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NUMERICAL INVESTIGATION OF CHAOS IN THE TIME-DELAY IKEDA LASER-RING CAVITY.

by

PAUL JENKINS
SCHOOL OF PHYSICAL SCIENCES
NATIONAL INSTITUTE for HIGHER EDUCATION DUBLIN IRELAND
and
DANIEL M. HEFFERNAN
SCHOOL OF PHYSICAL SCIENCES
NATIONAL INSTITUTE for HIGHER EDUCATION DUBLIN IRELAND
and
SCHOOL OF THEORETICAL PHYSICS,
DUBLIN INSTITUTE of ADVANCED STUDIES DUBLIN 4 IRELAND

Instabilities and chaos in a ring cavity -laser system containing a non-linear medium is investigated in detail. The system is modelled using the delay-difference equations obtained by Ikeda[1]:

$$E(t)=A+B*E(t-t_r)*\exp(i[\psi(t)-\psi_o]) \quad \ldots\ldots\ldots\ldots(1)$$

$$\gamma^{-1}d\psi/dt = -\psi(t)+|E(t-t_r)|^2 \quad \ldots\ldots\ldots\ldots(2)$$

where A is proportional to the amplitude of the incident field, B is a dissipation parameter defined by $B=R\exp(-\alpha L/2)$, α is the linear absorption rate. L is the length of the non-linear medium, E is a dimenionless complex electric field inside the cavity, ψ is the phase shift due to the medium. γ^{-1} is the inversion relaxion time of the medium, ψ_o is the mistuning of the medium of the cavity and t_r the round trip time.

It is assumed that there is significant detuning between the atomic and light frequenices so that the dispersive limit applies. Le

Berre et. al.[2] has solved similar ring cavity equations which allowed for both the dispersive and absortive limits.

Equations (1) and (2) were numerically integrated with the parameters B and Ψ_o set at 0.4 and 0.0 respectively. Figures 1-3 display the numerical computations, for three different values of A, after transients have decayed. Each figure displays ReE versus ImE and ReE versus Ψ_o, for three values of γt_r.

The 2d mapping has a period doubling route to chaos as A is increased. Equation (1) and (2) also have a period doubling route when γt_r is greater than 500. At low values of γt_r there is no period doubling as apparent from figure 3, where there is an attractor at γt_r=10 and there is period 16 at γt_r=1000. For all three values of A there is a smooth change from the attractor present at low values of γt_r, to the corresponding mapping attractor which manifests itself at γt_r>500 approximately. In figure 2 is the attractor at γt_r>500 fundamentally the same as the 2d mapping attractor for the same value of A? Le Berre et. al. have suggested that these two attractors are different because for the delay-differential equations (1) and (2) the dimension, as evaluated from the Kaplan-Yorke conjecture, goes to finity as γt_r does; whereas the dimension of the Ikeda-mappping attractor is less than 2.

We have computed the dimension of the attractor for γt_r in the interval [70,1500] and found the fractal dimension to be finite and less than 2. This proves that the two attractors are fundamentally the same, thus validating the use of the adabatic approximation. We use the method of Grassberger and Procaccia to obtain the correlation exponent ν. It has been proven that ν is a lower bound on the Hausdorff dimension[3]. In practice ν is obtained from the integral correlation function[4],

$$C_d(l)=\lim_{N\to\infty} 1/N^2 \text{ [number of pairs (n,m) with } \{\sum_{i=1}^{d}|X_{n+i}-X_{m+i}|^2\}^{1/2} < l] \ldots\ldots (3)$$

A time series was constructed from the real part of the electric field, denoted by $[x_i]^N_{i=1}$. In evaluating $C_d(l)$ we only used successive points separated by the round trip time t_r. Figures 4 and 5 display plots of $\ln C_d(l)$ versus $\ln l$ for a range of embedding dimensions d. Figure 4 is for the mapping with A=3.9 and figure 5 is for equations (1) and (2) with γt_r=200 and A=3.9. The correlation index ν was computed over the linear region and in table 1 we exhibit ν for various values of γt_r.

