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Quantization of Bosonic String Model in 26+2-dimensional Spacetime

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Abstract

We investigate the quantization of the bosonic string model which has a local $\mathrm{U}(1)_{\mathrm{V}} \times \mathrm{U}(1)_{\mathrm{A}}$ gauge invariance as well as the general coordinate and Weyl invariance on the world-sheet. The model is quantized by Lagrangian and Hamiltonian BRST formulations *á la* Batalin, Fradkin and Vilkovisky and noncovariant light-cone gauge formulation. Upon the quantization the model turns out to be formulated consistently in 26+2-dimensional background spacetime involving two time-like coordinates.

1 Introduction

It is the purpose of this paper to cast some further light upon constructions of theories involving two time coordinates. To consider the physics which has more than two time coordinates might be a clue to understand the origin of time and spacetime.

Several theories constructed on spacetime with two time coordinates are investigated from various viewpoints, such as F-theory [1], two-time physics [2] and 12-dimensional super Yang-Mills theory [3]. F-theory is proposed by Vafa as an extended concept of string theory and constructed by using field theory of super (2,2)-brane [4] with 10+2-dimensional background spacetime. The two-time physics is proposed by Bars as a device for searching a unified theory and developed by himself and his collaborators [5]. In this context, string-particle systems are proposed [6] from string theory point of view. By introducing constant null vectors in background spacetime into the formulation, the 12-dimensional super Yang-Mills theory [3] is also proposed.

Some years ago, one of the authors (Y.W.) had proposed a model which has a $\mathrm{U}(1)_\mathrm{V}\times\mathrm{U}(1)_\mathrm{A}$ gauge symmetry in two-dimensional spacetime [7]. The striking feature of this model is that there exists a negative norm state in two-dimensional spacetime as the same as string theories [7]. Using the $\mathrm{U}(1)_\mathrm{V}\times\mathrm{U}(1)_\mathrm{A}$ gauge symmetry he also proposed string models which have two time-like coordinates in ref. [8]. These models have the $\mathrm{U}(1)_\mathrm{V}\times\mathrm{U}(1)_\mathrm{A}$ gauge symmetry or a supersymmetric version of the $\mathrm{U}(1)_\mathrm{V}\times\mathrm{U}(1)_\mathrm{A}$ gauge symmetry on the two-dimensional world-sheet. The background spacetimes of the $\mathrm{U}(1)_\mathrm{V}\times\mathrm{U}(1)_\mathrm{A}$ bosonic and superstring model might be 26+2 and 10+2 dimensions, respectively. In ref. [9] manifest covariant formulations of the string models are given.

We in this paper further study the $\mathrm{U}(1)_\mathrm{V}\times\mathrm{U}(1)_\mathrm{A}$ string model. In particular, it would be obviously important to explicitly carry out the quantization, so that we can argue not only the critical dimension but also the mass spectrum at the quantum level. Since many concepts in string theories are presented in bosonic models, we focus our attentions on the bosonic $\mathrm{U}(1)_\mathrm{V}\times\mathrm{U}(1)_\mathrm{A}$ string model in this paper. A quantization of the superstring model based on our framework will be discussed in an additional work elsewhere [10].

The $\mathrm{U}(1)_\mathrm{V}\times\mathrm{U}(1)_\mathrm{A}$ string model is constructed as gauge field theory on two-dimensional world-sheet [8]. Although the similar models were investigated in refs. [6,11], an advantage of the formulation of our model is its manifest covariant expression in the background spacetime by using the $\mathrm{U}(1)_\mathrm{V}\times\mathrm{U}(1)_\mathrm{A}$ gauge symmetry [9], so that in this paper we can easily carry out the quantization with preserving the covariance. The $\mathrm{U}(1)_\mathrm{V}\times\mathrm{U}(1)_\mathrm{A}$ gauge symmetry is essential in our model. In constructing the covariant action, the generalized Chern-Simons action [12] proposed by Kawamoto and one of the authors (Y.W.) as a new type of topological action plays an important key role.

There are two remarks in quantizing our model. Firstly the action has a reducible symmetry which originally arises from symmetric structures of the generalized Chern-Simons action [13]. Secondly the gauge algebra is open. In the covariant BRST quantization of the system including reducible and open gauge symmetry, we need to use the formulations developed by Batalin, Fradkin and Vilkovisky [14, 15]. By adopting these methods we explicitly show the covariant quantizations are successfully carried out in the Lagrangian and the Hamiltonian formulations.

In order to treat the dynamics of our model more directly, we also quantize the model in noncovariant light-cone gauges. The suitable noncovariant gauge conditions can be imposed by residual symmetries of the $\mathrm{U}(1)_{\mathrm{V}} \times \mathrm{U}(1)_{\mathrm{A}}$ gauge symmetry and we can then solve all of the gauge constraints explicitly. We can also confirm that the existence of two time-like coordinates is not in conflict with the unitarity of the theory, since the two time-like coordinates are required by our "gauge" symmetry.

As an important feature of quantum string models, we can argue the critical dimension of the background spacetime. In usual bosonic string theories, the critical dimension is 25+1 [16, 17, 18], which is estimated by the BRST [19, 20] and the light-cone gauge formulation [21]. For our bosonic model, the critical dimension turns out to be 26+2. We obtain this result directly from both the BRST and the noncovariant light-cone gauge formulations.

This paper is organized as follows: We first introduce the $\mathrm{U}(1)_{\mathrm{V}} \times \mathrm{U}(1)_{\mathrm{A}}$ string model and explain semiclassical aspects of the model in Section 2. The preparation for the quantization is also given in this section. We present the covariant quantization based on the Lagrangian formulation in Section 3. In this section we investigate the perturbative aspect of the quantized model and determine the critical dimension of our $\mathrm{U}(1)_{\mathrm{V}} \times \mathrm{U}(1)_{\mathrm{A}}$ string model. In Section 4 the covariant quantization of the same model is carried out in the Hamiltonian formulation. By taking suitable gauge fixing conditions we reproduce the same gauge-fixed action in the Lagrangian formulation. We also obtain the BRST charge in this section. The quantization under noncovariant light-cone gauge fixing conditions is carried out in Section 5. We then study the symmetry of the background spacetime and obtain the same critical dimension by direct computation of the full quantum Poincaré algebra. We also present a mass-shell relation of the model and give low energy quantum states. Conclusions and discussions are given in the final section. Appendixes A and B contain our conventions. We also exhibit the BRST formulation of $\mathrm{U}(1)_{\mathrm{V}}\times\mathrm{U}(1)_{\mathrm{A}}$ model without two-dimensional gravity in Appendix C.

