UDC 004.942; 004.728

Guziy M.M., Ph. D.
Ignatov V.O., Doctor of Engineering, prof.
Wu Zijuan, Post-graduate

COMPARATIVE ANALYSIS OF MODELLING ADEQUACY OF THE NONSTATIONARY TRAFFIC IN TELECOMMUNICATION NETWORKS

National aviation university

In paper results of the comparative analysis of modeling adequacy of the nonstationary traffic in telecommunication networks are given. For optimum modeling dynamics of the traffic metrics of spaces Euclid and Gilbert are used. The algorithm and a technique of verification of optimum models are developed. It is shown, that modeling of the nonstationary traffic in space Gilbert is preferable as gives more exact models. Theoretical positions are illustrated by numerical examples and diagrams with use of system MathCAD.

Introduction. Problems of modeling, the analysis, synthesis of methods of optimum service of the traffic in telecommunication and computer networks become more and more actual in process of development of these networks, complication of their architecture, increase of number of levels in hierarchical structure of networks, transition to principles of construction of open information systems on the basis of seven levels reference model of standards OSI ISO, growth of a supply and demand in the market of methods of optimum synthesis of models of optimum service of the traffic at a transport level [1-3]. The hobby of some researchers models of the self-similar traffic [4] has resulted to that many real kinds of the periodic and nonstationary traffic remain outside of a field of vision, and, hence, behind frameworks of modeling and research. One of the main reasons of such state of affairs is complexity of the problems decision of modeling for nonstationary and nonlinear cases. In the known literature there are no results of the comparative analysis of models of the nonstationary traffic that in many respects is caused by weak development of mathematical methods of stochastic approximation of random nonstationary processes. The given work is directed on partial overcoming of the specified difficulties in the field of modeling and optimum service of the nonstationary traffic.

The purpose of work. The purpose of work is the comparative analysis of models of the nonstationary traffic in telecommunication and computer networks.

For achievement of the purpose the following tasks are put and solved: development of algorithm and a technique of the comparative analysis of adequacy of models of the non-stationary traffic in telecommunication and computer networks, a choice of the metrics of spaces of comparison of models and criteria of their optimality, a substantiation of reference models of the casual non-stationary traffic which the substantiation and a choice of criteria of an optimality of models of the non-stationary traffic, a substantiation of statement of a problem and construction of optimum models of the non-stationary traffic, the comparative analysis of adequacy of optimum models of the non-stationary traffic serve as base models of the comparative analysis of offered models of the real traffic, on the basis of the chosen parameters of adequacy, the decision of numerical examples, the formulation of conclusions by results of the comparative analysis of optimum models of the non-stationary traffic.

General statement of problems. In a role of the initial data are chosen initial polynomial representation of the reference non-stationary traffic with known Gaussian m dimension independent factors of decomposition. The basic methods of construction of the approached optimum models of the non-stationary traffic chooses a method of quantization of the traffic and Markova approximations of casual process of change of conditions of the traffic, method Ritz and a method of the maximal plausibility. As results of the decision of a problem results of the comparative analysis of adequacy of models of the non-stationary traffic constructed on the basis of metrics of spaces Euclid and Gilbert, and also conclusions on results of the analysis serve.

The decision of a problem. We shall start with development of algorithm and a technique of the comparative analysis of adequacy of models of the nonstationary traffic in telecommunication and computer networks. The logic analysis of a problem has shown that the operational basis of algorithm of the comparative analysis of adequacy of models of the nonstationary traffic should consist of 16 operations. Procedure of the comparative analysis of adequacy of models of the nonstationary traffic, as a matter of fact, is procedure of verification of models of the nonstationary traffic.

The general technique of verification of models of the nonstationary traffic offered by us is based on the following base algorithm of verification:

1. Normalization and reduction a dimensionless kind of a range of change of realizations the traffic envelope and its interval of the nonstationary. This operation allows to model "normalized dynamics of the traffic" in "a single square" with coordinates of tops: (0, 0), (0, 1), (1, 0), (1, 1).

