Constructing Fuzzy Time Series Model Using Combination of Table Lookup and Singular Value Decomposition Methods and Its Application to Forecasting Inflation Rate

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ABSTRACT

Fuzzy time series is a dynamic process with linguistic values as its observations. Modelling fuzzy time series data developed by some researchers used discrete membership functions and table lookup method from training data. This paper presents a new method to modelling fuzzy time series data combining table lookup and singular value decomposition methods using continuous membership functions. Table lookup method is used to construct fuzzy relations from training data. Singular value decomposition of firing strength matrix and QR factorization are used to reduce fuzzy relations. Furthermore, this method is applied to forecast inflation rate in Indonesia based on six-factors one-order fuzzy time series. This result is compared with neural network method and the proposed method gets a higher forecasting accuracy rate than the neural network method.

Keywords : Fuzzy time series, table lookup, singular value decomposition, inflation rate.

INTRODUCTION

Fuzzy time series is a dynamic process with linguistic values as its observations. Many researchers have developed fuzzy time series model. Song & Chissom (1993a) developed fuzzy time series model based on fuzzy relational equation using Mamdani's method. In their paper, determination of the fuzzy relation needs large computation. Meanwhile, Song & Chissom (1993b, 1994) also constructed first order fuzzy time series for time invariant and time variant cases. This model needs complex computation for fuzzy relational equation. Furthermore, to overcome the weakness of the model, Chen (1996) designed fuzzy time series model by clustering of fuzzy relations.

Hwang *et al*. (1998) constructed fuzzy time series model to forecast the enrollment in Alabama University. Fuzzy time series model based on heuristic model gives more accuracy than its model designed by some previous researchers (Huarng 2001). Forecasting for enrollment in Alabama University based on high order fuzzy time series resulted high prediction accuracy (Chen 2002). First order fuzzy time series model is also developed by some researchers (Sah & Degtiarev 2004, Chen & Hsu 2004).

Forecasting inflation rate in Indonesia by fuzzy model resulted more accuracy than that by regression method (Abadi *et al.* 2006). Following the above paper, Abadi *et al.* (2007) also constructed fuzzy time series model using table lookup method to forecast interest rate of Bank Indonesia certificate and the result gave high accuracy. Abadi *et al.* (2008a, 2008b) showed that forecasting inflation rate using singular value method had a higher accuracy than that using Wang's method.

Abadi *et al*. (2008c) constructed a generalization of table lookup method using firing strength of rules and applied it in financial problems. Fuzzy time series model based on a generalized Wang's method was designed and it was applied to forecast interest rate of Bank Indonesia certificate that gave a higher prediction accuracy than using Wang's method (Abadi *et al*. 2009a). Furthermore, forecasting interest rate of Bank Indonesia certificate based on multivariate fuzzy time series data was done by Abadi *et al*. (2009b). Kustono *et al*. (2006) applied neural network method to forecast interest rate of Bank Indonesia certificate.

In this paper, we will design fuzzy time series model combining table lookup method and singular value decomposition using continuous membership functions to improve the prediction accuracy. This method is used to forecast inflation rate in Indonesia.

METHODS

QR factorization and singular value decomposition

In this section, we will introduce *QR* factorization and singular value decomposition of matrix and its properties referred from Scheick (1997). Let *B* be *m* x *n* matrix and suppose $m \le n$. The *QR* factorization of *B* is given by $B = QR$, where Q is m x m orthogonal matrix and $R = [R_{11}\ R_{12}]$ is *m* x *n* matrix with *m* x *m* upper triangular matrix R_{11}. The *QR* factorization of matrix *B* always exists and can be computed by Gram-Schmidt orthogonalization. Any *m* x *n* matrix *A* can be expressed as

$$A = USV^T, \qquad (1)$$

where *U* and *V* are orthogonal matrices of dimensions *m* x *m*, *n* x *n* respectively, and *S* is *m* x *n* matrix whose entries are 0 except $s_{ii} = \sigma_i$ $i = 1, 2, ..., r$ with $\sigma_1 \ge \sigma_2 \ge ... \ge \sigma_r \ge 0$, $r \le \min(m, n)$. Equation (1) is called a singular value decomposition (SVD) of *A* and the numbers σ_i are called singular values of A. If U_i, V_i are columns of *U* and *V* respectively, then equation (1) can be written as $A = \sum_{i=1}^{r} \sigma_i U_i V_i^T$.

Let $\| A \|_F^2 = \sum_{i,j} a_{ij}^2$ be the Frobenius norm of *A*. Since *U* and *V* are orthogonal matrices, then $\| U_i \| = 1$ and $\| V_i \| = 1$. Hence $\| A \|_F^2 = \left\| \sum_{i=1}^{r} \sigma_i U_i V_i^T \right\|_F^2 = \sum_{i=1}^{r} \sigma_i^2$. Let $A = USV^T$ be SVD of *A*. For given $p \le r$, the optimal rank *p* approximation of *A* is given by $A_p = \sum_{i=1}^{p} \sigma_i U_i V_i^T$.

