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A Note on Supersymmetry Breaking in 1+1 – Dimensions

by

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Abstract

The spontaneous breakdown of supersymmetry for a single chiral superfield in 1+1 – dimensions is investigated and a number of puzzling features are resolved by noting that the loop expansion is not always valid. It is found that both broken and unbroken supersymmetry are stable with respect to radiative corrections.

1. Introduction.

The question of spontaneous breakdown of supersymmetry in 1+1 dimensions has been studied in a recent paper(1), at the end of which attention has been drawn to the puzzling fact that the computations of the effective potential with and without the Wess-Zumino dummy-field F do not always agree.

The aim of the present paper is to resolve this puzzle and also two other problems which arise in the analyses of ref. 1, namely, the apparent shift in the position of the potential minimum and the apparently complex nature of the effective potential in the cases that the supersymmetry is broken and unbroken at the classical level respectively. Such problems have been considered in general in a recent preprint(2) and here we consider them partly as an application of the results of that preprint and partly as a further elucidation of 1+1 supersymmetry breaking.

What we find is that all three problems mentioned have the same origin, namely the failure of the conventional loop expansion. This expansion is actually valid only when the classical field is a minimal (steepest descent) solution of the Euler-Lagrange equations, and in the supersymmetric model this is not always the case. For the dummy field F it is never the case and so this field must be eliminated before the effective potential techniques can be used. For the classically unbroken supersymmetry it is not the case for those values of the field for which the classical potential has negative curvature, but the situation can be retrieved by replacing the conventional loop expansion by an interpolated one. The interpolated expansion gives an effective potential which is not only real but positive for all values of the field, in accordance with the general principles of supersymmetry. Finally for classically broken supersymmetry the fermion field is massless and the loop expansion fails because of an infra-red divergence coming from this field. We propose an alternative approximation for the effective potential in this case, and in this approximation at least, the radiative corrections preserve both the spontaneous breakdown and the location of the potential minimum.

2. The Model.

The supersymmetric model considered here and in ref. 1 consists of a single chiral scalar(3) superfield $\Phi = (\varphi, \psi, F)$ whose components consist of one scalar field φ, one fermion field ψ and one dummy field F. The

Lagrangian is

$$-\mathcal{L} = \tfrac{1}{2}\left\{ (\partial\varphi)^2 + i\bar{\psi}\gamma\psi + F^2 - 2mFS(\varphi) - mS'(\varphi)\bar{\psi}\psi \right\}, \qquad (2.1)$$

where $S(\varphi)$ may be any well-behaved potential function, but which in practice is taken to be

$$S(\varphi) = \varphi^2 - b,$$

where the constant b may be either positive or negative. The classical Euler-Lagrange equations for F are clearly

$$F = mS, \qquad (2.2)$$

and hence, on elimination of F the Lagrangian (2.1) becomes

$$\mathcal{L}_\varphi = \tfrac{1}{2}\left\{ (\partial\varphi)^2 + i\bar{\psi}\gamma\psi - 2m\bar{\psi}\psi\varphi - m^2(\varphi^2 - b)^2 \right\}. \qquad (2.3)$$

Since a spontaneous breakdown of supersymmetry occurs[3] if, and only if, $F \neq 0$ at the potential minimum it is clear that (in contrast to reflexion symmetry) supersymmetry is broken or unbroken at the classical level according as $b \lessgtr 0$. Thus

$$b<0 \Rightarrow \text{broken supersymmetry}, \qquad b>0 \Rightarrow \text{unbroken supersymmetry}, \qquad (2.4)$$

at the classical level. The masses of the scalar and fermion fields become

$$m_\varphi^2 = 2m^2|b|,\ m_\psi^2 = 0, \qquad m_\varphi^2 = m_\psi^2 = 4m^2|b|, \qquad (2.5)$$

in the respective cases.

