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OPTIMAL OBSERVER-BASED CONTROL DESIGN FOR AIRCRAFT LATERAL MOTION

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The paper presents an alternative approach of observer-based flight control system design. This approach is based on the application of linear matrix inequality technique. The design procedure treats the design of both observer and controller by solving the set of linear inequalities simultaneously. The proposed approach is free of observer poles placement location. Simulation results demonstrate the validity and effectiveness of the proposed design approach

Key words: aircraft lateral motion; flight control system; linear matrix inequalities; observer design; state feedback; separation principle; state estimation; performance index

Introduction

The motion of aircraft is considered in a non-uniform atmosphere. Therefore, the use of control strategy is necessary for completing aircraft mission successful. The application of modern control theory requires all the states variables to be available. Thus, the control systems developed due to modern control theory application increase the complexity of the system. To overcome the requirement of complete state space vector measurements, observer-based control systems design have been considered. The development of observer-based control system reduces the requirement of full phase vector measurements. Observers avoid complexity to the system and require only computational resources [1]. The observer design was originally proposed in works [2-4]. Lately, the numbers of observer-based control system design approaches were proposed [5-7].

The design of observer-based flight control systems are successfully applied in the area of small Unmanned Aerial Vehicles (UAV), satisfying manifold requirements imposed on them [5, 8-9]. The observer-based control system design approaches were proposed in [5-7]. In [5] the design strategy involves observer design without reducing the robustness and performance of the system. The required level of performance and robustness is kept due to mixed $\mathbf{H}_2/\mathbf{H}_\infty$ optimization technique.

The survey on observer design is given in [10].

It is shown three main observer design results connected with reduced order, under separation principle and observers for input fault detection and identification.

The autopilot design is also may be performed basing on the available information about the output variables. This circumstance leads to the problem of static output feedback (SOF) controller design. The main advantage of SOF design is that it requires only available signals from the plant to be controlled. Unfortunately, the output feedback problem is much more difficult to solve in comparison to state feedback control problem [11].

The motivation for this research arises from a desire to reduce the number of sensors necessary for multivariable flight control system (FCS) design for civil aircraft. The research concerns on finding appropriate solution under linear matrix inequalities (LMIs) approach [12] for aircraft control during flight envelope.

It is known that the design procedure of observer deals with selecting desired region poles location. Moreover, the observer eigenvalues should be faster up to ten times in comparison to plant eigenvalues. It results in the observer sensitivity to noisy measurement, which is not desirable. To overcome this difficulty procedure of observer design is proposed basing on Lyapunov approach.

The main result of this paper is the FCS design via LMI technique, where the observer gains and controller structure are defined by solving the set of LMIs, simultaneously.

To demonstrate the validity and efficiency of the proposed approach, the lateral motion of the aircraft is considered as a case study.

Problem Statement

Let us consider a problem of FCS design with incomplete state vector measurement. The aircraft dynamics is represented by the following set of equations

$$\begin{cases} \dot{\mathbf{x}} = \mathbf{A}\mathbf{x} + \mathbf{B}\mathbf{u} \\ \mathbf{y} = \mathbf{C}\mathbf{x} \end{cases} \qquad \mathbf{x}(0) = \mathbf{x}_0, \qquad (1)$$

where $\mathbf{x} \in \mathbf{R}^{\mathbf{n}}$ is the state space vector; $\mathbf{u} \in \mathbf{R}^{\mathbf{m}}$ is the control vector; $\mathbf{y} \in \mathbf{R}^{\mathbf{p}}$ is the observation vector.

Besides that, the state space matrices of the controlled plant have the following dimensions $\mathbf{A} \in \mathbf{R}^{\mathbf{n}\times\mathbf{n}}, \mathbf{B} \in \mathbf{R}^{\mathbf{m}\times\mathbf{n}}, \mathbf{C} \in \mathbf{R}^{\mathbf{p}\times\mathbf{n}}$. It could be seen that number of measuring variables $\mathbf{p}$ is less than number of all phase coordinates, $\mathbf{n}$ The FCS design is utilized in terms of restricted number of state variables measurements. The autopilot design includes reconstruction of sate space vector by using a state observer.