Thus

$$\| A - A_p \|_F^2 = \left\| \sum_{i=1}^{r} \sigma_i U_i V_i^T - \sum_{i=1}^{p} \sigma_i U_i V_i^T \right\|_F^2,$$

$$= \left\| \sum_{i=p+1}^{r} \sigma_i U_i V_i^T \right\| = \sum_{i=p+1}^{r} \sigma_i^2$$

this means that A_p is the best rank *p* approximation of *A* and the approximation error depend only on the summation of the square of the rest singular values.

Designing fuzzy time series model using table lookup method

Let $Y(t) \subset R$, *t* = ..., -1, 0, 1, 2, ..., be the universe of discourse in which fuzzy sets $f_i(t)$ (i = 1, 2, 3,...) are defined. If $F(t)$ is the collection of $f_i(t)$, i = 1, 2, 3,..., then $F(t)$ is called fuzzy time series on $Y(t)$. Based on this definition, fuzzy time series $F(t)$ can be considered as a linguistic variable and $f_i(t)$ as the possible linguistic values of $F(t)$. The value of $F(t)$ can be different depending on time *t*. Therefore $F(t)$ is function of time *t*. Let $F_1(t)$ be fuzzy time series on $Y(t)$. If $F_1(t)$ is caused by $(F_1(t-1), F_2(t-1))$, $(F_1(t-2), F_2(t-2))$, . . . $(F_1(t-n), F_2(t-n))$, then *a* fuzzy logical relationship is presented by $(F_1(t-n), F_2(t-n)), \ldots, (F_1(t-2), F_2(t-2)),$ $(F_1(t-1), F_2(t-1)) \to F_1(t)$. The fuzzy logical relationship is called two-factors n-order fuzzy time series forecasting model, where $F_1(t), F_2(t)$ are called the main factor and the secondary factor fuzzy time series respectively. If a fuzzy logical relationship is presented as

$$\begin{aligned} &(F_1(t-n), F_2(t-n), ..., F_m(t-n)), ..., \\ &(F_1(t-2), F_2(t-2), ..., F_m(t-2)), \\ &(F_1(t-1), F_2(t-1), ..., F_m(t-1)) \to F_1(t), \end{aligned} \qquad (2)$$

then the fuzzy logical relationship is called m-factors n-order fuzzy time series forecasting model, where $F_1(t)$ is called the main factor fuzzy time series and $F_2(t), ..., F_m(t)$ are called the secondary factor fuzzy time series. The application of multivariate high order fuzzy time series can be found in Lee *et al*. (2006) and Jilani *et al*. (2007).

Like in modeling traditional time series data, training data are used to set up the relationship among data values at different times. In fuzzy time series, the relationship is different from that in traditional time series. In fuzzy time series, we exploit the past experience knowledge into the model. The experience knowledge has form "IF … THEN …". This form is called fuzzy rules. So fuzzy rules is the heart of fuzzy time series model. Furthermore, main step to modeling fuzzy time series data is to identify the training data using fuzzy rules.

Let $A_{1,k}(t-i), ..., A_{N_i,k}(t-i)$ be Ni fuzzy sets in fuzzy time series $F_k(t-i)$, *i* = 0,1, 2, 3,…, *n*, *k* = 1, 2, …, *m* and the membership functions of the fuzzy sets are continuous, normal and complete. The rule:

$$\begin{aligned} R^j &: IF\ (x_1(t-n)\ is\ A^j_{i_1,1}(t-n) \text{ and ...} \\ &\text{and } x_m(t-n) \text{ is } A^j_{i_m,m}(t-n)) \text{ and ...} \\ &\text{and } (x_1(t-1)\ is\ A^j_{i_1,1}(t-1) \text{ and ...} \\ &\text{and } x_m(t-1) \text{ is } A^j_{i_m,m}(t-1)) \text{ and ...} \\ &\text{and } (x_1(t-1)\ is\ A^j_{i_1,1}(t-1) \text{ and ...} \\ &\text{and } x_m(t-1) \text{ is } A^j_{i_m,m}(t-1)), \text{ THEN } x_1(t) \text{ is } A^j_{i_1,1}(t) \end{aligned} \qquad (3)$$

is equivalent to the fuzzy logical relationship (2) and vice versa. So (3) can be viewed as fuzzy relation in $U \times V$ where $U = U_1 \times ... \times U_{mn} \subset R^{mn}$, $V \subset R$ with $\mu_A(x_1(t-n),...,x_1(t-1),...,x_m(t-n),...,x_m(t-1)) = \mu_{A_{i_1,1}}(x_1(t-n))...\mu_{A_{i_1,1}}(x_1(t-1))...\mu_{A_{i_m,m}}(x_m(t-n)...\mu_{A_{i_m,m}}(t-1)$, where $A = A_{i_1,1}(t-n) \times ... \times A_{i_1,1}(t-1) \times ... \times A_{i_m,m}(t-n) \times ... \times A_{i_m,m}(t-1)$.

