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Abstract: Ramsey regulation, in the context of tariff rebalancing, is analyzed when the regulator is not fully informed about the cost structure of the firm. It is shown that even if the estimated relation between variable costs of the two goods produced is correct, errors regarding the composition of a given total cost between fixed and variable elements result in: (i) the price of the good with a higher (lower) elasticity of demand decreases (increases) as the estimated fixed cost is higher; and (ii) whatever mistake is made, i.e., under or over estimating fixed costs, the profits obtained by the regulated firm are lower than intended.

1. Introduction.

This paper presents briefly the problem that comes out when the composition of the total cost of a regulated multiproduct monopolist is unknown to the regulator in the context of tariff rebalancing. The structure of prices is chosen by the regulator to recover total cost minimizing welfare losses from marginal cost pricing, i.e., according to Ramsey pricing principles.[1] Unlike other forms of asymmetric information, which allow the regulated firm to obtain informational rents so discussed in the literature,[2] the disinformation of the regulator under Ramsey pricing could become a source of losses to the firm. Furthermore, as long as the demand functions for the various goods display different price-elasticities at equilibrium, the structure of prices that the regulator imposes upon the regulated firm differ under various alternative estimations of the composition of the observed total

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[1] The assumption underlying is that there are dynamic aspects (such as predatory pricing) that make it unadvisable to delegate the determination of the structure to the firm under a zero-profit constraint, **as** prices chosen would not be Ramsey in this case.

[2] For a recent and comprehensive treatment of regulation under asymmetric information see Laffont and Tirole (1993).

cost between fixed and variable components, even when the variable costs of the different products remain proportional in the different options. This question is particularly relevant in tariff rebalancing in telecommunications, for instance, where different estimations calculate fixed costs ranging from 20-30% (Burns (1994)) to 80% (Oftel (1993)),[3] and where current discussion exists regarding the inappropriateness of considering access costs as common costs (see Kahn y Shew (1987), and more recently Kaserman and Mayo (1994), Parsons (1994) and Gabel (1995)), each of them thus indicating very different price structures and possible losses to the regulated firm.

The paper is organized as follows. Section 2 presents the model. Section 3 presents a few numerical examples that illustrate the main results. Formal proofs are given in section 4, whereas section 5 closes with some comments.

2. The model.

Consider a benevolent regulator who, dealing with a two-products monopolistic firm, sets prices to maximize social welfare, defined as net consumer surplus, subject to the constraint of recovering **a** fixed cost α through linear prices. Assume for simplicity that both demands and costs are linear, and that they are also independent

[3] In the case of Oftel (1993), the figure corresponds to common or joint costs. However, as the allocation of common costs is necessarily arbitrary, they might be considered to be fixed as well, and they are indeed so regarding each service individually.

(up to the joint fixed cost in the last case). The problem then is the following:

$$\max_{q_1, q_2, \lambda} L = S(q_1) + S(q_2) - P_1(q_1)\,q_1 - P_2(q_2)\,q_2 + $$
$$+ \lambda\left[P_1(q_1)\,q_1 + P_2(q_2)\,q_2 - c_1 q_1 - c_2 q_2 - \alpha\right],$$

where

$$S(q_i) = \int_0^{q_i} P_i(q_i)\,dq_i, \quad \textbf{for } i=1,2$$

represents the (gross) consumer surplus and λ is the Lagrange multiplier.

Letting $P_i(q_i)=a_i-b_iq_i$, for $i=1,2$, the first-order conditions that characterize the solution of this problem are the following:

$$L_{q_1} = b_1 q_1 + \lambda\,[a_1 - 2b_1 q_1 - c_1] = 0,$$
$$L_{q_2} = b_2 q_2 + \lambda\,[a_2 - 2b_2 q_2 - c_2] = 0,$$
$$L_\lambda = (a_1 - b_1 q_1)\,q_1 + (a_2 - b_2 q_2)\,q_2 - c_1 q_1 - c_2 q_2 - \alpha = 0\,.$$

From the first two equations we obtain $q_1=q_2[b_2(a_1-c_1)]/[b_1(a_2-c_2)]$; replacing this equation into the third first-order condition, the value of q_2 that characterizes the solution is given by the following expression:

$$q_2^* = \frac{\dfrac{(a_2-c_2)}{b_2} + \left[\left(\dfrac{a_2-c_2}{b_2}\right)^2 - \dfrac{4b_1\alpha(a_2-c_2)^2}{b_2[b_2(a_1-c_1)^2+b_1(a_2-c_2)^2]}\right]^{1/2}}{2}.$$

Naturally, once q_2^* is computed from this equation for given values of α, a_i, b_i, and c_i, for $i=1,2$, q_1^*, P_1^* and P_2^* are immediately obtained, from which it can also be calculated the total cost (C), its composition between variable and fixed (VC/C), and the equilibrium price-elasticities of demand (η_1 and η_2). As it is well known in the literature, this is just an exercise in Ramsey pricing, where relative mark-ups are inversely related to the price-elasticity of the demands in a way that minimizes the welfare loss from the first best allocation (which we ruled out by imposing the self-financing constraint, as such allocation would require pricing at marginal cost and financing the fixed cost with a transfer collected somewhere else).