In this paper we develop the design procedure of full-order state observer design with further state feedback construction such that the performance of closed-loop system satisfies selected performance criterion. Thus, the FCS construction is performed under the well-known separation principle.

Observer-based Control System Design via Linear Matrix Inequality Approach

It is known that the observer estimates the state variables based on measurement of the output $\mathbf{y}$ and control $\mathbf{u}$ variables [2] – [4]. Let us consider the procedure of observer-based flight control system design under LMI approach.

Consider linear time-invariant system given by (1). Assume that the states $\mathbf{x}$ are approximated by the states $\hat{\mathbf{x}}$. The observer model takes into account feedback information about observation error and can be represented as

$$\begin{aligned} \dot{\hat{\mathbf{x}}}(t) &= \mathbf{A}\hat{\mathbf{x}}(t) + \mathbf{B}\mathbf{u}(t) + \mathbf{L}\left(\hat{\mathbf{y}}(t) - \mathbf{y}(t)\right) \\ &= \mathbf{A}\hat{\mathbf{x}}(t) + \mathbf{B}\mathbf{u}(t) + \mathbf{L}\mathbf{C}\left(\hat{\mathbf{x}}(t) - \mathbf{x}(t)\right), \end{aligned} \qquad (2)$$

where $\left(\mathbf{x}(t) - \hat{\mathbf{x}}(t)\right) = \mathbf{e}(t)$ is a difference between the real and estimated states (observation error); $\mathbf{L}$ is the observer gain matrix that has to be chosen such that the observation error approaches zero as time increases. From (1) and (2) the observation error equation dynamics takes the following form

$$\dot{\mathbf{e}}(t) = \left(\dot{\mathbf{x}}(t) - \dot{\hat{\mathbf{x}}}(t)\right);$$

$$\begin{aligned} \dot{\mathbf{x}}(t) - \dot{\hat{\mathbf{x}}}(t) &= \mathbf{A}\mathbf{x}(t) + \mathbf{B}\mathbf{u}(t) \\ &\quad - \left(\mathbf{A}\hat{\mathbf{x}}(t) + \mathbf{B}\mathbf{u}(t) + \mathbf{L}\mathbf{C}\left(\hat{\mathbf{x}}(t) - \mathbf{x}(t)\right)\right) \\ &= (\mathbf{A} + \mathbf{L}\mathbf{C})\mathbf{e}(t). \end{aligned} \qquad (3)$$

The error decays to zero if it is possible to find observer gain matrix $\mathbf{L}$ such that $(\mathbf{A} + \mathbf{L}\mathbf{C})$ is asymptotically stable. Moreover, the eigenvalues of $(\mathbf{A} + \mathbf{L}\mathbf{C})$ are the same as those of $(\mathbf{A} + \mathbf{L}\mathbf{C})^{\mathrm{T}} = \mathbf{A}^{\mathrm{T}} + \mathbf{C}^{\mathrm{T}}\mathbf{L}^{\mathrm{T}}$.

The final goal is to control the motion of the plant basing on the estimated states. Thus, for the state feedback control based on observed state variables $\hat{\mathbf{x}}$, namely

$$\mathbf{u} = \mathbf{K}\hat{\mathbf{x}}, \qquad (4)$$

where $\mathbf{K}$ is a constant state feedback gain matrix that assures that the system is asymptotically stable, the state equation becomes

$$\begin{aligned}\dot{\mathbf{x}}(t) &= \mathbf{A}\mathbf{x}(t)+\mathbf{B}\mathbf{K}\hat{\mathbf{x}}(t)=\mathbf{A}\mathbf{x}(t)\\ &+\mathbf{B}\mathbf{K}\left(\mathbf{x}(t)-\mathbf{e}(t)\right)=(\mathbf{A}+\mathbf{B}\mathbf{K})\mathbf{x}(t)-\mathbf{B}\mathbf{K}\mathbf{e}(t).\end{aligned} \quad (5)$$

Combining together (3) and (5), we obtain

$$\begin{bmatrix}\dot{\mathbf{x}}(t)\\ \dot{\mathbf{e}}(t)\end{bmatrix}=\begin{bmatrix}\mathbf{A}+\mathbf{B}\mathbf{K} & -\mathbf{B}\mathbf{K}\\ 0 & \mathbf{A}+\mathbf{L}\mathbf{C}\end{bmatrix}\begin{bmatrix}\mathbf{x}(t)\\ \mathbf{e}(t)\end{bmatrix}. \quad (6)$$

