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ON QUANTUM SYSTEMS OF PARTICLES WITH PAIR LONG-RANGE MAGNETIC INTERACTION IN ONE DIMENSION. EQUILIBRIUM.

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A b s t r a c t.

Quantum one-dimensional systems of particles interacting via a (singular) "collective" (depending on all the position vectors of particles) vector electromagnetic potential is considered in the thermodynamic limit.The Gibbs(grand-canonical) reduced density matrices for the Maxwell-Boltzmann statistics are computed in the limit for a pair interaction, generated by a pair magnetic scalar potential ϕ, which is a sum of a short-range, increasing and long-range decreasing potentials. The considered n-particle systems are integrable and have a trivial thermodynamics.

1 INTRODUCTION

One-dimensional(1-d) systems of spinless non-relativistic n-particles with a singular magnetic interaction are characterized by the "collective" vector electromagnetic potential $a_j(X_n), X_n = (x_1, ..., x_n) \in \mathbb{R}^n$, which depends on the differences $x_j - x_k$ of the position vectors of particles and has a mild singularity(in the neighborhood of hyperplane $x_j = x_k$ it behaves as $|x_j - x_k|^{-s}, s \geq 0$), and the Hamiltonian $\dot{H}_n$ defined on $C^\infty(\mathbb{R}_0^n)$, $\mathbb{R}_0^n = \mathbb{R}^n \setminus \bigcup_{j<k}(x_j = x_k)$,

$$\dot{H}_n = \frac{1}{2}\sum_{j=1}^{n}(p_j - a_j(X_n))^2, X_n = (x_1, ...x_n) \in \mathbb{R}^n, \tag{1.1}$$

$$a_j(X_n) \in C^\infty(\mathbb{R}_0^n), \quad p_j = i^{-1}\partial_j = i^{-1}\frac{\partial}{\partial x_j}.$$

We assume that the magnetic interaction is mediated through the pair magnetic (vector) potential $a(x) \in C^\infty(\mathbb{R}_0^n)$

$$a_j(X_n) = \sum_{k\neq j,k=1}^{n} a(x_j - x_k). \tag{1.2}$$

There exists a function(scalar magnetic potential) $\phi(x) \in C^\infty(\mathbb{R}_0^n)$ such that

$$a(x) = \partial\phi(x). \tag{1.3}$$

As a result

$$a_j(X_n) = \partial_j U(X_n), \quad x_j \neq x_k, \quad U(X_n) = \sum_{1\leq k<jy\leq n} \phi(x_j - x_k). \tag{1.4}$$

We call the magnetic interaction long-range if $\phi(x) \notin L^1(\mathbb{R}_0)$.

From the simple equality

$$p_j - \hat{a}_j = exp\{i\hat{U}_n\}p_j exp\{-i\hat{U}_n\}$$

it follows that

$$\dot{H}_n = exp\{i\hat{U}_n\}\dot{H}_n^0 exp\{-i\hat{U}_n\}, \tag{1.5}$$

where $\hat{U}_n$, $\hat{a}_j$ are operators of multiplication by functions $U(X_n)$, $a_j(X_n)$, respectively, and $\dot{H}_n^0$ is the minus one-half n-dimensional Laplacian restricted to $C_0^\infty(\mathbb{R}_0^n)$. In order to define evolution in the system one needs to consider a selfadjoint extention of the operator $\dot{H}_n$. From (1.5) it follows that the most simple extension H_n is given by

$$P_n^t = exp\{-tH_n\} = exp\{i\hat{U}_n\}exp\{-tH_n^0\}exp\{-i\hat{U}_n\}.$$

Let's consider the system in the interval [-L,L] with the Dirichlet boundary condition on its boundary, i.e.with the Hamiltonian $H_{n,L}$

$$P_{n,L}^t = exp\{-\beta H_{n,L}\} = exp\{i\hat{U}_n\}P_{0(n,L)}^t exp\{-i\hat{U}_n\}, \tag{1.6}$$

where the semigroup $P_{0(n,L)}^t$ is generated by the n-dimensional Laplacian with the Dirichlet boundary condition onthe boundary of [-L,L].

