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Gonzalo E. Fernández (UNLP)

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by

Gonzalo E. Fernández

University of Illinois at Urbana-Champaign

484 Wohlers Hall

1206 South Sixth St.

Champaign, IL 61820

e-mail: gefernan@uiuc.edu

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Abstract

This paper analyzes a model of strategic tax competition with mobile capital and mobile identical consumers. The results of the model are compared to the traditional strategic tax competition model with immobile population. In addition to the fiscal and pecuniary externalities present in the standard model, a new effect shows up in the mobility model to affect provision of the public good. As with the pecuniary externality, this new effect depends on whether the jurisdictions are net exporters or net importers of capital. Thus, in a symmetric set up, the mobility effect along with the pecuniary externality disappear, yielding unambiguous underprovision of the public good. While in the asymmetric case both models have the same qualitative results, the mobility model strengthens the effects of the pecuniary externality. The above results are obtained by comparing the form of the first-order conditions between the mobility and immobility cases. The remaining question is whether or not the equilibrium levels of the public goods conform to the predicted tendencies. This question is answered with an example. The results of this exercise show that when the jurisdiction is a net exporter of capital, the level of the public good is lower in the mobility case than in the immobility case. However, if the jurisdiction is a net importer of capital, the public good level is sometimes higher and sometimes lower in the mobility case, contrary to predictions.

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1   Introduction

Since the mid-80s, a huge public finance literature has focused on the fiscal interaction among governments due to tax-base mobility, which generates what is known as "tax competition". In tax competition models, the analysis investigates the distortions that a tax on mobile capital causes in an economy where the tax revenue is used to finance public expenditure. The problem arises when one jurisdiction raises its capital-tax rate in order to increase the level of the public good. The net-of-tax return in that jurisdiction then falls below that prevailing in other jurisdictions, and capital relocates to other communities until net returns are equalized. This capital relocation is perceived as a cost by the community, which tends to reduce the level of provision of the public good.

The first studies analyzed models within a purely competitive setup. In this framework, there are a large number of competing communities, each providing, to a fixed immobile population, a local public good financed with a tax on capital employed locally. The total capital in the economy is fixed, but capital is mobile among jurisdictions. Jurisdictions are small relative to the economy, so that they do not affect the net-of-tax return to capital. Strategic interaction is then absent (tax rates of other communities are irrelevant), and each jurisdiction chooses its tax rate taking capital's net return as parametric. This model is analyzed by Beck (1983), Wilson (1986), and Zodrow and Mieszkowski (1986).

A more recent literature has investigated models of tax competition when strategic interaction among communities is present. In this case, jurisdictions are large relative to the economy.

Then each jurisdiction, by changing its capital-tax rate, is able to modify the net-of-tax return to capital (the capital outflow, due to a higher tax rate, is large enough to depress the net return). Therefore, to choose their optimal tax rates, jurisdictions take into account interjurisdictional capital flows and their effects on the net return to capital, viewing tax rates chosen by other jurisdictions as parametric. Wildasin (1988) and Bucovetsky (1991) are examples of this kind of model.

In both kinds of models, equilibrium is inefficient, with the public good typically underprovided. This outcome is generated by the presence of two externalities. First, each jurisdiction ignores the positive externality it generates when it increases the capital tax rate (a higher tax rate causes capital to flow to other jurisdictions). The second externality appears when jurisdictions are large. In this case, the increase in the capital tax rate in a given jurisdiction depresses the earnings of all capital owners by lowering the net return to capital. When all jurisdictions are alike (symmetric case) this second externality vanishes. The reason is that each jurisdiction is then a zero exporter of capital. However, equilibrium tax rates are too low because each jurisdiction ignores the external gains from capital flows caused by an increase in its tax rate. Even though the capital stock ends up evenly divided among jurisdictions in the symmetric case, jurisdiction's fears of tax base flight induce low tax rates. In equilibrium these fears are misplaced, and the resulting tax rates are too low.

Most of these models consider the population of each jurisdiction as fixed. However, some recent articles include consumer mobility in their models. See Hoyt (1991a, 1993), Burbidge and Myers (1994), Henderson (1994), and Wilson (1997). These papers analyze the effect that different taxes have in models where land is present, and where local governments choose the fiscal variables. Brueckner (2000) analyzes a perfectly competitive model where jurisdictions are formed by profit-maximizing community developers and heterogeneous consumers sort according to their preferences.

The purpose of the present paper is to analyze a model of strategic tax competition with a mobile population, while attempting to maintain other features of the standard model. To do so, consumers are viewed as homogeneous, in contrast to Brueckner (2000). In addition, land plays no role in the model, in contrast to the papers mentioned above.

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One departure from the standard setup, however, is the use of the community-developer model, which follows Brueckner (2000). The reason for using this model is that, when populations are mobile, it is not clear what is the right objective function for the community in a utility-maximizing framework. Is it the common utility level of the consumers, both within the community and outside? Is it the community population times the utility (i.e. "total utility" for community residents)? These issues disappear with the developer model since total profit is the unambiguous objective.