Equation (6) describes the dynamics of the observed state feedback control system. The characteristic equation for the system is

$$\left|s\mathbf{I}-\mathbf{A}-\mathbf{B}\mathbf{K}\right|\left|s\mathbf{I}-\mathbf{A}-\mathbf{L}\mathbf{C}\right|=0.$$

It is possible to rewrite the system dynamics in terms of plant and observer states, respectively.

$$\begin{bmatrix}\dot{\mathbf{x}}(t)\\ \dot{\hat{\mathbf{x}}}(t)\end{bmatrix}=\begin{bmatrix}\mathbf{A} & \mathbf{B}\mathbf{K}\\ -\mathbf{L}\mathbf{C} & \mathbf{A}+\mathbf{B}\mathbf{K}+\mathbf{L}\mathbf{C}\end{bmatrix}\begin{bmatrix}\mathbf{x}(t)\\ \hat{\mathbf{x}}(t)\end{bmatrix}.$$

It is supposed also that the obtained solution given by (4) minimizes performance index as

$$\begin{aligned}J &= \int_0^{\infty}\left(\hat{\mathbf{x}}(t)^{\mathrm{T}}\mathbf{Q}\hat{\mathbf{x}}(t)+\mathbf{u}(t)^{\mathrm{T}}\mathbf{R}\mathbf{u}(t)\right)dt\\ &=\int_0^{\infty}\hat{\mathbf{x}}(t)^{\mathrm{T}}\left(\mathbf{Q}+\mathbf{K}^{\mathrm{T}}\mathbf{R}\mathbf{K}\right)\hat{\mathbf{x}}(t)\,dt,\end{aligned} \quad (7)$$

where $\mathbf{Q}$ and $\mathbf{R}$ are diagonal matrices, weighting each state and control variables, respectively. This cost depends on the trajectory, of $\hat{\mathbf{x}}(t)$, taken, such that the worst trajectory will correspond to the worst cost [13]. The state feedback problem is to select $\mathbf{K}$ for the desired closed-loop properties and minimize the upper bound $\mathbf{x}_0^{\mathrm{T}}(t)\mathbf{P}\mathbf{x}_0(t)$ of the cost function J in (7).

It is known that the observed-state feedback control system design consists of two stages: (1) design of state feedback control law assuming that all states are available; (2) design a state estimator to estimate states of the system. Replace the states in state feedback control law from stage (1) by the state estimates. Further, they can be combined to form the observed-state feedback control system. This principle of independent state feedback and observer design is referred to as separation principle. Moreover, the observer design deals with choice of poles location. They are usually chosen such that the observer response is much faster that the system response, but very fast observers possess with noise. The proposed approach solves the problem of observed-state feedback design under LMI technique. The main advantage of the proposed design procedure is that there is no need to define the observer poles location. The solution of this problem via LMIs gives the constant state feedback gain matrix $\mathbf{K}$ and observer gain $\mathbf{L}$ by solving the set of LMIs simultaneously. The proposed design procedure is very simple and utilizes Lyapunov approach.

The simultaneous observer and controller design can be formulated with following theorem.

Theorem. The observer-based system (6) is said to be statically stable via state feedback (4) if there exist matrices $\mathbf{X}_1=\mathbf{X}_1^{\mathrm{T}}>0$, $\mathbf{M}$, $\mathbf{X}_2=\mathbf{X}_2^{\mathrm{T}}>0$, $\mathbf{Z}$, and minimizes γ by satisfying the following conditions:

$$\begin{bmatrix}\mathbf{X}_1\mathbf{A}^{\mathrm{T}}+\mathbf{A}\mathbf{X}_1+\mathbf{M}^{\mathrm{T}}\mathbf{B}^{\mathrm{T}}+\mathbf{B}\mathbf{M} & \mathbf{X}_1\mathbf{Q}^{1/2} & \mathbf{M}^{\mathrm{T}}\mathbf{R}^{1/2}\\ \mathbf{Q}^{1/2}\mathbf{X}_1 & -\mathbf{I} & 0\\ \mathbf{R}^{1/2}\mathbf{M} & 0 & -\mathbf{I}\end{bmatrix}<0, \quad (8),$$