3. Functional Integration and the Dummy Field.

In this section we wish to consider the computation of the effective potential with and without the dummy field F. We recall[4] that for any Lagrangian $\mathcal{L}(\varphi,\eta)$ with scalar field φ and any other fields η, the effective potential is (minus) the constant field limit of the effective action, and the latter is just the Legendre transform of the Schwinger functional W(J) defined as

$$e^{W(J)} \quad \int d(\varphi,\eta)\, e^{\int dx[\mathcal{L}(\varphi,\eta) + J\varphi]} \qquad (3.1)$$

where N is a normalization factor and the x-integration is over Euclidean 2-space. Hence it will be sufficient to consider W(J), which for the supersymmetric Lagrangian (2.1) becomes

$$e^{W_F(J)} = N\int d\mu(\varphi)d\mu(F)\, e^{\frac{1}{2}\int dx[(\partial\varphi)^2 + F^2 - 2mFS]} = \int d\mu(\varphi)\, e^{\frac{1}{2}\int dx[(\partial\varphi)^2 - m^2S^2]} \int d\mu(F)\, e^{\frac{1}{2}\int dx(F-mS)^2} \qquad (3.2)$$

where we have completed the square in F and, for simplicity, have suppressed the fermion terms. If the F-measure is assumed to be translational-invariant the variable F-mS may be changed to F and then the F-integral becomes independent of S and cancels against a similiar term in the normalizing factor N, leaving

$$e^{W(J)} = N'\int d\mu(\varphi)\, e^{\frac{1}{2}\int dx[(\partial\varphi)^2 - m^2S^2]} \qquad (3.3)$$

The functional (3.3) is just the one corresponding to the Lagrangian (2.3) in which F has been eliminated, and thus the end-result with and without the dummy field F is the same.

We should like to emphasize, however, that in spite of this agreement the derivation (3.2)-(3.3) is at best formal, because, on account of the plus sign preceding F^2 in the exponential, the F-integral is actually divergent. Furthermore, this plus sign is not a matter of convention because it is necessary in order to have a minus sign for m^2S^2 and hence convergence of the φ-integral. (The best that one could do to improve the convergence would be to Wick-rotate to Minkowski-space where both the F and φ integrands become oscillatory)

Once the F-integral is divergent, the translational invariance of the measure no longer makes sense. Furthermore the classical EL-path $F=mS$ in (2.2) is maximal rather than minimal and the loop expansion around it is no longer valid. The formal expansion may give correct results in some cases (just as (3.3) comes out correctly) but it will not be a reliable tool for investigating problems such as the complexity of the effective potential, which, as we have seen in ref.2, depends crucially on the validity of the loop expansion. For this reason (except for illustrative purposes in section 5) we shall eliminate the dummy field F from the beginning, as in (2.3), and use only the convergent Schwinger functional W(J) as in (3.3) for the remaining two sections of this note.

4. Spontaneous Breakdown at Classical Level.

In section 2 we saw that when $b<0$ the model-Lagrangian

$$\mathcal{L} = \tfrac{1}{2}\left\{ (\partial\varphi)^2 + i\bar{\psi}\gamma\psi - 2m\bar{\psi}\psi\varphi - m^2(\varphi^2 + a^2)^2 \right\}, \qquad a^2 = |b|, \qquad (4.1)$$

suffered a spontaneous breakdown of supersymmetry at the classical level.

The question we wish to consider in this section is how the radiative corrections affect this breakdown. The usual computation of the one-loop contribution to the effective potential yields

$$\frac{1}{m^2}\overset{(1)}{V}_{\text{eff}} = \frac{1}{2}S^2 + \frac{\hbar}{8\pi}\left\{SS'' + (S'^2)\ln(S'^2) - (SS')'\ln(SS')'\right\}, \qquad S=\varphi^2+a^2. \qquad (4.2)$$

Since (4.2) is still spontaneously broken the authors of ref. 1 conclude that (at least to first order in $\hbar$) there is no restoration of supersymmetry. But since the term $(S')^2\ln(S')^2$ dominates, and is negative for small φ they also conclude that there is a shift in the position of the potential minimum. We disagree with this latter conclusion, on the grounds that the loop expansion is no longer valid for small φ.