The relevant question we are after here, though, is to see what is the effect of a different belief held by the regulator about the true composition of the total cost $C=\alpha+c_1q_1^*+c_2q_2^*$, when, furthermore, the relation between the variable costs (c_1/c_2) is common knowledge (i.e., this ratio is known to be equal to β). That is, we want to compare the price structure (P_1^* and P_2^*) when c_1 and c_2 change, and the value of α is adjusted in the amount of the estimated change in the variable cost (i.e., $\alpha'=\alpha-(c_2'-c_2)(\beta q_1^*+q_2^*)$). In that sense, our discussion applies very well to the context of tariff-rebalancing in the face of technological (or judgement) change regarding the composition of total costs.

3. Results.

We present the results of this exercise in table 1, where different examples are constructed for different values of c,, β, a,, a,, b_1

and b,. The true situation is called "situation 0", whereas situations 1 to 3 represent the solution to the problem when alternative beliefs are held by the regulator about the composition of total (observed) cost C,. Π_j denotes the benefit obtained by the firm in situation j, j=0,1,2,3, whereas $\Pi_{j/0}$ is the level of profits reached by the firm when the regulator believes the situation prevailing is j when the true one is 0.

Note that the "true" situation 0 is not necessarily the initial one. Given the regulated prices prevailing in the status-quo situation, quantities are given by demand functions, and different cost structures are compatible with the observed total cost C,, so that the initial situation (regarding technology) could correspond to either one of them and therefore need not be optimal. It is an important assumption, however, that the true and initial situations be characterized by C_0.[4]

[4] If the description of the regulatory setting had an initially unregulated monopolist choosing prices to maximize profits, and only then a regulator came in and observed its cost, he could also think of different technologies that were compatible with it. However, in the "true" situation 0 to which the regulator wants to go, the observed cost would be different (higher) than the observed cost of the unregulated monopolist. It can be shown with a simple example that beginning to regulate a monopolist with wrong estimations of the cost structure would have an impact on the quantities of both products and on profits that depends on the type of mistake made. (See the following footnote.) The examples and demonstrations below, though, refer to technologies that generate the same cost as that of situation 0, but not necessarily that of an initially unregulated monopolist.

Table 1: Examples 1 to 5

Example 1: $a_1=a_2=20$, $b_1=1$, $b_2=2$.

β = .5	Situation 3	Situation 0	Situation 1	Situation 2
c_1 c_2 α	2.2 4.4 3.354	2 4 10	1 2 43.23	0.5 1 59.846
q_1	17.66	17.59	17.27	17.13
q_2	7.74	7.82	8.18	8.35
P_1	2.34	2.41	2.73	2.87
P_2	4.52	4.36	3.64	3.31
C_j	76.27	76.46	76.87	76.76
Π_j	0.00	0.00	0.00	0.00
η_1	0.13	0.14	0.16	0.17
η_2	0.29	0.28	0.22	0.20
VC_j/C_j	0.96	0.87	0.44	0.22
$\Pi_{j/o}$	-0.02	0.00	-0.41	-0.89

Example 2: $a_1=30$, $a_2=20$, $b_1=1$, $b_2=2$.

β = .5	Situation 3	Situation 0	Situation 1	Situation 2
c_1 c_2 α	2.2 4.4 1.298	2 4 10	1 2 53.512	0.5 1 75.268
q_1	27.76	**27.69**	27.36	27.21
q_2	7.79	7.91	8.49	8.76
P_1	2.24	2.31	2.64	2.79
P_2	4.42	4.18	3.02	2.48
C_j	96.64	97.02	97.85	97.63
Π_j	0.00	0.00	0.00	0.00
η_1	0.08	0.08	0.10	0.10
η_2	0.28	0.26	0.18	0.14
VC_j/C_j	0.99	0.90	0.45	0.23
$\Pi_{j/o}$	-0.04	0.00	-0.83	-1.83

Table 1 (continuation)

Example 3: $a_1=a_2=20$, $b_1=1$, $b_2=2$.