The Gibbs or equilibrium grand canonical (the number of particles is not fixed) reduced density matrices(RDMs) for our system with the MB statistics are given by

$$\rho^L(X_m|Y_m) = \Xi_L^{-1}z^m\sum_{n\geq 0}\frac{z^n}{n!}\int_{[-L,L]^n} P_{(L)}^\beta(X_m, X_n'|Y_m, X_n')dX_n', \tag{1.7}$$

where Ξ_L coincides with the numerator in (1.7) for the case $m = 0$, z is the activity of particles, β is the inverse temperature, $P^{\beta}_{(L)}(X_n|Y_n)$ is the kernel of the operator $P^{\beta}_{n,L}$.

In this paper we calculate the RDMs in the limit $L \to \infty$ (thermodynamic limit) for the following choice of the scalar magnetic potential ($\phi_0 \in L^1$)

$$\phi(x) = \phi_0(x) + \phi^0(x) + \alpha_{-1}|x|^{-1} + \alpha^0 ln|x| = \phi_0(x) + \phi'(x), \quad \phi^0(x) = \sum_{n<\infty} \alpha_n |x|^n. \tag{1.8}$$

In our previous paper [SI] we calculated the limit for the case when all α, except α_1, are equal to zero.

For all systems with the pair magnetic interaction, generated by the vector magnetic potential satisfying (1.3), the following representation is valid

$$\rho^L(X_m|Y_m) = exp\{i[U(X_m) - U(Y_m)]\} z^m \prod_{k=1}^{m} P^{\beta}_{0(L)}(x_k|y_k) exp\{zG_L(X_m|Y_m)\}, \tag{1.9}$$

$$G_L(X_m|Y_m) = \int_{-L}^{L} \{exp\{i[\sum_{j=1}^{m} \phi(|x_j - x|) - \phi(|y_j - x|)]\} - 1\} P^{\beta}_{0(L)}(x|x) dx, \tag{1.10}$$

where $P^{\beta}_{0(L)}(x|y)$ is the kernel of the operator of the semigroup generated by the one-dimensional Laplacian on [-L,L] with the Dirichlet boundary condition.

The problem of the thermodynamic limit is solved by us by computing the asymptotic behavior in L of the function G_L. For long-range potentials it is not so evident, as for the case of short-range magnetic potential ϕ_0 (ϕ_0 is an integrable function) how to calculate it.

It turns out that for the case of increasing potentials the RDMs have unusual properties in the thermodynamic limit established earlier only for the 1-d Coulomb potential for MB statistics [1,3] and other statistics [2].

The proposed method does not permit to calculate RDMs in the thermodynamic limit for all the values of variables in the case the magnetic potential contains the logarithmic potential.

We formulate our general result in the end of the paper (Theorem 4.1). In the preceding paragraphs partial results are formulated in propositions.

2 Quadratic potential.

From (1.6-7) it follows that the RDMs are given by

$$\rho^L(X_m|Y_m) = \Xi_L^{-1} \sum_{n\geq 0} \frac{z^n}{n!} \int_{[-L,L]^n} dX'_n \exp\{i[U(X_m, X'_n) - U(Y_m, X'_n)]\} P^{\beta}_{0(L)}(X_m, X'_n|Y_m, X'_n),$$

where Ξ_L is the grand partition function, $P^{\beta}_{0(L)}(X_m|Y_m)$ is the kernel of the semigroup, whose infinitesimal generator concides with $H^0_{n,\Lambda}$, $-2H^0_{n,L}$ is the Laplacian in $[-L, L]^n$ with the Dirichlet boundary condition on the boundary $\partial[-L, L]^n$,

$$P^{\beta}_{0(L)}(X_n|Y_n) = \prod_{j=1}^{n} P^{\beta}_{0(L)}(x_j|y_j), \quad P^{\beta}_{0(L)}(x|y) = \int P^{\beta}_{x,y}(d\omega)\chi_L(\omega),$$

$P_{x,y}(d\omega)$ is the conditional Wiener measure and $\chi_L(\omega)$ is the characteristic function of the paths that are inside $[-L, L]$. From the equality

$$U(X_m, X'_n) = U(X_m) + \sum_{j=1}^{m}\sum_{k=1}^{n} \phi(x_j - x'_k) + U(X'_n)$$

we obtain

$$\rho^L(X_m|Y_m) = e^{-zL_P}\exp\{i[U(X_m)-U(Y_m)]\}P^\beta_{0(L)}(X_m|Y_m)\times$$

$$\times\sum_{n\geq 0}\frac{z^n}{n!}\int_{[-L,L]^n} dX'_n \prod_{k=1}^{n}\exp\{i\sum_{j=1}^{m}[\phi(x_j-x'_k)-\phi(y_j-x'_k)]\}P^\beta_{0(L)}(x'_k|x'_k),$$

where $L_P = \int_{-L}^{L} dx P^\beta_{0(L)}(x|x)$. Since the function under the sign of the integral factorizes we immediately obtain (1.9-10).