In order to see that the loop expansion fails we note that (4.2) can be written in the form

$$\frac{1}{m^2}\overset{(1)}{V}_{\text{eff}} = \frac{1}{2}a^2\left\{a^2+\frac{\hbar}{2\pi}\left[1-\ln(6\varphi^2+2a^2)\right]\right\}+\varphi^2\left\{a^2+\frac{\hbar}{4\pi}\left[1+2\ln(4\varphi^2)-3\ln(6\varphi^2+2a^2)\right]\right\}+\frac{1}{2}\varphi^4, \qquad (4.3)$$

which shows that Γ is an expansion in $\hbar\ln\varphi^2$ not $\hbar\varphi^2\ln\varphi^2$. It follows that the expansion is no longer valid when $\hbar\ln\varphi^2 > O(1)$ which happens for small φ. The situation is analogous to that encountered by Coleman and Weinberg[5] (CW) in their φ^4-model, for which the 1-loop effective potential is of the form

$$\overset{(1)}{V}_{\text{eff}}(\varphi) = \varphi^4\left[1+\hbar\ln(\varphi^2)\right]. \qquad (4.4)$$

The difficulty in both cases is due to the presence of massless fields (the φ-field itself for CW and the fermion field in the present instance) since if these fields were massive, with mass μ say, the infra-red divergent term $\ln(\varphi^2)$ would be replaced by $\ln(\varphi^2+\mu^2)$.

Because of the infra-red divergence it is not possible to determine whether or not the potential minimum has shifted at the one-loop level, and the question arises as to whether the one-loop computation can be improved so as to decide this question. In the CW case the one-loop computation (4.4) was improved to

$$\tilde{V}_{\text{eff}}(\varphi) = \varphi^4\left[1-\hbar\ln(\varphi^2)\right]^{-1}, \qquad (4.5)$$

by using the renormalization group and a dimensional argument. The effect of the improvement (4.5) is dramatic, since it changes the sign of $\tilde{V}_{\text{eff}}(\varphi)$ for small φ. In the present model the CW dimensional argument fails because of the presence of a massive field (mass-parameter m for the φ-field) in (4.1) but we suggest that an improvement on the one-loop formula (4.2) may be obtained as follows:

First we note that since the infra-red term $\ln(\varphi^2)$ in (4.2) is the only one to give a negative contribution to $V_{\text{eff}}(\varphi)$ we may concentrate on the infra-red limit. But in this limit the mass of the φ-field should not matter (note that the mass-parameter m does not occur in the $\ln(\varphi^2)$ term) and hence we should be able to approximate the Lagrangian (4.1) with one in which the φ-field is infinitely heavy and is independent of x. Such an approximate Lagrangian is obtained by setting $\varphi = \theta/ma$ and then letting $m\to\infty$, to get

$$\mathcal{L}_{IR} = \frac{1}{2}\bar{\psi}\gamma\psi + \frac{1}{a}\bar{\psi}\psi\theta - \theta^2, \qquad (4.6)$$

To obtain the effective potential V^{IR}_{eff} corresponding to (4.2) it is convenient to compute first the Schwinger function

$$Z_{IR} = e^{W_{IR}(J)} = \int d(\theta,\bar{\psi},\psi)\, e^{J\theta+\int dx\,\mathcal{L}_{IR}}, \qquad (4.7)$$

(recalling that $V^{IR}_{\text{eff}}(\theta)$ is its Legendre transform). A first integration of (4.7) may be obtained by integrating either θ or $(\bar{\psi},\psi)$. The integration over θ yields

$$e^{W_{IR}(J)} = \frac{e^{-\frac{J^2}{4}}}{a}\int d(\psi,\bar{\psi})\, e^{\int \frac{1}{2}\bar{\psi}\gamma\psi - \frac{J}{2a}\bar{\psi}\psi + \frac{1}{4a^2}(\bar{\psi}\psi)^2}, \qquad (4.8)$$

while the integration over $(\bar{\psi},\psi)$, which requires renormalization since $\psi(x)$ is a field, yields

$$e^{W_{IR}(\theta)} = \int d\theta\, e^{-\theta^2\left(1+\frac{\hbar}{8\pi a^2}\ln\frac{\theta^2}{a^2}\right)+J\theta} \quad \text{or} \quad \int d\mu(\theta)\, e^{J\theta}, \quad \text{where} \quad d\mu(\theta)= e^{-\theta^2\left(1+\frac{\hbar}{8\pi a^2}\ln\frac{\theta^2}{a^2}\right)}d\theta. \qquad (4.9)$$

The expression (4.8) shows that $W_{IR}(J)$ is the sum of all the connected vacuum graphs for a self-interacting 4-fermion theory with mass J/μ. Since the one-loop contribution reproduces the $\varphi^2 \ln(\varphi^2)$ in (4.1) and the higher 4-fermion loops resemble the bubble graphs of the CW correction, this result supports the plausibility of (4.6).

