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THE SQUEEZED QUANTUM STATE AT FINITE TEMPERATURE

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The methods of thermofield dynamics are used to study the properties of a squeezed quantum state which is in an environment maintained at finite temperature. A relationship is established between the squeezing parameter and temperature. In addition, the variances of certain quantum operators are obtained using a squeezed quantum state at finite temperature.

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I. INTRODUCTION

It is possible to produce systems where the variance of a certain quantum operator is below the standard quantum limit. This is accomplished with the preparation of a squeezed quantum state[1-7]. These states can be seen in a number of examples. They appear in the description of quantum noise in a dissipative quantum oscillator[8], in the quantum representation of a parametric amplifier[9], and in four wave laser mixing[10-12] where they have recently been observed. In addition, these states are believed to be important to the development of a laser interferometer[13] which could be used as a means of detecting gravitational radiation. It is expected that in the future these states will play an important role in communications with weak signals[14]. Most practical systems will usually be under the influence of thermal effects. It is with this prospect in mind that it is believed to be interesting to study the influence of temperature on the properties of a squeezed quantum state. This is accomplished with the the introduction of a finite temperature vacuum state[15] which allows the evaluation of ensemble averages of quantum operators as the matrix element formed with these finite temperature vacuum states. In this paper, the methods of thermofield dynamics are applied to a squeezed coherent quantum state and a finite temperature squeezed quantum state is found which acts as the new vacuum state in this representation. This new state can then be used to form matrix elements of quantum operators so as to obtain the ensemble averages of these operators when they are under the influence of a squeezed state.

II. THE SQUEEZED STATE AT FINITE TEMPERATURE

The squeezed coherent state at finite temperture is generated from the vacuum state $|0\rangle\otimes|\tilde{0}\rangle$ upon the application of the operator $K(\theta,\alpha,\xi)$ which is defined in terms of the squeezed state operator $S(\xi)$[16], the coherent state operator $D(\alpha)$[17], and the adjoint of the finite temperature operator $G(\theta)$[18] as $K(\theta,\alpha,\xi) = G^{\dagger}(\theta)D(\alpha)S(\xi)$ so that the finite temperature squeezed coherent state becomes

$$|\theta,\alpha,\xi\rangle = K(\theta,\alpha,\xi)|0\rangle\otimes|\tilde{0}\rangle. \tag{2.1}$$

The operators $S(\xi)$, $D(\alpha)$, and $G(\theta)$ are defined in terms of the boson

operators$A$,$A^{\dagger}$, $\tilde{A}$, and $\tilde{A}^{\dagger}$ as

$$S(\xi) = exp[(\xi^{*}A^{2} - \xi A^{\dagger 2})/2]$$

$$D(\alpha) = exp[\alpha A^{\dagger} - \alpha^{*}A] \tag{2.2}$$

$$G(\theta) = exp[\theta(A\tilde{A} - A^{\dagger}\tilde{A}^{\dagger})].$$

In these expressions both ξ and α are complex numbers, and theta is related to temperature through the expression

$$f^{1/2}(\beta) = sinh(\theta)$$

$$f(\beta) = (exp(\beta) - 1)^{-1} \tag{2.3}$$

$$\beta = \hbar\omega/KT$$

where $\hbar$ is Planck's constant, K is Boltzmann's constant, and ω is the angular frequency associated with a simple harmonic oscillator.

The operator $G(\theta)$ involves the commuting boson operators A and $\tilde{A}$ and their adjoints. When the adjoint of this unitary operator is applied to the vacuum state $|0\rangle\otimes|\tilde{0}\rangle$ which is annihilated by either A or $\tilde{A}$, one obtains a finite temperature vacuum state which is annihilated by the operator

$$A(\theta) = G^{\dagger}(\theta)AG(\theta). \tag{2.4}$$

The ensemble average of an operator $O(A,A^{\dagger})$ is found by forming the matrix element of this operator using the finite temperature vacuum state in the form

$$Tr(\rho O(A,A^\dagger)) = (\tilde{0}| \otimes (0|G(\theta)O(A,A^\dagger)G^\dagger(\theta)|0) \otimes |\tilde{0}). \qquad (2.5)$$

III. THE EVALUATION OF VARIANCES

The finite temperature squeezed state that has been introduced in the previous section can now be used to evaluate the variances of operators that are of physical interest. The conjugate operators Q, and P ,$[Q,P] = i$, associated with a simple harmonic oscillator are represented as

$$2^{1/2}Q = A + A^\dagger$$
$$2^{1/2}iP = A - A^\dagger \qquad (3.1)$$
$$H = N + 1/2$$

where H is the Hamiltonian and where $N = A^\dagger A$ is the number operator.