In this framework, developers maximize profit by choosing capital-tax rates and public good levels. Consumers are identical and each owns $\overline{k}$ units of capital. They are free to chose where to reside and where to invest their capital (it could be in a different jurisdiction). So developers compete strategically for capital, and they have to ensure that each consumer in the community reaches at least the level of utility that prevails in other jurisdictions.

The analysis of the equilibrium shows that, in addition to the two externalities explained above, there is a third effect that operates through population mobility. An explanation for this effect (in a context with two jurisdictions) is that when the tax rate in jurisdiction i increases, consumer income in jurisdiction j changes. The direction of this change depends on whether jurisdiction i is a net exporter or net importer of capital. In the first case, total income in jurisdiction j increases when the tax rate in jurisdiction i increases. Then the level of public good in jurisdiction i must be increased to keep the utility level in jurisdiction i equal to the utility level in the other jurisdiction. This increase in the level of public good is another cost of raising the tax rate. However, if jurisdiction i is a net importer, income in jurisdiction j falls when the tax rate in i is raised, and the public good level in jurisdiction i can be reduced. Therefore, the developer in jurisdiction i enjoys an added benefit from increasing the tax rate.

When jurisdictions are large enough and different (i.e. with different production functions), the results of the mobile population model differ from the results of the standard asymmetric tax competition model. The reason for this difference is the presence of the new mobility effect. If the community is a net exporter of capital, the extra cost imposed by the mobility of the population tends to generate greater underprovision than in the fixed-population setup.

In the symmetric case (when jurisdictions have the same production functions) the outcome

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of the model is equivalent to that in the standard fixed-population model. Since in equilibrium all jurisdictions are alike, each one is a zero of exporter of capital. Then the effect of the second externality explained above vanishes in both types of models. The new effect of free mobility also depends on the community being a net exporter or net importer of capital, so under symmetry it also is not present.

Section 2 presents the model and compares the results with those of the fixed-population tax competition model. Section 3 explores a particular example for the asymmetric case, and section 4 generalizes the model to J jurisdictions. Section 5 concludes.

2 The Model and Analysis of the Equilibrium

2.1 *The Model*

The economy is divided in two jurisdictions governed by community developers. The developers behave strategically, taking account of the effect of their tax-rate choices on the net return to capital. Competitive firms in each jurisdiction produce a numeraire private good, x_i, with a constant-returns technology. The production function for the private good in jurisdiction i is $F_i(K_i, N_i)$, where K_i gives the capital input in jurisdiction i and N_i is the labor input. Notice that since each individual inelastically supplies one unit of labor, N_i is also the population in jurisdiction i. Expressed in intensive form, the production function is $f_i(k_i) \equiv F_i(k_i, 1)$, where k_i equals capital per worker in jurisdiction i.

Community developers control provision of the public good, and they levy a tax per unit of capital to finance the provision of the good, with t_i denoting the tax rate in community i. Capital is freely mobile, so the net-of-tax return on capital, ρ, must be equal in both jurisdictions. That is,

$$f_1'(k_1) - t_1 = \rho \tag{1}$$

$$f_2'(k_2) - t_2 = \rho. \tag{2}$$

Let w_i denote the wage in jurisdiction i. Then,

$$w_1 = f_1(k_1) - k_1 f_1'(k_1) \tag{3}$$

$$w_2 = f_2(k_2) - k_2 f_2'(k_2). \tag{4}$$

The resource constraint is given by

$$\frac{N_1}{\overline{N}} k_1 + \frac{N_2}{\overline{N}} k_2 = \frac{\overline{K}}{\overline{N}}, \tag{5}$$

where $\overline{K}$ is the fixed total amount of capital in the economy and $\overline{N}$ is total population in the economy. Letting $\theta = \frac{N_1}{\overline{N}}$ denote the population share of jurisdiction 1, (5) can be written as

$$\theta k_1 + (1 - \theta)k_2 = \overline{k}. \tag{6}$$

Equations (1)-(4) and (6) determine k_1, k_2, w_1, w_2 and ρ as functions of t_1, t_2 and θ. Then community developers, through their choice of taxes, influence the levels of these variables. In particular, the effects of changing t_1 are[1]

$$\frac{\partial \rho}{\partial t_1} = - \frac{\theta f_2''(k_2)}{\theta f_2''(k_2) + (1 - \theta) f_1''(k_1)} < 0 \tag{7}$$

$$\frac{\partial k_1}{\partial t_1} = \frac{(1 - \theta)}{\theta f_2''(k_2) + (1 - \theta) f_1''(k_1)} < 0 \tag{8}$$

$$\frac{\partial k_2}{\partial t_1} = - \frac{\theta}{\theta f_2''(k_2) + (1 - \theta) f_1''(k_1)} > 0 \tag{9}$$

$$\frac{\partial w_1}{\partial t_1} = -k_1 f_1''(k_1) \frac{\partial k_1}{\partial t_1} = - \frac{(1 - \theta) k_1 f_1''(k_1)}{\theta f_2''(k_2) + (1 - \theta) f_1''(k_1)} < 0 \tag{10}$$

$$\frac{\partial w_2}{\partial t_1} = -k_2 f_2''(k_2) \frac{\partial k_2}{\partial t_1} = \frac{\theta k_2 f_2''(k_2)}{\theta f_2''(k_2) + (1 - \theta) f_1''(k_1)} > 0. \tag{11}$$

As indicated in the introduction, an increase in the tax rate in jurisdiction 1 lowers capital's net return ρ. Also, capital relocates, flowing from jurisdiction 1 to jurisdiction 2. This flow of capital causes wages to fall in the jurisdiction where the tax rate is increased and to rise in the other jurisdiction.

