

## Applying VNPSO Algorithm to Solve the Many-to-Many Hub Location-Routing Problem in a Large scale

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### Abstract

One way to increase the companies' performance and reducing their costs is to concern the transportation industry. Many-to-many hub location-routing problem (MMHLRP) is one of the problems that can affect the process of transportation costs. The problem of MMHLRP is one of the NP-HARD problems. Hence, solving it by exact methods is not affordable; however it was first solved by Benders decomposition algorithm. Modeling and the solving algorithm is able to solve the problem with 100 nodes. In this study, using VNPSO (a combination of the two methods VNS and PSO) was suggested to solve MMHLRP in large-scale. Given high similarity of the results obtained in small scale, using a random sample confirmed that the proposed method was able to solve problem MMHLRP with 300 nodes and acceptable accuracy and speed.

**Keywords:** Many-to-many hub location-routing problem (MMHLRP), VNPSO algorithm, Hub and Spoke networks.

### Introduction

In each country, goods transportation reflects its economic situation and the level of industrial development, so that transportation activities help various industries and businesses associated with the transportation. Transportation is an important component of the economy of each country and because of its fundamental role has great impact on the process of economic growth. So the industry is affecting the overall performance of the companies and their marginal costs. Transportation networks and supply chains, includes all types of transportation costs, and these costs have substantial role in the national economy. In the past two decades, due to development in telecommunications and transportation and logistics systems, different strategies have been devoted to these cases in which Hub and Spoke networks have had a special importance (Gelareh & Nickel, 2011). Since transportation is really important, the providers of goods transportation industry are constantly under pressure to reduce their costs (de Camargo, de Miranda, and Løkketangen, 2013).

Development of economic and social activities makes the need for rapid movement of people and goods, thereby increasing demand in the transportation industry inevitable. That is why the transportation industry suppliers are frequently trying to reduce their costs in order to attract customers. On the other hand this industry is considered as an important factor in the economic development of the community. Planning in transportation network determines the horizon for the future development of a company when the company should be able to compete in the market and also to meet customer demands in the best way. Strategies for these decisions need high costs, which are not easily reversible. So to be successful in the long-term plan, the company should be able to optimize their costs. Optimizing the costs include activities flowing widely in all fields of production, distribution and consumption of goods and services. It includes much transportation of large volumes of packages in the hub facility which creates the possibility of obtaining the economies scales and leads to the reduction in the transportation costs. Over the past decade the

problem of hub and spoke location especially in the field of transportation and posts have attracted the attention of many researchers. In 2010, it was shown in an article that HS networks are highly effective to reduce delivery times in postal shipping programs (Çetiner, Sepil, and Süral, 2010). In another article, a model for logistics network design, considering the network of HS issues combination, was offered and is mentioned in this study. The optimized network for postal facility can be designed (Jeong-Hun Lee, Ilkyeong Moon, 2014). Utilizing the HS network needs creating local tours as well as the current assumptions available in order to give services (O'Kelly, 1992). The flow routing in the network should also be considered in order to transport with the lowest cost. HS is a special part of the location problems. So all the decisions made should be coordination and in this way, many-to-many hub location-routing problem (MMHLRP) gets emerged which has a close relationship with location routing problem (LRP). The problem LRP should be defined as a series of problems in the field of location. This was firstly introduced by Watson et al. (Watson-Gandy & Dohrn, 1973). The aspects of the tour in the problem of LRP were also considered by Bruins in 1998 (Bruins, 1998).

Generally, facilities are not connected to each other in the problem LRP and there is no flow exchange between customers, while exchange flow between each pair of customers and hubs are the main features of the problem in the problem MMHLRP (de Camargo, de Miranda, and Løkketangen, 2013). However, in both of these issues, the number of vehicles available and the facility is not known in advance (Nagy & Salhi, 2007).