Let $F_1(t-1), F_2(t-1),...,F_m(t-1) \to F_1(t)$ be m-factors one-order fuzzy time series forecasting model. So $F_1(t-1), F_2(t-1),...,F_m(t-1) \to F_1(t)$ can be viewed as fuzzy time series forecasting model with m inputs and one output. In this paper, we will design m-factors one-order time invariant fuzzy time series model using table lookup and singular value decomposition methods. This method can be generalized to m-factors n-order fuzzy time series model. Table lookup method is used to construct fuzzy logical relationships and the singular value decomposition method is used to reduce unimportant fuzzy logical relationships.

Suppose we are given the following N training data: $(x_{1p}(t-1), x_{2p}(t-1),...,x_{mp}(t-1); x_{1p}(t))$, $p = 1,2,3,...,N$. Constructing fuzzy logical relationships from training data using the table lookup method is presented as follows:

Step 1. Define the universes of discourse for main factor and secondary factor. Let $U = [\alpha_1, \beta_1] \subset R$ be universe of discourse for main factor, $x_{1p}(t-1), x_{1p}(t) \in [\alpha_1, \beta_1]$ and V = $[\alpha_i, \beta_i] \subset R, i = 2,3,..., m$, be universe of discourse for secondary factors, $x_{ip}(t-1) \in [\alpha_i, \beta_i]$.

Step 2. Define fuzzy sets on the universes of discourse. Let $A_{1,k}(t-i),...,A_{N_i,k}(t-i)$ be N_i fuzzy sets in fuzzy time series $F_k(t-i)$. The fuzzy sets are continuous, normal and complete in $[\alpha_k, \beta_k] \subset R$, i =0,1, $k = 1,2,3,...,m$.

Step 3. Set up fuzzy relationships using training data. For each input-output pair $(x_{1p}(t-1), x_{2p}(t-1),...,x_{mp}(t-1); x_{1p}(t))$, determine the membership values of $x_{kp}(t-1)$ in $A_{i_k,k}(t-1)$ and membership values of $x_{1p}(t)$ in $A_{i_1,1}(t)$. Then, for each $x_{kp}(t-i)$, determine $A_{i_k^*,k}(t-i)$ such that $\mu_{A_{j^*,k}(t-i)}(x_{k,p}(t-i)) \geq \mu_{A_{j,k}(t-i)}(x_{k,p}(t-i))$, j = 1, 2, …, N_k. Finally, for each input-output pair, obtain a fuzzy logical relationship as $(A_{j_1^*,1}(t-1), A_{j_2^*,2}(t-1),...,A_{j_m^*,m}(t-1)) \to A_{i_1^*,1}(t)$.

If there are some fuzzy logical relationships having the same antecedent part but different consequent part, then the fuzzy logical relationships are called conflicting fuzzy relations. So we must choose one fuzzy logical relationship of conflicting group that has the maximum degree.

For a fuzzy logical relationship generated by the input-output pair $(x_{1p}(t-1), x_{2p}(t-1),...,x_{mp}(t-1); x_{1p}(t))$, we define its degree as

$$(\mu_{A_{j_1^*,1}(t-1)}(x_{1p}(t-1))\mu_{A_{j_2^*,2}(t-1)}(x_{2p}(t-1))...$$
$$\mu_{A_{j_m^*,m}(t-1)}(x_{mp}(t-1))\mu_{A_{i_1^*,1}(t)}(x_{1p}(t)).$$

From this step we have the following M collections of fuzzy logical relationships designed from training data:

$$R^l : (A^l_{j_1^*,1}(t-1), A^l_{j_2^*,2}(t-1),...,A^l_{j_m^*,m}(t-1)) \to A^l_{i_1^*,1}(t),$$
$l = 1, 2, 3, …, M.$ (4)

Step 4. Determine the membership function for each fuzzy logical relationship resulted in the Step 3. If each fuzzy logical relationship is viewed as a fuzzy relation in $U \times V$ with $U = U_1 \times ... \times U_m \subset R^m$, $V \subset R$, then the membership function for the fuzzy logical relationship (4) is defined by

$$\mu_{R^l}(x_{1p}(t-1), x_{2p}(t-1),...,x_{mp}(t-1); x_{1p}(t))$$
$$= \mu_{A_{j_1^*,1}(t-1)}(x_{1p}(t-1))\mu_{A_{j_2^*,2}(t-1)}(x_{2p}(t-1))...$$
$$\mu_{A_{j_m^*,m}(t-1)}(x_{mp}(t-1))\mu_{A^l_{i_1^*,1}(t)}(x_{1p}(t))$$

Step 5. For given fuzzy set input $A'(t-1)$ in input space U, establish the fuzzy set output $A'_l(t)$ in output space V for each fuzzy logical relationship (4) defined as

$$\mu_{A'_l}(x_1(t)) = \sup_{x \in U}(\mu_{A'}(x(t-1))\mu_{R^l}(x(t-1); x_1(t))),$$

where $x(t-1) = (x_1(t-1),...,x_m(t-1))$.