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[1]The effects of changing t_2 are analogous.

Turning to the remaining assumptions of the model, the private good can either be consumed directly as a private commodity, x, or used to produce a public good z at a constant cost of c per unit. Since z is a publicly provided private good, the cost of providing z_i units of the public good in jurisdiction i is $cN_i z_i$.

Consumers have identical preferences represented by a well-behaved utility function, $U(x_i, z_i)$. Private good consumption is equal to the consumer's wage income plus income from capital. Since it is assumed that ownership of the total stock of capital is equally shared among all individuals in the economy, utility can be written as $U(x_i, z_i) = U(w_i + \rho \overline{k}, z_i)$.

Developers collect taxes on capital, keeping any excess tax revenue as profits. So the objective function for jurisdiction i is $N_i(t_i k_i - c z_i)$. The developer's choice variables are t_i and z_i, the tax rate and public good level. In setting these variables, the developer takes into account the effects of his decision on capital usage, on wages, and on the after-tax return to capital. However, in maximizing profits, developers treat population as parametric, even though the population ultimately adjusts among jurisdictions to satisfy the equilibrium conditions. Consistent with their parametric view of population sizes, developers attempt to offer the inhabitants of their community the same utility level as residents of other communities. In doing this, they rule out the possibility of jurisdictional utility differentials that would lead to relocation of the population. Thus, the optimization problem for the developer in jurisdiction 1 is

$$\max_{(t_1, z_1)} \Pi_1 = max_{(t_1, z_1)} \{\theta \overline{N}(t_1 k_1 - c z_1)\}$$

subject to (1)-(4), (6), and the free mobility constraint

$$U(w_1 + \rho \overline{k}, z_1) = U(w_2 + \rho \overline{k}, z_2).$$

The developer of jurisdiction 2 faces a similar problem.

Taking into account equations (1)-(4) and (6), which determine k_1, k_2, w_1, w_2 and ρ as functions of t_1, t_2 and θ, the first-order conditions for the developer are

$$t_1: \quad \theta \overline{N}\left(k_1 + t_1 \frac{\partial k_1}{\partial t_1}\right) + \lambda \left[U_x(w_1 + \rho \overline{k}, z_1)\left(\frac{\partial w_1}{\partial t_1} + \overline{k}\frac{\partial \rho}{\partial t_1}\right) - U_x(w_2 + \rho \overline{k}, z_2)\left(\frac{\partial w_2}{\partial t_1} + \overline{k}\frac{\partial \rho}{\partial t_1}\right)\right] = 0$$
$$(12)$$

$$z_1: \quad -\theta\overline{N}c + \lambda U_z(w_1 + \rho\overline{k}, z_1) = 0 \tag{13}$$

$$\lambda: \quad U(w_1 + \rho\overline{k}, z_1) - U(w_2 + \rho\overline{k}, z_2) = 0, \tag{14}$$

where λ is the Lagrange multiplier for the "equal utility" constraint.

From equations (12) and (13) the following condition is obtained:

$$\frac{U_z(w_1 + \rho\overline{k}, z_1)}{U_x(w_1 + \rho\overline{k}, z_1)} = \frac{c}{k_1 + t_1\frac{\partial k_1}{\partial t_1}}\left\{k_1 + (k_1 - \overline{k})\left[1 + \frac{\theta}{(1-\theta)}\frac{U_x(w_2 + \rho\overline{k}, z_2)}{U_x(w_1 + \rho\overline{k}, z_1)}\right]\frac{\partial\rho}{\partial t_1}\right\}. \tag{15}$$

The interpretation of this condition is left until the next section.

Finally a free entry condition is imposed. It is assumed that there are many potential developers that may enter to compete for a particular jurisdiction if profits are positive. Then, in equilibrium, both jurisdictions earn zero profits.[2] Thus,

$$\Pi_1 = \theta\overline{N}(t_1 k_1 - cz_1) = 0. \tag{16}$$

Equations (14), (15), (16) and the analogous conditions for jurisdiction 2 determine the values of t_1, t_2, z_1, z_2, and θ.[3]

2.2 *Analysis of the Equilibrium*

In this section, the equilibrium conditions of the model are analyzed using as a benchmark the traditional strategic tax-competition model with fixed population. In that model, the government of each jurisdiction maximizes the utility of a representative individual taking into account the effects of its tax choice on ρ subject to a budget constraint.[4] Thus, the optimization problem is

$$\max_{t_1, z_1} U(x_1, z_1)$$

$$\text{s. t. } t_1 k_1 = cz_1 ,$$

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[2]This can be seen clearly in a model where developers bid for land to develop a jurisdiction. In this case, the presence of positive profits would cause bids for land to increase until profits are reduced to zero. For simplicity, however, land is not included in the model.

