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**OPTIMAL MANAGERIAL AND CONTROL VALUES FOR ACTIVE OPERATION**

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**Abstract**—Considered a method of determining the optimal managerial or control values for periodicity of the scheduled aircraft or aeronautical engineering maintenance and a linear inertness-less object of control ruled with a proportional governor developed on the basis of subjective entropy maximum principle in one-dimensional case. Mathematical models for the obtained preferences densities distributions for continuous alternatives are introduced. Calculation experiments are carried out. The necessary diagrams are plotted.

**Index Terms**—Active aircraft flight control; optimization; subjective entropy maximum principle; continuous alternative; preferences distribution density; variational problem; extremal; optimal choice.

## I. INTRODUCTION

Subjective preferences distributions of individuals responsible for making control and managerial decisions are very helpful and useful tool when choosing optimal solutions for a so-called active system functioning or operation [1].

Such problems have been considered for operation of active transport systems in conditions of multi-alternativeness and uncertainty in the manuscript of the Dissertation [2], its Abstract [3], reported consequentially at related International Conferences [4] – [7], and they make their roots out of the newly emerging powerful scientific stream “Subjective Analysis” [8] – [10]. The core of the theoretical researches is the postulated variational, optimization principle which got the name of Subjective Entropy Maximum Principle (SEMP).

On the basis of the principle it is possible to obtain the individual preferences functions of the subjects (active elements of the corresponding control or managerial systems) in the explicit, the so-called canonical view. These functions are deemed to be the optimal controlling functions that take into account psychological properties of the responsible for making decisions persons.

The young theory [2] – [10] has some valuable conceptual results for solving the problem of the optimal choice. Nevertheless, it needs to make a few ideological steps forward with the proof in the view of at least calculation modeling.

One of such problem settings is the actual problem of the optimal periodicity of the scheduled aircraft or aeronautical engineering maintenance [1, pp. 170–172]. The other urgent problem is the optimal control value determination in case of an active system operation, which behavior is described as a linear inertness-less object of control [11, pp. 160–165] (for example, navigation and motion

when the object functioning). Moreover, the control system is equipped with a proportional governor [2] – [4], [11].

## II. SOLUTION OF THE PROBLEM

## A. Maintenance Optimal Periodicity

According to [1, pp. 170–172] at the work of a product at a random moment of time  $\tau$  there happens a damage, the further development of which leads to the failure at the moment of time  $t$ . Maintenance has to be performed in the interval  $(\tau, t)$  at the moment  $t_p$ .

The optimal time interval  $t_p^*$ , accepting, for instance, the law of the damages appearance times distribution, as well as of their development times to the failure happening as the exponential ones with the corresponding intensities of  $\lambda_1$  and  $\lambda_2$ , for the scheduled maintenance works performance will be found at the maximum of the probability of the conjoint events of the damage occurrence and failure not happening realization by the formula of [1, P. 171]:

$$t_p^* = \frac{(\ln \lambda_1 - \ln \lambda_2)}{(\lambda_1 - \lambda_2)}. \quad (1)$$

The optimal value of (1) can be obtained with the use of SEMP [12].

Supposedly, the subjective preferences entropy functional has the view of

$$\Phi_\pi = -\sum_{i=1}^2 \pi_i(\lambda_i) \ln \pi_i(\lambda_i) + t_p^* \sum_{i=1}^2 \pi_i(\lambda_i) \lambda_i + \gamma \left[ \sum_{i=1}^2 \pi_i(\lambda_i) - 1 \right], \quad (2)$$

where  $\pi_i(\cdot)$  are preferences functions;  $\gamma$  is the normalizing coefficient.

The first member of equation (2) is the subjective entropy of the preferences.

The necessary conditions for the functional (1) extremum existence

$$\frac{\partial \Phi_{\pi}}{\partial \pi_i(\cdot)} = 0 \quad (3)$$

in accordance with [12] yield

$$\frac{\partial \Phi_{\pi}}{\partial \pi_i(\cdot)} = -\ln \pi_i(\cdot) - 1 + t_p^* \lambda_i + \gamma = 0, \quad \forall i \in \overline{1, 2}. \quad (4)$$

This inevitably means in turn

$$\ln \pi_1(\cdot) - t_p^* \lambda_1 = \gamma - 1 = \ln \pi_2(\cdot) - t_p^* \lambda_2. \quad (5)$$

$$\ln \pi_1(\cdot) - \ln \pi_2(\cdot) = t_p^* (\lambda_1 - \lambda_2). \quad (6)$$

$$t_p^* = \frac{[\ln \pi_1(\cdot) - \ln \pi_2(\cdot)]}{(\lambda_1 - \lambda_2)}. \quad (7)$$

Thus, we have got the law of subjective conservatism [12] if the values of parameters  $t_p^*$ ,  $\lambda_1$ , and  $\lambda_2$  are given.