The expression (4.9) is more useful than (4.8) for practical purposes since it expresses $W_{IR}(J)$ in terms of an ordinary integral. It is not possible to express the integral in terms of more elementary functions but the integral from is already sufficient to show that $W_{IR}(J)$ is convex, in which case its Legendre transform $V^{IR}_{eff}(\bar\varphi)$ is automatically convex. Indeed from (4.9) we have

$$\frac{\partial W_{IR}}{\partial J} = \bar\varphi = \int d\theta\, j_\mu(\theta)\, e^{J\theta} \quad \text{and} \quad \frac{\partial^2 W_{IR}}{\partial J^2} = e^{-2W_{IR}}\left\{ \int \theta^2 j_\mu(\theta) e^{-J\theta} \int j_\mu(\theta) e^{-J\theta} - \left[\int \theta j_\mu(\theta) e^{-J\theta}\right]^2 \right\}, \tag{4.10}$$

which shows that $\bar\varphi = 0$ when $J=0$ and that W_{IR} is convex for all J. Since $W_{IR}(J)$ is manifestly positive, it follows from (4.10) that the Legendre transform $V^{IR}_{eff}(\bar\varphi)$ is positive and convex for all $\bar\varphi$, with minimum at $\bar\varphi = 0$. Thus in the present approximation at least, both the symmetry breaking and the position of the potential minimum are left unchanged by the radiative corrections. A numerical computation of $V^{IR}_{eff}(\bar\varphi)$ from (4.9) verifies and illustrates this result (Fig. 1).

5. Reality and Positivity of the Unbroken Effective Potential.

Let us now consider the case ($b > 0$) when the supersymmetry is unbroken at the classical level. The model-Lagrangian may then be written as

$$\mathcal{L} = \tfrac{1}{2}(\partial\varphi)^2 + \tfrac{i}{2}\bar\psi\partial\!\!\!/\psi - m\bar\psi\varphi\psi - \tfrac{m^2}{2}(\varphi^2 - a^2)^2, \qquad \text{where} \qquad a^2 = b. \tag{5.1}$$

Both the φ-field and ψ-field have mass $2ma$ and hence there is no infra-red problem for the loop-expansion. However, in this case the loop expansion has another problem, namely, that it becomes complex for some values of the field. Indeed the conventional one-loop expression

$$\frac{1}{m^2}\Gamma^{(1)}(\varphi) = \tfrac{1}{2}(\varphi^2 - a^2)^2 + \hbar\,\gamma(\varphi), \quad \text{where} \quad \gamma(\varphi) = \frac{1}{4\pi}\left\{(\varphi^2 - a^2) + 4\varphi^2 \ln(4\varphi^2) - (3\varphi^2 - a^2)\ln(3\varphi^2 - a^2)\right\}, \tag{5.2}$$

is manifestly complex for $\varphi^2 < a^2/3$. The original puzzle of ref. 1 stemmed from the fact that (5.2) did not agree with the result obtained using the

dummy-field F, for which (in the numerical computation at any rate) no complexity appears. Actually, the expression obtained using F is ambiguous because the one-loop Euler-Lagrange equation

$$F = (\varphi^2 - a^2) - \frac{\hbar}{4\pi}\ln(4\varphi^2 + 2F), \tag{5.3}$$

admits both a perturbative complex solution

$$F \simeq (\varphi^2 - a^2) - \frac{\hbar}{4\pi}\ln(3\varphi^2 - a^2), \tag{5.4}$$

and a non-perturbative (and therefore inadmissable) real solution

$$F \simeq -2\varphi^2 + e^{-\frac{4\pi(a^2 - 3\varphi^2)}{\hbar}}, \qquad (\varphi^2 \le a^2). \tag{5.5}$$

and what happens is that the computer picks up the real non-perturbative solution for $\varphi^2 \le a^2$. But since (5.3) is itself perturbative only the perturbative solution makes sense, and hence the effective potential computed from (5.3) should be complex. This agrees with the result obtained when F is first eliminated and thus resolves this particular puzzle.

