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DISCONNECTED NON-MAXIMAL STABILITY GROUPS AND HORIZONTAL SYMMETRY*

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ABSTRACT

We study the (2,2) representation of SU(3) and show that in this case non-maximal and disconnected stability subgroups exist. From this particular example we extract a general rule for obtaining non-maximal stability groups. The resulting principle is applied to SO(16) Grand Unified Theory. We build a model with four left-handed and four right-handed families and with W_4 (Weyl Group of SU(4)) as the discrete horizontal symmetry group.

In this talk, we are going to correct or, at least, modify the following two statements:

1. "... all discrete symmetries (to describe the generations) appear completely arbitrary and artificial."[1]
2. For irreducible representations and renormalizable Higgs potentials the stability groups of the potential minimum are maximal little groups. (Michel conjecture[2])

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We will see that, first, there is more room than the Michel conjecture would allow for,[3] and, that, second, the Weyl group appears naturally as a discrete symmetry group.

<u>A non-maximal stability group.</u> Let us consider the 27-dimensional representation of SU(3) which can be realized as a traceless symmetric 8x8 matrix M satisfying the subsidiary condition $d^a_{bc}M_{ac} = 0$, where d is the Gell-Mann totally symmetric invariant tensor. The group acts on M by conjugation. The important property of this representation is that it is contained twice in its symmetric product. That is, given the $\underline{27}$ M, there are two independent ways to construct a $\underline{27}$ quadratic in M, $M \vee M$ and $M \triangledown M$, say.

If for a given little group the invariant vector M is unique (up to normalization) then

$$\mathring{M} \vee \mathring{M} = a(\mathring{M})\ \mathring{M}, \quad \mathring{M} \triangledown \mathring{M} = b(\mathring{M})\ \mathring{M}, \quad \mathrm{tr}(\mathring{M})^2 = 1,$$

must hold. In particular, these equations hold (with different constants a and b, as a calculation shows) for the maximal little groups SO(3), U(2) and $W_3 \wedge (U(1) \times U(1))$ which can be shown to have a unique invariant vector each. If for a given little group the invariant vectors form a one-parameter family (up to normalization) these equations do not have to hold. We know, however, that

$$\cos t\ \mathring{M} \vee \mathring{M} + \sin t\ \mathring{M} \triangledown \mathring{M} = s\ \mathring{M}, \quad \mathrm{tr}(\mathring{M})^2 = 1, \qquad (*)$$

holds for suitable s and t. Within this category falls the little group $W_2 \wedge (U(1) \times U(1))$ whose invariant vectors can be shown to form a one-parameter family.

Furthermore, we can choose s and t such that eq. (*) is satisfied for one and only one of the four little groups mentioned above. In particular, we can choose s and t such that only for

$W_2 \wedge (U(1) \times U(1))$ the potential

$$V = \mathrm{tr}(\cos t\, M \vee M + \sin t\, M \triangledown M - s\, M)^2 + (\mathrm{tr}\, M^2 - 1)^2$$

is zero. Since we can prove that SO(3), U(2) and $W_3 \wedge (U(1) \times U(1))$ are the only maximal little groups in our case we have constructed a counter-example to the Michel conjecture. Of course, with a different choice of s and t we can pick any other group out of our four candidates.

Notice that we have not proven that $W_2 \wedge (U(1) \times U(1))$ is the only stability group of the potential minimum. Because we have a complete list of maximal groups we know, however, that whatever other stability groups there might be they are certainly not maximal. In this respect (the completeness of the list of maximal groups) our counter-example differs from the first counter-example for continuous Lie groups found by Abud, Anastaze, Eckert and Ruegg.[4)]

<u>The Weyl group as discrete symmetry group.</u> In our special case, we have seen that the Weyl group appears naturally. To show that this is no special feature of SU(3) we go on to discuss other examples. The obvious next step is to study SU(4) gauge theory with the 84-dimensional representation which is the analogue of the <u>27</u> of SU(3). Also the <u>84</u> of SU(4) admits two independent symmetric algebras which again gives us a chance to construct models with unbroken symmetries which would not be allowed according to the Michel conjecture.

To know which symmetries we could pick out we have to count the number of parameters of the family of invariant vectors. If the number of parameters is less or equal to one we can find a potential whose absolute minimum is the corresponding little group. Our analysis yields that among others we can pick out $W_4 \wedge U^3(1)$ which has a unique invariant vector and is therefore a maximal little group.

Because $SU(2) \times (W_2 \wedge U^2(1))$ has a one-parameter family of invariant vectors we can also pick out this symmetry which could serve as a mini-model with a vertical non-abelian symmetry SU(2), a horizontal discrete symmetry W_2 and two U(1) symmetries.

After we have studied SU(4) we go on to discuss SO(16). The idea is to allow for a vertical SO(10) symmetry and to use our knowledge of SO(6), which is equivalent to SU(4), to produce the horizontal symmetries. Obviously, SO(10) and SO(6) fit nicely into SO(16). The Higgs field we choose is, of course, the analogue of the ones discussed above, namely the $\underline{5304}$. The $\underline{5304}$ is again contained twice in its symmetric product. So, because $SO(10) \times (W_4 \wedge U^3(1))$ has a one-parameter family of invariant vectors we can pick it out as the unbroken symmetry. With the fermions in the spinor representation Δ^{-}_{16} of SO(16) we have a model with 4 generations Δ^{-}_{10} and 4 generations Δ^{+}_{10}, the vertical group SO(10), the horizontal discrete group W_4 and some additional U(1)'s. It is worth noting that the 4-dimensional representation in question is irreducible with respect to W_4.

The above model is not yet satisfactory, especially because it allows for an SO(16) - invariant mass-term at the grand unification scale and has too many generations. However, our discussion has already brought into focus some general group theoretical principles which should be relevant in many different contexts.

REFERENCES

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