The ensemble average of an operator evaluated between a squeezed quantum state is found upon forming the matrix element of the operator using the finite temperature squeezed coherent state $|\theta\alpha\xi)$ to write

$$(\xi\alpha\theta|O(A,A^\dagger)|\theta\alpha\xi) = (\tilde{0}| \otimes (0|K^\dagger(\theta,\alpha,\xi)O(A,A^\dagger)K(\theta,\alpha,\xi)|0) \otimes |\tilde{0}). \qquad (3.2)$$

Since $K(\theta,\alpha,\xi)$ is unitary, these matrix elements can be evaluated upon using the transformation

$$K^\dagger(\theta,\alpha,\xi)AK(\theta,\alpha,\xi) = cosh(\theta)[Acosh|\xi| - e^{i\phi}sinh|\xi|A^\dagger + \alpha] + sinh(\theta)\tilde{A}^\dagger, \qquad (3.3)$$

its adjoint,and the fact that both A and $\tilde{A}$ annihilate the zero temperature vacuum state $|0) \otimes |\tilde{0})$. Before presenting the results for the variances of different operators, it is useful to introduce the self-adjoint operators Y_1 and Y_2 defined by the equations

$$Y_1 + iY_2 = (Q + iP)e^{-i\phi/2}$$
$$Y_1 - iY_2 = (Q - iP)e^{i\phi/2}. \qquad (3.4)$$

The variance in an operator $O(A, A^\dagger)$ is defined by the equation

$$\sigma^2(O)_{\theta\alpha\xi} = (\xi\alpha\theta|O^2|\theta\alpha\xi) - (\xi\alpha\theta|O|\theta\alpha\xi)^2. \qquad (3.5)$$

Following the method of evaluation described above, one obtains the results

$$2\sigma^2(Y_1)_{\theta\alpha\xi} = (1 + f(\beta))e^{-2|\xi|} + f(\beta)$$
$$2\sigma^2(Y_2)_{\theta\alpha\xi} = (1 + f(\beta))e^{2|\xi|} + f(\beta). \qquad (3.6)$$

In a similar manner the variances for Q, P, and H are found to be

$$\sigma^2(Q)_{\theta\alpha\xi} = (1+f(\beta))[sinh^2|\xi| - cos\phi sinh|\xi|cosh|\xi| + 1/2] + f(\beta)/2$$

$$\sigma^2(P)_{\theta\alpha\xi} = (1+f(\beta))[sinh^2|\xi| + cos\phi sinh|\xi|cosh|\xi| + 1/2] + f(\beta)/2$$

$$\begin{aligned}\sigma^2(H)_{\theta\alpha\xi} &= [(1+f(\beta))(sinh^2|\xi| + |\alpha|^2) + f(\beta)] \\ &\times [(1+f(\beta))(sinh^2|\xi| + |\alpha|^2) + f(\beta) + 1] \\ &- (1+f(\beta))^2|\alpha|^4. \end{aligned} \tag{3.7}$$

The results for the various special cases can be easily obtained when any of the parameters θ, α , or ξ has the value zero.

IV. THE RELATIONSHIP BETWEEN SQUEEZING AND TEMPERATURE

It is of interest to establish a relationship between the value of the squeezing parameter ξ and the temperature T. When both θ and ξ are zero, one finds the standard quantum limits

$$\sigma(Q) = \sigma(P) = \sigma(Y_1) = \sigma(Y_2) = 2^{-1/2}. \tag{4.1}$$

If, returning to (3.6), it is required that the variance in Y_1 has the standard quantum limit value, then the relationship between temperature and the squeezing parameter is expressed as

$$\hbar\omega/KT = ln(1 + coth|\xi|). \tag{4.2}$$

It is now easily seen that in the limit $\xi \to \infty$, the standard quantum limit is maintained for temperatures which satisfy the relationship

$$\hbar\omega/KT = ln2. \tag{4.3}$$

For the operators considered, it is known that the variances in conjugate variables must satisfy the Weyl inequality which for Y_1 and Y_2 becomes

$$\sigma(Y_1)\sigma(Y_2) \geq 1/2. \tag{4.4}$$

In the coherent state standard quantum limit both the variances are equal and the inequality becomes an equality. It is possible using a squeezed state to find variances below this limit in one of these variables; however, the variance in the second variable becomes greater than the standard quantum limit. It is still possible to maintain the variance in one of these variables, e.g. Y_1, at the standard quantum limit if one has the relationship (4.2).

V. DISCUSSION

The results for the variances given in (3.6) and (3.7) are important for measurements on systems where the quantities to be observed are of the order of magnitude of quantum fluctuations. If a system is prepared in a squeezed state it is possible to obtain variances below the standard coherent state quantum limit; however, if the mechanism which is used to generate the squeezed state is affected by thermal fluctuations, then the squeezed state variance can increase. At a characteristic temperature represented by the relationship (4.2), the standard quantum limit is restored. In the limit when the squeezing parameter becomes infinite, the characteristic temperature is found from (4.3). This is the temperature at which the mean number of quanta as determined from the number operator is equal to one. In addition, it is the characteristic temperature for a quantum limited amplifier[19]. A squeezed state affected by thermal noise could result from a parametric amplifier of the type described in [9] which is operating in a heat bath. The results for the variances of a system which produces a coherent state at zero temperature but which is under the influence of thermal fluctuations can be found from (3.6) and (3.7) in the limit of vanishing ξ. A system of this type could be realized with a classical current [20] in a low temperature heat bath.

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