#### Statement of problem and literature review

Many-to-many hub location-routing problem (MMHLRP) was first introduced by Nagy and Salhi (Nagy & Salhi, 1998). In their study, mathematical models were applied for this problem and its relationship with the other related problems was studied as well. The study was done to find a way to reduce the costs. However, some constraints were increasingly appeared due to an increase in the number of the customers and this is why this model can be difficult even in small scale in which the model was solved by an heuristic method. In this problem, there are lots of people who are going to send each other some goods. In one estate, it is assumed that each customer sends different goods to the others which are correspondent to the movement of the flow among the customers. In this problem, the route of internal hubs was assumed to be direct, while there are some pauses in the routes between customer and hubs. However, the cost of transportation in the hub- communication was not considered. It was stated that location and routing problem is an approach to locate the facilities and many-to-many hub location-routing problem is an approach to locate the hub. The problem proposed by Nagy and Salhi in 1998, was solved by two other studies with different and new method. Julia (2014) presented the model of location-routing problem along with internal transportation in a situation with multi goods and solved it through genetic algorithm. The transportation processes inside hubs were taken into the consideration in this problem and the model mathematical formula with linear constrains were also considered. Many-to-many hub location-routing problem was developed by Ricardo et al in 2013. They offered a new mathematical model which is a combination of two known formulas: unique hub location problem and itinerant seller problem (de Camargo, de Miranda, and Løkketangen, 2013). The model of the problem after combining the two formulas is as follow:

$$\min \sum_{k \in N} a_k z_{kk} + \sum_{i \in N} \sum_{\substack{k \in N \\ k \neq i}} \hat{c}_{ik} z_{ik} + \sum_{l \in V} \sum_{(u,v) \in A} \sum_{k \in N} \check{c}_{uv} y_{uv}^{kl} + \sum_{l \in V} \sum_{k \in N} \dot{c}_l q_{kl} + \sum_{i \in N} \sum_{\substack{j \in N \\ i < j}} \sum_{k \in N} \sum_{\substack{m \in N \\ k \neq m}} \check{c}_{ij}^{km} x_{ij}^{km} \quad (1)$$

$$s. t.: \sum_{k \in N} z_{ik} = 1 \quad \forall i \in N \quad (2)$$

$$z_{ik} \leq z_{kk} \quad \forall i, k \in N: i \neq k \quad (3)$$

$$\sum_{m \in N} x_{ij}^{km} = z_{ik} \quad \forall i, j, k \in N: i < j \quad (4)$$

$$\sum_{k \in N} x_{ij}^{km} = z_{jm} \quad \forall i, j, m \in N: i < j \quad (5)$$

$$\sum_{(u,v) \in A} y_{uv}^{kl} = p_u^{kl} \quad \forall u, k \in N, \quad l \in V \quad (6)$$

$$\sum_{(u,v) \in A} y_{uv}^{kl} = p_v^{kl} \quad \forall v, k \in N, \quad l \in V \quad (7)$$

$$y_{uv}^{kl} \leq q_{kl} \quad \forall k \in N, (u, v) \in A, l \in V \quad (8)$$

$$q_{kl} \leq q_{k(l-1)} \quad \forall k \in N, l \in V: l > 1 \quad (9)$$

$$\sum_{l \in V} p_t^{kl} = z_{tk} \quad \forall t, k \in N: k \neq t \quad (10)$$

$$\sum_{(u,v) \in A} \lambda_{uv} y_{uv}^{kl} \leq T \quad \forall k \in N, l \in V \quad (11)$$

$$f_{uv}^{klt} \leq y_{uv}^{kl} \quad \forall (u, v) \in A, k, t \in N, l \in V: k \neq t, k \neq v, u \neq t \quad (12)$$

$$\sum_{(k,v) \in A} f_{uv}^{klt} = p_t^{kl} \quad \forall k, t \in N, l \in V: k \neq t \quad (13)$$

$$\sum_{(u,t) \in A} f_{ut}^{klt} = p_t^{kl} \quad \forall k, t \in N, l \in V: k \neq t \quad (14)$$

$$\sum_{(u,t) \in A} f_{uv}^{klt} = \sum_{(v,u) \in A} f_{vu}^{klt} \quad \forall v, k, t \in N, l \in V: k \neq t, v \neq k, v = t \quad (15)$$