[3]Notice that there are five unknowns and five equations since equation (14) is repeated.

[4]See Wildasin (1988), Hoyt (1991b).

where k_1 is determined from (1), (2), and (6), and where $x_1 = w_1 + \rho\overline{k}$. The first order condition for this problem is

$$\frac{U_z(w_1 + \rho\overline{k}, z_1)}{U_x(w_1 + \rho\overline{k}, z_1)} = \frac{c}{k_1 + t_1\frac{\partial k_1}{\partial t_1}}\left[k_1 + (k_1 - \overline{k})\frac{\partial\rho}{\partial t_1}\right]. \qquad (17)$$

The comparison between models focuses on the provision of the public good. The relevant conditions that need to be compared are then equations (15) and (17).

Equation (17) shows that the marginal benefit differs from the marginal cost of production of the public good (c).[5] This is due to the presence of two externalities. The first is known in the tax-competition literature as a fiscal externality. In (17), jurisdiction 1 ignores the benefit in jurisdiction 2 from the relocation of capital when its tax rate increases. This effect is captured by the denominator expression on the RHS of (17), which is less than unity, tending to make the RHS greater than c. With the MRS tending to be greater than c, the fiscal externality tends to make equilibrium tax rates too low.

The second externality is known as a pecuniary externality.[6] This externality shows up when the jurisdictions are big enough to influence the "terms-of-trade" by changing ρ when they change their tax rates. As explained above, the direction of this effect, which is captured by the second term in brackets in (17), depends on whether the jurisdiction is a net importer or net exporter of capital. When the jurisdiction is a net importer of capital (when $k_i > \overline{k}$), the community benefits from a lower value of ρ. The government then has an extra incentive to increase the tax rate, and as a result, overprovision of the public good may occur. This follows because the second term in (17) is negative when $k_i > \overline{k}$, tending to decrease the RHS expression below c. On the other hand, net-exporter jurisdictions are harmed by the lower value of ρ caused by a higher capital tax. This effect aggravates underprovision of the public good (in this case, the second term in (17) is positive, reinforcing the tendency of the RHS to exceed c).

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[5]The marginal benefit is the sum of the resident's marginal willingness to pay for one more unit of z, $\sum U_z/U_x = N_1 U_z/U_x$. Since z is a publicly provided private good, the marginal cost of producing z for N_1 residents is cN_1. The N_1 then cancels out in (17).

[6]See DePater and Myers (1994).

These two externalities are also present in the mobility model. However, a third effect appears in equation (15), captured by the term involving the ratio of the U_x's. To see the origin of this term, note that when the tax rate is incremented in jurisdiction 1, consumer income in jurisdiction 2 $(w_2 + \rho\overline{k})$ changes. Using (7) and (11), this change is given by

$$\frac{\partial(w_2 + \rho\overline{k})}{\partial t_1} = \frac{\theta(k_2 - \overline{k})f_2''}{\theta f_2'' + (1-\theta)f_1''} \begin{cases} > 0 & \text{if jurisdiction 1 is a net exporter} \\ < 0 & \text{if jurisdiction 1 is a net importer.} \end{cases}$$

If jurisdiction 1 is a net exporter of capital, then $w_2 + \rho\overline{k}$ increases when t_1 increases. Utility then rises in jurisdiction 2, and z_1 must increase to maintain the utility equality. But this increase in z_1 reduces the developer's profit, lowering the benefit of raising t_1. So the mobility effect reinforces the effect of the pecuniary externality, strengthening the tendency of the net-exporter jurisdiction to keep the tax rate low, aggravating underprovision of the public good. However, if community 1 is a net importer, then $w_2 + \rho\overline{k}$ decreases with an increase of t_1, and z_1 can be reduced. As noted above, in this case the pecuniary externality from raising t_1 is beneficial, and the mobility effect adds another benefit. This strengthens the forces causing the net importer to raise its tax rate, increasing the tendency toward overprovision of the public good.

In the symmetric case, where production functions in both jurisdictions are identical, jurisdictions do not export or import capital in equilibrium. Therefore, like the pecuniary externality, the mobility effect vanishes in this case. Since $k_1 = k_2 = \overline{k}$ holds in equilibrium, (15) reduces to

$$\frac{U_z(w_1 + \rho\overline{k}, z_1)}{U_x(w_1 + \rho\overline{k}, z_1)} = \frac{c}{k_1 + t_1\frac{\partial k_1}{\partial t_1}}k_1 = \frac{c}{1 + (t_1/k_1)\frac{\partial k_1}{\partial t_1}}. \tag{18}$$

However, inspection of (17) shows that this equation also reduces to (18) in the symmetric case. Therefore, provided that communities are symmetric, the strategic equilibrium is unaffected by the presence of population mobility. This is an important result because it shows that, in the symmetric case, the main conclusions of the standard strategic model are robust to the presence of population mobility. Summarizing the preceding discussion yields

Proposition 1 *(i) In the symmetric case, where production functions are the same in both jurisdictions, the equilibrium in the mobile population model is the same as the equilibrium in the utility-maximizing model with fixed (and equal) populations.*

(ii) In the asymmetric case, when production functions are different across jurisdictions, the mobile population model predicts underprovision of the public good in the jurisdiction that is a net exporter of capital, but over or underprovision may occur in the net-importer jurisdiction. These results are the same as in the standard model.