Step 6. Find out fuzzy set $A'(t)$ as the combination of M fuzzy sets $A'_1(t), A'_2(t), A'_3(t),...,A'_M(t)$ defined as

$$\mu_{A'(t)}(x_1(t)) = \max_{l=1}^{M}(\mu_{A'_1(t)}(x_1(t),...,\mu_{A'_M(t)}(x_1(t)))$$
$$= \max_{l=1}^{M}(\sup_{x \in U}(\mu_{A'}(x(t-1))\mu_{R^l}(x(t-1); x_1(t)))$$
$$= \max_{l=1}^{M}(\sup_{x \in U}(\mu_{A'}(x(t-1))\prod_{f=1}^{m}\mu_{A_{i_f,f}(t-1)}(x_f(t-1))\mu_{A^l_{i_1,1}}(x_1(t)))).$$

Step 7. Calculate the forecasting outputs. Based on the Step 6, if fuzzy set input $A'(t-1)$ is given, then the membership function of the forecasting output $A'(t)$ is

$$\mu_{A'(t)}(x_1(t)) =$$
$$\max_{l=1}^{M}(\sup_{x \in U}(\mu_{A'}(x(t-1))\prod_{f=1}^{m}\mu_{A_{i_f,f}(t-1)}(x_f(t-1))\mu_{A^l_{i_1,1}}(x_1(t)))). \quad (5)$$

Step 8. Defuzzify the output of the model. If the aim of output of the model is fuzzy set, then we stop at the Step 7. We use this step if we want the real output. For example, if fuzzy set input $A'(t-1)$ is given with Gaussian membership function $\mu_{A'(t-1)}(x(t-1)) = \exp(-\sum_{i=1}^{m}\frac{(x_i(t-1)-x_i^*(t-1))^2}{a_i^2})$,

then the forecasting real output using the Step 7 and center average defuzzifier is

$$x_1(t) = f(x_1(t-1),...,x_m(t-1)) = \frac{\sum_{j=1}^{M} y_j \exp(-\sum_{i=1}^{m}\frac{(x_i(t-1)-x_i^{*j}(t-1))^2}{a_i^2+\sigma_{i,j}^2})}{\sum_{j=1}^{M}\exp(-\sum_{i=1}^{m}\frac{(x_i(t-1)-x_i^{*j}(t-1))^2}{a_i^2+\sigma_{i,j}^2})} \quad (6)$$

where y_j is center of the fuzzy set $A_{i_1,1}^j(t)$.

The procedure to design fuzzy time series model using table lookup method is shown in Figure 1.

Reducing unimportant fuzzy logical relationships using SVD-QR factorization method

If the number of training data is large, then the number of fuzzy logical relationships may be large too. So increasing the number of fuzzy logical relationships will add the complexity of computation. To overcome the complexity of computation, we will apply singular value decomposition method to reduce the fuzzy logical relationships using the following steps.

Step 1. Set up the firing strength of the fuzzy logical relationship (4) for each training datum (x;y) = $(x_1(t-1), x_2(t-1),...,x_m(t-1); x_1(t))$ as follows

$$L_1(x;y) = \frac{\prod_{f=1}^{m}\mu_{A_{i_f,f}(t-1)}(x_f(t-1))\mu_{A_{i_1,1}^l}(x_1(t))}{\sum_{k=1}^{M}\prod_{f=1}^{m}\mu_{A_{i_f,f}(t-1)}(x_f(t-1))\mu_{A_{i_1,1}^k}(x_1(t))}$$

Step 2. Construct N x M matrix

$$L = \begin{pmatrix} L_1(1) & L_2(1) & \cdots & L_M(1) \\ L_1(2) & L_2(2) & \cdots & L_M(2) \\ \vdots & \vdots & \vdots & \vdots \\ L_1(N) & L_2(N) & \cdots & L_M(N) \end{pmatrix}.$$

Step 3. Compute singular value decomposition of L as $L = USV^T$, where U and V are N x N and M x M orthogonal matrices respectively, S is N x M matrix whose entries $s_{ij} = 0, i \neq j$, $s_{ii} = \sigma_i$ $i = 1, 2, \ldots, r$ with $\sigma_1 \geq \sigma_2 \geq ... \geq \sigma_r \geq 0$, $r \leq \min(N, M)$.

Step 4. Determine the number of fuzzy logical relationships that will be taken as k with $k \leq \text{rank}(L)$.

Step 5. Partition V as $V = \begin{pmatrix} V_{11} & V_{12} \\ V_{21} & V_{22} \end{pmatrix}$, where V_{11} is k x k matrix, V_{21} is (M-k) x k matrix, and construct $\bar{V}^T = \begin{pmatrix} V_{11}^T & V_{21}^T \end{pmatrix}$.