$$x_{ij}^{km} \geq 0 \quad \forall i, j, k, m \in N: i < j \quad (16)$$

$$f_{uv}^{klt} \geq 0 \quad \forall (u, v) \in A, t, k \in N, l \in V: v \neq k, u \neq t, k \neq t \quad (17)$$

$$z_{ik} \in \{0,1\} \quad \forall i, k \in N \quad (18)$$

$$q_{kl} \in \{0,1\} \quad \forall k \in N, l \in V \quad (19)$$

$$p_t^{kl} \in \{0,1\} \quad \forall k, t \in N, l \in V: k \neq t \quad (20)$$

$$y_{uv}^{kl} \in \{0,1\} \quad \forall (u, v) \in A, k \in N: l \in V \quad (21)$$

In the above model, the objective function (1) consists of minimizing the costs of installation, management, conducting of local tours, allocating vehicles to each hub and transportation costs within each hub. Constraints (2) - (5) represents centers installation and allocating the non- hub nodes to the hub nodes. Constraints (6) - (15) form local tours and the relationship 16 to 21 show the variables are non- negative and integer (de Camargo, de Miranda, and Løkketangen, 2013). Ricardo et al (2013) solved their proposed model by using Benders decomposition algorithm introduced in 1962 to solve mixed integer problems (Benders, 1962). The main problem is divided into two pure and linear integer problems by his algorithm. Pure integer problem is called main problem (MP) and linear integer is called sub-problem (SP). This method is

solved through repetition form and dependent on the main and sub problem. Sub-problem includes the continuous variables and constraints related to it while main problem includes an integer and continuous variables which connects the two problems to each other. Optimal solution provides the main problem a lower bound (LB) through considering integer values of variables and related constraints and freezing the integer values. A dual is solved for the sub-problem by the obtained solution through solving the main problem or freezing the integer. An upper bound (UB) can be obtained for general purpose of the problem by this solution. Dual solution of sub problems is used to make a Benders incision. In the next iteration, this incision has been added to the main problem and a new lower bound was obtained by solving the problem to ensure that it is not worse than the current low bound. So the main problem and sub problem are repeatedly solved until UB and LB get converged to an optimal solution and a termination condition is reached when the distance between upper bound (UB) and lower bound (LB) gets less than a small number. Ricardo et al (2013) divide the Benders decomposition algorithm into two sub-problems of possible transportation. A number of specimens with dimensions of 100 knots can be solved by the proposed algorithm. Benders decomposition algorithm has been used for various hub problems (de Sá, de Camargo, de Miranda, 2011 & 2013). It can also be used in some other location problems (FazelZarandi, 2010; Haghghat, 2015). Several exact algorithms have been introduced and developed to solve location-routing problem, but the use of these algorithms are limited to examples in small and medium scale (Akca, Berger, Ralphs, 2009; Contardo, Cordeau, Gendron, 2013). Heuristic approaches aimed at finding near optimal solutions are used in the examples with large and medium-scale (Vidal, Crainic, Gendreau, Prins, 2013; Kim, Li a, Johnson, 2013). In this study, to solve large-scale mathematical programming model by Ricardo et al (2013), a proposed algorithm was presented. Heuristic algorithms are proper strategies to solve location and routing problems, because these problems are kind of NP-HARD problems.

### The proposed VNPSO algorithm

Heuristic algorithm a combination of VNS and PSO algorithm is explained in this section. Figure 1 shows a flow chart of the VNPSO algorithm. In the proposed algorithm, all the solutions generated by the PSO algorithm, are improved by VNS algorithm. Particle Swarm Optimization (PSO) algorithm was first introduced by using the previous experiences developed by writers on modeling the collective behavior visible in many types of birds in 1995 (Eberhart & Kennedy, 1995). This algorithm is a method of minimization used to deal with issues which are best answered by a single point or a surface in n-dimensional space. Variable neighborhood search algorithm (VNS) was designed by Bremberg and Mladenovi in 1996 ( Hansen & Mladenovi, 1998 & 1999). VNS is a new meta-heuristic algorithm based on neighborhood systematic change during the search process. All the answers produced by PSO were improved by applying the proposed algorithm. In other words, VNS algorithm is put in parallel with PSO algorithm and whenever better answer is not offered for evaluation function by PSO algorithm then it gets into the VNS algorithm and selects one of the VNS neighborhoods and compares evaluation function. VNPSO Meta-heuristic method expressed in this article was inspired by VNS method. In The algorithm of PSO, if the velocity of a particle gets reduced to the amount of VC, the new velocity is devoted by using the following equation (Liu1, Abraham, Choi, and Hwan Moon, 2006)