2. A choice of the model dimension of n that is numbers of discrete conditions of the nonstationary traffic. This number shows, on how much quantums the individual square for step approximation of the nonstationary traffic shares and generally is the essential characteristic of the model dimension, and also its accuracy.

3. Definition of the dimensionless normalized intensities of changes of the traffic conditions. These of intensities are defined as intensities of crossing by the traffic of the given levels of quantization.

4. Construction of logical-mathematical model of dynamics of the traffic as the column as which tops conditions of the traffic serve, and the directed edges - arrows with the instruction of the directions changes of conditions the traffic.

5. Drawing up of differential equations Kolmogorov-Chepmen concerning the probabilities of the traffic conditions describing dynamics of the traffic, on logical-mathematical model of the traffic dynamics. B.V.Vasil'eva's known rule for drawing up of the differential equations on column [1] is used.

6. A choice of entry conditions system for the decision of problem Cauchy with the help of the made differential equations of dynamics of the traffic.

7. Direct transformation Laplace system of the differential equations in system of the algebraic equations concerning images of the conditions probabilities of the traffic at entry conditions of item 6.

8. The decision of the received system of the linear algebraic equations concerning images of the conditions probabilities of the traffic method Gauss or a method of a matrix system to a triangular kind in view of a normalization condition for unknown images.

9. Transformation by a method of uncertain factors of images to a kind of the sums of the simple rational fractions convenient for search of originals in return transformation Laplace.

10. Return transformation Laplace for the received images of probabilities.

11. Use of probabilities of conditions and quantized values of the traffic for definition of a population mean and a dispersion bending around nonstationary Gaussian the traffic.

12. A choice of metrics spaces of comparison models, criteria of optimality, optimization quantized values of the traffic with the purpose of achievement the best approximation of the modeling and reference moments of the traffic. Comparison and optimization of models are carried out with use of metrics Euclid (a discrete case) and metrics Gilbert (a continuous case).

13. Specification by an iterative

graphic method of optimum values normalized intensities.

14. A choice of criteria adequacy and quantitative measurements of adequacy of compared models to the reference traffic by these criteria.

15. The comparative analysis of results of use of various models and the reference traffic, revealing of new laws of optimum modeling.

16. The formulation of conclusions by results of the analysis and comparisons of characteristics modeling and reference traffic.

By this technique with a required level of detailed elaboration of steps 1-16 algorithms in the further are solved the problems of the comparative analysis of models designated above and the reference traffic. For check of adequacy of models to the reference traffic dependences of parameters accuracy of modeling on parameters of models are used.

The basic features of a technique and base algorithm of a choice optimum quantized values we shall show for parallel consideration of two cases of modeling: discrete and continuous. These cases are different, as it was already marked, a choice of the metrics space comparison of reference and modeling characteristics (see item 12). In a discrete case the metrics space Euclid is used, in a continuous case the metrics space Gilbert is used.

Let's apply consistently base algorithm of the decision of problems of verification models at use of two conditions of the nonstationary traffic, we shall receive the following results.

1. Normalization and reduction a dimensionless kind of a range of change of realizations bending around the traffic and its interval nonstationary.

On fig. 1 "the individual square" for research of adequacy of modeling the first order moments- population means nonstationary, growing on an interval nonstationary, Gaussian traffic is shown. The continuous line represents the normalized dimensionless population mean $m(t_k)$ the reference traffic submitted as normalized on normalized interval nonstationary [0,1] parabola with casual independent Gaussian in factors:

$$m(t_k) = t_k \cdot (2 - t_k), \qquad (1)$$

where t_k – *k-th* moment of time from the dimensionless normalized interval [0,1] times nonstationary. Stroke - dotted represent normalized dimensionless population means $M(t_k)$ the modeling traffic with optimized in space of Euclid quantized values of the traffic (fig. 1a) and with optimized in space of Gilbert quantized values of the traffic submitted in same normalized Gilbert space:

$$M(t_k) = Z_2 - (Z_2 - Z_1) \cdot P_{10} \cdot e^{-\eta 1 \cdot t_k}, \qquad (2)$$

where Z_1, Z_2 - quantized values of the traffic,

P_{10} - initial value of the first condition probability of the traffic,

$\eta 1$ - intensity of change of the traffic for the first condition.

