ , in terms of the constituent masses is given by (Paczynski 1971)

$$R_L = 0.46a \left( 1 + \frac{M_{\text{NS}}}{M_G} \right)^{-1/3}, \quad (1)$$

where  $M_G$  is the mass of the giant and  $a$  is the orbital separation. If we combine equation (1) with Kepler's third law and set  $R_G = R_L$  (i.e. the giant fills its Roche lobe), we obtain an expression for the orbital period

$$P = 20 G^{-1/2} R_G^{3/2} M_G^{-1/2}. \quad (2)$$

Note that  $P$  is independent of the mass of the NS. Near the end of the mass transfer phase, the envelope of the giant is very tenuous and embraces a mass substantially smaller than that of the core,  $M_c$ ; then  $M_G \simeq M_c$ . Therefore, since  $R_G$  is a nearly unique function of  $M_c$ , the final orbital period at the termination of the mass transfer can be written as

$$P \simeq 20 G^{-1/2} R_G^{3/2}(M_c) M_c^{-1/2}. \quad (3)$$

In order to test the relation given by equation (3) against the observed set of binary pulsars containing low-mass WDs, it is important to establish an accurate theoretical core mass–radius relation ( $M_c - R_G$ ). Rappaport et al. (1995) have performed a systematic study of the core mass–radius relation from a series of single star evolutionary calculations. They covered a range of giant masses between  $0.8$  and  $2.0 M_{\odot}$  and different chemical compositions. They fitted the relation  $M_c - R_G$  with the empirical formula

$$R_G(m_c) = \left[ R_0 \frac{m_c^{4.5}}{1 + m_c^4} + 0.5 \right] R_{\odot} \quad (4)$$

where  $m_c = M_c/M_{\odot}$  and  $R_0$  is an adjustable constant that for the case of Population I objects takes the value of  $R_0 = 5500$ . Now, by combining the core mass–radius relation (equation 4) and

equation (3), and setting  $M_c = M_{\text{WD}}$ ,  $M_{\text{WD}}$  being the mass of the WD that has been the core of the giant before envelope dissipation, we obtain

$$P \simeq 0.374 \left[ R_0 \frac{m_{\text{WD}}^{4.5}}{1 + m_{\text{WD}}^4} + 0.5 \right]^{3/2} m_{\text{WD}}^{-1/2} d \quad (5)$$

where  $m_{\text{WD}} = M_{\text{WD}}/M_{\odot}$ .

Since then, other relations have been presented. Tauris & Savonije (1999) give the relation

$$m_{\text{WD}} = \left( \frac{P}{b} \right)^{1/a} + c, \quad (6)$$

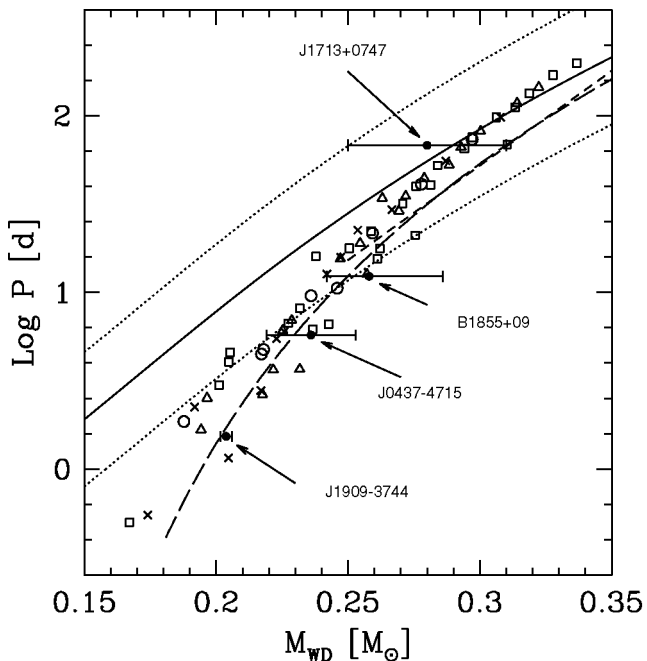
where, for the case of Population I, the authors find  $a = 4.50$ ,  $b = 1.2 \times 10^5$ ,  $c = 0.120$ , with  $P$  and  $b$  expressed in days. This fit is valid for  $0.18 \leq m_{\text{WD}} \leq 0.45$ . Also, Nelson, Dubeau & MacCannell (2004) stated the relation as

$$P = 0.1042 Z^{0.3} 10^{(10.7 m_{\text{WD}})} d, \quad (7)$$

where  $Z$  is the metal content of the donor star. In this case, the fit is valid for  $m_{\text{WD}} \geq 0.25$ .