$$v_{ij}(t) = w\hat{v} + c_1r_1(x_{ij}^\#(t-1) - x_{ij}(t-1)) + c_2r_2(x_j^*(t-1) - x_{ij}(t-1)) \quad (22)$$

$$\hat{v} = \begin{cases} v_{ij} & \text{if } |v_{ij}| \geq v_c \\ u(-1,1) v_{max}/\rho & \text{if } |v_{ij}| < v_c \end{cases} \quad (23)$$

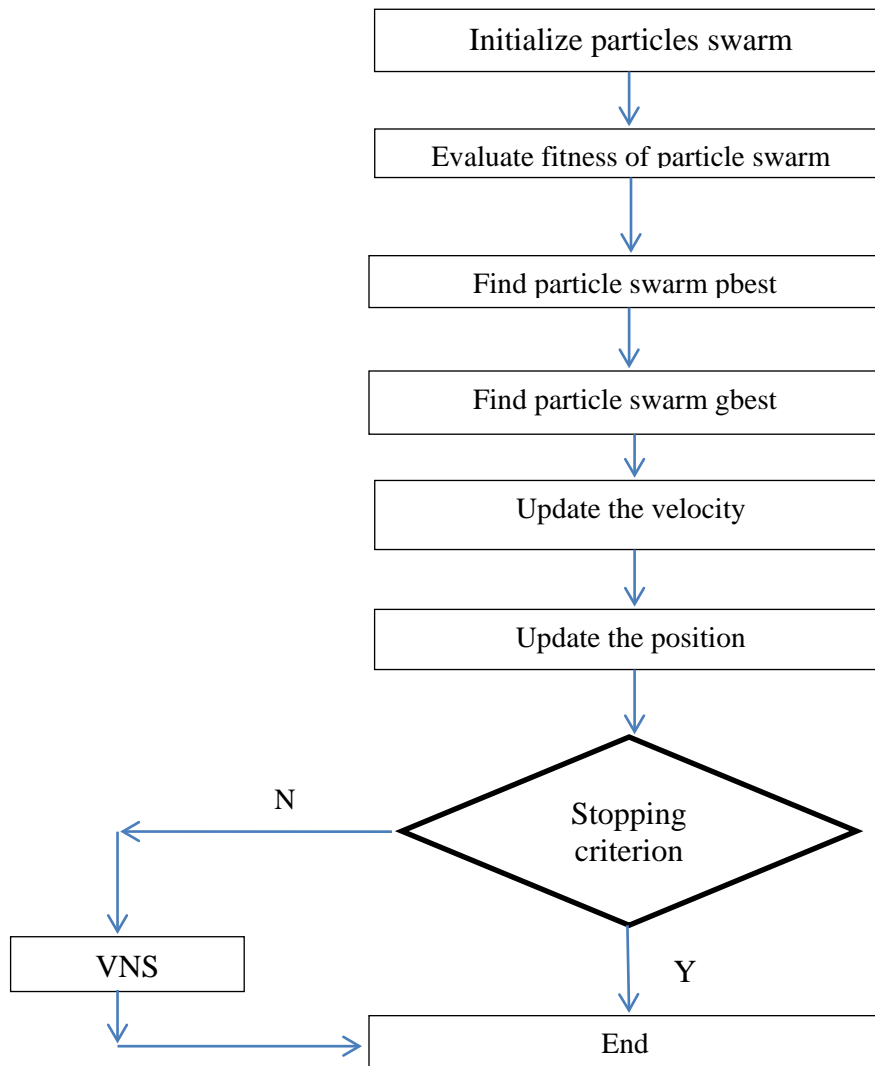
In fact, PSO algorithm is used for problems with continuous space and given that the chromosomes discussed in this paper consists of two sections in which the first section is integer and the second section is binary, it is attempted to control the speed in the interval (0.1) in the proposed hybrid algorithm in order to define the use of the relation of location-based particles as zero and one. For this purpose, the following formula is used to convert binary numbers:

$$x_{ij}(t+1) = \begin{cases} 0 & \text{if } r_i(t) \geq f(v_{ij}(t)) \\ 1 & \text{if } r_i(t) < f(v_{ij}(t)) \end{cases}$$

$$f(v_{ij}(t)) = \frac{1}{1 + e^{-v_{ij}(t)}}$$

$r_i(t)$ : a random number for particle  $i$  in zero and one intervals

$v_{ij}(t)$ : velocity of particle  $i$ -th in the  $j$ -th place



**Figure 1: Flow chart of VNPS algorithm**

### Algorithm 1. Variable Neighborhood Particle Swarm Optimization Algorithm

01. Initialize the size of the particle swarm  $n$ , and other parameters.
02. Initialize the positions and the velocities for all the particles randomly.
03. Set the flag of iterations without improvement  $Nohope = 0$ .
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04. While (the end criterion is not met) do
05.  $t = t + 1$ ;
06. Calculate the fitness value of each particle;
07.  $x^* = argmin_{i=1}^n (f(x^*(t - 1)), f(x_1(t)), f(x_2(t)), \dots, f(x_i(t)), \dots, f(x_n(t)))$ ;
08. If  $x^*$  is improved then  $Nohope = 0$ , else  $Nohope = Nohope + 1$ .
09. For  $i = 1$  to  $n$
10.  $x_i^\#(t) = argmin_{i=1}^n (f(x_i^\#(t - 1)), f(x_i(t)))$
11. For  $j = 1$  to  $d$
12. If  $Nohope < 10$  then
13. Update the  $j$ -th dimension value of  $x_i$  and  $v_i$
14. according to Eqs
 
$$v_{ij}(t) = wv_{ij}(t - 1) + c_1r_1(x_{ij}^\#(t - 1) - x_{ij}(t - 1)) + c_2r_2(x_j^*(t - 1) - x_{ij}(t - 1))$$

$$x_{ij}(t) = x_{ij}(t - 1) + v_{ij}(t)$$
15. else
16. Update the  $j$ -th dimension value of  $x_i$  and  $v_i$
17. according to Eqs(1),(2)
18. Next  $j$
19. Next  $i$
20. End While.

**Figure 2: VNPSO Meta-heuristic algorithm**

#### Computational results

In this section, VNPSO algorithm performance was evaluated for a number of sample problems in terms of solution quality and solution time. To evaluate the performance of the proposed solution, some tests were conducted on a random sample data. The data is generated randomly with monotonous distribution. Using a random sample of the numerous problems, the comparison of the proposed method with Benders decomposition method (in GAMS software with solver (cplex)), can be performed only for problems small sizes. The proposed algorithm in Matlab 7.5 programming environment was implemented using Matlab 7.5 software toolbox. All of these algorithms have been implemented on a PC with a processor specification CORE i7, 64 -bit, 3 GHz, 6 GB RAM, Windows 7. Typical problems are created randomly with uniform distribution with two small and large groups. Exact solution and obtaining optimal solutions except for a bunch of little things shown in table 1, is not possible for other problems while the results of the proposed algorithm shows the quality of the algorithm in order to find an appropriate answer to the problem under study (with a maximum of 20.37 % error) in a reasonable time (less than a minute). The

results of table 2 shows the optimum solution of the proposed meta-heuristic algorithm has significantly better performance than Benders decomposition algorithm in large-scale.