Deeper sense of all parameters of expression (2) will be opened later at construction of dynamics model of the traffic.

2. A choice of dimension of model *n*, in other words, numbers of discrete conditions of the nonstationary traffic. For an illustration of a technique and the general algorithm the minimal dimension of model is chosen: $n = 2$. At $n = 2$ there are only two conditions of the traffic. They are defined by two quantums of traffic Δ_1, Δ_2 and three levels of quantization of the traffic: $z_0 = 0$, $z_1 = 0,5$, $z_2 = 1$. These values participate in definition of intensity of the traffic and optimum quantized values of traffic Z_1, Z_2 for two conditions. The elementary model is the least exact, reflects the worst case of modeling and gives pessimistic estimations.

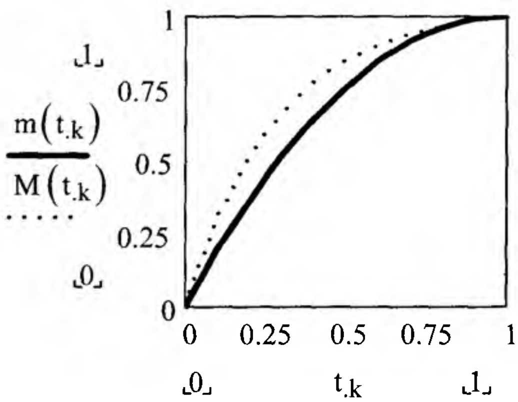
When quantization of the traffic carry out the following nonlinear transformations of the traffic, all values of the traffic, which get in the first interval of quantization

$$\Delta_1 = z_1 - z_0, \qquad (3)$$

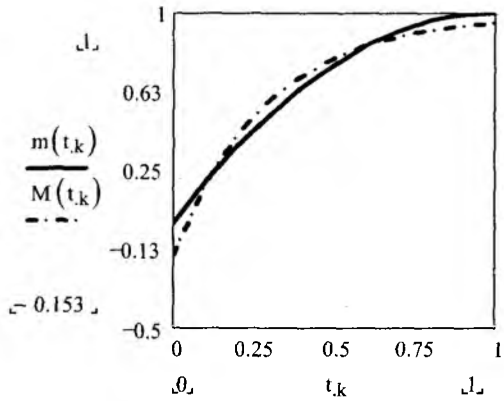
replace with one quantized value Z_1. All values of the traffic, which get in the second interval of quantization

$$\Delta_2 = z_2 - z_1, \qquad (4)$$

replace with one quantized value Z_2.



a. A discrete case



b. A continuous case

Fig.1. An illustration of normalization values of the traffic and an interval of time nonstationary

Thus, in the space quantization of continual changes of the traffic is replaced with discrete space with final accounting number of conditions. In result continuous monotonous change of the nonstationary traffic is replaced with a step line which consists from quantized constant values of the traffic on intervals of quantization. In the further, quantized values of the traffic are considered as realization of the discrete traffic which take place with probabilities of discrete conditions of the traffic.

3. Definition dimensionless normalized intensities changes of conditions by the traffic.

At $n = 2$ there is only one change of the traffic intensity - intensity $\eta 1$ transition of the traffic from the first condition to the second. By definition $\eta 1$ there is a size, return to average time before crossing by a population mean of the reference traffic of the first level of quantization. More precisely the physical sense $\eta 1$ can be defined as: an intensity $\eta 1$ change of the traffic on the first interval is normalized on length of first interval $\Delta 1$ speed v_1 changes of the traffic:

$$\eta 1 = v_1/\Delta_1 = 1/t_1, \qquad (5)$$

For definition $\eta 1$ it is necessary to solve be relative t the equation

$$t \cdot (2-t) = z_1 \left| \begin{matrix} \text{solve}, t \\ \text{float}, 4 \end{matrix} \right. \to \begin{pmatrix} .2929 \\ 1.707 \end{pmatrix} \qquad (6)$$

The decision of the equation (6), "with an accuracy of fourth mark after a point", is shown in system MathCAD for a case, when the first level of quantization $z_1 = 0.5$. As the second root of a parabola does not belong to the normalized interval, as the decision the first root $t_1 = 0.2929$ serves.