In case

$$\pi_i(\lambda_i) = x \lambda_i, \quad (8)$$

where  $x$  – unknown, uncertain multiplier in type of the Lagrange one, we obtain with the help of the procedure considered through (2) – (8) the needed optimal periodicity (1).

Indeed, substituting for their values equations (8) into expression (7) it yields

$$t_p^* = \frac{\ln \frac{x \lambda_1}{x \lambda_2}}{(\lambda_1 - \lambda_2)}, \quad (9)$$

finally, equation (1).

The sense of the uncertain multiplier  $x$  becomes obvious with the use of the normalizing condition of the initial functional (2). That is

$$x \lambda_1 + x \lambda_2 = 1. \quad (10)$$

$$x = \frac{1}{(\lambda_1 + \lambda_2)}. \quad (11)$$

Here, the cognitive function has the view of

$$t_p^* \left( \frac{\sum_{i=1}^2 \lambda_i^2}{\sum_{i=1}^2 \lambda_i} \right). \quad (12)$$

Moreover, in case expressed with (1) – (12) the optimal value of the sought maintenance interval  $t_p^*$  has been got for the given values of preferences  $\pi_i(\cdot)$  and at this the optimum of  $t_p^*$  makes the preferences of  $\pi_i(\cdot)$  also be optimal for the objective functional (2).

### B. Control Problem

When we deal with a linear inertness-less object of control its behavior is described with the equation of [11, P. 162, (1)]. A proportional governor in the control system means [11, P. 163, (2)]. As a result, increasing the governor augmentation coefficient  $k_p$  it is possible to decrease the control system error  $\varepsilon$ . As the increase of  $k_p$  cannot be unlimited. We come to the problem of its optimal value.

Let losses due to  $\varepsilon$  are proportional to  $|\varepsilon|$  with the coefficient  $C_{\varepsilon}$  and depend on time  $t$ . The cost of coefficient  $k_p$  increases nonlinearly with power index  $n$  and coefficient  $C_{k_p}$ . Total expenses [2]-[4]

$$C_{\varepsilon} |\varepsilon(t, k_p)| + C_{k_p} k_p^n. \quad (13)$$

For any fixed moment, consider the variation of the parameter  $k_p$  as a continuous alternative. On the basis of SEMP [5] – [10] we take the expression (13) as a cognitive function. The control purpose functional will be compiled as, [2] – [4]:

$$\Phi_{\pi} = \int_0^{1000} \left\{ -\pi_t(k_p) \ln \pi_t(k_p) - \beta \pi_t(k_p) \left[ C_{\varepsilon} |\varepsilon_t(k_p)| + C_{k_p} k_p^n \right] \right\} dk_p + \gamma \left[ \int_0^{1000} \pi_t(k_p) dk_p - 1 \right] - \ln \Delta k_p \quad (14)$$

where  $\pi_t(k_p)$  are preferences functions distribution density for the point of time  $t$  as a function of  $k_p$  which value is the continuous alternative varying in  $[0 \dots 1000]$ ;  $\beta$ ,  $\gamma$  are structural parameters;  $\Delta k_p$  is the degree of accuracy at the subjective entropy determination. The extremal of  $\pi_t(k_p)$ , as a solution of the Euler–Lagrange equation, takes the view

$$\pi_t(k_p) = \frac{\exp \left\{ -\beta \left[ C_{\varepsilon} |\varepsilon_t(k_p)| + C_{k_p} k_p^n \right] \right\}}{\int_0^{1000} \exp \left\{ -\beta \left[ C_{\varepsilon} |\varepsilon_t(k_p)| + C_{k_p} k_p^n \right] \right\} dk_p}. \quad (15)$$

The normalizing condition satisfies. Calculation results [4] shown in Fig. 1 are in the corresponding scale and similar for each separate time.