**Table 1: The results of Benders algorithm in compare to VNPSO algorithms for random problems in small- scale**

Problem parameter										Bender decomposition		VNPSO		Recommended solution efficiency %	
node	Number of vehicle	T the maximum time allowed for the tours	Alpha	the traveling time of arc $(u, v)$ $\hat{\lambda}_{uv}$	flow demand from client i to customer i $w_i$	fixed cost of installing a hub at node k $a_k$	cost of handling the incoming and outgoing demands of client I by hub k $\hat{c}_{ik}$	the transportation cost of demands for the inter-hub connection $\hat{c}_{ij}^{km}$	unit transportation cost of inter-hub connection $c_{ij}^{km}$	the cost of traveling by a vehicle $c_{uv}$	Time (seconds)	Best Cost	Time (seconds)	Best cost	
10	3	100	0.01	Randi ([3 6])	Randi ([100 300])	Randi ([2000 3000])	Randi ([200 300])	Randi ([200 300])	randi([5 15])	Randi ([300 600])	75.704	4790	27.756	5690	84.18
10	5	100	0.01	Randi ([3 6])	Randi ([100 300])	Randi ([2000 3000])	Randi ([200 300])	Randi ([200 300])	randi([5 15])	Randi ([300 600])	126.941	4790	32.849	6015	79.63
10	3	100	0.001	Randi ([3 6])	Randi ([100 300])	Randi ([2000 3000])	Randi ([200 300])	Randi ([200 300])	randi([5 15])	Randi ([300 600])	73.503	4772	33.690	5027	94.92
10	3	200	0.0001	Randi ([3 6])	Randi ([100 300])	Randi ([2000 3000])	Randi ([200 300])	Randi ([200 300])	randi([5 15])	Randi ([300 600])	73.540	4770	30.298	4864	98.06
20	4	200	0.01	Randi ([3 6])	Randi ([100 300])	Randi ([2000 3000])	Randi ([200 300])	Randi ([200 300])	randi([5 15])	Randi ([300 600])	•	•	57.075	11592	

• shows that the algorithm is not able to solve the problem in these dimensions

The solution proposed in this study has been used for different number of nodes and was compared with the results of previous studies on the Benders decomposition algorithm. As shown in Table 1, GAMS software and Benders algorithm are not able to solve the model due to the memory constrains for the nodes more than 10. While the values of VNPSO and Benders algorithms are similar to each other in an acceptable level (with a maximum of 20.37 % error). This similarity is listed in a separate column to the ratio of percentage. According to the time of the solution of proposed algorithm (with less than a minute), it is observed that this algorithm has better performance in compare to Benders decomposition algorithm (with more than two minutes for some cases). According to the above table, solving the problem in large-scale is very difficult, time consuming and somewhat impossible. Hence, trying to find meta-heuristic answers seems feasible and ideal. By examining Table 2 it can be seen that the time required to solve the problem is increased by the enlargement of the aspects of the problem (increasing the number of nodes). To increase the amount of time needed to solve the problem expresses the fact that for large-scale problems, which are not practically possible to solve by the exact algorithm it is needed to go to the meta-heuristic algorithms. However, mixed meta-heuristic algorithm is able to solve large problems in appropriate time.

Table 2: The results of VNPSO algorithms computational for random problems in large-scale