Hence, required dimensionless intensity for a case $z_1 = 0.5$, $t_1 = 0.2929$ is equal

$$\eta 1 = 1/t_1 = 1/0.2929 = 3.414.$$

4. Construction of logical-mathematical model of dynamics of the traffic as the column as which tops conditions of the traffic serve, and the directed edges - arrows with the instruction of directions of change of conditions traffic. At $n = 2$ the most simple kind has form:

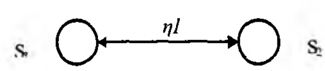


Fig. 2. Graf as logical-mathematical model of dynamics of the traffic

On fig. 2 the following designations are accepted: S_1, S_2 - the first and second conditions of the traffic, $\eta 1$ - intensity transition of the traffic from the first condition to the second. Probabilities of a presence of the traffic at the moment of time t in conditions S_1, S_2 are designated through $P_1(t)$, $P_2(t)$. After quantization of value of the traffic for these two conditions are equal, accordingly, to two quantized values Z_1, Z_2 which appear with probabilities $P_1(t)$, $P_2(t)$. As events, which consist in hit of value of the traffic in this or that interval of quantization, form full group of events, for probabilities of conditions of the traffic fairly a normalization condition:

$$P_1(t) + P_2(t) = 1. \qquad (7)$$

5. Drawing up of differential equations Kolmogorov-Chepmen concerning the probabilities of conditions of the traffic describing dynamics of the traffic, on

logical-mathematical model of dynamics of the traffic.

We use Vasil'eva B.V.'s rule for drawing up of the differential equations on the column fig. 2. We shall receive:

$$\frac{dP_1(t)}{dt} = -\eta_1 P_1(t) \quad (8)$$

,

$$\frac{dP_2(t)}{dt} = \eta_1 P_1(t) \quad (9)$$

.

As probabilities of conditions are connected by conditions normalizations (7), it is necessary to solve system from two equations: the differential equation (8) and the algebraic equation (7):

$$\frac{dP_1(t)}{dt} = -\eta_1 P_1(t), \quad P_1(t) + P_2(t) = 1. \quad (10)$$

6. A choice of system of entry conditions for the decision of problem Cauchy with the help of the made differential equations of dynamics of the traffic.

The assumption that at the initial moment of time t_0 value of the traffic can be in any interval is natural. General view entry conditions therefore are fair:

$$P_1(t_0) = P_{10}, \; P_2(t_0) = P_{20}. \quad (11)$$

7. Direct transformation Laplace system of the differential equations in the system of the algebraic equations concerning images of probabilities of conditions traffic at entry conditions (11).

Applying to system (10) direct transformation Laplace by known rules [2], we shall receive

$$s P_1(s) - P_{10} = -\eta_1 P_1(s), \quad P_1(s) + P_2(s) = 1/s \quad (12)$$

.

8. The decision of the received system of the algebraic equations concerning images of probabilities of conditions traffic method Gauss, a method of data of a matrix system to a triangular kind or a method of substitution in view of a condition normalization for unknown images.

Solving a system from two algebraic equations (12) concerning unknown images $P_1(s)$, $P_2(s)$ probabilities of conditions S_1, S_2 a method of substitution in the second equation of the image of probability of the first condition, we shall find

$$P_1(s) = P_{10}/(s + \eta_1) \quad (13)$$

,

$$P_2(s) = \frac{1}{s} - \frac{P_{10}}{(s + \eta_1)} \quad (14)$$

9. Transformation by a method of uncertain factors of images to a kind of the sums of the rational fractions convenient for search of originals in return transformation Laplace.