$$\begin{bmatrix}\gamma & \mathbf{x}_0^{\mathrm{T}}\\ \mathbf{x}_0 & \mathbf{X}_1\end{bmatrix}\geq 0 \quad (9),$$

$$\mathbf{A}^{\mathrm{T}}\mathbf{X}_2+\mathbf{X}_2\mathbf{A}+\mathbf{C}^{\mathrm{T}}\mathbf{Z}^{\mathrm{T}}+\mathbf{Z}\mathbf{C}<0,\ \mathbf{X}_2=\mathbf{X}_2^{\mathrm{T}}>0. \quad (10)$$

Proof. Let $\mathbf{V}_1(\mathbf{x},t)=\mathbf{x}(t)\mathbf{P}_1\mathbf{x}^{\mathrm{T}}(t)$ with $\mathbf{P}_1=\mathbf{P}_1^{\mathrm{T}}>0$ be a candidate Lyapunov function. The closed loop system (6) preserves stability and minimizes performance index (7) if:

$$\dot{\mathbf{V}}_1(\mathbf{x},t)+\mathbf{x}^{\mathrm{T}}(t)\mathbf{Q}\mathbf{x}(t)+\mathbf{u}^{\mathrm{T}}(t)R\mathbf{u}(t)<0. \quad (11)$$

The condition (11) leads to the following inequality:

$$\mathbf{x}^{\mathrm{T}}(t)\left\{\mathbf{A}^{\mathrm{T}}\mathbf{P}_1+\mathbf{P}_1\mathbf{A}+\mathbf{K}^{\mathrm{T}}\mathbf{B}^{\mathrm{T}}\mathbf{P}_1+\mathbf{P}_1\mathbf{B}\mathbf{K}+\mathbf{Q}+\mathbf{K}^{\mathrm{T}}\mathbf{R}\mathbf{K}\right\}\times\mathbf{x}(t)<0.$$

Pre-multiplying and post-multiplying right and left sides above written inequality by $\mathbf{P}^{-1}$:

$$\mathbf{P}_1^{-1}\mathbf{A}^{\mathrm{T}}+\mathbf{A}\mathbf{P}_1^{-1}+\mathbf{P}_1^{-1}\mathbf{K}^{\mathrm{T}}\mathbf{B}^{\mathrm{T}}+\mathbf{B}\mathbf{K}\mathbf{P}_1^{-1} + \mathbf{P}_1^{-1}\mathbf{Q}\mathbf{P}_1^{-1}+\mathbf{P}_1^{-1}\mathbf{K}^{\mathrm{T}}\mathbf{R}\mathbf{K}\mathbf{P}_1^{-1}<0. \quad (12)$$

Let us define the following change of variables $\mathbf{X}_1=\mathbf{P}_1^{-1}, \mathbf{M}=\mathbf{K}\mathbf{P}_1^{-1}, \mathbf{K}=\mathbf{M}\mathbf{P}_1$ and rewrite inequality (12) as

$$\mathbf{X}_1\mathbf{A}^{\mathrm{T}}+\mathbf{A}\mathbf{X}_1+\mathbf{M}^{\mathrm{T}}\mathbf{B}^{\mathrm{T}}+\mathbf{B}\mathbf{M}+\mathbf{X}_1\mathbf{Q}\mathbf{X}_1+\mathbf{X}_1\mathbf{K}^{\mathrm{T}}\mathbf{R}\mathbf{K}\mathbf{X}_1<0 \quad (13)$$

By applying Shur's Lemma to inequality (13), it is possible to rewrite as linear matrix inequality:

$$\begin{bmatrix} \mathbf{X}_1\mathbf{A}^{\mathrm{T}}+\mathbf{A}\mathbf{X}_1+\mathbf{M}^{\mathrm{T}}\mathbf{B}^{\mathrm{T}}+\mathbf{B}\mathbf{M} & \mathbf{X}_1\mathbf{Q}^{1/2} & \mathbf{M}^{\mathrm{T}}\mathbf{R}^{1/2} \\ \mathbf{Q}^{1/2}\mathbf{X}_1 & -\mathbf{I} & 0 \\ \mathbf{R}^{1/2}\mathbf{M} & 0 & -\mathbf{I} \end{bmatrix}<0.$$