(iii) The form of the first-order conditions suggests that, in the mobility case, the tendency to underprovide the public good is strengthened for the net-exporter jurisdiction, and that any tendency to overprovide the public good is strengthened for the net-importer jurisdiction.

While the first and second parts of Proposition 1 show that the same qualitative results apply to the models with and without population mobility, the third part is a statement based on a comparison of the form of the first-order conditions for the two types of models. However, as is well known from research in other areas of public economics, a comparison of the optimality *rules* across models does not necessarily translate into a straightforward comparison of the *levels* of the choice variables.[7]

To see this issue in the present contest, suppose that the allocation of capital per worker happened to be the same in the mobility and immobility cases, yielding the same values of k_1 and k_2 for these cases, as well as the same values of w_1 and w_2. Suppose further that $k_1 < k_2$, so that community 1 is the net exporter. Then, consider the question of whether the mobility equilibrium could have the same values of z_1 and z_2 as the immobility equilibrium. For this to be true, the values of t_1 and t_2 would have to be the same across equilibria, also implying a common value of ρ. Under these conditions, however, it is easily seen that the MRS expression in (15), which is equal to that in (17) given equality of arguments, is less than the RHS expression in (15). This conclusion follows because of the presence of the extra positive term on the RHS of (15) when community 1 is a net exporter. With the MRS less than the RHS expression in (15), the implication is that z_1 is too large, suggesting that its value must be reduced relative to the value in the immobility equilibrium to satisfy the optimality condition for the mobility case. Thus, the extent of underprovision in the net-exporting community should be larger in the mobility case. Reversing this argument for the case where community 1 is a net importer, it

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[7]See Atkinson and Stern (1974).

follows that the level of z_1 should be larger in the mobility case for a net-importer community. If the public good is overprovided in that community, this worsens the extent of overprovision. If underprovision occurs, however, the conclusion is that underprovision is less severe.

This discussion, however, is based on an assumption (identical capital stocks per worker in both equilibria) that typically will not be satisfied. As a result, the above predictions are only suggestive and may not actually hold. To investigate this issue further, the next section presents numerical examples comparing equilibria in the mobility and immobility cases.

3   An Example

To explore in more detail the differences between these two models in the asymmetric case (where production functions differ across jurisdictions), a particular example is presented. In the example, production functions are assumed to be quadratic, so that

$$f_1(k_1) = (V - bk_1)k_1 \tag{19}$$

$$f_2(k_2) = (W - dk_2)k_2. \tag{20}$$

Then, the return to capital in each jurisdiction is given by

$$V - 2bk_1 = \rho + t_1 \tag{21}$$

$$W - 2dk_2 = \rho + t_2, \tag{22}$$

and the return to labor by

$$w_1 = bk_1^2 \tag{23}$$

$$w_2 = dk_2^2. \tag{24}$$

The resource constraint is again

$$\theta k_1 + (1 - \theta)k_2 = \overline{k}, \tag{25}$$

where θ again is the share of the population in jurisdiction 1 and $\overline{k}$ is per capita capital in the economy. The consumer utility function is assumed to be linear, so that

$$U(x_i, z_i) = x_i + az_i. \tag{26}$$

This assumption, which is highly restrictive, is necessary to generate numerical solutions to the equilibrium conditions. Linearity yields a constant MRS equal to a on the LHS's of both (15) and (17), while also generating a unitary value for the ratio of U_x's in (15). These simplifications facilitate a solution to the respective equilibrium systems, which remain highly nonlinear.

From (21), (22), and (25) k_1, k_2, and ρ are obtained as functions of t_1, t_2 and θ:

$$k_1 = \frac{d\bar{k} + (1-\theta)(t_2 - t_1 + V - W)/2}{d\theta + b(1-\theta)} \tag{27}$$

$$k_2 = \frac{b\bar{k} - \theta(t_2 - t_1 + V - W)/2}{d\theta + b(1-\theta)} \tag{28}$$

$$\rho = V - t_1 - \frac{2b(d\bar{k} + (1-\theta)(t_2 - t_1 + V - W)/2)}{d\theta + b(1-\theta)}. \tag{29}$$

Solving the developer's optimization problem for this example, the equilibrium conditions (14), (15), and (16) (along with the analogous conditions for jurisdiction 2) reduce to

$$w_1 + az_1 = w_2 + az_2 \tag{30}$$

$$a = \frac{c}{k_1 + t_1 \frac{\partial k_1}{\partial t_1}}\left[k_1 + (k_1 - \bar{k})\frac{1}{(1-\theta)}\frac{\partial \rho}{\partial t_1}\right] \tag{31}$$

$$a = \frac{c}{k_2 + t_2 \frac{\partial k_2}{\partial t_2}}\left[k_2 + (k_2 - \bar{k})\frac{1}{\theta}\frac{\partial \rho}{\partial t_2}\right] \tag{32}$$

$$\theta\bar{N}(t_1 k_1 - cz_1) = 0 \tag{33}$$

$$(1-\theta)\bar{N}(t_2 k_2 - cz_2) = 0. \tag{34}$$

Equations (23), (24), (27)-(34), constitute a system of ten equations and ten unknowns (k_1, k_2, ρ, w_1, w_2, t_1, t_2, z_1, z_2, and θ) that can be solved for different values of the parameters of the model (c, a, V, W, b, and d).