Images (13), (14) in a considered simple case already represent the sums of simple rational fractions, therefore intermediate transformations it is not required.

10. Return transformation Laplace for the received images of probabilities.

Applying return transformation Laplace to images (13), (14) [2], we shall receive

$$P_1(t) = P_{10} e^{-\eta_1 t} \quad (15)$$

,

$$P_2(t) = 1 - P_{10} e^{-\eta_1 t} \quad (16)$$

.

11. Use of probabilities conditions and quantized values of the traffic for definition of a population mean and a dispersion bending around nonstationary Gaussian the traffic.

Let's use known formulas for a population mean and a dispersion of a discrete random variable and we shall take into account also that probabilities of conditions because of nonstationary the traffic depend on time. We shall receive

$$M(t) = \sum_{i=1}^{2} Z_i P_i(t) = Z_2 + (Z_1 - Z_2) P_{10} e^{-\eta_1 t} \quad (17)$$

,

$$D(t) = \sum_{i=1}^{2} Z_i^2 P_i(t) - M^2(t) \quad (18)$$

.

The formula (17) evidently shows, how quantization and Markov approximation result in models of a population mean of the nonstationary traffic as a linear combination an exhibitor. From the formula (17) follows, that in maintenance of adequacy of the modeling traffic to the reference traffic essential value has a correct choice of quantized values. Further for optimization of a choice quantized values the method of maximal plausibility (MMP) which at normal multivariate distribution of the traffic results in use of a method of least squares (MLS) is used. Comparison (17) and (2) the reasons of modeling representation of the first initial moment of the nonstationary traffic as a linear combination the exhibitor completely opens.

We shall use the metrics of the Euclid's space then the square of distance between the first initial moments of the reference and model of the nonstationary traffic can be submitted as [4]

$$\varepsilon(Z_1,Z_2)^2=\sum_{t_k=0}^{1}[t_k\cdot(2-t_k)-[Z_2-(Z_2-Z_1)\cdot P_{10}\cdot e^{-\eta t_k}]]^2, \quad (19)$$

where Z_1 and Z_2 - the traffic quantization values,

t_k $(2-t_k)$ - normalized on an interval of comparison the standard of the traffic,

η - intensity of change of the nonstationary traffic,

t_k – the k - th moment of the traffic supervision time,

P_{10} - probability of the traffic presence in the first condition at the initial moment of time.

When we use the metrics of the Gilbert's space then the square of distance between the first initial moments of the reference and model of the nonstationary traffic can be submitted as [4]

$$\varepsilon(Z_1,Z_2)^2=\int_0^1[[t_k\cdot(2-t_k)- -[Z_2-(Z_2-Z_1)\cdot P_{10}\cdot e^{-\eta\cdot t_k}]]^2\,dt_k, \quad (20)$$

From formulas (19), (20) follows, that with a choice of quantized values Z_1 and Z_2 it is possible to provide the best approximation of the initial moments of the reference traffic. Similar expression can be written down concerning the second central moments of the reference and model of the nonstationary traffic. For a case of the Gaussian multivariate distribution the optimum choice of the first two moments of the traffic model the method of optimization of quantized values and their dispersions allows to solve a problem of the best approximation of the normal density of distribution of the reference nonstationary traffic and the normal density distribution of the model of the nonstationary traffic.

Let's illustrate the specified opportunity with the problem decision of the best approximation of a population mean of the reference nonstationary traffic by a population mean of the model of the traffic. Considering (19), (20) as optimization criteria of quantized values in a nonlinear bidimentional problem of optimization without restrictions, we use a classical method of search of the optimal quantized values.