Once again, using the Schur complement, the cost

$$\mathbf{x}_0^{\mathrm{T}}\mathbf{P}_1\mathbf{x}_0=\mathbf{x}_0^{\mathrm{T}}\mathbf{X}_1^{-1}\mathbf{x}_0\le\gamma,$$

is expressed as the LMI

$$\begin{bmatrix} \gamma & \mathbf{x}_0^{\mathrm{T}} \\ \mathbf{x}_0 & \mathbf{X}_1 \end{bmatrix}\ge 0.$$

This part of the proof considers the design stage (1) according to the separation principle. The second part of the proof considers stage (2) of the design procedure connected with observer construction.

Let $V_2(\mathbf{e}(t),t)=\mathbf{e}(t)\mathbf{P}_2\mathbf{e}^{\mathrm{T}}(t)$ with $\mathbf{P}_2=\mathbf{P}_2^{\mathrm{T}}>0$ be a candidate Lyapunov function. The observer gains can be found if the following inequality is hold:

$$\mathbf{e}^{\mathrm{T}}(t)\left\{(\mathbf{A}+\mathbf{L}\mathbf{C})^{\mathrm{T}}\mathbf{P}_2+\mathbf{P}_2(\mathbf{A}+\mathbf{L}\mathbf{C})\right\}\mathbf{e}(t)<0,$$

or

$$\mathbf{A}^{\mathrm{T}}\mathbf{P}_2+\mathbf{P}_2\mathbf{A}+\mathbf{C}^{\mathrm{T}}\mathbf{L}^{\mathrm{T}}\mathbf{P}_2+\mathbf{P}_2\mathbf{L}\mathbf{C}<0.$$

The use of the following change of variables $\mathbf{X}_2=\mathbf{P}_2$, $\mathbf{P}_2\mathbf{L}=\mathbf{Z}$ reduces to the next LMIs:

$$\mathbf{A}^{\mathrm{T}}\mathbf{X}_2+\mathbf{X}_2\mathbf{A}+\mathbf{C}^{\mathrm{T}}\mathbf{Z}^{\mathrm{T}}+\mathbf{Z}\mathbf{C}<0,$$

$$\mathbf{X}_2=\mathbf{X}_2^{\mathrm{T}}>0.$$

Thus, the observer gains can be evaluated as

$$\mathbf{L}=\mathbf{X}_2^{-1}\mathbf{Z}.$$

The design procedure reduces to solving the set of inequalities (8)–(10) simultaneously. The resulting observer-based control system operates with desired level of performance and brings minimum to (7).

Case Study

The state space linearized lateral model of large four-fanjet Boeing 747 aircraft flying about equilibrium point (Mach=0.650) is used as a case study.

The main geometrical characteristics of the given aircraft are:

– wing reference area, S = 5500 ft^2;
– wing span, b = 195.68 ft;
– mean geometric chord, $\bar{c}=27.31\,ft$;

The moments of inertia:

$$I_x=18.2\times10^6\,slug-ft^2;$$

$$I_y=33.1\times10^6\,slug-tf^2;$$

$$I_z=0.97\times10^6\,slug-ft^2;$$

The state space vector of Boeing 747 lateral channel comprises the following variables: $\mathbf{x}=[b \quad p \quad r \quad j \quad y]^{\mathrm{T}}$ where β is a sideslip angle, p is a roll rate, r is a yaw rate, φ is a roll angle, ψ is yaw angle. The control input vector $\mathbf{u}=[d_a \quad d_r]^{\mathrm{T}}$ is represented by aileron and rudder deflections, respectively.