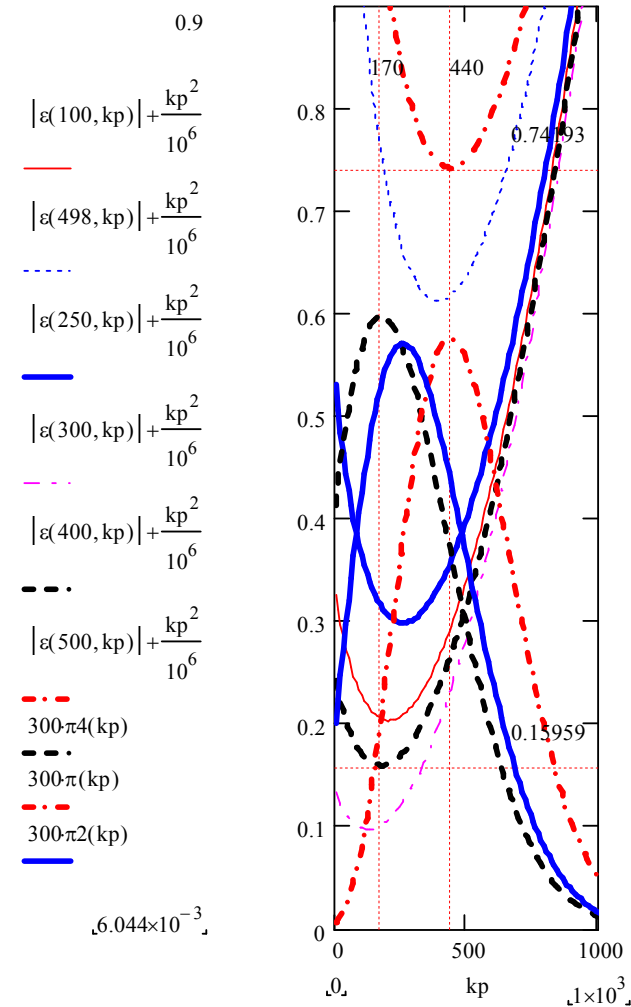


Fig. 1. Optimal value of the governor augmentation coefficient

The developed approach (1) – (12) gives the way to discover the analog to the reverse temperature in the Gibbs distribution with the use of the proposed optimization principle. The concept described with formulae (13) – (15) yields the optimal value for  $k_p$  and maximal value of  $\pi_i(k_p)$  (15) (see Fig. 1).

### III. CONCLUSIONS

From the presented theoretical methods (1) – (12) it is possible to optimize the periodicity of the aircraft or aeronautical engineering scheduled maintenance  $t_p^*$ , formula (1). On the basis of the speculations (13) – (15) illustrated with the examples (see Fig. 1), we can conclude that the preferences distributions densities in the view of  $\pi_i(k_p)$  (15), for each of the predetermined specified

time moments, obtained with the use of SEMP, provide a quantitative model of a qualitative function for an aircraft active navigation and motion adaptive and optimal control system.

In the prospect researches such preferences functions are worth of application to other problems with multidimensional continuous alternatives.

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#### **А. В. Гончаренко. Оптимальні значення керування для активної експлуатації**

Розглянуто, розроблений на основі принципу максимуму суб'єктивної ентропії, метод визначення оптимальних значень керування для періодичності регламентного обслуговування літаків або авіаційної техніки та лінійного безінерційного об'єкту, керованого дією пропорційного регулятора, в одновимірному випадку. Введено математичні моделі для отриманих розподілів щільностей переваг неперервних альтернатив. Виконано розрахункові експерименти. Побудовано необхідні діаграми.

**Ключові слова:** активне керування польотом літака; оптимізація; принцип максимуму суб'єктивної ентропії; неперервна альтернатива; щільність розподілу переваг; варіаційна задача; екстремаль; оптимальний вибір.

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#### **А. В. Гончаренко. Оптимальные значения управления для активной эксплуатации**

Рассмотрен, разработанный на основе принципа максимума субъективной энтропии, метод определения оптимальных значений управления для периодичности регламентного обслуживания самолетов или авиационной техники и линейного безинерциального объекта, управляемого действием пропорционального регулятора, в одномерном случае. Введены математические модели для полученных распределений плотностей предпочтений непрерывных альтернатив. Выполнены расчетные эксперименты. Построены необходимые диаграммы.

**Ключевые слова:** активное управление полетом самолета; оптимизация; принцип максимуму субъективной энтропии; непрерывная альтернатива; плотность распределения предпочтений; вариационная задача; экстремаль; оптимальный выбор.

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