Quantised relativistic membranes and non-perturbative checks of gauge/gravity duality

Veselin G. Filev [1] **and Denjoe O'Connor**[2]

[1] Institute of Mathematics and Informatics,
Bulgarian Academy of Sciences, Acad. G. Bonchev Str., 1113 Sofia, Bulgaria
[2] School of Theoretical Physics, Dublin Institute for Advanced Studies,
10 Burlington Road, Dublin 4, Ireland.

E-mail: denjoe@stp.dias.ie, vfilev@math.bas.bg

Abstract. We test the background geometry of the BFSS model using a D4–brane probe. This proves a sensitive test of the geometry and we find excellent agreement with the D4–brane predictions based on the solution of a membrane corresponding to the D4–brane propagating on this background.

1. Introduction

From the perspective of a quantised eleven dimensional supermembrane it is perhaps natural to argue that there ought to be a dual gravitational background to the matrix regulated supermembrane proposed by Banks et al [1] as a non-perturbative formulation of M-theory. However, it is important to test this hypothesis as far as possible. In recent years several non-perturbative tests of this hypothesis have been performed [2, 3, 18, 5, 6, 7, 8, 9] at the level of comparing the expectation value of the microscopic Hamiltonian with the energy prediction coming from the black hole background. These are increasingly sensitive tests and the agreement found between theory and simulation in the most recent, and currently most precise test, by Berkowitz et al [9] while impressive is open to question. They observe that if they fit their data "by a single powerlaw $E_0(T) = aT^p$, then $E_0(T) = (3.13 \pm 0.03)T^{2.02\pm0.03} \simeq \pi T^2$ describes our continuum large-N data very well ($\chi^2/DOF = 7.7/5$) in the whole temperature range." While their precision tests are still rather convincing it is important to have other tests that check the geometry in more detail. This can be done, just as one does in practice in the everyday world, by using probes which are sensitive to the geometry yet do not in turn change it significantly.[1] This corresponds to adding $M5$-membrane density to the BFSS matrix model – the Berkooz-Douglas (BD) matrix model. From a string theory point of view the BD model describes the low energy regime of the D0/D4–brane intersection. It is a flavoured $1 + 0$ dimensional gauge theory with $\mathcal{N} = 8$ supersymmetry and is the dimensional reduction of $4 - d$ $\mathcal{N} = 2$ supersymmetric Yang–Mills theory. This makes the BD model particularly interesting in the context of holographic studies of flavour dynamics, since the $\mathcal{N} = 16$ supersymmetric Yang–Mills theory which it probes has a well know gravitation dual background.

---

[1] A similar study using probe D0-branes has been performed in ref. [10].

The proposal that there is a gravitational dual to gauged matter systems represents a dramatic insight into certain non-perturbative phenomena. In the original formulation of Maldacena [11], the duality relates string theory in the $AdS_5 \times S^5$ background space-time to the large N limit of $3 + 1$ dimensional $\mathcal{N} = 4$ Supersymmetric Yang-Mill theory living on the asymptotic boundary of the AdS_5 space-time. This idea has inspired numerous extensions of the duality with ever increasing phenomenological relevance, currently ranging from heavy ion collisions to condensed matter physics. In this paper we are interested in holographic flavour dynamics–the generalisation of the AdS/CFT correspondence to flavoured gauge theories.

The first such generalisation was proposed by Karch and Katz [12] , who introduced a probe D7–brane to the $AdS_5 \times S^5$ supergravity background. On the field theory side this corresponds to introducing an $\mathcal{N} = 2$ fundamental hypermultiplet in the quenched approximation. The classical dynamics of the probe brane is governed by an effective Dirac-Born-Infeld action. Remarkably the AdS/CFT dictionary relates the classical properties of the brane to quantum vacuum expectation values in the dual flavoured gauge theory. One such quantity is the fundamental condensate of the theory, which is encoded in the classical profile of the probe brane near the asymptotic boundary. In refs. [13] and [14] the finite temperature set-up has been considered. The authors uncovered a first order meson melting phase transition corresponding to a topology change transition of the possible D7–brane embeddings. In ref. [15] these studies have been extended to the general Dp/Dq–brane system and certain universal properties of the corresponding holographic gauge theories have been uncovered. In fact the BD matrix model is dual to the D0/D4–brane system, which falls into the same universality class as the phenomenologically relevant D3/D7–brane system.

Another attractive feature of the BD matrix model is that it is $1 + 0$ dimensional supersymmetric Yang–Mills theory, which makes it super renormalizable and thus accessible with lattice simulations. Therefore, the BD model can serve as a bridge between lattice gauge theory and holography allowing a direct precision test of the gauge/gravity duality with flavours. This was our main motivation to study the D0/D4 system and the BD matrix model [16].