This problem we shall solve in the following statement: expression of optimization criterion quantized values Z_1, Z_2 as functional (19) or (20) is known, the classical method of search of a minimum (19) or (20) requires to find Z_{1opt}, Z_{2opt}, delivering a minimum functional (1) (a discrete case) or functional (20) (a continuous case):

$$\min_{Z_1,Z_2}\varepsilon^2(Z_1,Z_2)=\varepsilon^2_{\min}(Z_{1opt},Z_{2opt}), \quad (21)$$

Differentiating functionals (19), (20) on parameters Z_1, Z_2, we shall receive two equations of optimization concerning values Z_1, Z_2:

1. For a discrete case:

$$\sum_{t_k=0}^{1}[[t_k\cdot(2-t_k)-[Z_2-(Z_2-Z_1)\cdot P_{10}\cdot e^{-\eta t_k}]]*(P_{10}\cdot e^{-\eta t_k})=0, \quad (22)$$

$$\sum_{t_k=0}^{1}[[t_k\cdot(2-t_k)-[Z_2-(Z_2-Z_1)*P_{10}\cdot e^{-\eta\cdot t_k}]]\cdot(1-P_{10}\cdot e^{-\eta\cdot t_k})=0. \quad (23)$$

2. For a continuous case:

$$\int_0^1[t_k\cdot(2-t_k)-[Z_2-(Z_2-Z_1)\cdot P_{10}\cdot e^{-\eta t_k}]](P_{10}\cdot e^{-\eta t_k})dt_k=0, \quad (24)$$

$$\int_0^1[t_k\cdot(2-t_k)-[Z_2-(Z_2-Z_1)*P_{10}\cdot e^{-\eta\cdot t_k}]]\cdot(1-P_{10}\cdot e^{-\eta\cdot t_k})dt_k=0 \quad (25)$$

Let's result these equations in an initial form of system from two linear algebraic equations, we shall receive

$$a_{11}Z_1+a_{12}Z_2=b_1, \quad (26)$$

$$a_{21}Z_1+a_{22}Z_2=b_2. \quad (27)$$

where elements of matrix $\boldsymbol{A}$ and vector $\boldsymbol{B}$ of this system

$$A:=\begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix}, \quad (28)$$

$$B:=\begin{pmatrix} b_1 \\ b_2 \end{pmatrix}. \quad (29)$$

are defined by the following formulas:

1. For a discrete case:

$$a11 = \sum_{t_k=0}^{1} (P_{10} \cdot e^{-\eta \cdot t_k})^2 \quad , \tag{30}$$

$$a21 = \sum_{t_k=0}^{1} P_{10} \cdot e^{-\eta \cdot t_k} \cdot (1 - P_{10} \cdot e^{-\eta \cdot t_k}) \quad .$$

$$a12 = \sum_{t_k=0}^{1} (1 - P_{10} \cdot e^{-\eta \cdot t_k}) \cdot P_{10} \cdot e^{-\eta \cdot t_k} \quad , \tag{31}$$

$$a22 = \sum_{t_k=0}^{1} (1 - P_{10} \cdot e^{-\eta \cdot t_k})^2 \quad .$$

$$b1 = \sum_{t_k=0}^{1} [t_k(2 - t_k)] \cdot P_{10} \cdot e^{-\eta \cdot t_k} \quad , \tag{32}$$

$$b2 = \sum_{t_k=0}^{1} [t_k(2 - t_k)] \cdot (1 - P_{10} \cdot e^{-\eta \cdot t_k}) \quad .$$

2. For a continuous case:

$$a_{11} = \int_0^1 (P_{10} \cdot e^{-\eta \cdot t_k})^2 dt_k \, , \tag{33}$$

$$a_{21} = \int_0^1 (P_{10} \cdot e^{-\eta \cdot t_k}) \cdot (1 - P_{10} \cdot e^{-\eta \cdot t_k}) dt_k \, .$$

$$a_{12} = \int_0^1 [(1 - P_{10} \cdot e^{-\eta \cdot t_k}) \cdot (P_{10} \cdot e^{-\eta \cdot t_k})] dt_k \tag{34}$$

$$a_{22} = \int_0^1 (1 - P_{10} \cdot e^{-\eta \cdot t_k})^2 dt_k$$

$$b1 = \int_0^0 [t_k(2 - t_k)] \cdot (P_{10} \cdot e^{-\eta \cdot t_k}) dt_k \tag{35}$$

$$b2 = \int_0^0 [t_k(2 - t_k)] \cdot (1 - P_{10} \cdot e^{-\eta \cdot t_k}) dt_k$$