Step 6. Apply QR-factorization to $\bar{V}^T$ and find M x M permutation matrix E such that $\bar{V}^T E = QR$, where Q is k x k orthogonal matrix, R = [R_{11} R_{12}], R_{11} is k x k upper triangular matrix.

Step 7. Assign the position of entries one's in the first k columns of matrix E that indicate the position of the k most important fuzzy logical relationships.

Step 8. Construct fuzzy time series forecasting model (5) or (6) using the k most important fuzzy logical relationships.

RESULTS AND DISCUSSION

Figure 2 shows the procedure to design fuzzy time series model using combination of table lookup and *SVD* methods. The method is applied to forecast the inflation rate in Indonesia based on six-factors one-order fuzzy time series model. The main factor is inflation rate and the secondary factors are the interest rate of Bank Indonesia certificate, interest rate of deposit, money supply, total of deposit and exchange rate. The data of the factors are taken from January 1999 to February 2003. The data from January 1999 to January 2002 are used to training and the data from February 2002 to February 2003 are used to testing.

In this paper, we will predict the inflation rate of k^{th} month using data of inflation rate, interest rate of Bank Indonesia certificate, interest rate of deposit, money supply, total of deposit and exchange rate of (k-1)th month. The universes of discourse of interest rate of Bank Indonesia certificate, interest rate of deposit, exchange rate, total of deposit, money supply, inflation rate are defined as [10, 40], [10, 40], [6000, 12000], [360000, 460000], 40000, 90000], [-2, 4] respectively.

We define fuzzy sets that are continuous, complete and normal on the universe of discourse such that the fuzzy sets can cover the input spaces. We define sixteen fuzzy sets $B_1, B_2,..., B_{16}$, sixteen fuzzy sets $C_1, C_2,..., C_{16}$, twenty five fuzzy sets $D_1, D_2,..., D_{25}$, twenty one fuzzy sets $E_1, E_2,..., E_{21}$, twenty one fuzzy sets $F_1, F_2,..., F_{21}$, thirteen fuzzy sets $A_1, A_2,..., A_{13}$ on the universes of discourse of the interest rate of Bank Indonesia certificate, interest rate of deposit, exchange rate, total of deposit, money supply, inflation rate respectively. All fuzzy sets are defined by Gaussian membership function. Then, we set up fuzzy logical relationships based on training data resulting 36 fuzzy relations in the form:

$$(B^{l}_{j_2}(t-1), C^{l}_{j_3}(t-1), D^{l}_{j_4}(t-1), E^{l}_{j_5}(t-1),$$
$$F^{l}_{j_6}(t-1), A^{l}_{j_1}(t-1)) \to A^{l}_{j_*}(t)$$

Table 1. Six-factors one-order fuzzy logical relationship groups for inflation rate using table lookup method.

rule	$((x_2(t-1), x_3(t-1), x_4(t-1), x_5(t-1), x_6(t-1), x_1(t-1)) \to x_1(t)$						
1	(B14,	C14,	D13,	E12,	F2,	A11)	→ A8
2	(B15,	C14,	D12,	E13,	F2,	A8)	→ A5
3	(B15,	C14,	D12,	E13,	F3,	A5)	→ A4
4	(B14,	C13,	D10,	E15,	F2,	A4)	→ A4
5	(B10,	C11,	D9,	E16,	F2,	A4)	→ A4
6	(B7,	C8,	D4,	E13,	F2,	A4)	→ A3
7	(B4,	C5,	D5,	E13,	F2,	A3)	→ A3
8	(B3,	C3,	D7,	E11,	F3,	A3)	→ A4
9	(B3,	C2,	D11,	E10,	F4,	A4)	→ A5
10	(B3,	C2,	D5,	E4,	F4,	A5)	→ A5
11	(B3,	C2,	D7,	E9,	F4,	A5)	→ A8
12	(B2,	C2,	D5,	E6,	F8,	A8)	→ A8
13	(B2,	C2,	D7,	E7,	F5,	A8)	→ A5
14	(B2,	C2,	D7,	E7,	F5,	A5)	→ A4
15	(B2,	C1,	D7,	E7,	F5,	A4)	→ A6
16	(B1,	C1,	D9,	E7,	F5,	A6)	→ A7
17	(B2,	C1,	D11,	E8,	F6,	A7)	→ A6
18	(B2,	C1,	D12,	E4,	F7,	A6)	→ A8
19	(B3,	C1,	D13,	E3,	F7,	A8)	→ A6
20	(B3,	C2,	D10,	E2,	F7,	A6)	→ A4
21	(B3,	C2,	D12,	E3,	F8,	A4)	→ A7
22	(B3,	C2,	D15,	E6,	F8,	A7)	→ A8
23	(B3,	C2,	D15,	E7,	F8,	A8)	→ A9
24	(B3,	C2,	D15,	E7,	F14,	A9)	→ A6
25	(B3,	C2,	D15,	E9,	F9,	A6)	→ A7
26	(B3,	C3,	D16,	E11,	F9,	A7)	→ A7
27	(B4,	C3,	D19,	E13,	F9,	A7)	→ A6
28	(B4,	C3,	D24,	E15,	F10,	A6)	→ A7
29	(B4,	C3,	D21,	E14,	F10,	A7)	→ A8
30	(B4,	C3,	D23,	E14,	F11,	A8)	→ A9
31	(B5,	C3,	D15,	E10,	F12,	A9)	→ A5
32	(B5,	C3,	D12,	E10,	F13,	A5)	→ A6
33	(B5,	C5,	D16,	E12,	F13,	A6)	→ A6
34	(B5,	C5,	D19,	E16,	F12,	A6)	→ A8
35	(B5,	C5,	D19,	E17,	F14,	A8)	→ A8
36	(B5,	C5,	D19,	E18,	F16,	A8)	→ A9