Our studies provide a highly non-trivial test of the AdS/CFT correspondence with matter. The results provide substantial evidence for the validity of the holographic approach to flavour dynamics. They also test the predicted D0-brane geometry in a highly non-trivial way since as the fundamental mass is varied, the D4-brane probes the radial dependence of this dual geometry. The agreement we find with the predictions from the D4-brane probe embedded in the dual D0 black hole geometry is remarkable. Although it is not a mathematical proof, we believe that the remarkable agreement between theory and simulation, which we uncovered is due to the cancellation of α' corrections in the black hole embedding.

2. Holographic flavours in one dimension
In this section we review the description of the D0/D4–brane system in the quenched approximation adapting the general discussion of references [15] and [17].

2.1. D0-brane background
In the near horizon limit the D0-brane background is given by the metric:

$$
\begin{aligned}
ds^2 &= -H^{-\frac{1}{2}} f \, dt^2 + H^{\frac{1}{2}} \left(\frac{du^2}{f} + u^2 \, d\Omega_8^2 \right) , \\
e^\Phi &= H^{\frac{3}{4}} , \qquad C_0 = H^{-1} ,
\end{aligned}
\tag{1}
$$

where $H = (L/u)^7$, $f(u) = 1 - (u_0/u)^7$ is the blackening factor, Φ is the dilaton field and C_0 is the only component of the RR one form coupled to the D0-branes . Here u_0 is the radius of the

horizon and the length scale L can be expressed in terms of string theory units as:

$$L^7 = 60\,\pi^3\,g_s\,N_c\,\alpha'^{7/2} \ , \tag{2}$$

where N_c is the number of D0–branes corresponding to the rank of the gauge group of the dual field theory[2]. According to the general gauge/gravity duality [17], the Yang-Mills coupling of the corresponding dual gauge theory is given by:

$$g_{\text{YM}}^2 = g_s\,(2\pi)^{-2}\,\alpha'^{-3/2} \ . \tag{3}$$

The Yang-Mills coupling is dimensionful and the corresponding dimensionless effective coupling runs with the energy scale according to:

$$g_{\text{eff}}^2 = \lambda\,U^{-3} \ , \tag{4}$$

where $\lambda = g_{\text{YM}}^2\,N_c$ is the t'Hooft coupling. The supergravity background can be trusted if both the curvature and the dilaton are small, which leads to the restriction [17]:

$$1 \ll g_{\text{eff}} \ll N_c^{\frac{4}{7}} \ . \tag{5}$$

and the theory is strongly coupled in this regime. From equations (1) and (4) it follows that the upper bound in equation (5) can be violated at low energies (small radial distances) when the dilaton blows, however at finite temperature and fixed 'tHooft coupling, g_{eff} peaks at the black hole horizon and the bound $\lambda/T^3 \ll N_c^{8/7}$ is satisfied in the large N limit. At high energies (large radial distances) the curvature of the background grows, while the effective coupling decreases. As a result the lower bound in (5) is violated at energies higher than approximately $\lambda^{1/3}$ and hence α' corrections are increasingly important at large radial distances.

Finally, the Hawking temperature of the background is given by:

$$T = \frac{7}{4\,\pi\,L}\left(\frac{u_0}{L}\right)^{\frac{5}{2}} \tag{6}$$

and is identified with the temperature of the dual gauge theory.

2.2. Flavour D4-branes

To introduce matter in the fundamental representation we consider the addition of N_f D4-branes to the D0-brane background. In the probe approximation $N_f \ll N_c$, the dynamics of the D4-branes is governed by the Dirac-Born-Infeld, which in the absence of a background B-field is given by:

$$S_{\text{DBI}} = -N_f\,T_4 \int d^4\xi\,e^{-\Phi}\,\sqrt{-\det||G_{\alpha,\beta} + (2\pi\alpha')F_{\alpha,\beta}||} \ , \tag{7}$$

where $G_{\alpha,\beta}$ is the induced metric and $F_{\alpha,\beta}$ is the $U(1)$ gauge field of the D4-brane, which we will set to zero. The D4-brane tension is given by:

$$T_4 = \frac{\mu_4}{g_s} = \frac{1}{(2\pi)^4\,\alpha'^{5/2}\,g_s} \ . \tag{8}$$

The D4-brane embedding that we consider extends along the radial and time directions and wraps a three sphere, S^3, in the directions transverse to the D0-brane. To parametrise it let us split the unit S^8 in the metric (1) into:

$$d\Omega_8^2 = d\theta^2 + \cos^2\theta\,d\Omega_3^2 + \sin^2\theta\,d\Omega_4^2 \ . \tag{9}$$

---

[2] Note that we will abbreviate N_c to N when the context is clear.