It is considered operating mode with true airspeed at V_t = 67.4 m/s. The linear model in the state space is represented by the matrices $[\mathbf{A},\mathbf{B}]$:

$$\mathbf{A}=\begin{bmatrix} -2.1219 & 0 & -1.0000 & 0.1455 & 0 \\ -5.4500 & -1.4700 & 0.2560 & 0 & 0 \\ 1.8200 & -0.0214 & -0.3440 & 0 & 0 \\ 0 & 1.0000 & 0 & 0 & 0 \\ 0 & 0 & 1.0000 & 0 & 0 \end{bmatrix};$$

$$\mathbf{B} = \begin{bmatrix} 0 & 0.0213 \\ 0.3720 & 0.3180 \\ 0.0371 & -0.9700 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}.$$

The output vector of measured variables is given as follows $\mathbf{y}_{est} = [p \quad r \quad j \quad y]^{\mathrm{T}}$. Thus, the observation matrix has the following structure:

$$\mathbf{C} = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}.$$

Disturbance, $\boldsymbol{\upsilon}$ affecting the lateral motion of the aircraft involves the following components: the sideslip angle, β, roll rate, p and the yaw rate, r, so that $\boldsymbol{\upsilon} = \left[\beta_g \;\; p_g \;\; r_g\right]^{\mathrm{T}}$. In order to simulate the atmospheric turbulence the Dryden filter is used [13]. It is considered that aircraft flies at moderate turbulence.

The transfer functions of forming filter according to standard MIL–F–8785C [13], [14] used in simulation to account external disturbances have the following structure:

$$H_v(s) = s_v \sqrt{\frac{L_v}{pV}} \cdot \frac{1 + \frac{\sqrt{3} L_v}{V} s}{\left(1 + \frac{L_v}{V} s\right)^2}; \qquad H_r(s) = \frac{\mp \frac{s}{V}}{(1 + (\frac{3b}{pV}) s)} \cdot H_v(s). \qquad H_p(\mathrm{s}) = \sigma_w \sqrt{\frac{0.8}{V}} \frac{\left(\frac{\pi}{4b}\right)^{1/6}}{L_w^{1/3} \left[1 + \left(\frac{4b}{\pi V}\right) \mathrm{s}\right]}.$$

The transfer function of forming filter along the variable v is possible to rewrite in terms of sideslip angle, β according to the phase vector [13], [15]. Thus, for small angles

$$\beta = \frac{\nu}{U_0}, \text{ where } U_0 = V_t.$$

Parameters appearing in the transfer functions of the forming filters are given as follows [13], [14]:

b = 59.64 m; $L_v = L_w$ = 533.3 m;

$\sigma_v = \sigma_w$=1.542 m/s.

The variable b represents the aircraft wingspan. The variables L_v, L_w represent the turbulence scale lengths. The variables σ_v, σ_w represent the turbulence intensities. The computation of these values depends on the altitude at which the aircraft is flying, wing span and type of turbulence according to standard MIL–F–8785C [14].

The weighting matrices $\mathbf{Q}$, $\mathbf{R}$ in (7) have the structure:

$$\mathbf{Q} = \mathrm{diag}([9.6012 \quad 1.5456 \quad 0.6600 \quad 0.0030 \quad 4.500]);$$

$$\mathbf{R} = \mathrm{diag}([8 \quad 0.4]).$$

By applying proposed approach of observer based controller design under LMI-technique, the state feedback gain matrix $\mathbf{K}$ and observer gains $\mathbf{L}$ are found. Their numerical values are given below:

– state feedback gain matrix:

$$\mathbf{K} = \begin{bmatrix} 0.0867 & -0.0359 & -0.0432 & -0.0145 & -0.0225 \\ -0.4487 & 0.8621 & 4.9057 & 0.3549 & 6.9358 \end{bmatrix},$$

– observer gain matrix:

$$\mathbf{L} = \begin{bmatrix} 6.4457 & -0.4478 & -0.1573 & 1.1063 \\ 2.3130 & 1.8097 & 0.3608 & 1.9318 \\ -2.9231 & -0.0123 & 1.2201 & -7.3520 \\ -1.3611 & -1.2200 & -0.5000 & 0.0000 \\ -1.9033 & 6.3425 & -0.0000 & -0.5000 \end{bmatrix}.$$

Table 1 reflects standard deviations of the aircraft outputs.

Table 1. Standard deviations of Boeing 747 outputs in a stochastic case

Plant	Standard Deviation (Lateral Channel)				
	s_b, °	σ_p, °/sec	σ_r, °/sec	s_j, °	s_y, °
V = 67.4 m/s	0.0004	0.0007	0.0176	0.0727	0.0065

Performance indices of closed loop system with observed state feedback in a loop are given in Table 2.