$\beta = 1$	Situation 3	Situation 0	Situation 1	Situation 2
c_1	2.2	2	1	0.5
c_2	2.2	2	1	0.5
α	4.713	10	36.433	49.649
q_1	17.62	17.62	17.62	17.62
q_2	8.81	8.81	8.81	8.81
P_1	2.38	2.38	2.38	2.38
P_2	2.38	2.38	2.38	2.38
C_j	62.87	62.87	62.87	62.87
Π_j	0.00	0.00	0.00	0.00
η_1	0.13	0.13	0.13	0.13
η_2	0.13	0.13	0.13	0.13
VC_j/C_j	0.93	0.84	0.42	0.21
$\Pi_{j/0}$	0.00	0.00	0.00	0.00

Example 4: $a_1=10$, $a_2=15$, $b_1=1$, $b_2=2$.

$\beta = 1$	Situation 3	Situation 0	Situation 1	Situation 2
c_1	2.2	2	1	0.5
c_2	2.2	2	1	0.5
α	7.311	10	23.447	30.171
q_1	7.38	7.42	7.60	7.69
q_2	6.05	6.03	5.91	5.89
P_1	2.62	2.58	2.40	2.31
P_2	2.89	2.94	3.17	3.27
C_j	36.86	36.89	36.97	36.95
Π_j	0.00	0.00	0.00	0.00
η_1	0.36	0.35	0.31	0.30
η_2	0.24	0.24	0.27	0.28
VC_j/C_j	0.80	0.73	0.37	0. 18
$\Pi_{j/0}$	-0.003	0.00	-0.07	-0.16

Table 1 (continuation)

Example 5: $a_1=15$, $a_2=10$, $b_1=1$, $b_2=2$.

$\beta = 1$	Situation 3	Situation 0	Situation 1	Situation 2
c_1	2.2	2	1	0.5
c_2	2.2	2	1	0.5
α	6.779	10	26.107	34.161
q_1	12.34	12.32	12.23	12.19
q_2	3.76	3.79	3.93	3.99
P_1	2.66	2.68	2.77	2.81
P_2	2.48	2.42	2.14	2.01
C_j	42.19	42.21	42.27	42.25
Π_j	0.00	0.00	0.00	0.00
η_1	0.22	0.22	0.23	0.23
η_2	0.33	0.32	0.27	0.25
VC_j/C_j	0.84	0.76	0.38	0.19
$\Pi_{j/0}$	-0.002	0.00	-0.06	-0.12

The conclusions obtained upon observation of these examples are the following ones:

1. Different beliefs held by the regulator about the composition of the observed total cost between fixed and variable cost, being correct about the relative marginal costs between different products, generate different price structures with Ramsey pricing whenever the equilibrium price-elasticity of demand differs among goods.

2. When the believed fixed cost increases, the price of the good whose demand is less elastic increases, and the price of the other good decreases.

3. Whenever the regulator is mistaken -i.e., either if he over or under estimates fixed costs-, the profits of the regulated firm are negative.[5]

4. Naturally, as a mistake is made, the sum of consumers' surplus and the firm's profit is lower than the one under correct Ramsey pricing. This is true even if there are distributive concerns that indicate departures from pure Ramsey pricing, as those considerations are properly accounted for by incorporation of the distributive characteristic of the different goods[6] instead of a miscalculation of the composition of total cost.

5. Changes in prices when estimated fixed costs go from 20% to 80%, depending on the characteristics of the demands, may be significant (up to 30%, comparing situations 0 and 2 in Example 1, for instance), whereas induced losses could be higher than 1% of total cost.

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[5] If a monopolist was initially unregulated, where the demand functions are those corresponding to example 1, and the technology is the one that corresponds to situation 0 in the same case, the quantities that maximize profits are $q_1^*=9$ and $q_2^*=4$, resulting in $C^*=44$ and profits (Π) of 103. Observing q_1^*, q_2^* and C^*, the regulator sets new prices, with the following results: if he is right about the technology (i.e., $c_1=2$, $c_2=4$ and $\alpha=10$), profits are zero as described in situation 0 (in particular $C_0=76,46$); if he overestimates fixed costs compatible with q_1^*, q_2^* and C^* (but not with C_0, i.e., $c_1=1$, $c_2=2$ and $\alpha=27$), both prices are lower than those of situation 0 and the regulated profits would be negative (Π=-17.98); finally, if he underestimates fixed costs ($c_1=2.2$, $c_2=4.4$ and $\alpha=6.6$), prices are higher and profits are positive (Π=3,18).

[6] See Feldstein (1972).

4. Discussion of Results 1 to 3.

Proof of Result 1. We can see from the first two first-order conditions that q_1/q_2 varies with c,, when $c_1=\beta c_2$, with the sign of $a_1-\beta a_2$, according to the following expression:

$$\frac{\partial(\frac{q_1}{q_2})}{\partial c_2} = \frac{b_2\,(a_1-\beta a_2)}{b_1\,(a_2-c_2)^2},$$

Therefore, whenever a, is different than βa_2, q_1/q_2 changes for different values of c,, which means that P_1/P_2 changes too. This proves result 1.