Let's assume that $\phi(x) = \alpha_2 x^2$. Then

$$G_L(X_m|Y_m) = \int_{-L}^{L} exp\{i\alpha_2[\sum_{j=1}^{m}(x_j-x)^2-(y_j-x)^2]\}P^\beta_{0(L)}(x|x)dx - L_P. \tag{2.1}$$

As a result

$$G_L(X_m|Y_m) = exp\{i\alpha_2\sum_{j=1}^{m}(x_j^2-y_j^2)\}\int_{-L}^{L} exp\{i2\alpha_2\sum_{j=1}^{m}(x_j-y_j)x\}P^\beta_{0(L)}(x|x)dx - L_P.$$

It is clear that if

$$\sum_{j=1}^{m}(x_j-y_j)=0, \quad \sum_{j=1}^{m}(x_j^2-y_j^2)=2\pi l, \ \in \mathbb{Z}^+ \tag{2.2}$$

then $G_L(X_m|Y_m)=0$. If one of the conditions is not satisfied then $|G_{0L}(X_m|Y_m)| < L_P$. It follows from the fact that $P^\beta_{0(L)}(x|x)\to(2\pi\beta)^{-\frac{1}{2}}$ when $L\to\infty$ and the equality

$$\int_{-L}^{L} exp\{i2\alpha_2\sum_{j=1}^{m}(x_j-y_j)x\}dx = (2\alpha_2\sum_{j=1}^{m}(x_j-y_j))^{-1}sin(L2\alpha_2\sum_{j=1}^{m}(x_j-y_j)), x_j\neq y_j.$$

So we proved the following proposition

P r o p o s i t i o n (2.1)

The thermodynamic limit of the RDMs for pair quadratic potential is zero if one of the equalities (2.2) is not satisfied. If both are true then

$$\rho(X_m|Y_m) = \lim_{L\to\infty}\rho^L(X|Y_m) = z^m exp\{i[U(X_m)-U(Y_m)]\}\prod_{k=1}^{m}P^\beta_0(x_k-y_k), \tag{2.3}$$

where $P^\beta_0(x) = (2\pi\beta)^{-\frac{1}{2}}\exp\{-\frac{x^2}{2\beta}\}$.

3 Logarithmic and Coulomb potentials.

For the function G_L the following representation is valid

$$G_L(X_m|Y_m) = G^0_L(X_m,Y_m) + G'_L(X_m,Y_m), \tag{3.1}$$

$$G^0_L(X_m|Y_m) =$$

$$= \int_{-L}^{L}\{exp\{i[\sum_{j=1}^{m}\phi_0(|x_j-x|)-\phi_0(|y_j-x|)]\}-1\}exp\{i[\sum_{j=1}^{m}\phi'(|x_j-x|)-\phi'(|y_j-x|)]\}P^\beta_{0(L)}(x|x)dx, \tag{3.2}$$

$$G'_L(X_m|Y_m) = \int_{-L}^{L} \{exp\{i[\sum_{j=1}^{m} \phi'(|x_j - x|) - \phi'(|y_j - x|)]\} - 1\} P^{\beta}_{0(L)}(x|x)dx. \tag{3.3}$$

Let $l^+_m = \max(x_j, y_j), j = 1, ..., m$. Then decomposing the interval $[-L, L]$ into union of two intervals $[-l^+_m, l^+_m], [-L, L]\backslash[-l^+_m, l^+_m]$ we obtain

$$G'_L(X_m|Y_m) = \int_{-l^+_m}^{l^+_m} \{exp\{i\alpha^0[\sum_{j=1}^{m}(\phi'|x_j - x| - \phi'|y_j - x|]\} - 1\} P^{\beta}_{0(L)}(x|x)dx + G^-_L(X_m|Y_m) =$$

$$= G^+_L(X_m|Y_m) + G^-_L(X_m|Y_m), \tag{3.4}$$

$$G^-_L(X_m|Y_m) = \int_{l^+_m < |x| \le L} \{exp\{i\alpha^0[\sum_{j=1}^{m}(\phi'|x_j - x| - \phi'|y_j - x|]\} - 1\} P^{\beta}_{0(L)}(x|x)dx.$$