The optimality conditions for the immobility model are given by (17) and the analogous conditions for the other jurisdiction. In this example, these conditions are

$$a = \frac{c}{k_1 + t_1 \frac{\partial k_1}{\partial t_1}} \left[k_1 + (k_1 - \overline{k}) \frac{\partial \rho}{\partial t_1} \right] \tag{35}$$

$$a = \frac{c}{k_2 + t_2 \frac{\partial k_2}{\partial t_2}} \left[k_2 + (k_2 - \overline{k}) \frac{\partial \rho}{\partial t_2} \right]. \tag{36}$$

Simultaneously solving equations (27)-(29), (35), and (36), the equilibrium values for k_1, k_2, t_1, t_2, and ρ are obtained as functions of the parameters of the problem (c, a, V, W, b, d, and θ[8]). All the other variables (z_1, z_2, w_1, w_2, x_1, x_2) are solved for by substitution.

To compare the two models, the following exercise is carried out. First, the mobility model is solved for given values of the parameters. Among other values, the equilibrium population share θ is obtained. Then this equilibrium θ value is taken as a parameter in solving the immobility model. In this way, both models have the same population distribution.

Solutions are obtained using Mathematica for the following parameter values: $c = 1$, $b = d = 1$, $V = 4$, $W = 4.01$. Note that $V < W$ implies that jurisdiction 2 is the more productive. The parameter a is changed from 1.1 to 1.5 to see what happens when the MRS is increased.

It is important to explain why $W = 4.01$ is chosen. After running some simulations for several values of W, it was found that the results of the mobility model are very sensitive to the difference between V and W. Incrementing this difference (holding the value of a constant) by more than 0.01128 leads to nonexistence of equilibrium.[9]

The results are presented in Tables 1-6. In Table 1, the case where $a = 1.1$ is shown. For the mobility model, the equilibrium values for θ, t_1, t_2, and ρ, are 0.209155, 0.272817, 0.412331, and 1.62476, respectively. The level of capital per worker in jurisdiction 1 (k_1) is 1.05121 and level of capital per worker in jurisdiction 2 (k_2) is 0.986456. Public good levels are 0.286788 for community 1 and 0.406746 for community 2. Finally, equilibrium values for wages, level of private good, and utilities in both communities are shown in the last six rows. The third column

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[8]In this model the population share is fixed.

[9]Calculations for $W = 4.01127$ are available upon request. The results are not presented in the paper because they follow the same pattern as those for $W = 4.01$.

displays the resulting values of the same variables for the immobility model, when $a = 1.1$ and θ is fixed at 0.209155.

The Tables show that, under the above parameters, jurisdiction 1 is the net importer of capital and jurisdiction 2 the net exporter. Then, as the theory predicts, the tax rate on capital in the net-importer jurisdiction is lower than in the net-exporter jurisdiction, and this is true for both models.

Comparing the results for both models, it can be observed that for the net-exporter jurisdiction, the level of public good is always (for any value of a) smaller in the mobility case than in the immobility case. So the prediction of section 2.1 seems to hold. Then, looking at the net-importer jurisdiction, one would expect to find that the level of public good is always greater in the mobility model. However, this is not the case. For example, for values of $a \leq 1.3$, the level of public good in the net-importer jurisdiction is lower in the mobility case. This outcome contradicts the prediction made above, and the reason is that, rather than being the same in both equilibria, the values of k_1 and k_2 are different.

Two additional observations can be made. First, notice that the capital level in the net-importer jurisdiction is always lower in the mobility case than in the immobility case. On the other hand, the capital level in the net-exporter jurisdiction is always higher under free mobility. This suggest that free population mobility smooths capital differences among jurisdictions.

Finally, looking at the resulting utility levels for the immobility model, it can be seen that the small community inhabitants are always better off. This result was shown by Bucovetsky (1991), who demonstrated that when jurisdictions are different and use quadratic production functions, people in the small jurisdiction reach a higher level of utility than people in the big jurisdiction.

4 Generalization of the model

Now consider a model with J jurisdictions. For each jurisdiction i, $k_i(\theta_1, ..., \theta_J, t_1, ..., t_J)$, $w_i(\theta_1, ..., \theta_J, t_1, ..., t_J)$, and ρ are determined by the following $2J + 1$ equations:

$$f_i'(k_i) = \rho + t_i, \qquad i = 1, ..., J \tag{37}$$

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$$w_i = f_i(k_i) - k_i f_i'(k_i), \qquad i = 1, ..., J \tag{38}$$

$$\sum_{i=1}^{J} \theta_i k_i = \overline{k}. \tag{39}$$

Differentiating equations (37) and (39) with respect to t_i yields

$$\frac{\partial k_i}{\partial t_i} = \frac{\sum_{j \neq i}(\theta_j/f_j''(k_j))}{f_i''(k_i)\sum_{j=1}^{J}(\theta_j/f_j''(k_j))} < 0 \tag{40}$$

$$\frac{\partial k_j}{\partial t_i} = - \frac{\theta_i}{f_i''(k_i)f_j''(k_j)\sum_{j=1}^{J}(\theta_j/f_j''(k_j))} > 0, \qquad \forall j \neq i \tag{41}$$

$$\frac{\partial \rho}{\partial t_i} = - \frac{\theta_i/f_i''(k_i)}{\sum_{j=1}^{J}(\theta_j/f_j''(k_j))} < 0. \tag{42}$$