Table 1 presents all fuzzy logical relationships. To know the k most important fuzzy logical relationships, we apply the singular value decomposition method to matrix L of firing strength of the fuzzy logical relationships in Table 1 for each training datum,

where $L = \begin{pmatrix} L_1(1) & L_2(1) & \cdots & L_{36}(1) \\ L_1(2) & L_2(2) & \cdots & L_{36}(2) \\ \vdots & \vdots & \vdots & \vdots \\ L_1(36) & L_2(36) & \cdots & L_{36}(36) \end{pmatrix}$.

There are thirty four nonzero singular values of L. The distribution of the singular values of L can be seen in Figure 3. The singular values of L decrease strictly after the first twenty nine singular values (Figure 3). Based on properties of SVD, the error of training data can be decreased by taking more singular values of L but this may cause increasing error of testing data.

The error of training data depends on the summation of the square of the rest singular values. So we must choose the appropriate k singular values. In this paper, we choose the first eight, twenty, and twenty nine singular values. To get permutation matrix E, we apply the QR factorization and then assign the position of entries one's in the first k columns of matrix E that indicate the position of the k most important fuzzy logical relationship. The mean square error (MSE) of training and testing data from the different number of reduced fuzzy logical relationships are shown in Table 2.

The mean square error (MSE) of training and testing data from the different number of reduced fuzzy logical relationships are shown in Table 2.

Table 2. Comparison of MSE of training and testing data using the different methods.

Method	Number of fuzzy relations	MSE of training data	MSE of testing data
Proposed method	8	0.485210	0.66290
	20	0.312380	0.30173
	29	0.191000	0.21162
Table lookup method	36	0.063906	0.30645
Neural network	-	0.757744	0.42400

Based on Table 2, fuzzy time series model (5) or (6) designed by table lookup method gives the smallest error of training data. If we use 29 fuzzy logical relationships, then we have the smallest error of testing data.