The function G^+_L converges to the function G^+

$$G^+(X_m|Y_m) = (2\pi\beta)^{-\frac{1}{2}} \int_{-l^+_m}^{l^+_m} \{exp\{i\alpha^0[\sum_{j=1}^{m}(\phi'|x_j - x| - \phi'|y_j - x|]\} - 1\}dx. \tag{3.5}$$

Let's assume that $\phi(x) = \phi'(x) = \alpha^0 ln|x|$, utilize the expansion

$$ln(1 - a) = \sum_{n>0} \frac{(-a)^n}{n}, a < 1, \quad a = \frac{x_j}{x}, \frac{y_j}{x}$$

and substitute it into G^-_L, using $|1 - \frac{x_j}{x}| = 1 - \frac{x_j}{x}$

$$G^-_L(X_m|Y_m) = \int_{l^+_m < |x| \le L} \exp\{i\alpha^0[\sum_{j=1}^{m} \frac{x_j - y_j)}{x}]\{exp\{i\alpha^0 \sum_{j=1}^{m} \sum_{s>1}[(\frac{x_j}{x})^s - (\frac{y_j}{x})^s]\} - 1\} P^{\beta}_{0(L)}(x|x)dx+$$

$$+\tilde{G}^0_L(X_m|Y_m) = \tilde{G}^-_L(X_m|Y_m) + \tilde{G}^0_L(X_m|Y_m),$$

$$\tilde{G}^0_L(X_m|Y_m) = \int_{l^+_m < |x| \le L} \exp\{i\alpha^0 \sum_{j=1}^{m}(\frac{x_j - y_j}{x})\} - 1\} P^{\beta}_{0(L)}(x|x)dx.$$

Here we added $+1 - 1$ to the first exponent.The function $\tilde{G}^-_L$ tends to $\tilde{G}^-$ in the limit of increasing L

$$\tilde{G}^-(X_m|Y_m) = (2\pi\beta)^{-\frac{1}{2}} \int_{|x| > l^+_m} \exp\{i\alpha^0[\sum_{j=1}^{m} \frac{x_j - y_j}{x}]\{exp\{i\alpha^0 \sum_{j=1}^{m} \sum_{s>1} s^{-1}[(-\frac{x_j}{x})^s - (-\frac{y_j}{x})^s]\} - 1\}dx, \tag{3.6}$$

since expanding the exponent, containing the sum over s under the sign of the integral, it can be seen that all the integrals are finite and the sum is convergent uniformly in x_j, y_j taking values in a compact set. Function $\tilde{G}^0_L = 0$ if the first equality in (2.2) is satisfied. It diverges otherwise. So we proved the following proposition.

P r o p o s i t i o n (3.1)
The thermodynamic limit of the RDMs for pair logarithmic potential exists if the first equality (2.2) is satisfied and is given by

$$\rho(X_m|Y_m) = z^m exp\{i[U(X_m) - U(Y_m)]\} \prod_{k=1}^{m} P_0^\beta(x_k - y_k) \exp\{G^+(X_m|Y_m) + \tilde{G}^-(X_m|Y_m)\}.$$

Now let's put $\phi(x) = \frac{\alpha - 1}{|x|}$. Then using the expansion $(1 - \frac{x_j}{x})^{-1} = \sum_{s \geq 0} (\frac{x_j}{x})^s$ and repeating the same operation as for the case of the logarithmic potential we arrive at the following proposition

P r o p o s i t i o n (3.2)
The thermodynamic limit of the RDMs for the potential $\phi(x) = \frac{\alpha - 1}{|x|}$ exists and is given by

$$\rho(X_m|Y_m) =$$

$$= z^m exp\{i[U(X_m) - U(Y_m)]\} \prod_{k=1}^{m} P_0^\beta(x_k - y_k) \exp\{G^+(X_m|Y_m) + G^-(X_m|Y_m)\},$$

where

$$G^-(X_m|Y_m) = (2\pi\beta)^{-\frac{1}{2}} \int_{|x| > l_m^+} \{exp\{i\frac{\alpha_{-1}}{|x|} \sum_{j=1}^{m} \sum_{s>0} [(\frac{x_j}{x})^s - (\frac{y_j}{x})^s]\} - 1\} dx. \quad (3.7)$$