2 $\mathrm{U}(1)_{\mathrm{V}}\times\mathrm{U}(1)_{\mathrm{A}}$ bosonic string model

2.1 Classical action and its symmetries

The $\mathrm{U}(1)_{\mathrm{V}}\times\mathrm{U}(1)_{\mathrm{A}}$ bosonic string model [8], described by two-dimensional field theory, consists of D scalar fields $\xi^I(x)$, an Abelian gauge field $A_m(x)$ and the metric $g_{mn}(x)$. The two-dimensional spacetime coordinates are x^m $(m=0,\ 1)$ and the signature of metric is $(-,+)$. Our conventions are given in Appendix A. The scalar fields $\xi^I(x)$ are considered to be string coordinates in D-dimensional flat background spacetime with the background metric:

$$\eta_{IJ} \ = \ \eta^{IJ} \ = \ \begin{cases} -1 & (I=J=0) \\ 1 & (I=J=i,\ \ i=1,\,2,\,\ldots,\,D-3) \\ -1 & (I=J=\hat{0}) \\ 1 & (I=J=\hat{1}) \\ 0 & (\text{otherwise}) \end{cases} \tag{2.1}$$

The indices I and J run through $0,1,2,\ldots,D{-}3,\hat{0},\hat{1}$. As we will explain, the unitarity as a two-dimensional field theory requires two negative signatures to the background metric η_{IJ}, because the $\mathrm{U}(1)_{\mathrm{A}}$ gauge transformation as well as the general coordinate transformations removes a negative norm state. At the quantum level the absence of conformal anomaly requires $D{=}28$, however, we need not specify the value of D at the classical level.

The covariant action of the present model [9] is

$$S=\int\mathrm{d}^2x\,\sqrt{-g}\left(-\frac{1}{2}\,g^{mn}\partial_m\xi^I\partial_n\xi_I+\tilde{A}^m\phi_I\partial_m\xi^I\right)+S_{\mathrm{GCS}}, \tag{2.2}$$

where

$$g(x)=\det g_{mn}(x),\qquad \sqrt{-g(x)}\,\tilde{A}^m(x)=\varepsilon^{mn}A_n(x).$$

The action S_{GCS} is the generalized Chern-Simons action which is formulated in two-dimensional spacetime [12]

$$S_{\mathrm{GCS}}=\int\mathrm{d}^2x\,\sqrt{-g}\left(\tilde{B}^{mI}\partial_m\phi_I-\frac{1}{2}\tilde{C}\phi^I\phi_I\right), \tag{2.3}$$

where

$$\sqrt{-g(x)}\,\tilde{B}^{mI}(x) = \varepsilon^{mn} B_n^I(x)\,, \qquad \sqrt{-g(x)}\,\tilde{C}(x) = \tfrac{1}{2}\,\varepsilon^{mn} C_{mn}(x)\,.$$

The fields $\phi^I(x)$, $B_m^I(x)$ and $C_{mn}(x)$ are introduced for the purpose that the action S has the covariant form in the background spacetime. A derivation of the action (2.3) from the original generalized Chern-Simons action has been given in the paper [9].

The action (2.2) is invariant under the following $\mathrm{U(1)_V \times U(1)_A}$ gauge transformations,

$$\begin{aligned}
\delta\xi^I &= v'\phi^I, \\
\delta\tilde{A}^m &= \frac{\varepsilon^{mn}}{\sqrt{-g}}\partial_n v + g^{mn}\partial_n v', \\
\delta\tilde{B}^{mI} &= -\,v\frac{\varepsilon^{mn}}{\sqrt{-g}}\partial_n\xi^I + v' g^{mn}\partial_n\xi^I, \\
\delta\tilde{C} &= \partial_m v'\tilde{A}^m - v'\nabla_m\tilde{A}^m, \\
\delta\phi^I &= \delta g_{mn} = 0,
\end{aligned} \tag{2.4}$$

where the parameters $v(x)$ and $v'(x)$ correspond to the vector $\mathrm{U}(1)$ transformation "$\mathrm{U(1)_V}$" and the axial vector $\mathrm{U}(1)$ transformation "$\mathrm{U(1)_A}$", respectively. Since the generalized Chern-Simons action is invariant under nontrivial gauge transformations, the action (2.2) is also invariant under these gauge transformations with gauge parameters $u^I(x)$ and $\sqrt{-g(x)}\,\tilde{w}^m(x) \equiv \varepsilon^{mn} w_n(x)$,

$$\begin{aligned}
\delta\tilde{B}^{mI} &= \frac{\varepsilon^{mn}}{\sqrt{-g}}\partial_n u^I - \tilde{w}^m\phi^I, \\
\delta\tilde{C} &= \nabla_m\tilde{w}^m, \\
\delta\xi^I &= \delta\tilde{A}^m = \delta\phi^I = \delta g_{mn} = 0.
\end{aligned} \tag{2.5}$$

The action (2.2) is invariant under the general coordinate transformations and the Weyl transformation

$$\begin{aligned}
\delta\xi^I &= k^n\partial_n\xi^I, \\
\delta\tilde{A}^m &= k^n\partial_n\tilde{A}^m - \partial_n k^m\tilde{A}^n + 2s\tilde{A}^m, \\
\delta\tilde{B}^{mI} &= k^n\partial_n\tilde{B}^{mI} - \partial_n k^m\tilde{B}^{nI} + 2s\tilde{B}^{mI}, \\
\delta\phi^I &= k^n\partial_n\phi^I, \\
\delta\tilde{C} &= k^n\partial_n\tilde{C} + 2s\tilde{C}, \\
\delta g_{mn} &= k^l\partial_l g_{mn} + \partial_m k^l g_{ln} + \partial_n k^l g_{ml} - 2s g_{mn},
\end{aligned} \tag{2.6}$$

where $k^n(x)$ are parameters for the general coordinate transformations and $s(x)$ is a scaling parameter for the Weyl transformation.