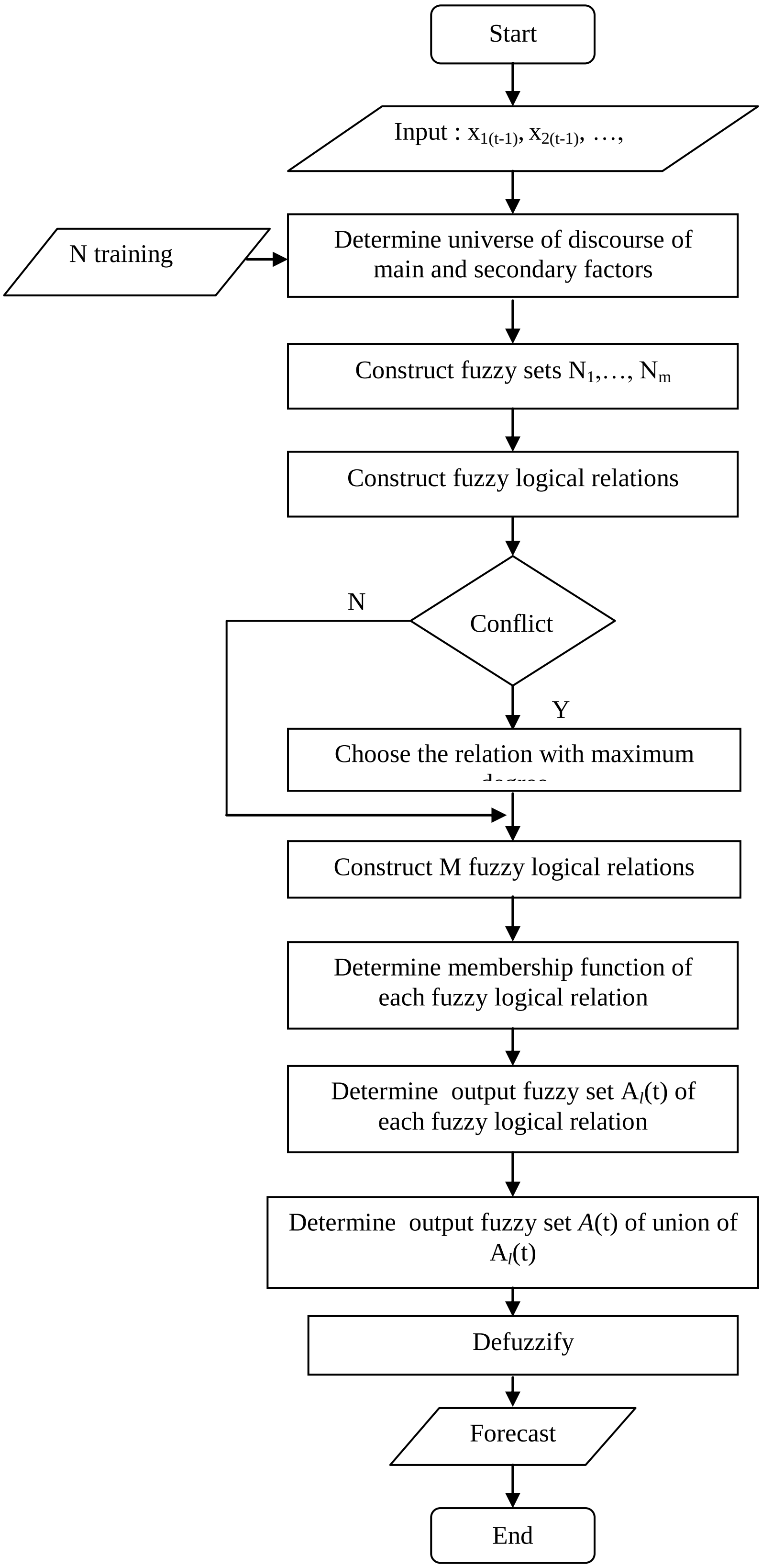


Figure 1. The procedure of forecasting fuzzy time series data using table lookup method.