The result that the one-loop effective potential is complex whether or not F is eliminated raises another problem, however, because, by <u>definition</u>, the effective potential should be real. Indeed since the Schwinger functional W(J), defined as

$$e^{W(J)} = \int d(\theta\psi, \bar\psi)\, e^{\int dx(\mathcal{L} + J\varphi)}, \tag{5.6}$$

is manifestly real (in Euclidean space) and V_{eff} is just its Legendre transform, V_{eff} should obviously be real.

This second problem is actually not a new one, and, as has been shown in ref.2, it occurs whenever the classical potential is not convex (as in (5.1). The point is that the loop-expansion for W(J) is an expansion of the form

$$e^{W(J)} = e^{-\Omega(V(\mathring\varphi) + J\mathring\varphi)} \int d\varphi\, e^{\int \frac{1}{2}(\partial\varphi)^2 - \frac{1}{2}V''(\mathring\varphi)\varphi^2 + \dots} = e^{-\Omega(V(\mathring\varphi) + J\mathring\varphi + \hbar\gamma(\mathring\varphi) + O(\hbar^2))}, \qquad \Omega = \int d^2x, \tag{5.7}$$

around a point $\mathring\varphi$ which is tacitly assumed to be a <u>minimal</u> (steepest descent) solution of the classical Euler-Lagrange equations (with potential $V(\varphi) + J\varphi$). But, in the non-convex case, there will always be values of $\mathring\varphi$ for which $(V(\mathring\varphi) + J\mathring\varphi)'' \equiv V''(\mathring\varphi)$ is negative. For these values of $\mathring\varphi$ the path will be maximal, not minimal, and the integral in (5.7) will not converge.

The way out of the difficulty is to use instead of the formal expression (5.7) the expansion about the true minimum of $V(\varphi)+J\varphi$ for each J. However, this computation is not trivial because, for quartic non-convex potentials at least, there exists always a critical value J_c of J for which $V(\varphi)+J\varphi$ becomes a spontaneously broken potential (with its degenerate minima at $\varphi=\varphi_1$ and φ_2 say). Then for $J \gtrsim J_c + O(\hbar)$ the absolute minima are at the analytic continuations of φ_1 and φ_2 respectively, while for $J=J_c+O(\hbar)$ there are two absolute minima and both must be used in (5.2). Thus for $J \simeq J_c$ one obtains an interpolated loop expansion. The general procedure has been given in ref. 2 and here we concentrate on the special case (5.1).

Because the potential $m^2(\varphi^2-a^2)^2$ in (5.1) is already spontaneously broken we have $J_c=0$ and $(\varphi_1,\varphi_2)=(a,-a)$ in this special case. For $J>O(\hbar)$ it is easy to see that the minimum $\mathring{\varphi}(J)$ of $V(\varphi)+J\varphi$ is unique and that $\mathring{\varphi}(J)>a$. Hence for $J>O(\hbar)$ we have from (5.2)

$$W(J) \simeq V(\varphi)+J\varphi+\gamma(\varphi), \qquad \bar{\varphi}=\frac{\partial W}{\partial J}\simeq\varphi, \tag{5.8}$$

where

$$2m^2\mathring{\varphi}(\mathring{\varphi}^2-a^2)+J=0 \qquad (\Rightarrow \mathring{\varphi}(J)\geq a), \tag{5.9}$$

and hence

$$V^{(1)}_{eff}(\bar{\varphi})=V(\bar{\varphi})+\gamma(\bar{\varphi}), \qquad \text{where} \quad \bar{\varphi}>a. \tag{5.10}$$

Here $\varphi(J)$ is the minimal solution of (5.9) and $\gamma(\varphi)$ is the formal expression given in (5.2). Similarly, for $J<O(\hbar)$ we obtain

$$V_{eff}(\bar{\varphi}) = V(\bar{\varphi})+\gamma(\bar{\varphi}) \qquad \bar{\varphi}<-a. \tag{5.11}$$