4 Polynomial potential.

Let's consider the case $\phi(x) = \alpha_{2r} x^{2r}, 2 < r \in \mathbb{Z}^+$. Then the function G_L is given by

$$G_L(X_m|Y_m) =$$

$$= exp\{i\alpha_{2r} \sum_{j=1}^{m} (x_j^{2r} - y_j^{2r})\} \int_{-L}^{L} exp\{i2\alpha_2 \sum_{j=1}^{m} \sum_{s=1}^{2r} (-1)^s C_{2r}^s (x_j^{2r-s} - y_j^{2r-s}) x^s\} P_{0(L)}^\beta (x|x) dx - L_P.$$

It's evident that if the equalities are satisfied for $l \in \mathbb{Z}^+$

$$\sum_{j=1}^{m} (x_j^s - y_j^s) = 0, \quad s = 1, ..., 2r - 1, \quad \sum_{j=1}^{m} (x_j^{2r} - y_j^{2r}) = 2\pi l, \quad (4.1)$$

then $G_L = 0$ and the RDMs in the thermodynamic limit are given by (2.3). If one of them is not true the $G_L + L_P$ is either bounded(oscillating) in L or strictly less than L_P. So for the case $\phi(x) = \sum_{s=1}^{r} \alpha_{2s} x^{2s}$ proposition (2.1) holds.

Now, let $\phi(x) = \phi^0(x) = \sum_{s<\infty} \alpha_s |x|^s$ and $l_m^- = \min(x_j, y_j)$. Then

$$G_L(X_m|Y_m) = \hat{G}_L(X_m|Y_m) + (\int_{-L}^{l_m^-} + \int_{l_m^+}^{L}) exp\{i \sum_{j=1}^{m} [\phi^0(x_j - x) - \phi^0(y_j - x)]\} P_{0(L)}^\beta (x|x) dx -$$

$$-(L_P - \int_0^{l_m^+} P_{0(L)}^\beta (x|x) dx + \int_0^{l_m^-} P_{0(L)}^\beta (x|x) dx),$$

$$\hat{G}_L(X_m|Y_m) = \int_{l_m^-}^{l_m^+} exp\{i \sum_{j=1}^{m} [\phi^0(x_j - x) - \phi^0(y_j - x)]\} P_{0(L)}^\beta (x|x) dx - \int_0^{l_m^+} P_{0(L)}^\beta (x|x) dx + \int_0^{l_m^-} P_{0(L)}^\beta (x|x) dx).$$

So if the conditions (4.1) are satisfied $G_L - \hat{G}_L = 0$ and the following proposition is true.

P r o p o s i t i o n (4.1)
The thermodynamic limit of the RDMs for the potential $\phi(x) = \sum_{s<\infty} \alpha_s |x|^s$ exists and is zero if one of the equalities (4.2) is not satisfied. If all are true then

$$\rho(X_m|Y_m) = \lim_{L\to\infty} \rho^L(X|Y_m) = z^m exp\{i[U(X_m) - U(Y_m)]\} \prod_{k=1}^{m} P_0^\beta(x_k - y_k) e^{\hat{G}^0(X_m|Y_m)}, \qquad (4.2)$$

where

$$\hat{G}^0(X_m|Y_m) = (2\pi\beta)^{-\frac{1}{2}} [\int_{l_m^-}^{l_m^+} exp\{i \sum_{j=1}^{m} [\phi_-^0(x_j - x) - \phi_-^0(y_j - x)]\} dx - l_m^+ + l_m^-],$$

$$\phi_-^0(x) = \sum_{s<\infty} \alpha_{2s+1} |x|^{2s+1}.$$

Here we used the equality $\hat{G} = \hat{G}^0$, decomposed ϕ^0 into sum of ϕ_+^0 and ϕ_-^0 and added $+1-1$ to the exponent containing ϕ_-^0.

In general case the function G'_L has the following representation

$$G'_L(X_m|Y_m) = \sum_{s=0}^{2} G_{sL}(X_m|Y_m),$$

$$G_{1L}(X_m|Y_m) = \int_{-L}^{L} exp\{i[\sum_{j=1}^{m} \phi^0(|x_j - x|) - \phi^0(|y_j - x|)]\} exp\{i\alpha_{-1} [\sum_{j=1}^{m} |x_j - x|^{-1} - |y_j - x|^{-1}]\} \times$$

$$\times \{exp\{i\alpha^0 [\sum_{j=1}^{m} ln(|x_j - x|) - \ln(|y_j - x|)]\} - 1\} P_{0(L)}^\beta(x|x) dx,$$

$$G_{2L}(X_m|Y_m) =$$

$$= \int_{-L}^{L} exp\{i[\sum_{j=1}^{m} \phi^0(|x_j - x|) - \phi^0(|y_j - x|)]\} \{\{exp\{i\alpha_{-1} [\sum_{j=1}^{m} |x_j - x|^{-1} - |y_j - x|^{-1}]\} - 1\} P_{0(L)}^\beta(x|x) dx,$$

$$G_{0L}(X_m|Y_m) = \int_{-L}^{L} exp\{i[\sum_{j=1}^{m} \phi^0(|x_j - x|) - \phi^0(|y_j - x|)]\} P_{0(L)}^\beta(x|x) dx.$$