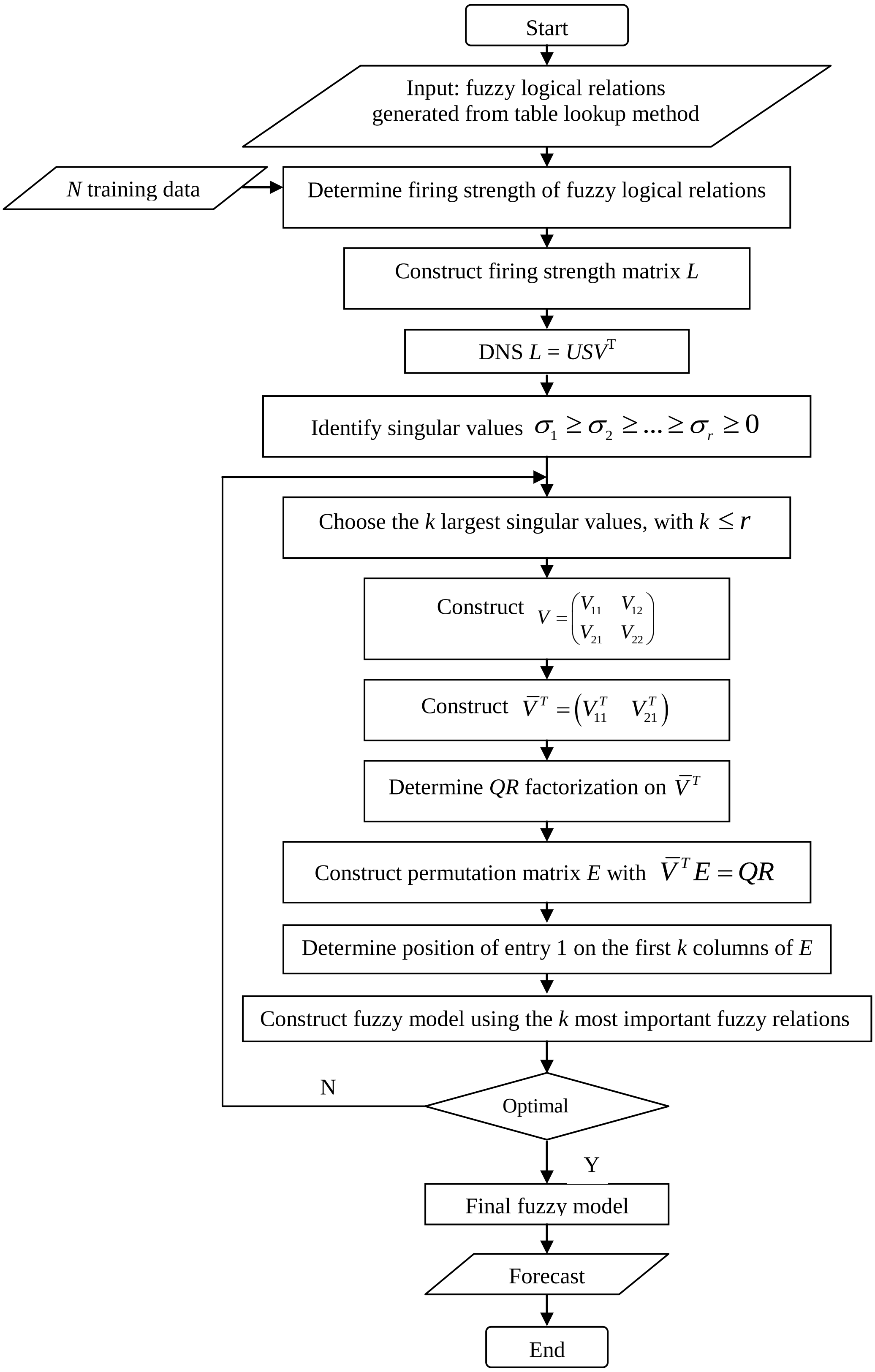


Figure 2. The procedure of forecasting fuzzy time series data using combination of table lookup and *SVD- QR* factorization methods.

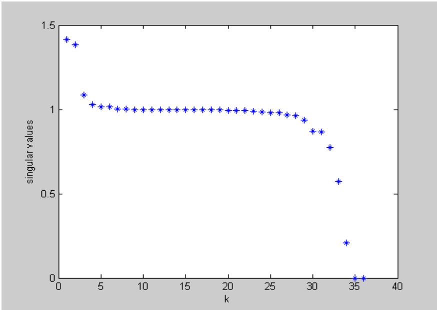
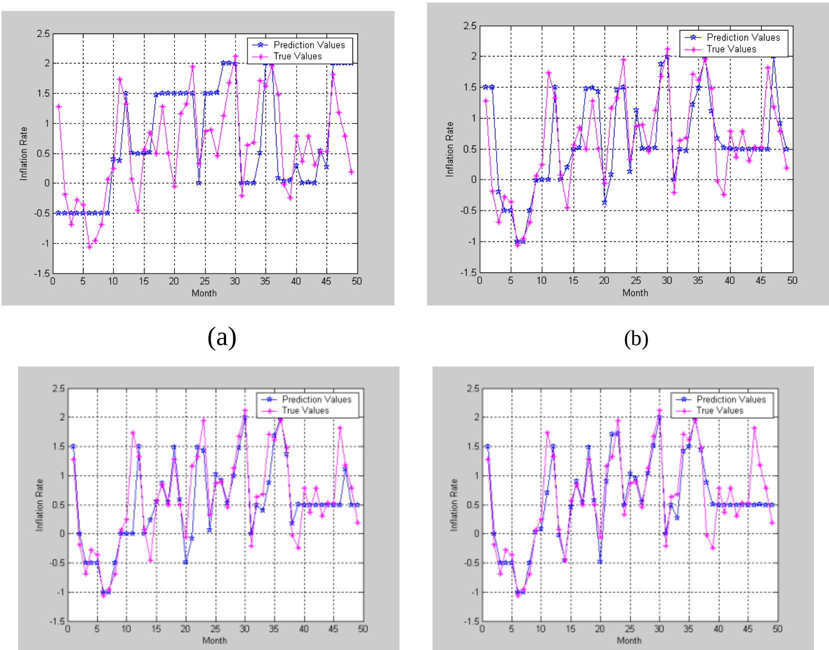


Figure 3. Distribution of singular values of matrix L.



(a) (b)

(c) (d)

Figure 4. Prediction and true values of inflation rate using proposed method: (a) eight fuzzy logical relationships, (b). twenty fuzzy logical relationships, (c) twenty nine fuzzy logical relationships, (d) thirty six fuzzy logical relationships.

So to forecasting inflation rate, fuzzy time series model (5) or (6) designed by 29 fuzzy logical relationships gives a better accuracy than that designed by 8 and 20 fuzzy logical relationships. Furthermore, Table 2 shows that in predicting inflation rate, the proposed method results a better accuracy than the neural network method.

Figure 4(c) illustrates that prediction accuracy is improved by choosing 29 fuzzy logical relationships. The plotting of prediction and true values of inflation rate using different number of fuzzy logical relationships is shown in Figure 4.

CONCLUSION

In this paper, we have presented a new method to design fuzzy time series model. The method combines the table lookup and *SVD-QR* factorization methods. Based on the training data, the table lookup method is used to construct fuzzy logical relationships and we apply the singular value decomposition and *QR*-factorization methods to the firing strength matrix of the fuzzy logical relationships to remove the less important fuzzy logical relationships. The proposed method is applied to forecast the inflation rate. Furthermore, predicting inflation rate using the proposed method gives more accuracy than that using the table lookup and neural network methods. The precision of forecasting depends also to taking factors as input variables and the number of defined fuzzy sets. In the next work, we will design how to select the important input variables to improve prediction accuracy.

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