Proof of Result 2. Accordingly, using those two first-order conditions, it can be shown that $a_1>\beta a_2$ is equivalent (in this linear setting) to $\eta_1<\eta_2$, indicating that q_1/q_2 increases with c_2 whenever $\eta_1<\eta_2$, and, equivalently, that P_1/P_2 increases when c_2 decreases under this same condition. Finally, if the two prices changed in the same direction, the budget constraint would not be respected, as total income and total cost would move in opposite directions, whatever technology is in place (i.e., if prices increase, total income also increases -with price-elasticities lower than 1-, but since both quantities decrease, total cost decreases).[7] This proves result 2' above.

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[7] Even if the price-elasticities were higher than one, provided that regulated prices are under the monopoly level, marginal income is lower than marginal cost for both products, meaning that an increase in both prices would generate a reduction in production with a higher reduction in cost than in income.

Proof of Result 3. Regarding result 3, note that total cost in situation 0 is given by $C_0=\alpha_0+c_1^0q_1^0+c_2^0q_2^0$, and by construction, if technology is that corresponding to situation j, that same cost, of producing the same output, is given by $C_0=\alpha_j+c_1^jq_1^0+c_2^jq_2^0$, where by construction $\alpha_j-\alpha_0=q_1^0(c_1^0-c_1^j)+q_2^0(c_2^0-c_2^j)$, as was previously pointed out using different notation. Also, if prices are regulated in a Ramsey manner for situation j, total cost in that situation is given by $C_j=\alpha_j+c_1^jq_1^j+c_2^jq_2^j$, whereas if the true technology is that corresponding to situation 0, total cost $C_{j/0}$ (i.e., the cost of producing Ramsey output designed for situation j when the true technology is 0) is given by $\alpha_0+c_1^0q_1^j+c_2^0q_2^j$. Thus, the expression for $C_j-C_{j/0}$, replacing $\alpha_j-\alpha_0$ above, and since $c_1=\beta c_2$, is given by

$$C_j-C_{j/0} = (c_2^j-c_2^0)\,[\beta\,(q_1^j-q_1^0) + (q_2^j-q_2^0)]\ .$$

Note that, taking infinitesimal changes (denoting differentials with "d"), the above expression indicates that $C_j-C_{j/0}<0$ if and only if $\beta(dq_1/dc_2)+dq_2/dc_2<0$. From the previous result we know that $dq_1/dc_2>0$ and $dq_2/dc_2<0$ when $a_1>\beta a_2$ (case 1), $dq_1/dc_2<0$ and $dq_2/dc_2>0$ when $a_1<\beta a_2$ (case 2), and $dq_1/dc_2=dq_2/dc_2=0$ when $a_1=\beta a_2$ (case 3). Therefore, since values of β lower than a_1/a_2 multiply the positive term in case 1, and values of β higher than a_1/a_2 multiply the negative term in case 2, a continuity argument, using case 3 where $\beta=a_1/a_2$ precisely balances the two terms of opposite sign, yields $C_j-C_{j/0}<0$ for all β different than a_1/a_2. Finally, since (regulated) income is the same in both situations (as prices are regulated assuming that situation j prevails), result ***3*** is obtained: independently of the direction of the error in the estimation made by the regulator, if such mistake is made, the level of profits of the regulated firm (with respect to the one intended) is negative.

5. Concluding Remarks.

The results of this paper indicate that a sounded regulatory practice facing a multiproduct firm could be to rely on it for the estimation of fixed versus variable costs, although no implication was derived regarding the relative variable costs themselves. This is so because the firm has no incentive to misrepresent the true figures, conducing then to the determination of prices that maximize social welfare.

Nevertheless, that delegation to the firm is not free of problems. First, as was mentioned in the introduction, there are dynamic (strategic) considerations of the firm that might induce it to report costs that generate a lower price for a product subject to potential entry. Second, if the asymmetric information extends to the demand curve, where the belief held by the regulator about its position and shape is incorrect, and this is known by the firm, there would be incentives to induce a price structure with a lower (higher) price for the good with price-elasticity higher (lower) than the one believed by the regulator, as the resulting income would then be higher than expected by the regulator.'

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[8] It is true that these features could be contemplated by the regulator in order to "correct" the choices made by the regulated firm. The modelling of this interaction would require some sort of signaling game in which the regulator extracts information from the firm as an application of the Revelation Principle. Following this

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attempt, however, is left for future research.