In Fig. 5, we plot the  $P - M_{\text{WD}}$  relation for some of our models, where we also include the relations given in equation (5) for Population I with their error bars given by Rappaport et al. (1995) and also that of Tauris & Savonije (1999) (equation 6) and Nelson et al. (2004) (equation 7). As can be seen in Fig. 5, our evolutionary calculations agree with the prediction that in wide binaries the  $P - M_{\text{WD}}$  relation is fairly independent of the value of the initial NS mass.

In Fig. 5, we have not included some of our models. The criterion to include a model was simply if the position in the  $P - M_{\text{WD}}$  plane is fairly independent of time on a reasonably large time interval.



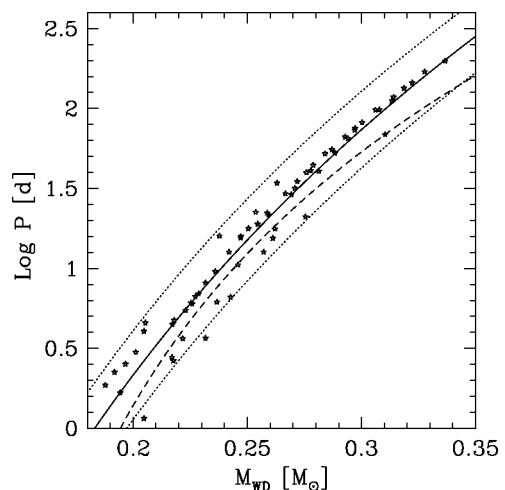
**Figure 5.** The  $P - M_{\text{WD}}$  relation for the binary systems presented in this work. Circles, crosses, triangles and squares depict systems with accreting NS of initial mass of 0.80, 1.00, 1.20 and 1.40  $M_{\odot}$  respectively. In addition, we plot with solid line the relation given by Rappaport et al. (1995) for Population I, with their error bars (dot lines), the relation of Tauris & Savonije (1999) with long-dashed line and the relation of Nelson et al. (2004) with short-dashed line. Also we have included the WD masses and orbital periods cited in Table 1 with the corresponding error bars.

For example, as stated above, some of our models evolve to ultra-compact systems with masses of only a few per cent of the solar mass. Even for the dimmest considered models, they are on a RLOF episode and thus move on the aforementioned plane. In any case, it is clear that these objects are quite different from those that represent our main interest. Notably, there are another kind of objects that do form a HeWD but on a very tight orbit. Data related to these objects are presented in Table 2 with numbers in italics. These systems are subject to strong orbital evolution due to gravitational wave radiation. As they are not on a RLOF episode, they evolve downwards vertically. Thus, in studying the  $P - M_{\text{WD}}$  relation we shall consider systems with a period  $P > 0.25$  d. For systems with  $P < 1$  d we considered the value of  $P$  at 13 Gyr, while for others this is an irrelevant detail.

Now we shall present a fit of our results in the  $P - M_{\text{WD}}$  plane. Notably, the values of  $\log m_{\text{WD}}$  have an approximate linear dependence upon  $\log P$ . Thus, we propose a linear fit by least squares. The fit we find is

$$P = B (m_{\text{WD}})^A d, \quad (8)$$

where  $A = 8.7078$ ,  $B = 2.6303 \times 10^6$  (see Fig. 6). In this figure, we also included, with dotted lines, the uncertainty associated with this fit corresponding to  $1\sigma$  deviation for the coefficients  $A$  and  $\log B$  for which  $(A, B) = (8.4948, 3.5372 \times 10^6)$  (upper curve) and  $(8.9208, 1.9559 \times 10^6)$  (lower curve). This relation is very similar to that of Tauris & Savonije (1999) (equation 6), although it accommodates to periods slightly longer. In any case, the differences between their fit (equation 6) and ours (equation 8) are smaller than the uncertainty in our fit. Thus, we consider that the agreement is fairly good. On the contrary, our fit (equation 8) is notably different from the one presented by Rappaport et al. (1995). While for  $m_{\text{WD}} \approx 0.3$  our calculations are in good agreement with their fit, this is not the case for lower WD mass values. This result is not surprising, simply because the fit presented by Tauris & Savonije (1999) is based on full binary evolution calculations, while that of Rappaport et al. (1995) relies on single stellar evolution results. Finally, in the range of masses  $m_{\text{WD}} > 0.25$ , the agreement between the relation of Nelson et al. (2004) (equation 7) and ours is also good although it is a bit poorer than for the case of the others analysed previously.



**Figure 6.** The fit of our results, performed with a linear function (equation 8) in the plane  $\log P - \log(M_{\text{WD}})$  denoted with a solid line. Upper and lower limits, showing the uncertainty inherent to our fit, are given with dotted lines. Short-dashed line represents the  $P - M_{\text{WD}}$  relation found by Tauris & Savonije (1999).