Thus for $|J|>O(\hbar)$ the usual one-loop expansion is valid, but $\bar{\varphi}$ is limited to the range $|\bar{\varphi}|>a$ For $J\simeq O(\hbar)$ on the other hand, the minima of $V(\varphi)+J\varphi$ are approximately at the minima of $V(\varphi)$ namely at $\varphi=\pm a$ and since these minima are degenerate both minima must be used in (5.3). We then have

$$e^{W(J)} \simeq e^{-\Omega(Ja+\gamma(a))}+e^{-\Omega(-Ja+\gamma(a))}, \tag{5.12}$$

where in the second-step we have used the reflexion symmetry of (5.1) to set $\gamma(-a)=\gamma(a)$ but (for pedagogical reasons) have not yet used the supersymmetry to set $\gamma(a)=0$. From (5.12) we easily find that

$$\bar{\varphi}=\frac{\partial W}{\partial J}\simeq a\tanh\left(\frac{Ja}{\hbar}\Omega\right). \tag{5.13}$$

Eq. (5.13) implies that $-a<\bar{\varphi}<a$ and hence, comparing with (5.10) and (5.11) we see that it completes the range of $\bar{\varphi}$. From (5.12) and (5.13) we then have

$$V_{eff}(\bar{\varphi})=\gamma(a)+\frac{1}{2\Omega}\left\{\left(\frac{a+\bar{\varphi}}{a}\right)\ln\left(\frac{a+\bar{\varphi}}{a}\right)+\left(\frac{a-\bar{\varphi}}{a}\right)\ln\left(\frac{a-\bar{\varphi}}{a}\right)\right\}, \quad \text{for} \quad -a<\bar{\varphi}<a, \tag{5.14}$$

where in the second equality we have used the supersymmetric property $\gamma(a)=0$ which follows from (5.2). Furthermore, since terms of order Ω^{-1} have been neglected in computing $\gamma(\bar{\varphi})$ in (5.10) and (5.11) consistency requires that we neglect them in (5.14) also. Thus in the limit $\Omega\to\infty$ we have

$$\begin{aligned} V_{eff}(\varphi) &= \tfrac{1}{2}m^2(\varphi^2-a^2)^2+\frac{\hbar}{4\pi}\left\{(\varphi^2-a^2)+4\varphi^2\ln 4\varphi^2-(3\varphi^2-a^2)\ln[2(3\varphi^2-a^2)]\right\}, \\ V_{eff}(\varphi) &= 0, \quad \text{for } |\varphi|\geq a \quad \text{and} \quad |\varphi|\leq a \quad \text{respectively,} \end{aligned} \tag{5.15}$$

as the final expression for the effective potential. This expression is manifestly real and positive and is plotted in Fig. 2.

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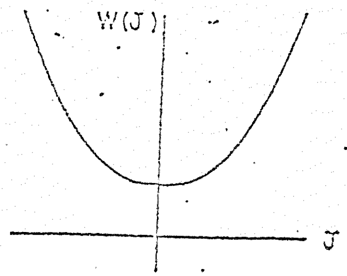

Fig. 1a

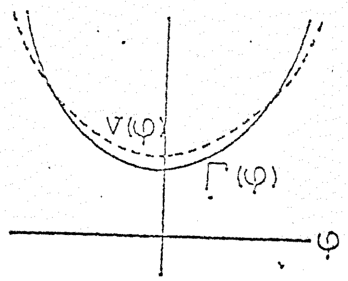

Fig. 1b

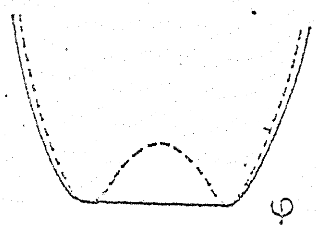

Figure Captions

Fig. 1a. W_{IR} for broken supersymmetry.

Fig. 1b. V_{cl} (dashed line) and V_{eff}^{IR} (solid line) for broken supersymmetry.

Fig. 2. V_{cl} (dashed line) and V_{eff} (solid line) for unbroken supersymmetry.