Here it is worth to mention about some algebraic structures of the symmetry. The first is the reducibility of the symmetry. The system is on-shell reducible because the gauge transformations (2.5) have on-shell invariance under the following transformations of the gauge parameters with a reducible parameter $w'(x)$,

$$\begin{aligned} \delta' u^I &= w' \phi^I, \\ \delta' \tilde{w}^m &= \frac{\varepsilon^{mn}}{\sqrt{-g}} \partial_n w'. \end{aligned} \tag{2.7}$$

Since the transformations (2.7) are not reducible anymore, the action (2.2) is called a first-stage reducible system. The on-shell reducibility is the characteristic feature of the gauge symmetry (2.5) for the generalized Chern-Simons action and the quantization of such a system has been discussed in the previous works [13]. The second is that the gauge algebra is open. This means that the gauge algebra closes only when the equations of motion are satisfied. Actually, a direct calculation of the commutator of two gauge transformations on $\tilde{B}^{mI}(x)$ leads to

$$[\delta_1, \delta_2] \tilde{B}^{mI} = \cdots - (v_1' v_2 - v_2' v_1) \frac{\varepsilon^{mn}}{\sqrt{-g}} \partial_n \phi^I,$$

where the dots $(\cdots)$ contain terms of the usual "structure constants" of the gauge algebra. From the points of view of these structures of the gauge symmetry we adopt the Batalin-Fradkin-Vilkovisky formulation [14, 15] which allows us to deal with reducible and open gauge symmetries to obtain covariant gauge-fixed theories.

2.2 Semiclassical aspects

Before getting into the quantization of the system, we present semiclassical aspects of the action (2.2) by eliminating gauge fields through their equations of motion. Indeed, this manipulation might be helpful to understand the heart of the model.

First, equations of motion for the fields $\tilde{B}^{mI}(x)$ and $\tilde{C}(x)$ give constraints

$$\begin{aligned} \partial_m \phi_I &= 0, \\ \phi^I \phi_I &= 0. \end{aligned} \tag{2.8}$$

The nontrivial solution for these constraints is possible if the background spacetime metric includes two time-like signatures (2.1). In the light-cone notation*, one of the interesting

*We use a convention of the light-cone coordinates for the background spacetime as $x^I = (x^\mu, x^{\hat{+}}, x^{\hat{-}})$ where $x^{\hat{\pm}} = \frac{1}{\sqrt{2}}(x^{\hat{0}} \pm x^{\hat{1}})$ and the index μ runs through $0, 1, \ldots, D-3$.

solutions, which is naturally related with the usual string action, is $\phi^{\hat{-}}(x) = \phi^{\mu}(x) = 0$ and $\phi^{\hat{+}}(x) = $ const.. By substituting this solution into the classical action (2.2), the action S becomes

$$S = \int \mathrm{d}^2 x \sqrt{-g} \Big(-\frac{1}{2} g^{mn} \partial_m \xi^I \partial_n \xi_I - \tilde{A}^m \phi^{\hat{+}} \partial_m \xi^{\hat{-}} \Big). \tag{2.9}$$

In the action (2.9) a relation $\partial_m \xi^{\hat{-}}(x) = 0$ is given by the equation of motion for $\tilde{A}^m(x)$. Then, the final form of the action becomes the usual string action

$$S = \int \mathrm{d}^2 x \sqrt{-g} \Big(-\frac{1}{2} g^{mn} \partial_m \xi^{\mu} \partial_n \xi_{\mu} \Big). \tag{2.10}$$

Thus, the string action (2.10) is regarded as a gauge-fixed version of the action (2.2). The scalar fields $\phi^I(x)$ play an important role for the covariant formulation of the $\mathrm{U}(1)_{\mathrm{V}} \times \mathrm{U}(1)_{\mathrm{A}}$ string model in the background spacetime which involves two time-like coordinates.†

From the above manipulation it is suggested that the critical dimension of the background spacetime is defined as $D-3 = 25$, *i.e.* $D = 28$. However the dimensions D should be determined in the quantum analysis as we will investigate on this paper. We also want to emphasize that the quantization will be carried out with preserving D-dimensional covariance.

2.3 Preparation for the quantization

In order to carry out the quantization of the model smoothly, we here introduce new D scalar fields $\bar{\phi}^I(x)$ by replacing $\tilde{B}^{mI}(x)$ as

$$\tilde{B}^{mI} \longrightarrow \tilde{B}^{mI} - g^{mn} \partial_n \bar{\phi}^I.$$

Because of the above replacement, a new gauge symmetry with a gauge parameter $u'^I(x)$,

$$\begin{aligned} \delta \tilde{B}^{mI} &= g^{mn} \partial_n u'^I, \\ \delta \bar{\phi}^I &= u'^I, \end{aligned} \tag{2.11}$$

appears. Then, the action (2.2) is modified to

$$\begin{aligned} S = \int \mathrm{d}^2 x \sqrt{-g} \Big(&-\frac{1}{2} g^{mn} \partial_m \xi^I \partial_n \xi_I - g^{mn} \partial_m \bar{\phi}^I \partial_n \phi_I \\ &+ \tilde{A}^m \phi_I \partial_m \xi^I + \tilde{B}^{mI} \partial_m \phi_I - \frac{1}{2} \tilde{C} \phi^I \phi_I \Big). \end{aligned} \tag{2.12}$$

†Using this method, the noncovariant quantization of the models with an extra time coordinate was done in [6, 11]. Their models are similar to our model, but do not contain the $\mathrm{U}(1)_{\mathrm{V}} \times \mathrm{U}(1)_{\mathrm{A}}$ gauge symmetry.

Together with the gauge symmetry (2.5) the new gauge symmetry (2.11) constructs another $\mathrm{U(1)_V \times U(1)_A}$ gauge symmetry on the gauge fields $\tilde{B}^{mI}(x)$. In particular, these $\mathrm{U(1)_V \times U(1)_A}$ gauge symmetries turn out to be helpful for the covariant quantizations.

