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Rotating black holes in conformal gravity

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Abstract

It is a long time since the motion of astronomical objects has been explained in the framework of Newtonian gravity. The great success of Newton's law of universal gravitation in planetary motion persuaded astronomers to use this regime as a viable framework in the larger scales. Meanwhile, whenever a deviation of observed motions from expected ones were observed, the immediate question came up: should such anomalies be considered as incompleteness of laws of gravitation or as indication of the existence of unseen objects? In following we explain rotating black holes and solving this metric's black hole with conformal gravity. Firstly explaining problem with DM then introducing conformal transformation and conformal gravity, in the last step we solve a metric for rotating black hole in the presence and absence of matter.

Keywords: Rotation, Gravity, Black Holes, Conformal Gravity

Introduction

During the last century Einstein gravity (EG) was one the corner of theoretical physics. Despite of the success in explanation of various gravitational phenomena in nature, there are some unsolved basic problems such as singularity problem, black hole physics,… and most importantly quantum theory of gravity. There was an enormous effort in these lines to solve such problems but up to now, it has not been obtained a complete theory of gravity. There was an enormous effort in these lines to solve such problems butit has not been obtained a complete theory of gravity.

One of such alternative theories of gravity is Conformal Gravity(CG)(Maldacena, 1997), a gravitational theory which is based on a large symmetry principle known as conformal symmetry. Intuitively, beside of local Lorentz symmetry, it also has an scaling symmetry in which the physics is invariant under the rescaling the metric as $g_{\mu\nu} \longrightarrow e^{\Omega(x)} g_{\mu\nu}$. Due to such interesting symmetry CG has some special properties such that it may consider as an alternative theory of EG for solving those old problems. For example, since it does not suffer from infinity problem because and CG has a well behaved quantum nature. In addition it admits the common EG vacuum solutions and so it agree whit Newtonian gravity at solar scale. Also it produce a linearity growing potential which may explain galactic rotation curve without considering dark matter. It was further argued that its cosmological solution explain accelerating universe without using dark energy.

In contrast with all these interesting good news of CG there are some bad phenomenological and formal things which need to be solved. In particular, in Stabile (2013); Rose (2013);Benitez (2013) and Oda (2013), it was claim that the weak field limit of CG is not agree with solar system observation specially with light deflection. More importantly, it is believed that most of hire derivative gravities face with tachyonic or ghost fields in the theory (Klusoň et al., 2014). Also only the null geodesics are physically meaningful because the point particle lagrangian is not conformaly invariant (Dea, 2013; Bars et al., 2014).

However, recently, there were a new attraction to conformal gravity and gravities related to it [maldacena, hoe, massive gravity, …]. For example, it was argued that EG can be obtained from CG by appropriate boundary conditions or ghost field may disappear in conformably extended of EG [hope].

Another approach for solving the ghost problem was suggested in [Manheinn]. A different scenario was also suggested for solving these problem by extending CG theory to a scalar-tensor theory (Ghosh& Pranzetti, 2014;de Freitas & Reall, 2014). Due to such observations and in continue of the works [Verbein] were some solutions of CG theory was found we are interested on finding rotating solutions of CG.

Materials and Methods

For solving the rotating black holes first of all we talk about the theory of Conformal Gravity(CG) and write the Lagrangian and action of this theory then we introduce our metric and calculate the coefficient of this metric in CG and show the difference between our answer in EG and CG.

Conformal Gravity

Conformal Gravity(CG) (Mannheim, 2006) is one of many possible extensions of Einstein Gravity(GR), which due to some interesting symmetry properties may have a good framework for solving some old problems of GR such as singularity, dark energy and etc. (Mannheim& O'Brien, 2013).

The starting point is the following action

$$S_{CG} = -\alpha_g \int d^4x(-g)^{1/2} C_{\mu\nu\lambda\theta}C^{\mu\nu\lambda\theta} + S_M$$
$$= -2\alpha_g \int d^4x(-g)^{1/2}\left(\left(R_{\mu\nu}\right)^2 - \frac{1}{3}R^2\right) + S_M \qquad (1)$$

Where

$$C_{\lambda\mu\nu\kappa} = R_{\mu\nu\lambda\theta} + \frac{1}{6}R\left[g_{\mu\lambda}g_{\nu\theta} - g_{\mu\theta}g_{\lambda\nu}\right] - \frac{1}{2}\left[g_{\mu\lambda}R_{\nu\theta} - g_{\mu\theta}R_{\lambda\nu} - g_{\nu\lambda}R_{\mu\theta} + g_{\mu\theta}R_{\nu\lambda}\right] \qquad (2)$$

is the Weyl conformal tensor. The pure gravity part of the action is invariant under the general scaling $g_{\mu\nu} \longrightarrow e^{\Omega(x)}g_{\mu\nu}$. As a result the overall coupling of the theory $-\alpha_g$ is dimensionless which seems is a good news for UV finiteness of the theory.