Our embedding now extends along t and Ω_3 and has a non-trivial profile in the (u, θ) plane, which we parametrise as $(u, \theta(u))$. Next we Wick rotate the action (7) and periodically identify time with period $\beta = 1/T$. Using equation (1) we obtain:

$$S_{\text{DBI}}^{E} = \frac{N_f\,\beta}{8\,\pi^2\,\alpha'^{5/2}\,g_s} \int du\, u^3 \cos^3\theta(u)\, \sqrt{1 + u^2\, f(u)\, \theta'(u)^2}\ . \tag{10}$$

In the limit of zero temperature ($u_0 \to 0$) the regular solution to the equation of motion for $\theta(u)$ is given by $u \sin\theta = m$, where the constant m is proportional to the bare mass of the flavours [12], [15]. At finite temperature the separation $L(u) = u \sin\theta(u)$ has a non-trivial profile reflecting the non-vanishing condensate of the theory. To analyse this case it is convenient to define dimensionless radial coordinate $\tilde{u} = u/u_0$. At large $\tilde{u}$ the general solution $\theta(\tilde{u})$ has the expansion:

$$\sin\theta = \frac{\tilde{m}}{\tilde{u}} + \frac{\tilde{c}}{u^3} + \dots . \tag{11}$$

Holography relates the dimensionless constants $\tilde{m}, \tilde{c}$ to the bare mass and condensate of the theory via [15][3]:

$$
\begin{aligned}
m_q &= \frac{u_0\,\tilde{m}}{2\pi\alpha'} = \left(\frac{120\,\pi^2}{49}\right)^{1/5} \left(\frac{T}{\lambda^{1/3}}\right)^{2/5} \lambda^{1/3}\,\tilde{m}\ , \\
\langle \mathcal{O}_m \rangle &= -\frac{N_f\,u_0^3}{2\,\pi\,g_s\,\alpha'^{3/2}}\,\tilde{c} = \left(\frac{2^4\,15^3\,\pi^6}{7^6}\right)^{1/5} N_f\,N_c \left(\frac{T}{\lambda^{1/3}}\right)^{6/5} (-2\,\tilde{c})\ .
\end{aligned} \tag{12}
$$

Note that equation (11) implies that the D7-branes are described by a one parameter family of embeddings (parametrised by $\tilde{m}$). In the case of the D3/D7 system this is natural due to the scaling symmetry (everything depends on the dimensionless ratio m_q/T), but is this consistent with the D0/D4 system, which has a dimensionful 'tHooft coupling? Indeed, the D0/D4 system has a 'tHooft coupling λ of dimension three suggesting that there are two independent dimensionless parameters $m_q/\lambda^{1/3}$ and $T/\lambda^{1/3}$. However, while the holographic set-up allows rescaling of the radial coordinate by u_0 and the description of D7-brane embeddings by the single parameter $\tilde{m}$, as can be seen from the first equation (12) we have $\tilde{m} \sim (m_q/\lambda^{1/3})(T/\lambda^{1/3})^{-2/5}$. As a result the condensate in the second equation in (12) is indeed a function of the two dimensionless parameters $(T/\lambda^{1/3}, m_q/\lambda^{1/3})$ which is consistent with dimensional analysis and is in contrast to the D3/D7 system, where the condensate depends on the dimensionless parameter m_q/T.

3. Testing the correspondence
In this section we compare the result of the lattice simulations of the model to the predictions of gauge gravity duality. Our main focus is the fundamental condensate of the theory. As definition of the condensate we use the derivative of the free energy of the theory with respect to the bare mass parameter m_q. Note that on the lattice we use a dimensionless mass parameter m and temperature $\tilde{T}$ given by:

$$m = \frac{m_q}{\lambda^{1/3}}\ , \qquad \tilde{T} = \frac{T}{\lambda^{1/3}}\ , \tag{13}$$

where λ is the dimensionful 'tHooft coupling. Note also that in 1+0 dimensions the fundamental condensate $\langle \mathcal{O}_m \rangle$ is dimensionless. The AdS/CFT dictionary equations (12) can be easily