In addition to the gauge symmetry (2.4)-(2.6) and (2.11), the action (2.12) is also invariant under the following global transformations,

$$\begin{aligned}
\delta\xi^I &= \omega^I{}_J\xi^J + a^I, \\
\delta\tilde{A}^m &= r\tilde{A}^m + \sum_{i=1}^{2g}\alpha_i h^m_{(i)}, \\
\delta\phi^I &= -r\phi^I + \omega^I{}_J\phi^J, \\
\delta\bar{\phi}^I &= r\bar{\phi}^I + \omega^I{}_J\bar{\phi}^J, \\
\delta\tilde{B}^{mI} &= r\tilde{B}^{mI} + \omega^I{}_J\tilde{B}^{mJ} + \sum_{i=1}^{2g}(\beta^I_i + \alpha_i\xi^I)h^m_{(i)}, \\
\delta\tilde{C} &= 2r\tilde{C}, \\
\delta g_{mn} &= 0.
\end{aligned} \tag{2.13}$$

In the transformation (2.13) the parameters $\omega_{IJ} = -\omega_{JI}$, a^I and r are global parameters for the D-dimensional Lorentz transformation, the translation and the scale transformation, respectively. The functions $h^m_{(i)}(x)$ are harmonic functions which satisfy $\nabla_m h^m_{(i)}(x) = \varepsilon_{mn}\nabla^m h^n_{(i)}(x) = 0$ ($i = 1, 2, \ldots, 2g$; g = genus of two-dimensional spacetime) and α_i and β^I_i are global parameters.

3 Covariant quantization in the Lagrangian formulation

In this section we consider the covariant quantization of the action (2.12). As we explained in the previous section, the action has first-stage reducible and open gauge symmetries. In order to quantize the action we adopt the field-antifield formulation *á la* Batalin-Vilkovisky.

In the construction of Batalin-Vilkovisky formulation [14], ghost and ghost for ghost fields according to the reducibility condition and corresponding each antifields are introduced. The Grassmann parities of the antifields are opposite to those of the corresponding fields. If a field has ghost number n, its antifield has ghost number $-n-1$. We denote a set of fields and their antifields by $\Phi^A(x)$ and $\Phi^*_A(x)$, respectively,

$$\begin{aligned}
\Phi^A(x) &= \Big(\varphi^i(x), \mathcal{C}^{a_0}_0(x), \mathcal{C}^{a_1}_1(x), \ldots, \mathcal{C}^{a_N}_N(x)\Big), \\
\Phi^*_A(x) &= \Big(\varphi^*_i(x), \mathcal{C}^*_{0,a_0}(x), \mathcal{C}^*_{1,a_1}(x), \ldots, \mathcal{C}^*_{N,a_N}(x)\Big).
\end{aligned}$$

The fields $\varphi^i(x)$ are classical fields, on the other hand, the fields $\mathcal{C}_n^{a_n}(x)$ $[n = 0, 1, \ldots, N]$ are ghost and ghost for ghost fields corresponding to N-th reducible conditions. The classical fields $\varphi^i(x)$ and the ghost fields $\mathcal{C}_n^{a_n}(x)$ have the ghost number 0 and $n+1$, respectively. Then a minimal action $S_{\text{min}}(\Phi, \Phi^*)$ is determined by solving the following master equation,

$$\Big(S_{\text{min}}(\Phi, \Phi^*), S_{\text{min}}(\Phi, \Phi^*)\Big) = 0, \tag{3.1}$$

with the boundary conditions

$$S_{\text{min}}(\Phi, \Phi^*)\Big|_{\Phi^*=0} = S_{\text{classical}}(\varphi), \tag{3.2a}$$

$$\frac{\delta_{\text{L}}\delta_{\text{R}} S_{\text{min}}(\Phi, \Phi^*)}{\delta \mathcal{C}_n^{a_n} \delta \mathcal{C}^*_{n-1, a_{n-1}}}\Bigg|_{\Phi^*=0} = R^{a_{n-1}}_{n, a_n}(\Phi), \qquad (n = 0, 1, \ldots, N). \tag{3.2b}$$

Here the antibracket is defined by

$$(X, Y) \equiv \frac{\delta_{\text{R}} X}{\delta \Phi^*_A}\frac{\delta_{\text{L}} Y}{\delta \Phi^A} - \frac{\delta_{\text{R}} X}{\delta \Phi^A}\frac{\delta_{\text{L}} Y}{\delta \Phi^*_A}. \tag{3.3}$$

In this notation, $\mathcal{C}^*_{-1, a_{-1}}(x) \equiv \varphi^*_i(x)$ are the antifields of the classical fields $\varphi^i(x)$. The terms $R^{a_{-1}}_{0, a_0}(\Phi)$ and $R^{a_{n-1}}_{n, a_n}(\Phi)$ represent the gauge transformations and the n-th reducibility transformations, respectively. The master equation is solved order by order with respect to the ghost number. The BRST transformations of fields and antifields are given by

$$s\Phi^A = \Big(S_{\text{min}}, \Phi^A\Big), \qquad s\Phi^*_A = \Big(S_{\text{min}}, \Phi^*_A\Big). \tag{3.4}$$

Eqs. (3.1) and (3.4) assure that the BRST transformation is nilpotent and the minimal action is invariant under the BRST transformation*.

Now let us consider to construct the minimal action $S_{\text{min}}(\Phi, \Phi^*)$ of the model. For simplicity of calculation we first redefine field variables as $\hat{g}^{mn}(x) \equiv \sqrt{-g(x)}\, g^{mn}(x)$, $\hat{A}^m(x) \equiv \sqrt{-g(x)}\, \tilde{A}^m(x)$, $\hat{B}^{mI}(x) \equiv \sqrt{-g(x)}\, \tilde{B}^{mI}(x)$ and $\hat{C}(x) \equiv \sqrt{-g(x)}\, \tilde{C}(x)$. Using these new field variables, the classical action (2.12) is rewritten as

$$\begin{aligned} S_{\text{classical}} = \int \mathrm{d}^2x \bigg(&- \frac{1}{2}\hat{g}^{mn}\partial_m \xi^I \partial_n \xi_I - \hat{g}^{mn}\partial_m \bar{\phi}^I \partial_n \phi_I \\ &+ \hat{A}^m \phi_I \partial_m \xi^I + \hat{B}^{mI}\partial_m \phi_I - \frac{1}{2}\hat{C}\phi^I \phi_I + (\hat{g}+1)\hat{Z} \bigg), \end{aligned} \tag{3.5}$$

where the scalar density field $\hat{Z}(x)$ is a multiplier whose equation of motion compensates $\hat{g}(x) \equiv \det \hat{g}^{mn}(x) = -1$ [19]. We also redefine the gauge transformations (2.4)-(2.6) and

*Our convention for the Leibniz rule of the BRST operation is given by $s(XY) = (sX)Y + (-)^{|X|}X(sY)$, where $|X|$ is a Grassmann parity of field X.