Now the problem for a particular developer i is to

$$\max_{(t_i, z_i)} \Pi_i = max_{(t_i, z_i)}\{\theta_i \overline{N}(t_i k_i - cz_i)\}$$

s. t. (37) - (39), and

$$U(w_i + \rho\overline{k}, z_i) = U(w_j + \rho\overline{k}, z_j), \qquad \forall j \neq i \ .$$

The first order conditions of this problem are:

$$t_i: \quad \theta_i \overline{N}\left(k_i + t_i \frac{\partial k_i}{\partial t_i}\right) + \sum_{j \neq i} \lambda_j^i \left[U_x(w_i + \rho\overline{k}, z_i)\left(\frac{\partial w_i}{\partial t_i} + \overline{k}\frac{\partial \rho}{\partial t_i}\right) - U_x(w_j + \rho\overline{k}, z_j)\left(\frac{\partial w_j}{\partial t_i} + \overline{k}\frac{\partial \rho}{\partial t_i}\right)\right] = 0 \tag{43}$$

$$z_i: \quad -\theta_i \overline{N}c + \sum_{j \neq i} \lambda_j^i U_z(w_i + \rho\overline{k}, z_i) = 0 \tag{44}$$

$$\lambda_j^i: \quad U(w_i + \rho\overline{k}, z_i) - U(w_j + \rho\overline{k}, z_j) = 0, \qquad j = 1, ..., i-1, i+1, ..., J, \tag{45}$$

where the λ_j^i's are the J-1 Lagrange multipliers for jurisdiction i.

These J+1 conditions plus the zero profit condition for jurisdiction i ($t_i k_i - c z_i = 0$) determine the values of z_i, t_i, θ_i and the J-1 Lagrange multipliers as a function of taxes, public goods, and population shares of the other J-1 jurisdictions. Since there are J jurisdictions facing the same maximization problem, we have $2J + J^2$ unknowns ($z_1, ..., z_J$, $t_1, ..., t_J$, $\theta_1, ..., \theta_J$, $\lambda_2^1, ..., \lambda_J^1, ..., \lambda_1^j, ..., \lambda_{j-1}^j, \lambda_{j+1}^j, ..., \lambda_J^j, ..., \lambda_1^J, ..., \lambda_{J-1}^J$) and $2J + J^2$ equations.[10]

An equation analogous to (15) can be obtained for the general case from (43) and (44). This is

$$\frac{U_z(w_i + \rho\overline{k}, z_i)}{U_x(w_i + \rho\overline{k}, z_i)} = \frac{c}{k_i + t_i\frac{\partial k_i}{\partial t_i}}\left\{k_i + (k_i - \overline{k})\frac{\partial\rho}{\partial t_i} - \frac{1}{\left(\sum_{j\neq i}\lambda_j^i\right)}\left[\sum_{j\neq i}\lambda_j^i(k_j - \overline{k})\frac{U_x(w_j + \rho\overline{k}, z_j)}{U_x(w_i + \rho\overline{k}, z_i)}\frac{\partial\rho}{\partial t_i}\right]\right\}. \tag{46}$$

Now the extra term in the general model is more complicated than in the 2-jurisdiction model because of the presence of the different λ's. But in the symmetric case (where the production function is the same in all jurisdictions), $k_i = \overline{k}$ holds for all i, and the second and third terms in braces in (46) disappear, yielding (18) for $i = 1, ..., J$.[11] This confirms the first part of Proposition 1 for the generalized case. However, the second and third parts of the proposition do not follow.

5 Conclusion

In this paper, a model of strategic tax competition with mobile capital and mobile identical consumers is analyzed. The results of the model are compared to the traditional strategic tax competition model with immobile population.

As is well known, the presence of two externalities (a fiscal externality and a pecuniary externality) affects provision of the public good in the standard model. In the mobility model, in addition to those externalities, a new effect shows up. As with the pecuniary externality, this new effect depends on whether the jurisdictions are net exporters or net importers of capital.

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[10]Notice that some restrictions are repeated, then some Lagrange multipliers are equal, e.g. $\lambda_j^i = \lambda_i^j$. So there are $\binom{J}{2} = \frac{J!}{2!(J-2)!} = \frac{J(J-1)}{2}$ different restrictions, and the number of unknowns and equations is $3J + \frac{J(J-1)}{2}$.

[11]Notice that, the fact that $\theta_i = \frac{1}{J} = \theta$ holds in the symmetric case can be used along with (40) to yield $\frac{\partial k_i}{\partial t_i} = \frac{1-\theta}{f''(\overline{k})} = \frac{J-1}{J}\frac{1}{f''(\overline{k})}$. Equation (46) then collapses to $\frac{U_z}{U_x} = \frac{c}{\overline{k}+t_i(J-1)/Jf''(\overline{k})}\overline{k}$.