---

[3] Note that our expressions differ slightly from the ones presented in [15] due to the different choice of radial variable.
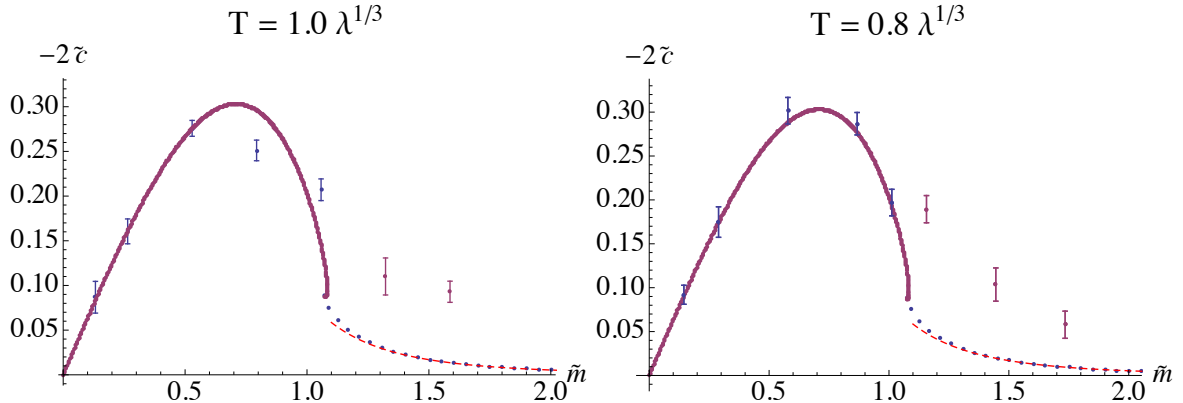
Figure 1. Plots of the condensate versus bare mass parameter curve for $N = 10$, $\Lambda = 16$ and two different temperatures. *left:* For temperature $T = 1.0\,\lambda^{1/3}$ the curve shows excellent agreement at small masses, but deviates quickly from the theoretical curve at greater masses. *right:* The curve at temperature $T = 0.8\,\lambda^{1/3}$ exhibits excellent agreement throughout the whole range of masses corresponding to the black hole phase (blue error bars). Similarly to the higher temperature curve there is a significant deviation from the theoretical curve for the Minkowski phase (red error bars).

rewritten in terms of the lattice dimensionless quantities. One gets:

$$
m = \left(\frac{120\,\pi^2}{49}\right)^{1/5} \tilde{T}^{2/5}\,\tilde{m}\;,
$$

$$
\langle \mathcal{O}_m \rangle = \left(\frac{2^4\,15^3\,\pi^6}{7^6}\right)^{1/5} N_f\,N_c\,\tilde{T}^{6/5}\,(-2\,\tilde{c})\;.
\tag{14}
$$

Equations (14) can now be used to scale the dependence $\langle \mathcal{O}_m \rangle$ versus m obtained from computer simulations and compare to the dependence $-2\,\tilde{c}$ versus $\tilde{m}$ obtained from holography. The resulting plots for two temperatures are presented in figure 1. The plots are for matrix size $N = 10$ and lattice spacing $\Lambda = 16$.

The left plot corresponds to temperature $T = 1.0\,\lambda^{1/3}$. One can observe excellent agreement between the gauge gravity duality and lattice simulations at small masses ($\tilde{m} < 1$). However, for greater masses there is a significant deviation from the theoretical curve. The right plot corresponds to temperature $T = 0.8\,\lambda^{1/3}$. The excellent agreement between gauge/gravity predictions and lattice simulations extends for the whole range of masses within the deconfined (black hole) phase (blue error bars). In the deconfined (Minkowski) phase there are still significant deviations from the theoretical curve. These results, we argue [16], indicate that the α' corrections to the supergravity background affect the black hole and Minkowski D4-brane embeddings differently. All black hole embeddings reach the horizon and as a result experience similar curvature effects for different values of the mass parameter therefore, the α' corrections largely cancel when one takes a derivative with respect to the mass to calculate the condensate. In contrast, Minkowski embeddings close at different radial distances above the horizon depending on the mass parameter. As a result the effect of the α' corrections depends strongly on the mass and contributes to the calculation of the condensate. The overall better agreement of the lower temperature curve to the theoretical predictions is another signature that the observed deviations at large masses are due to α' corrections as opposed to lattice effects, although at sufficiently high masses ($|m^a| \lesssim 1/a$) lattice effect also become significant.

4. Conclusion

In this paper we review the precision test of holographic flavour dynamics discussed in detail in [16]. We focused on the study of a one-dimensional flavoured Yang-Mills theory holographically dual to the D0/D4–brane intersection, also known as the Berkooz–Douglas matrix model. We considered a lattice discretisation of the model and the super-renormalizability of the model ensures that in the continuum limit, supersymmetry is broken only by the effect of finite temperature, which enabled us to simulate it on a computer.