node	Number of vehicle	T the maximum time allowed for the tours	Alpha	Problem parameter								VNPSO	
				$\lambda_{uv}$ the traveling time of arc (u, v)	$w_{ij}$ flow demand from client i to customer j	$a_k$ fixed cost of installing a hub at node k	$\hat{c}_{ik}$ cost of handling the incoming and outgoing demands of client I by hub k	$\hat{c}_{ij}^{km}$ the transportation cost of demands for the inter-hub connection	$c_{km}$ unit transportation cost of inter-hub connection	$c_{uv}''$ the cost of traveling by a vehicle	Time (seconds)	Best cost	
10	3	100	0.01	randi([3 6])	Randi ((100 300))	Randi ((2000 3000))	Randi ((200 300))	Randi ((200 300))	Randi ((200 300))	Randi ((15 15))	Randi ((300 600))	27.756	5690
20	4	200	0.01	randi([3 6])	Randi ((100 300))	Randi ((2000 3000))	Randi ((200 300))	Randi ((200 300))	Randi ((200 300))	randi([5 15])	Randi ((300 600))	57.075	11592
40	6	200	0.01	randi([3 6])	Randi ((100 300))	Randi ((2000 3000))	Randi ((200 300))	Randi ((200 300))	Randi ((200 300))	randi([5 15])	Randi ((300 600))	147.937	28400
60	8	200	0.01	randi([3 6])	Randi ((100 300))	Randi ((2000 3000))	Randi ((200 300))	Randi ((200 300))	Randi ((200 300))	randi([5 15])	Randi ((300 600))	265.219	53651
80	10	200	0.01	randi([3 6])	Randi ((100 300))	Randi ((2000 3000))	Randi ((200 300))	Randi ((200 300))	Randi ((200 300))	randi([5 15])	Randi ((300 600))	428.266	86752
100	12	200	0.01	randi([3 6])	Randi ((100 300))	Randi ((2000 3000))	Randi ((200 300))	Randi ((200 300))	Randi ((200 300))	randi([5 15])	Randi ((300 600))	643.412	127657
120	14	300	0.01	randi([3 6])	Randi ((100 300))	Randi ((2000 3000))	Randi ((200 300))	Randi ((200 300))	Randi ((200 300))	randi([5 15])	Randi ((300 600))	1032.788	176092
140	16	300	0.01	randi([3 6])	Randi ((100 300))	Randi ((2000 3000))	Randi ((200 300))	Randi ((200 300))	Randi ((200 300))	randi([5 15])	Randi ((300 600))	1377.239	233840
160	18	300	0.01	randi([3 6])	Randi ((100 300))	Randi ((2000 3000))	Randi ((200 300))	Randi ((200 300))	Randi ((200 300))	randi([5 15])	Randi ((300 600))	1816.831	297919
180	20	300	0.01	randi([3 6])	Randi ((100 300))	Randi ((2000 3000))	Randi ((200 300))	Randi ((200 300))	Randi ((200 300))	randi([5 15])	Randi ((300 600))	2386.524	371595
200	22	500	0.01	randi([3 6])	Randi ((100 300))	Randi ((2000 3000))	Randi ((200 300))	Randi ((200 300))	Randi ((200 300))	randi([5 15])	Randi ((300 600))	2831.768	450701
220	24	500	0.01	randi([3 6])	Randi ((100 300))	Randi ((2000 3000))	Randi ((200 300))	Randi ((200 300))	Randi ((200 300))	randi([5 15])	Randi ((300 600))	3601.374	540557
240	26	500	0.01	randi([3 6])	Randi ((100 300))	Randi ((2000 3000))	Randi ((200 300))	Randi ((200 300))	Randi ((200 300))	randi([5 15])	Randi ((300 600))	4275.118	636903
260	28	600	0.01	randi([3 6])	Randi ((100 300))	Randi ((2000 3000))	Randi ((200 300))	Randi ((200 300))	Randi ((200 300))	randi([5 15])	Randi ((300 600))	5289.502	742624
280	30	800	0.01	randi([3 6])	Randi ((100 300))	Randi ((2000 3000))	Randi ((200 300))	Randi ((200 300))	Randi ((200 300))	randi([5 15])	Randi ((300 600))	6560.524	854074
300	32	800	0.01	randi([3 6])	Randi ((100 300))	Randi ((2000 3000))	Randi ((200 300))	Randi ((200 300))	Randi ((200 300))	randi([5 15])	Randi ((300 600))	7784.737	973625

**Conclusion and suggestions**

This study presents the use of VNPSO algorithms to solve many-to-many hub location-routing problem (MMHLRP) aimed at minimization of total costs of the system including the total cost of installation , administration , enforcement , local tours , devoting vehicles to hubs, and transportation cost of internal hubs. Due to high computational complexity of the problem, using the exact solution, especially for large scale problems in a reasonable computational time is not possible. Therefore, in this study, an approximate solution algorithm is developed based on particle swarm optimization algorithm. The efficiency of the proposed algorithm is compared with Benders decomposition algorithm using numerous samples of the problems created randomly. Numerical results indicate that the proposed algorithm has better performance in large- scale problems. In future research, the issue of development for other conditions that may exist in the industry, would be very useful. In this regard, it is recommended to develop the problem for the case where there is uncertainty or the possibility of sending a number of means of transportation at any time for a supplier.



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