(2.11) in terms of these new field variables,

$$
\begin{aligned}
\delta\xi^I &= k^n\partial_n\xi^I + v'\phi^I, \\
\delta\hat{A}^m &= \partial_n(k^n\hat{A}^m) - \partial_n k^m\hat{A}^n + \varepsilon^{mn}\partial_n v + \hat{g}^{mn}\partial_n v', \\
\delta\phi^I &= k^n\partial_n\phi^I, \\
\delta\bar{\phi}^I &= k^n\partial_n\bar{\phi}^I + u'^I, \\
\delta\hat{B}^{mI} &= \partial_n(k^n\hat{B}^{mI}) - \partial_n k^m\hat{B}^{nI} + \varepsilon^{mn}\partial_n u^I + \hat{g}^{mn}\partial_n u'^I \\
&\quad - (\varepsilon^{mn}v - \hat{g}^{mn}v')\partial_n\xi^I - \hat{w}^m\phi^I, \\
\delta\hat{C} &= \partial_n(k^n\hat{C}) + \partial_n\hat{w}^n + \partial_n v'\hat{A}^n - v'\partial_n\hat{A}^n, \\
\delta\hat{g}^{mn} &= \partial_l(k^l\hat{g}^{mn}) - \partial_l k^m\hat{g}^{ln} - \partial_l k^n\hat{g}^{ml}, \\
\delta\hat{Z} &= \partial_n(k^n\hat{Z}),
\end{aligned}
\tag{3.6}
$$

where we denote $\hat{w}^m(x) \equiv \sqrt{-g(x)}\,\tilde{w}^m(x)$. The gauge transformation of the multiplier field $\hat{Z}(x)$ is required to keep the action (3.5) invariant under the general coordinate transformations. The reducibility condition (2.7) is expressed by

$$
\begin{aligned}
\delta' u^I &= w'\phi^I, \\
\delta'\hat{w}^m &= \varepsilon^{mn}\partial_n w'.
\end{aligned}
\tag{3.7}
$$

The classical fields $\varphi^i(x)$ consist of $\xi^I(x)$, $\phi^I(x)$, $\bar{\phi}^I(x)$, $\hat{A}^m(x)$, $\hat{B}^{mI}(x)$, $\hat{C}(x)$, $\hat{g}^{mn}(x)$ and $\hat{Z}(x)$. Here we introduce the ghost fields $a(x)$, $a'(x)$, $b^I(x)$, $b'^I(x)$, $\hat{c}^m(x)$ and $d^m(x)$ corresponding to the gauge parameters $v(x)$, $v'(x)$, $u^I(x)$, $u'^I(x)$, $\hat{w}^m(x)$ and $k^m(x)$ and a ghost for ghost field $f(x)$ to the reducible parameter $w'(x)$. The ghost fields and the ghost for ghost field are fermionic and bosonic, respectively. Since the $\mathrm{U}(1)_\mathrm{V}\times\mathrm{U}(1)_\mathrm{A}$ model is a first-stage reducible system, the boundary conditions (3.2b) with $n = 0, 1$ correspond to the gauge transformations (3.6) and the reducibility conditions (3.7), respectively. It is straightforward to solve the master equation perturbatively in the order of antifields [22, 23],

$$
\begin{aligned}
S_{\mathrm{min}} &= S_{\mathrm{classical}} \\
&\quad + \int \mathrm{d}^2x \Big\{ -\xi^*_I(\,d^n\partial_n\xi^I + a'\phi^I\,) \\
&\qquad - \hat{A}^*_m\Big(\partial_n(d^n\hat{A}^m) - \partial_n d^m\hat{A}^n + \varepsilon^{mn}\partial_n a + \hat{g}^{mn}\partial_n a'\Big) \\
&\qquad - \phi^*_I(\,d^n\partial_n\phi^I\,) \\
&\qquad - \bar{\phi}^*_I(\,d^n\partial_n\bar{\phi}^I + b'^I\,)
\end{aligned}
$$

$$
\begin{aligned}
&- \hat{B}^*_{mI}\Big(\partial_n(d^n \hat{B}^{mI}) - \partial_n d^m \hat{B}^{nI} + \varepsilon^{mn}\partial_n b^I + \hat{g}^{mn}\partial_n b'^I \\
&\qquad - (\varepsilon^{mn} a - \hat{g}^{mn} a')\partial_n \xi^I - \hat{c}^m \phi^I + \frac{1}{2}(f + aa')\varepsilon^{mn}\hat{B}^{*I}_n \Big) \\
&- \hat{C}^*\Big(\partial_n(d^n \hat{C}) + \partial_n \hat{c}^n + \partial_n a' \hat{A}^n - a'\partial_n \hat{A}^n \Big) \\
&- \frac{1}{2}\hat{g}^*_{mn}\Big(\partial_k(d^k \hat{g}^{mn}) - \partial_k d^m \hat{g}^{kn} - \partial_k d^n \hat{g}^{mk} \Big) \\
&- \hat{Z}^*\Big(\partial_n(d^n \hat{Z}) \Big) \\
&+ a^*(\, d^n\partial_n a \,) + a'^*(\, d^n\partial_n a' \,) + b^*_I(\, d^n\partial_n b^I + f\phi^I \,) + b'^*_I(\, d^n\partial_n b'^I \,) \\
&+ \hat{c}^*_m\Big(\partial_n(d^n \hat{c}^m) - \partial_n d^m \hat{c}^n + (\varepsilon^{mn} a - \hat{g}^{mn} a')\partial_n a' + \varepsilon^{mn}\partial_n f \Big) \\
&+ d^*_m(\, d^n\partial_n d^m \,) \\
&- f^*(\, d^n\partial_n f \,)\Big\}.
\end{aligned}
\tag{3.8}
$$