Table 1

Estimates of the correlation exponent ν for Ikeda's equations with $A=3.9$, $B=0.4$, $\psi_o=0.0$.

t_r	ν
2d map	1.34 +/- 0.06
1500	1.45 +/- 0.30
500	1.31 +/- 0.08
200	1.31 +/- 0.15
100	1.35 +/- 0.09
70	1.37 +/- 0.08

In conclusion we have found that when γt_r is greater than 500 (approximately) we have period doubling and that the attractor is fundamemtally the same as the Ikeda mapping attractor. This implies that in the adiabatic limit the physics is essentially unchanged i.e. the adiabatic approximation is a valid approximation.

Acknowledgements

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References

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(2) M. Le Berre, E. Ressayre, A. Tallet and H. M. Gibbs, Phys. Rev. Lett. 56, 274 (1986).

(3) P. Grassberger and I. Procaccia, Physica 9D, 189 (1983).

(4) P. Grassberger and I. Procaccia, Phy. Rev. A 28, 2591 (1983).

Figure 1

$A=2.155 \quad B=0.4 \quad \psi_0=0.0$

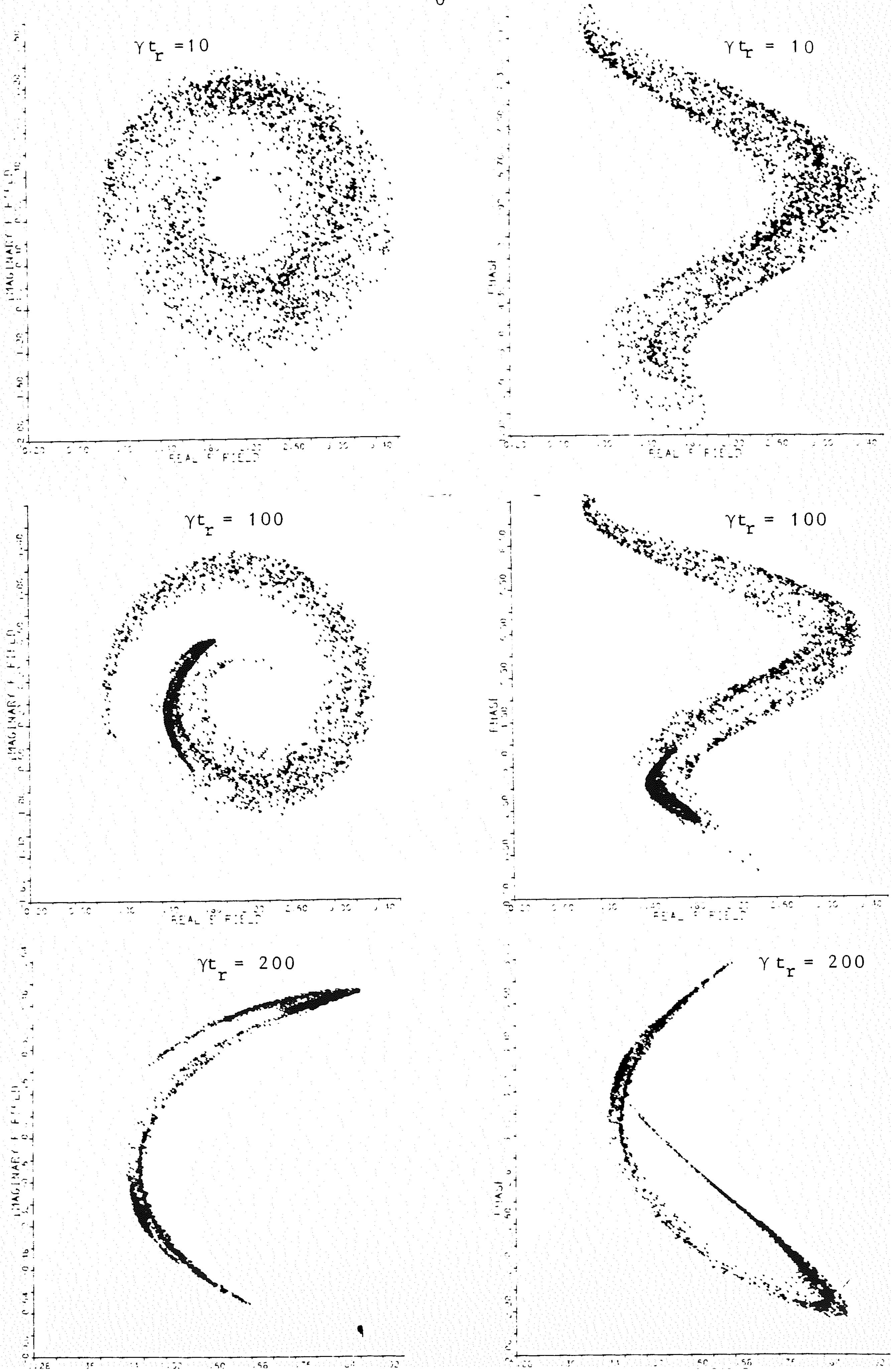

Figure 2

A=3.9 B=0.4 ψ_0=0.0

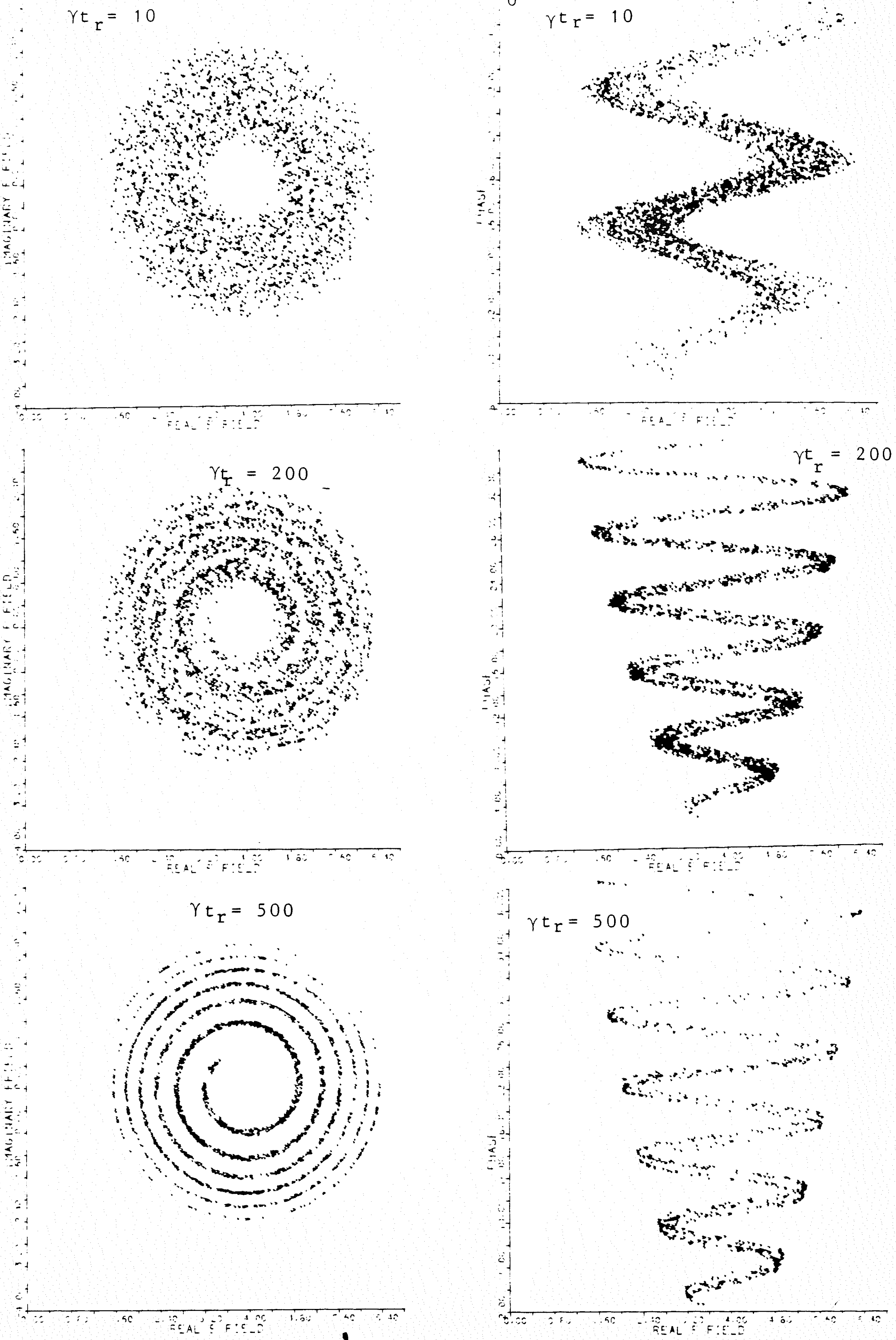

Figure 3

$A=4.265 \quad B=0.4 \quad \psi_0=0.0$

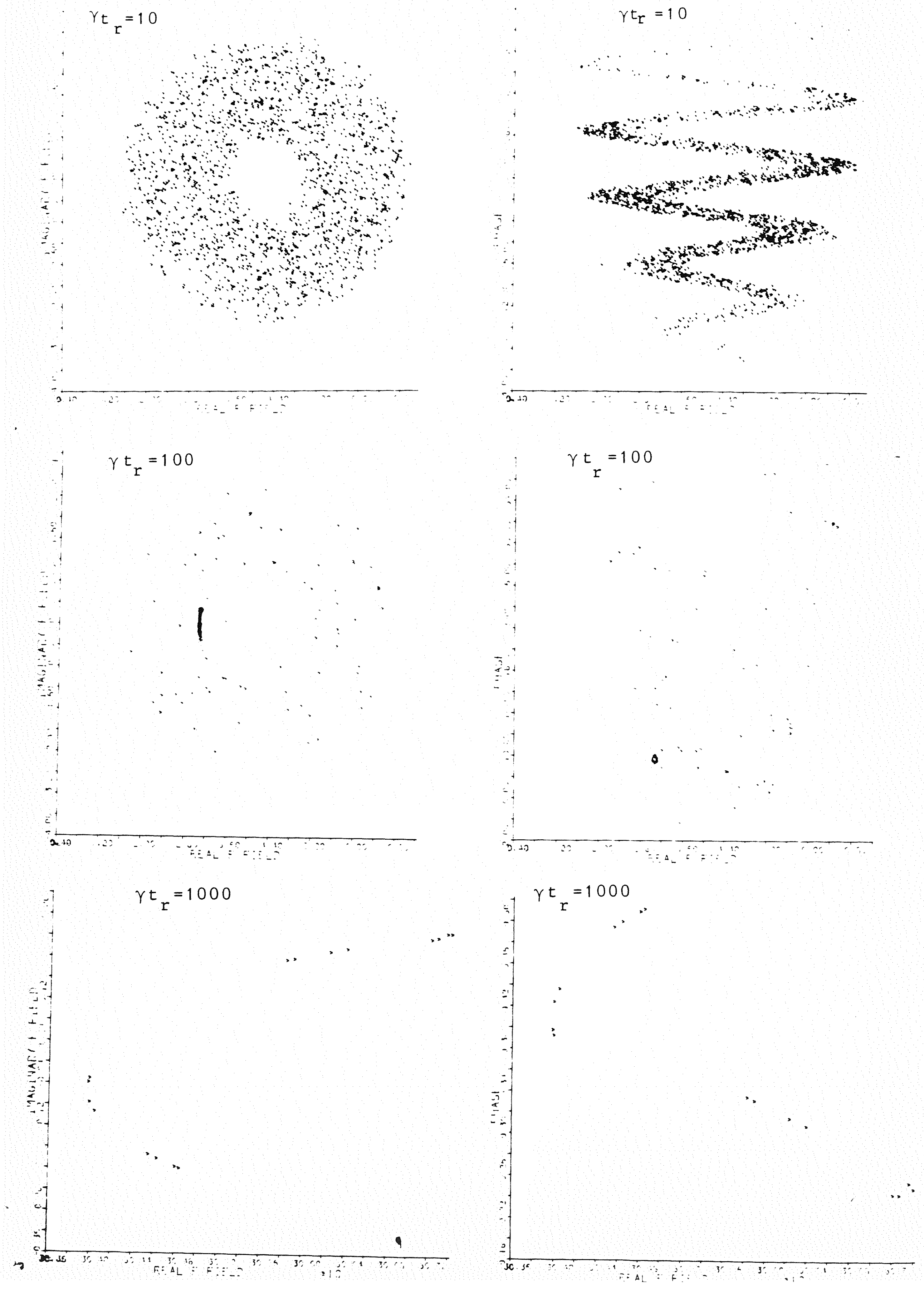

Figure 4

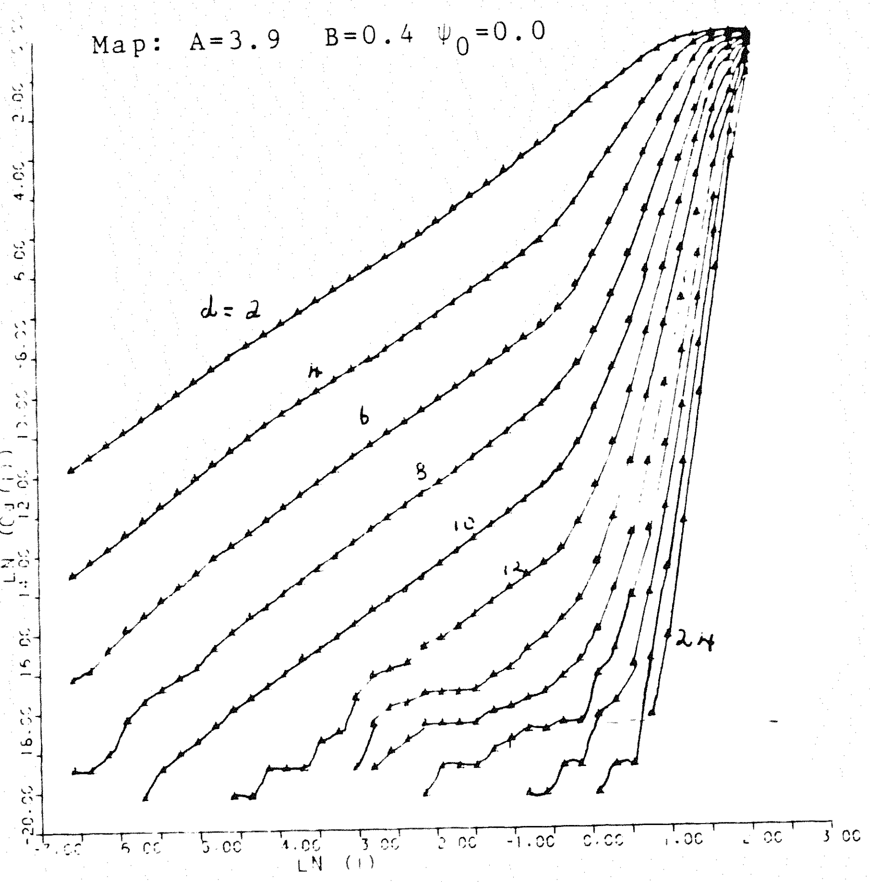

Figure 5