Using Kramer's rule, we shall receive optimum quantized values of controlled variables

$$Z_{10} := \frac{|A1|}{|A|} \, , \quad Z_{20} := \frac{|A2|}{|A|} \, , \tag{36}$$

where |A|, |A1|, |A2| - determinants of the equations system (26), (27).

Substituting these values in functional (19) or (20), we shall find its minimal value at optimum quantized values (36).

Example 1. We shall show feature of definition optimum quantized values and minimal root-mean-square deviations of the moments of the first order nonstationary parabolic traffic at the following initial data: $P_{10} = 0.95$, $P_{20} = 0.05$, $\eta_1 = 3.414$.

Using formulas (30) - (32), we shall find parameters of system of the algebraic equations for a discrete case

$a11 = 0.903 \quad a12 = 0.078 \quad b1 = 0.031$

$a21 = 0.078 \quad a22 = 0.941 \quad b2 = 0.969$

Using formulas (33) - (35), we shall find parameters of system of the algebraic equations for a continuous case

$a_{11} = 0.132 \quad a_{12} = 0.137 \quad b_1 = 0.108$

$a_{21} = 0.137 \quad a_{22} = 0.594 \quad b_2 = 0.559$

Substituting these values in formulas for determinants, we shall find their values for a discrete case

$|A| \text{ float},7 \rightarrow .8440187 = 0.844$

$|A1| \text{ float},7 \rightarrow -.4593524\text{e-}1 = -0.046$

$|A2| \text{ float},7 \rightarrow .8727696 = 0.873$

Substituting these values in formulas for determinants, we shall find their values for a continuous case

$|A| \text{ float}, 7 \rightarrow 5.961473 \cdot 10^{-2} = 0.06$

$|A1| \text{ float}, 7 \rightarrow -1.267710 \cdot 10^{-2} = -0.013$

$|A2| \text{ float}, 7 \rightarrow 5.905071 \cdot 10^{-2} = 0.059$

We use these values of determinants and we shall find under formulas (36) required optimum quantized values. In system MathCAD the decision looks like:

1. For a discrete case

$$\text{lsolve}(A,B) \text{ float},7 \rightarrow \begin{pmatrix} -.2126505 \\ .9905389 \end{pmatrix} = \begin{pmatrix} -0.213 \\ 0.991 \end{pmatrix}$$

$$Z_{10} := -0.213 \tag{37}$$

$$Z_{20} := .990538$$

2. For a continuous case

$$\text{lsolve}(A,B) \text{ float},7 \rightarrow \begin{pmatrix} -.5442444\text{e-}1 \\ 1.034064 \end{pmatrix} = \begin{pmatrix} -0.054 \\ 1.034 \end{pmatrix}$$

$$Z_{10} := -0.054 \tag{38}$$

$$Z_{20} := 1.034$$

Substituting optimum quantized values Z_{10}, Z_{20} in the formula (21), we shall find the minimal values target functional and root-mean-square errors of modeling of a population mean of the reference traffic:

1. For a discrete case

$$\varepsilon^2_{\min}(Z_1, Z_2) = 1.624 \times 10^{-7} \, ,$$

$$\sigma_{min} = \sqrt{1.624 \times 10^{-7}} = 0.04\% \tag{40}$$

2. For a continuous case

$$\varepsilon^2_{min}(Z_1, Z_2) = 2{,}587 \times 10^{-3} \, ,$$

$$\sigma_{min} = \sqrt{2.587 \times 10^{-3}} = 0{,}05086 = 5{,}086\% \tag{41}$$

Let's define the minimal value of average factor of a variation of an error on an interval nonstationary:

1. For a discrete case

$$V_{min} = \frac{\sigma_{min}}{m_0} = \frac{0.04\%}{0.667} = 0.06\% \qquad (41)$$

2. For a continuous case

$$V_{min} = \frac{\sigma_{min}}{m_0} = \frac{5{,}086}{0{,}667} = 7{,}6252\% \qquad (42)$$

where average value (a constant component) normalized traffic on an interval nonstationary is defined under the formula

$$m0 = \int_0^1 [t_k \cdot (2 - t_k)] dt_k \, . \qquad (43)$$

On fig. 3 results of modeling of a population mean of the reference nonstationary traffic $m(t_k)$ by model $M(Z_1, Z_2, t_k)$ optimized in space Gilbert with parameters (38), and a parameter of accuracy of the modeling (42) optimized in space Euclid by model $M1(t_k)$, and also model $M2(t_k)$ in which in a role quantized values average values of quantums are used are shown. Comparing curves, it is uneasy to notice, that model M optimized in space Gilbert $M(Z_1, Z_2, t_k)$ with parameters (38), and a parameter of accuracy of modeling (42) yields the best results of modeling of the reference traffic. It speaks that optimization of model in space Euclid is deceptive. Because of that in functional (19) the limited number of points of comparison is taken into account, considerably smaller value of factor of a variation of an error of modeling (39) in comparison with a margin error modeling in space Gilbert (40), approximately on two order turns out. At the same time fig. 3 evidently illustrates that the model optimized in space Gilbert approximates the reference traffic much better.

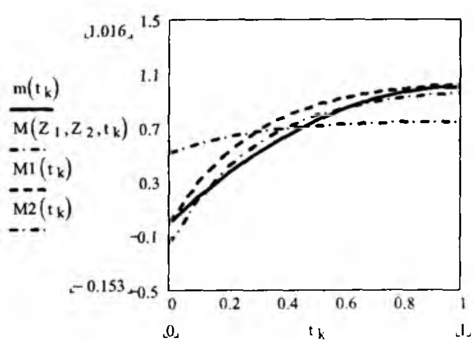


Fig. 3. Comparison of models of the nonstationary traffic

Conclusions

1. Application of quantization and Markov approximations allows building adequate models of the nonstationary traffic submitted initial polynomial by decomposition with casual factors.

2. Use even the elementary model with two conditions of the traffic allows to approximate a population mean of the nonstationary traffic submitted as a normalized parabola with two casual parameters.

3. Offered the system of parameters and a technique of verification of models nonstationary traffic with various number of conditions of the traffic allow to carry out verification of models and to use quantitative estimations of a degree of adequacy of models to the real non-stationary traffic.

References

1. Игнатов В.А., Жуков И.А., Гузий Н.Н. Данилина Г.В. Моделювання перехідних режимів трафіку комп'ютерної мережі. Информационные технологии и безопасность. – Сборник научных трудов. – К.: Национальная академия наук Украины, Институт проблем регистрации информации, 2006. – Вып.9. – С. 84 – 87.

2. Гузій М.М., Даниліна Г.В., Ігнатов В.О., Милокум Я.В. Методи і алгоритми оптимального управління трафіком в обчислювальних мережах. Проблеми інформатизації та управління.– К.: НАУ, 2006.– Вип.17. – С. 32-37.

3. Гузій М.М., Даниліна Г.В., Ігнатов В.О. Мачалін І.О. Марковські рандомізовані моделі. Вісник Житомирського державного технологічного університету,Житомир 2006.-Вип.IV (39)-С.179-184.

4. Koucheryavy Y., Harju J., A novel approach for self-similar traffic prediction// Proceedings of the St. Petersburg Regional International Teletraffic Seminar, St. Petersburg, Russia, January 29 - February 1, 2002, pp. 172 - 179.

5. Гузій М.М., Даниліна Г.В., Ігнатов В.О. Оцінювання ефективності використання ліній зв'язку в обчислювальних мережах. Проблеми інформатизації та управління. - К.: НАУ, 2006.– Вип.18. – С.54-59.