The gauge degrees of freedom are fixed by introducing a nonminimal action which must be added to the minimal one and choosing a suitable gauge-fixing fermion. We here choose the orthonormal gauge condition $\hat{g}^{mn}(x) = \eta^{mn}$ for the world-sheet metric. The $\mathrm{U(1)_V \times U(1)_A}$ gauge parameters $v(x)$, $v'(x)$ and the global parameter α_i make us possible to choose the gauge $\hat{A}^m(x) = 0$. In the same way, we can choose the gauge $\hat{B}^{mI}(x) = 0$ by using the parameters $u^I(x)$, $u'^I(x)$ and β^I_i. We also fix the gauge $\hat{C}(x) = \hat{C}_0$, where $\hat{C}_0$ is a constant parameter, by using the gauge parameter $\hat{w}^m(x)$. In addition to these gauge fixing procedure, we also impose the condition $\partial_m(\hat{g}^{mn}(x)\varepsilon_{nk}\hat{c}^k(x)) = 0$ to fix the residual gauge degrees of freedom from the reducibility condition. In order to adopt all of these gauge fixing conditions, we introduce the nonminimal action S_{nonmin},

$$
S_{\text{nonmin}} = \int \mathrm{d}^2 x \left(\varepsilon^{mn}\hat{\bar{a}}^*_m Z^a_n + \varepsilon^{mn}\hat{\bar{b}}^{*I}_m Z^b_{nI} + \bar{c}^* Z^c + \frac{1}{2}\bar{d}^{*mn} Z^d_{mn} - \bar{f}^* c' \right), \tag{3.9}
$$

and the gauge-fixing fermion Ψ,

$$
\Psi = \int \mathrm{d}^2 x \left(\varepsilon_{mn}\hat{\bar{a}}^m \hat{A}^n + \varepsilon_{mn}\hat{\bar{b}}^m_I \hat{B}^{nI} + c(\hat{C} - \hat{C}_0) + \bar{d}_{mn}\hat{g}^{mn} + \bar{f}\partial_m(\hat{g}^{mn}\varepsilon_{nk}\hat{c}^k) \right), \tag{3.10}
$$

where we require traceless conditions

$$
\eta^{mn}\bar{d}_{mn} = \eta_{mn}\bar{d}^{*mn} = \eta^{mn} Z^d_{mn} = \eta_{mn} Z^{d*mn} = 0. \tag{3.11}
$$

The antighost fields $\hat{\bar{a}}_m(x)$, $\hat{\bar{b}}^I_m(x)$, $c(x)$, $c'(x)$ and $\bar{d}_{mn}(x)$ are fermionic fields, and the auxiliary fields $Z^a_m(x)$, $Z^b_{mI}(x)$, $Z^c(x)$, $Z^d_{mn}(x)$ and $\bar{f}(x)$ are bosonic ones. The ghost numbers of the fields are as follows:

$$
\begin{array}{ll}
\bar{f} & (\text{ghost number} = -2) \\
\hat{a}^m, \quad \hat{b}_I^m, \quad c, \quad c', \quad \bar{d}_{mn} & (\text{ghost number} = -1) \\
\xi^I, \quad \phi^I, \quad \bar{\phi}^I, \quad \hat{A}^m, \quad \hat{B}^{mI}, \quad \hat{C}, \quad \hat{g}^{mn}, & \\
Z_m^a, \quad Z_{mI}^b, \quad Z^c, \quad Z_{mn}^d, \quad \hat{Z} & (\text{ghost number} = 0) \\
a, \quad a', \quad b^I, \quad b'^I, \quad \hat{c}^m, \quad d^m & (\text{ghost number} = 1) \\
f & (\text{ghost number} = 2)
\end{array}
$$

The BRST transformations of $\Phi^A(x)$ and $\Phi_A^*(x)$ are given by

$$
s\Phi^A = \left(S_{\min} + S_{\text{nonmin}}, \Phi^A\right), \qquad s\Phi_A^* = \left(S_{\min} + S_{\text{nonmin}}, \Phi_A^*\right). \tag{3.12}
$$

Therefore, the BRST transformations of fields $\Phi^A(x)$ are

$$
\begin{aligned}
s\xi^I &= d^n \partial_n \xi^I + a'\phi^I, \\
s\hat{A}^m &= \partial_n(d^n \hat{A}^m) - \partial_n d^m \hat{A}^n + \varepsilon^{mn}\partial_n a + \hat{g}^{mn}\partial_n a', \\
s\phi^I &= d^n \partial_n \phi^I, \\
s\bar{\phi}^I &= d^n \partial_n \bar{\phi}^I + b'^I, \\
s\hat{B}^{mI} &= \partial_n(d^n \hat{B}^{mI}) - \partial_n d^m \hat{B}^{nI} + \varepsilon^{mn}\partial_n b^I + \hat{g}^{mn}\partial_n b'^I \\
&\quad - (\varepsilon^{mn} a - \hat{g}^{mn} a')\partial_n \xi^I - \hat{c}^m \phi^I + (f + aa')\varepsilon^{mn} \hat{B}_n^{*I}, \\
s\hat{C} &= \partial_n(d^n \hat{C}) + \partial_n \hat{c}^n + \partial_n a' \hat{A}^n - a' \partial_n \hat{A}^n, \\
s\hat{g}^{mn} &= \partial_k(d^k \hat{g}^{mn}) - \partial_k d^m \hat{g}^{kn} - \partial_k d^n \hat{g}^{mk}, \\
s\hat{Z} &= \partial_n(d^n \hat{Z}), \\
sa &= d^n \partial_n a, \\
sa' &= d^n \partial_n a', \\
sb^I &= d^n \partial_n b^I + f\phi^I, \\
sb'^I &= d^n \partial_n b'^I, \\
s\hat{c}^m &= \partial_n(d^n \hat{c}^m) - \partial_n d^m \hat{c}^n + (\varepsilon^{mn} a - \hat{g}^{mn} a')\partial_n a' + \varepsilon^{mn}\partial_n f, \\
sd^m &= d^n \partial_n d^m, \\
sf &= d^n \partial_n f,
\end{aligned} \tag{3.13a}
$$

and

$$
s\hat{a}^m = \varepsilon^{mn} Z_n^a, \qquad sZ_m^a = 0,
$$

$$
\begin{aligned}
s\hat{b}^m_I &= \varepsilon^{mn} Z^b_{nI}, & sZ^b_{mI} &= 0,\\
sc &= Z^c, & sZ^c &= 0,\\
s\bar{d}_{mn} &= Z^d_{mn}, & sZ^d_{mn} &= 0,\\
s\bar{f} &= c', & sc' &= 0.
\end{aligned}
\tag{3.13b}
$$