The following representation is valid

$$G_{sL}(X_m|Y_m) = G_{sL}^+(X_m|Y_m) + G_{sL}^-(X_m|Y_m),$$

where, as in the third paragragh, + and - index correspond to the integration over $[-l_m^+, l_m^+]$ and its complement, respectively.

By analogy with $G^-, \tilde{G}^-$ of the previous paragragh, the functions G_2^-, $\tilde{G}_1^-$ are defined

$$G_2^-(X_m|Y_m) =$$

$$= (2\pi\beta)^{-\frac{1}{2}} \int_{|x|>l_m^+} exp\{i[\sum_{j=1}^{m} \phi^0(|x_j - x|) - \phi^0(|y_j - x|)]\} \{exp\{i \frac{\alpha_{-1}}{|x|} \sum_{j=1}^{m} \sum_{s>0} [(\frac{x_j}{x})^s - (\frac{y_j}{x})^s]\} - 1\} dx,$$

$$\tilde{G}_1^-(X_m|Y_m) =$$

$$= (2\pi\beta)^{-\frac{1}{2}} \int_{|x|\geq l_m^+} exp\{i[\sum_{j=1}^{m} \phi^0(|x_j - x|) - \phi^0(|y_j - x|)]\} exp\{i\alpha_{-1} [\sum_{j=1}^{m} |x_j - x|^{-1} - |y_j - x|^{-1}]\} \times$$

$$\times \exp\{i\alpha^0[\sum_{j=1}^{m} \frac{x_j - y_j}{x}]\}\{exp\{i\alpha^0 \sum_{j=1}^{m}\sum_{s>1} s^{-1}[(-\frac{x_j}{x})^s - (-\frac{y_j}{x})^s]\} - 1\}dx.$$

The function G_L^0 tends to G^0

$$G^0(X_m|Y_m) =$$

$$= (2\pi\beta)^{-\frac{1}{2}} \int_{-\infty}^{\infty} \{exp\{i[\sum_{j=1}^{m} \phi_0(|x_j - x|) - \phi_0(|y_j - x|)]\} - 1\}exp\{i[\sum_{j=1}^{m} \phi'(|x_j - x|) - \phi'(|y_j - x|)]\}dx. \quad (4.3)$$

Combining all the above arguments we see that the following theorem is valid.

THEOREM 4.1

Let $\alpha^0 = 0$. The thermodynamic limit of the RDMs exists and is zero if one of the equalities (4.2) is not satisfied. If all are true then

$$\rho(X_m|Y_m) = z^m exp\{i[U(X_m) - U(Y_m)]\} \prod_{k=1}^{m} P_0^{\beta}(x_k - y_k)\times$$

$$\times \exp\{\hat{G}^0(X_m|Y_m) + G^0(X_m|Y_m) + G_2^+(X_m|Y_m) + G_2^-(X_m|Y_m)\}. \quad (4.4)$$

If $\alpha^0 \neq 0$ then the thermodynamic limit of the RDMs exists if all the equalities (4.2) are satisfied and is given by

$$\rho(X_m|Y_m) = z^m exp\{i[U(X_m) - U(Y_m)]\} \prod_{k=1}^{m} P_0^{\beta}(x_k - y_k) \exp\{\hat{G}^0(X_m|Y_m) + G^0(X_m|Y_m)\}\times$$

$$\times \exp\{G_2^+(X_m|Y_m) + G_2^-(X_m|Y_m) + G_1^+(X_m|Y_m) + \tilde{G}_1^-(X_m|Y_m)\}, \quad (4.5)$$

REMARK.

The behavior of the RDMs in the thermodynamic limit for the pair scalar magnetic potentials growing at infinity (when there is no logarithmic term) is quite strange. They are almost everywhere zero if an algebra of observables includes only smooth functions. But if one adds to the algebra point measures then the average on the algebra, produced by the RDMs, is not zero.

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