The simulation results of the closed loop system taking into account the influence of the random wind, simulated according to the standard Dryden model of turbulence confirm the efficiency of proposed approach. Results of the simulation are shown in Figure.

Table 2. Performance indices of closed-loop system

Performance Index	**Plant**
	V = 67.4 m/s
H_2-norm	0.5952
H_∞-norm	1.1172

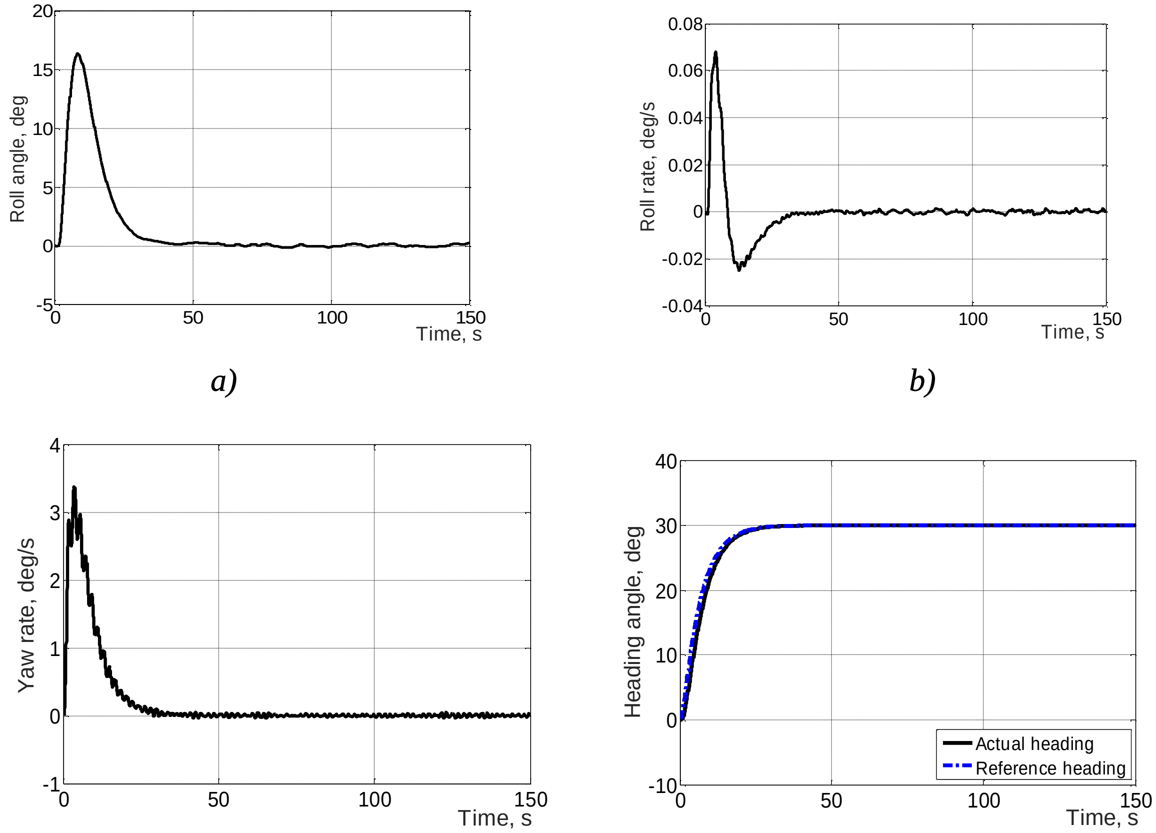

Fig. 1. Simulation results of lateral motion control in the presence of external disturbances:
a) is the roll angle, deg; *b)* is the roll rate, deg/s; *c)* is the yaw rate, deg/s; *d)* is the heading angle, deg

Conclusions

As far as the incomplete state space vector is available for measuring, the flight control system for aircraft can be easily designed by applying observer. Thus, the unavailable states can be suitable approximated by restored states. The proposed solution is very simple and uses Lyapunov approach. The proposed design procedure can be solved efficiently by applying LMI optimization technique. The main advantage of the proposed approach is that there is no need to define the region of observer poles placement. The proposed approach permits to define the observer gains and state feedback gain matrix directly from set of LMIs, simultaneously.

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