After varying the action with respect to the metric and one obtains the equation of motion as

$$4\alpha_g W^{\mu\nu} = T_M^{\mu\nu} \qquad (3)$$

where $T_M^{\mu\nu}$ is the Bach tensor and is defines as

$$W^{\mu\nu} = \frac{1}{3}\nabla_\mu\nabla_\nu R - \nabla_\lambda\nabla^\lambda R_{\mu\nu} + \frac{1}{6}\left(R^2 + \nabla_\lambda\nabla^\lambda R - 3(R_{k\theta})^2\right)g_{\mu\nu} + 2R^{k\theta}R_{\mu\kappa\nu\theta}$$
$$-\frac{2}{3}RR_{\mu\nu} \qquad (4)$$

in addition, one finds that the matter part of the action should also respect to the scaling symmetry because the left hand side of is traceless so the matter part of the action should have a traceless energy-momentum tensor. Fortunately, by introducing a conformal coupling term for the scalar mass term the standard model lagrangian is also conformably invariant (Mannheim, 2012).

In particular we have

$$\mathcal{L} = \frac{1}{2}\left(D_\mu\phi\right)^+\left(D^\mu\phi\right) - \frac{1}{12}R|\phi|^2 - \frac{\lambda}{4}|\phi|^4 - \frac{1}{4}F_{\mu\nu}^a F^{a\mu\nu} \qquad (5)$$

where $D_\mu = \nabla_\mu - ieA_\mu^a T_a$ and $F_{\mu\nu}^a$ is the lie algebra valued field strength tensor of gauge field. The equation of motion for these fields are

$$D_\mu D^\mu \phi + \lambda|\phi|^2\phi + \frac{R}{6}\phi = 0 \quad (6)$$

$$D_\mu F^{a\mu\nu} = -\frac{ie}{2}(\phi^+ T^a(D^\nu\phi) - (D^\nu\phi)^+ T^a \phi) \quad (7)$$

we also obtain

$$T_M^{\mu\nu} = \frac{1}{6}\left(g^{\mu\nu}\nabla^\lambda\nabla_\lambda|\phi|^2 - \nabla^\mu\nabla^\nu|\phi|^2 - G^{\mu\nu}|\phi|^2\right) \quad (8)$$

where $G^{\mu\nu}$ is Einstein tensor.

Various Abelian and non-abelian, spherical or non-spherical solutions of the above equation of motion has been studied in Mannheim& Kazanas, (1991). For example for a spherically symmetric solution

$$ds^2 = B(r)dt^2 - {dr^2}/{B(r)} - r^2(d\theta^2 + sin^2\theta d\phi^2) \quad (9)$$

one find

$$B(r) = c_0 + c_1 r + \frac{c_2}{r} + c_3 r^2. \quad (10)$$

Rotating Black Holes

In this section, firstly, we obtain the slowly rotating solutions for pure conformal gravity and then by considering the matter part of the action we try to solve the equation of motion for a charged slowly rotating black hole.

Pure Gravity

Let us consider the following line element around a rotating black hole

$$ds^2 = B(r)dt^2 - \frac{dr^2}{B(r)} - r^2 d\theta^2 - r^2 sin^2\theta\left(d\phi - \frac{N(r)}{r}dt\right)^2 \quad (12)$$

So metric matrices is equal to

The non vanishing components of Bach tensor read as $W^{\theta\phi}$, W^{rr},W^{tt},$W^{\phi t}$,$W^{\theta\theta}$,$W^{\phi\phi}$ which their answers are attached in the end of article.