Thus, in a symmetric set up, the mobility effect along with the pecuniary externality disappear, yielding unambiguous underprovision of the public good.

While in the asymmetric case both models have the same qualitative results (underprovision of the public good if the jurisdiction is a net exporter of capital, and over or underprovision in the net-importer case), the mobility model strengthens the effects of the pecuniary externality. Then, if the jurisdiction is a net exporter (net importer) of capital, the tendency for underprovision (overprovision) is reinforced by the presence of population mobility.

The above results are obtained by comparing the form of the first-order conditions between the mobility and immobility cases. The remaining question is whether or not the equilibrium levels of the public goods conform to the predicted tendencies. This question revisits the "rules vs. levels" issue seen elsewhere in public economics. The question is answered with an example using quadratic production functions and linear utility functions. The results of this exercise show that when the jurisdiction is a net exporter of capital, the level of the public good is lower in the mobility case than in the immobility case (as Proposition 1 suggests). However, if the jurisdiction is a net importer of capital, the public good level is sometimes higher and sometimes lower in the mobility case, contrary to predictions.

Finally, the model is generalized for J jurisdictions. As in the standard model, underprovision of the public good is the outcome of the mobility model in the symmetric case. But nothing can be said in the asymmetric case, since the form of the first-order conditions are more complicated than in the 2-jurisdiction case.

Table 1.

Model	Mobility	Immobility
a	1.1	1.1
θ	0.209155	0.209155
t_1	0.272817	0.314504
t_2	0.412331	0.625519
ρ	1.62476	1.44744
k_1	1.05121	1.11903
k_2	0.986456	0.968521
z_1	0.286788	0.351939
z_2	0.406746	0.605828
w_1	1.10504	1.25223
w_2	0.973095	0.938033
x_1	2.7298	2.69967
x_2	2.59786	2.38547
$U(x_1, z_1)$	3.04527	3.0868
$U(x_2, z_2)$	3.04527	3.05188

Table 2.

Model	Mobility	Immobility
a	1.15	1.15
θ	0.381831	0.381831
t_1	0.474312	0.476533
t_2	0.562672	0.604445
ρ	1.47725	1.45058
k_1	1.02422	1.03644
k_2	0.98504	0.977489
z_1	0.4858	0.493898
z_2	0.554254	0.590838
w_1	1.04903	1.07421
w_2	0.970304	0.955485
x_1	2.52628	2.52479
x_2	2.44755	2.40606
$U(x_1, z_1)$	3.08495	3.09277
$U(x_2, z_2)$	3.08495	3.08553

Table 3.

Model	Mobility	Immobility
a	1.2	1.2
θ	0.42823	0.42823
t_1	0.630743	0.627786
t_2	0.701136	0.723361
ρ	1.33473	1.32328
k_1	1.01727	1.02446
k_2	0.987069	0.981677
z_1	0.641636	0.643142
z_2	0.69207	0.710107
w_1	1.03484	1.04952
w_2	0.974305	0.96369
x_1	2.36957	2.3728
x_2	2.30904	2.28697
$U(x_1, z_1)$	3.13952	3.14457
$U(x_2, z_2)$	3.13952	3.1391

Table 4.

Model	Mobility	Immobility
a	1.3	1.3
θ	0.462055	0.462055
t_1	0.896639	0.892368
t_2	0.95002	0.961054
ρ	1.08002	1.07606
k_1	1.01167	1.01578
k_2	0.989978	0.986442
z_1	0.907103	0.90645
z_2	0.940499	0.948024
w_1	1.02348	1.03181
w_2	0.980056	0.973068
x_1	2.1035	2.10787
x_2	2.06008	2.04913
$U(x_1, z_1)$	3.28273	3.28625
$U(x_2, z_2)$	3.28273	3.28156

Table 5.

Model	Mobility	Immobility
a	1.4	1.4
θ	0.474847	0.474847
t_1	1.12073	1.1169
t_2	1.16574	1.17296
ρ	0.860884	0.858911
k_1	1.00919	1.01209
k_2	0.991688	0.989064
z_1	1.13103	1.1304
z_2	1.15605	1.16013
w_1	1.01846	1.02433
w_2	0.983445	0.978248
x_1	1.81935	1.88324
x_2	1.84433	1.83716
$U(x_1, z_1)$	3.46279	3.4658
$U(x_2, z_2)$	3.46279	3.46134

Table 6.

Model	Mobility	Immobility
a	1.5	1.5
θ	0.481255	0.481255
t_1	1.31371	1.31042
t_2	1.35371	1.35904
ρ	0.670728	0.669546
k_1	1.00778	1.01002
k_2	0.992781	0.990707
z_1	1.32393	1.32355
z_2	1.34394	1.34641
w_1	1.01562	1.02014
w_2	0.985614	0.9815
x_1	1.68635	1.68969
x_2	1.65634	1.65105
$U(x_1, z_1)$	3.67225	3.67501
$U(x_2, z_2)$	3.67225	3.67066

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