#### 4 APPLICATION TO MSP-WD SYSTEMS

In previous works (Benvenuto & De Vito 2005; Benvenuto et al. 2006), we have tried to identify possible binary system progenitors for some of the best observed MSP-WD systems (PSR J0437–4715, PSR J1713+0747 and PSR B1855+09). In those papers, we assumed a canonical value for the initial NS mass and varied the donor mass, orbital period and the value of  $\beta$  in order to account for the main observed characteristics (masses of the components, orbital period and, if available, the evolutionary status of the WD) of each system.

Let us now revisit this problem employing the set of models we present in this paper. Here, for the cases of PSR J1713+0747, PSR B1855+09 and PSR J1909–3744 we do not try to perform a detailed fit to the observed data as done in the aforementioned papers but only bracket plausible solutions. Some of them are possible because of the relaxation of the initial canonical NS mass value. In Table 4 we list, for given initial donor and NS masses, the initial period interval for which we expect plausible solutions for the observed parameters of the quoted systems. We remind the reader that these results are restricted to the case of  $\beta = 0.5$  and  $\alpha = 1$ .

For the case of PSR J1713+0747, here we do not find any solution corresponding to the case of  $(M_{\text{NS}})_i = 1.4 M_{\odot}$  as in Benvenuto et al. (2006). In that case, we found adequate configurations for  $\beta \lesssim 0.1$  but here, after RLOF episodes the NS becomes too massive. For the case of the best observed system, PSR J1909–3744, we also find plausible solutions but only for a  $(M_{\text{NS}})_i$  value well below  $1.4 M_{\odot}$ .

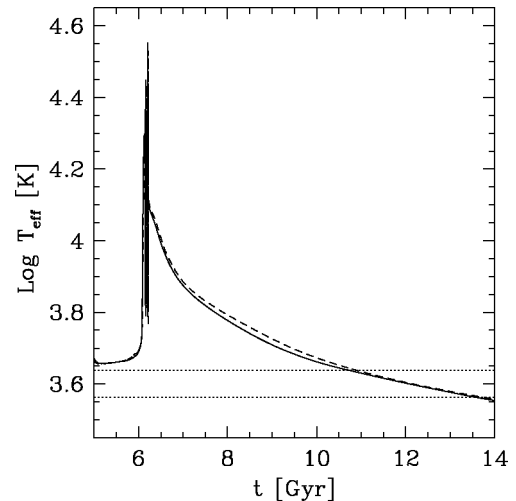
Let us perform a deeper analysis for the PSR J0437–4715 system. For this case we compute further evolutionary sequences, not included in Table 2, for which we allow for different values of  $\beta$  (although we still consider  $\alpha = 1$ ). We find two plausible solutions (see Table 5) that account for the main observed characteristic

**Table 4.** Some tentative initial conditions for the systems presented in Table 1, deduced from the results given in Table 2. The correct solution for each system should fall near the initial mass values and inside the period intervals listed below. Here, we considered donor stars with solar metallicity and for the orbital evolution we set  $\beta = 0.5$  and  $\alpha = 1$ . From left to right we list the system (pulsar) name, the plausible interval of initial orbital periods and the initial masses for the normal donor and accreting NS.

Name	$P_i$ (d)	$M_i$ ( $M_{\odot}$ )	$(M_{\text{NS}})_i$ ( $M_{\odot}$ )
PSR J1713+0747	6.00–12.00	1.00	1.00
PSR B1855+09	1.00–1.50	3.00	1.20
PSR J1909–3744	1.00–1.50	2.50	1.00

**Table 5.** Some possible initial conditions for the system containing PSR J0437–4715 and their main characteristics after evolution. Both systems correspond to an initial orbital period of  $P_i = 1$  d and an initial donor mass of  $1.25 M_{\odot}$ . From left to right we list the initial mass of the accreting NS, the value of  $\beta$ , the age and luminosity of the WD when its effective temperature is  $T_{\text{eff}} = 4000$  K, the final donor and NS masses, and the final orbital period. For further discussion see the main text.

$(M_{\text{NS}})_i$ ( $M_{\odot}$ )	$\beta$	$t$ (Gyr)	$\text{Log}(L/L_{\odot})$	$M_{\text{WD}}$ ( $M_{\odot}$ )	$M_{\text{NS}}$ ( $M_{\odot}$ )	$P$ (d)
1.20	0.25	12.081	–3.95	0.2235	1.4563	5.638
1.00	0.50	12.171	–3.93	0.2222	1.5131	5.059



**Figure 7.** The evolution of the effective temperature corresponding to the donor stars of the systems described in Table 5. Solid (dashed) line corresponds to the case of  $(M_{\text{NS}})_i = 1.2 M_{\odot}$  and  $\beta = 0.25$  [ $(M_{\text{NS}})_i = 1.0 M_{\odot}$  and  $\beta = 0.50$ ]. Horizontal dotted lines indicate the uncertainty in the effective temperature of the WD remnant. Evidently, both objects have a very similar behaviour and have acceptable effective temperatures at an age interval of 10–13 Gyr.