Solving the six differential equations simultaneously, one may obtain the following solution

$$B(r) = C_1 + \frac{1}{3}\frac{C_1^2 - 1}{C_2 r} + C_2 r + C_3 r^2 \quad (13)$$

$$N(r) = C_4 \quad (14)$$

The radius of the horizon r_+ is given by the condition $B(r) = 0$ which is a cubic algebric equation. We solve this metric with Einstein equation and gain

$$B(r) = 1 + \frac{C_2}{r}$$

$$N(\mathrm{r}) = \mathrm{C}_1 \quad (15)$$

Gravity with Matter

As we said, in conformal gravity with varying action respect to the metric we gain

$$4\alpha_g W^{\mu\nu} = T_M^{\mu\nu} \quad (16)$$

In the existence of matter $T_M^{\mu\nu} \neq 0$ so we have

$$T_{\mu\nu} = \nabla_\mu \varphi \nabla_\nu \varphi - \frac{1}{2} g_{\mu\nu} g^{\rho\sigma} \nabla_\rho \varphi \nabla_\sigma \varphi - g_{\mu\nu} \varphi + \frac{1}{6}\left(\nabla^\lambda \nabla_\lambda \varphi^2 - \nabla_\mu \nabla_\nu \varphi^2 - G_{\mu\nu} \varphi^2\right) \quad (17)$$

Among all the higher order gravitational theories (Setare eta l., 2013; Butter et al., 2014), CG is unique in the sense that it is based on an additional symmetry principle. The conformal symmetry imposes severe limitations on the allowed matter sources. When matter is described in terms of a Lagrangian, it is very much constrained, but the Abelian and non-Abelian (n generators T^a) Higgs models are essentially still consistent with the conformal symmetry provided the scalar field “mass term” is replaced with the appropriate “conformal coupling” term which introduces a non minimal coupling to the Ricci scalar R. The matter Lagrangian which we will use here is therefore

$$\mathcal{L}_m = \frac{1}{2}\left(\nabla_\mu \phi\right)^+ (\nabla^\mu \phi) - \frac{1}{12} R|\phi|^2 - \frac{\lambda}{4}|\phi|^4 - \frac{1}{4} F^a_{\mu\nu} F^{a\mu\nu} \quad (18)$$

The resulting field equation is

$$\nabla_\mu \nabla^\mu \varphi - \lambda \varphi^3 + \frac{R}{6}\varphi = 0 \quad (19)$$

By solving this field equation with Solving the six differential equations simultaneously

$$4\alpha_g W^{\varphi t} = T_M^{\varphi t}, 4\alpha_g W^{rr} = T_M^{rr}, 4\alpha_g W^{\varphi\varphi} = T_M^{\varphi\varphi}$$
$$4\alpha_g W^{\theta r} = T_M^{\theta r}, 4\alpha_g W^{t\varphi} = T_M^{t\varphi}, 4\alpha_g W^{r\theta} = T_M^{r\theta}$$
$$4\alpha_g W^{tt} = T_M^{tt}, 4\alpha_g W^{\theta\theta} = T_M^{\theta\theta} \quad (20)$$

We have

$$N(r) = C_4 \,, B(r) = C_1 + {}^1/_3 \frac{C_1^2 - 1}{3C_3 r} + C_2 r^2 + C_3 r \,, \varphi(r) = 0 \quad (21)$$

And

$$N(r) = 0 \,, B(r) = C_1 + {}^1/_3 \frac{C_1^2 - 1}{3C_3 r} + C_2 r^2 + C_3 r \,, \varphi(r) = \pm 1/3 \frac{\sqrt{6}}{\sqrt{\lambda r}} \quad (22)$$

Results

The answer for B(r), coefficient of time shows the added sentences in comparison with solving this metric in EG, in other word by solving this metric in EG without matter we have

$$B(r) = 1 + \frac{C_2}{r}$$

However solving this metric in CG gives us

$$B(r) = C_1 + \frac{1}{3}\frac{C_1^2 - 1}{C_2 r} + C_2 r + C_3 r^2$$

Conclusion

As we have seen above, by solving metric with conformal gravity we gain B(r) different from solving this metric in Einstein equation, we can say that this added sentences can replaced instead of DM. In this situation because B(r) is potential of black hole we can say that we cover DM with conformal gravity and we do not use this vague matter.

In future research we can work to find a suitable Lagrangian as a geometric resolution of dark matter, and compare the results of field equations with astronomical and cosmological observations.

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