The antifields are eliminated by using equations $\Phi^*_A(x) = \delta_{\mathrm{L}}\Psi/\delta\Phi^A(x)$. Then the gauge-fixed action is given by

$$
\begin{aligned}
S_{\text{gauge-fixed}} &= S_{\min} + S_{\text{nonmin}}\Big|_{\Phi^*=\frac{\delta\Psi}{\delta\Phi}}\\
&= \int \mathrm{d}^2x \Big\{ -\frac{1}{2}\hat{g}^{mn}\partial_m\xi^I\partial_n\xi_I - \hat{g}^{mn}\partial_m\bar{\phi}^I\partial_n\phi_I\\
&\qquad + \hat{A}^m\phi_I\partial_m\xi^I + \hat{B}^{mI}\partial_m\phi_I - \frac{1}{2}\hat{C}\phi^I\phi_I + (\hat{g}+1)\hat{Z}\\
&\qquad + \varepsilon_{mk}\hat{a}^k\Big(\partial_n(d^n\hat{A}^m) - \partial_n d^m\hat{A}^n + \varepsilon^{mn}\partial_n a + \hat{g}^{mn}\partial_n a'\Big)\\
&\qquad + \varepsilon_{mk}\hat{b}^k_I\Big(\partial_n(d^n\hat{B}^{mI}) - \partial_n d^m\hat{B}^{nI} + \varepsilon^{mn}\partial_n b^I + \hat{g}^{mn}\partial_n b'^I\\
&\qquad\qquad - (\varepsilon^{mn}a - \hat{g}^{mn}a')\partial_n\xi^I - \hat{c}^m\phi^I - \frac{1}{2}(f+aa')\hat{b}^{mI}\Big)\\
&\qquad - c\Big(\partial_n(d^n\hat{C}) + \partial_n\hat{c}^n + \partial_n a'\hat{A}^n - a'\partial_n\hat{A}^n\Big)\\
&\qquad - \frac{1}{2}\Big(\bar{d}_{mn} - \partial_m\bar{f}\varepsilon_{nl}\hat{c}^l - \partial_n\bar{f}\varepsilon_{ml}\hat{c}^l\Big)\Big(\partial_k(d^k\hat{g}^{mn}) - \partial_k d^m\hat{g}^{kn} - \partial_k d^n\hat{g}^{mk}\Big)\\
&\qquad + \varepsilon_{ml}\hat{g}^{lk}\partial_k\bar{f}\Big(\partial_n(d^n\hat{c}^m) - \partial_n d^m\hat{c}^n + (\varepsilon^{mn}a - \hat{g}^{mn}a')\partial_n a' + \varepsilon^{mn}\partial_n f\Big)\\
&\qquad - \hat{A}^m Z^a_m - \hat{B}^{mI}Z^b_{mI} + (\hat{C}-\hat{C}_0)Z^c + \frac{1}{2}\hat{g}^{mn}Z^d_{mn} - \partial_m(\hat{g}^{mn}\varepsilon_{nk}\hat{c}^k)\,c'\Big\}\\
&= \int \mathrm{d}^2x \Big\{ -\frac{1}{2}\hat{g}^{mn}\partial_m\xi^I\partial_n\xi_I - \hat{g}^{mn}\partial_m\bar{\phi}^I\partial_n\phi_I - \hat{g}^{mn}\partial_m\bar{f}\partial_n f\\
&\qquad - \Big(\hat{a}^m - \varepsilon^{mn}\partial_n(\bar{f}a) + \bar{f}\hat{g}^{mn}\partial_n a' - \hat{b}^m_I\xi^I\Big)\Big(\partial_m a + \varepsilon_{mk}\hat{g}^{kn}\partial_n a'\Big)\\
&\qquad - \hat{b}^m_I\Big(\partial_m(b^I + a\xi^I) + \varepsilon_{mk}\hat{g}^{kn}\partial_n(b'^I + a'\xi^I)\Big)\\
&\qquad - \hat{c}^m\Big(\partial_m c + \varepsilon_{mk}\hat{g}^{kn}\partial_n(c' - d^n\partial_n\bar{f})\Big) + \hat{g}^{mn}\bar{d}_{mk}\partial_n d^k - \frac{1}{2}\hat{g}^{mn}d^k\partial_k\bar{d}_{mn}\\
&\qquad - 2a\hat{b}^m_I\partial_m\xi^I + \varepsilon_{mn}\hat{b}^m_I\hat{c}^n\phi^I + \frac{1}{2}(f+aa')\varepsilon_{mn}\hat{b}^m_I\hat{b}^{nI}\\
&\qquad - \hat{A}^m\Big(Z^a_m - \phi_I\partial_m\xi^I - \varepsilon_{mn}d^k\partial_k\hat{a}^n - \partial_m d^k\varepsilon_{kn}\hat{a}^n + c\partial_m a' + \partial_m(ca')\Big)\\
&\qquad - \hat{B}^{mI}\Big(Z^b_{mI} - \partial_m\phi_I - \varepsilon_{mn}d^k\partial_k\hat{b}^n_I - \partial_m d^k\varepsilon_{kn}\hat{b}^n_I\Big)\\
&\qquad + \hat{C}\Big(Z^c - \frac{1}{2}\phi^I\phi_I - d^n\partial_n c\Big) - \hat{C}_0 Z^c\\
&\qquad + \frac{1}{2}\hat{g}^{mn}Z^d_{mn} + (\hat{g}+1)\Big(\hat{Z} - \varepsilon^{mn}\partial_m(\bar{f}a')\partial_n a'\Big)\Big\}.
\end{aligned}
\tag{3.14}
$$

Here, it should be noted that one can remove a BRST exact term $-\hat{C}_0 Z^c(x) = -s(\hat{C}_0 c(x))$ from the above action. An influence of the parameter $\hat{C}_0$ disappears at the quantum level.