of the system. Both of them correspond to an initial donor mass of  $1.25 M_{\odot}$  and an initial period of 1 d. Regarding  $(M_{\text{NS}})_i$  and  $\beta$  the values are  $1.2 M_{\odot}$  and  $0.25$  or  $1.0 M_{\odot}$  and  $0.50$ , respectively. These binary systems provide correct masses (both values fall inside the corresponding error bars) and a very approximate orbital period.<sup>3</sup> Let us compare the effective temperature of the computed WDs with the observed value of  $T_{\text{eff}} = 4000 \pm 350$  K (Bell, Bailes & Bessell 1993). In Fig. 7, we show the evolution of the effective temperature of the donor star for both systems described in Table 5 together with the observed values’ interval. We find it possible for the WD to evolve to observed conditions within a time interval of 10–13 Gyr, shorter than (but of the order of) the age of the Universe. Remarkably, this corresponds to 4–7 Gyr after RLOF episodes, in nice agreement with the usual expectation that this should be comparable to the characteristic time-scale of pulsar rotation braking<sup>4</sup>  $\tau = 0.5P/\dot{P} \simeq 5$  Gyr, observed for PSR J0437–4715. It is interesting to note that the viable solutions presented in Table 5 correspond to very different values of  $(M_{\text{NS}})_i$  and  $\beta$ . The main difference between these evolved systems is the final value of  $M_{\text{NS}}$  but, unfortunately, the large uncertainty in the determination of  $M_{\text{NS}}$  inhibits us to restrict the space of parameters any further.

The situation is more promising for the case of PSR J1909–3744 system, whose mass determinations are far more accurate (see Table 1). In principle, this system offers an excellent opportunity to determine the initial configuration more accurately and even to find *mean* values for the parameters  $\alpha$  and  $\beta$ .

In this section, we have described possible solutions for the primordial configuration of binary systems that evolved to account for the best observed MSP–WD pairs. Performing a deeper analysis, considering variation of all parameters of the calculations (masses of

<sup>3</sup> Trying to fit the orbital period more accurately to the observed value does not change the presented values significantly. Thus, we do not perform a fine-tuning of the orbital period.

<sup>4</sup> For the case of PSR J0437–4715, the period derivative is  $\dot{P} = 5.72906 \times 10^{-20}$  (see e.g. van Straten et al. 2001).



the components, orbital period,  $\beta$  and  $\alpha$ ) is a very time-consuming exercise. We shall defer such an analysis for a future publication. In any case, we consider that the results presented here justify an effort in such direction.

## 5 DISCUSSION AND CONCLUSIONS

In this paper, we perform a set of binary evolution calculations assuming an initial configuration of a normal, solar composition, donor star in orbit together with a NS. In doing so, we consider a variety of values for the masses of the donor and NS stars as well as for the initial orbital period. These values are selected in order to consider systems that evolve to ultra-compact systems or to MSP–HeWD pairs. In most of the calculations, we considered that the NS accretes, at most, half of the matter lost by the donor star and that the material ejected from the pair carries away the specific angular momentum of the NS. While one of the main reasons for constructing this set of calculation is to provide a reference frame to analyse the initial configurations of the best observed WD–MSP systems, in particular those for which it has been possible to detect the Shapiro delay, here we pay special attention on testing the dependence of the evolution of these binary systems with the initial NS mass value. Also, we study the relation between the final orbital period and the mass of the HeWD remnant.

We find that the evolution of systems with a given orbital period and initial mass of the normal donor star heavily depends on the value of the NS mass. For example, we find cases for which, while with an initially light NS the system evolves to an ultra-compact configuration, if the NS is more massive it gives rise to a well-detached HeWD–NS pair. Also, as expected, we find divergent mass transfer rates (a CE episode) especially for the case of initially light NSs.

Our calculations show that the final orbital period–HeWD mass relation is insensitive to the initial NS mass value, as already claimed by Rappaport et al. (1995). In any case, we find some systematic departure from the relation proposed by them, especially for the case of low-mass HeWDs ( $M_{\text{WD}} < 0.25 M_{\odot}$ ). This occurs because for the systems that give rise to such objects, the onset of the initial mass transfer episode occurs before the star becomes a red giant (as assumed in Rappaport et al. 1995). The best fit to our results corresponds to equation (8). Among the period–WD mass relations available in the literature, we find a much better agreement of our results with that presented by Tauris & Savonije (1999).

Employing the set of evolutionary sequences given in this paper, we also present preliminary indications of the interval of initial periods, for fixed donor and NS initial masses, inside which there are plausible initial configuration for the binary systems listed in Table 1. In particular, we explore the case of the PSR J0437–4715 system, showing that there is more than one acceptable solution.

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