Our results for the condensate versus bare mass curve (which is universal for different temperatures) show an excellent agreement with holography in the regime of small bare masses and at lower temperature this agreement extends to the whole range of masses in the deconfined phase. We believe that this agreement can be explained by a cancellation of the α' corrections to the condensate for black hole embeddings (deconfined phase). This allows a direct comparison between computer simulations and AdS/CFT predictions at relatively high temperatures compared to similar studies of the pure BFSS matrix model. Furthermore, this remarkable agreement (in the black hole phase) is obtained without any parameter fitting in contrast to the analogous studies of the BFSS matrix model [18], where the authors performed a fit to estimate the α' corrections to the internal energy. We believe that it is the cancellation mechanism described in section 3, which allows this highly non-trivial test of the gauge/gravity correspondence. The improved agreement as the temperature is reduced is also in accord with this interpretation.

References

[1] T. Banks, W. Fischler, S. H. Shenker and L. Susskind, "M theory as a matrix model: A Conjecture," Phys. Rev. D **55**, 5112 (1997) [hep-th/9610043].

[2] K. N. Anagnostopoulos, M. Hanada, J. Nishimura and S. Takeuchi, Phys. Rev. Lett. **100** (2008) 021601 doi:10.1103/PhysRevLett.100.021601 [arXiv:0707.4454 [hep-th]].

[3] S. Catterall and T. Wiseman, Phys. Rev. D **78** (2008) 041502 doi:10.1103/PhysRevD.78.041502 [arXiv:0803.4273 [hep-th]].

[4] M. Hanada, Y. Hyakutake, J. Nishimura and S. Takeuchi, Phys. Rev. Lett. **102** (2009) 191602 doi:10.1103/PhysRevLett.102.191602 [arXiv:0811.3102 [hep-th]].

[5] M. Hanada, Y. Hyakutake, G. Ishiki and J. Nishimura, Science **344** (2014) 882 doi:10.1126/science.1250122 [arXiv:1311.5607 [hep-th]].

[6] D. Kadoh and S. Kamata, arXiv:1503.08499 [hep-lat].

[7] V. G. Filev and D. O'Connor, JHEP **1605** (2016) 167 doi:10.1007/JHEP05(2016)167 [arXiv:1506.01366 [hep-th]].

[8] M. Hanada, Y. Hyakutake, G. Ishiki and J. Nishimura, Phys. Rev. D **94** (2016) no.8, 086010 doi:10.1103/PhysRevD.94.086010 [arXiv:1603.00538 [hep-th]].

[9] E. Berkowitz, E. Rinaldi, M. Hanada, G. Ishiki, S. Shimasaki and P. Vranas, Phys. Rev. D **94** (2016) no.9, 094501 doi:10.1103/PhysRevD.94.094501 [arXiv:1606.04951 [hep-lat]].

[10] E. Rinaldi, E. Berkowitz, M. Hanada, J. Maltz and P. Vranas, arXiv:1709.01932 [hep-th].

[11] J. M. Maldacena, Int. J. Theor. Phys. **38**, 1113 (1999) [Adv. Theor. Math. Phys. **2**, 231 (1998)] doi:10.1023/A:1026654312961 [hep-th/9711200].

[12] A. Karch and E. Katz, JHEP **0206**, 043 (2002) doi:10.1088/1126-6708/2002/06/043 [hep-th/0205236].

[13] J. Babington, J. Erdmenger, N. J. Evans, Z. Guralnik and I. Kirsch, Phys. Rev. D **69**, 066007 (2004) doi:10.1103/PhysRevD.69.066007 [hep-th/0306018].

[14] C. Hoyos-Badajoz, K. Landsteiner and S. Montero, JHEP **0704**, 031 (2007) doi:10.1088/1126-6708/2007/04/031 [hep-th/0612169].

[15] D. Mateos, R. C. Myers and R. M. Thomson, JHEP **0705**, 067 (2007) doi:10.1088/1126-6708/2007/05/067 [hep-th/0701132].

[16] V. G. Filev and D. O'Connor, JHEP **1605** (2016) 122 doi:10.1007/JHEP05(2016)122 [arXiv:1512.02536 [hep-th]].

[17] N. Itzhaki, J. M. Maldacena, J. Sonnenschein and S. Yankielowicz, Phys. Rev. D **58**, 046004 (1998) doi:10.1103/PhysRevD.58.046004 [hep-th/9802042].

[18] M. Hanada, Y. Hyakutake, J. Nishimura and S. Takeuchi, Phys. Rev. Lett. **102**, 191602 (2009) [arXiv:0811.3102 [hep-th]].