In order to simplify the form of the action, let us redefine some of fields as follows:

$$
\begin{aligned}
Z^a_m - \phi_I \partial_m \xi^I - \varepsilon_{mn} d^k \partial_k \hat{a}^n - \partial_m d^k \varepsilon_{kn} \hat{a}^n + c\partial_m a' + \partial_m (ca') &\to Z^a_m, \\
Z^b_{mI} - \partial_m \phi_I - \varepsilon_{mn} d^k \partial_k \hat{b}^n_I - \partial_m d^k \varepsilon_{kn} \hat{b}^n_I &\to Z^b_{mI}, \\
Z^c - \frac{1}{2}\phi^I \phi_I - d^n \partial_n c &\to Z^c, \\
\hat{Z} - \varepsilon^{mn} \partial_m (\bar{f} a') \partial_n a' &\to \hat{Z}, \\
\hat{a}^m - \varepsilon^{mn} \partial_n (\bar{f} a) + \bar{f} \hat{g}^{mn} \partial_n a' - \hat{b}^m_I \xi^I &\to \hat{a}^m, \\
b^I + a\xi^I &\to b^I, \\
b'^I + a'\xi^I &\to b'^I, \\
c' - d^n \partial_n \bar{f} &\to c'.
\end{aligned}
$$

Under these field redefinitions, the action (3.14) is modified to

$$
\begin{aligned}
S_{\text{gauge-fixed}} = \int \mathrm{d}^2 x \Big\{ & -\frac{1}{2}\hat{g}^{mn} \partial_m \xi^I \partial_n \xi_I - \hat{g}^{mn} \partial_m \bar{\phi}^I \partial_n \phi_I - \hat{g}^{mn} \partial_m \bar{f} \partial_n f \\
& - \hat{a}^m (\partial_m a + \varepsilon_{mk} \hat{g}^{kn} \partial_n a') - \hat{b}^m_I (\partial_m b^I + \varepsilon_{mk} \hat{g}^{kn} \partial_n b'^I) \\
& - \hat{c}^m (\partial_m c + \varepsilon_{mk} \hat{g}^{kn} \partial_n c') + \hat{g}^{mn} \bar{d}_{mk} \partial_n d^k - \frac{1}{2}\hat{g}^{mn} d^k \partial_k \bar{d}_{mn} \\
& - 2a \hat{b}^m_I \partial_m \xi^I + \varepsilon_{mn} \hat{b}^m_I \hat{c}^n \phi^I + \frac{1}{2}(f + aa') \varepsilon_{mn} \hat{b}^m_I \hat{b}^{nI} \\
& - \hat{A}^m Z^a_m - \hat{B}^{mI} Z^b_{mI} + \hat{C} Z^c + \frac{1}{2}\hat{g}^{mn} Z^d_{mn} + (\hat{g}+1)\hat{Z} \Big\}. \qquad (3.15)
\end{aligned}
$$

The BRST transformations (3.13a) and (3.13b) also become

$$
\begin{aligned}
s\xi^I &= d^n \partial_n \xi^I + a'\phi^I, \\
s\phi^I &= d^n \partial_n \phi^I, \\
s\bar{\phi}^I &= d^n \partial_n \bar{\phi}^I + b'^I - a'\xi^I, \\
sf &= d^n \partial_n f, \\
s\bar{f} &= d^n \partial_n \bar{f} + c', \\
sa &= d^n \partial_n a, \\
sa' &= d^n \partial_n a', \\
sb^I &= d^n \partial_n b^I + (f - aa')\phi^I, \\
sb'^I &= d^n \partial_n b'^I,
\end{aligned}
$$

$$
\begin{aligned}
sc &= d^n\partial_n c + \frac{1}{2}\phi^I\phi_I + Z^c,\\
sc' &= d^n\partial_n c',\\
sd^m &= d^n\partial_n d^m,\\
s\hat{a}^m &= \partial_n(d^n\hat{a}^m) - \partial_n d^m\hat{a}^n + \varepsilon^{mn}(\phi_I\partial_n\xi^I - \partial_n\phi_I\xi^I) - a'\hat{b}^m_I\phi^I\\
&\quad - (\varepsilon^{mn}c - \hat{g}^{mn}c')\partial_n a' - \varepsilon^{mn}\partial_n(ca' + c'a) + \varepsilon^{mn}(Z^a_n - Z^b_{nI}\xi^I),\\
s\hat{b}^m_I &= \partial_n(d^n\hat{b}^m_I) - \partial_n d^m\hat{b}^n_I + \varepsilon^{mn}\partial_n\phi_I + \varepsilon^{mn}Z^b_{nI},\\
s\hat{c}^m &= \partial_n(d^n\hat{c}^m) - \partial_n d^m\hat{c}^n + (\varepsilon^{mn}a - \hat{g}^{mn}a')\partial_n a' + \varepsilon^{mn}\partial_n f,\\
s\bar{d}_{mn} &= Z^d_{mn},\\
s\hat{A}^m &= \partial_n(d^n\hat{A}^m) - \partial_n d^m\hat{A}^n + \varepsilon^{mn}\partial_n a + \hat{g}^{mn}\partial_n a',\\
s\hat{B}^{mI} &= \partial_n(d^n\hat{B}^{mI}) - \partial_n d^m\hat{B}^{nI} + \varepsilon^{mn}\partial_n b^I + \hat{g}^{mn}\partial_n b'^I\\
&\quad - \varepsilon^{mn}(a\partial_n\xi^I + \partial_n(a\xi^I)) - \hat{g}^{mn}\partial_n a'\xi^I - \hat{c}^m\phi^I - (f + aa')\hat{b}^{mI},\\
s\hat{C} &= \partial_n(d^n\hat{C}) + \partial_n\hat{c}^n + \partial_n a'\hat{A}^n - a'\partial_n\hat{A}^n,\\
s\hat{g}^{mn} &= \partial_k(d^k\hat{g}^{mn}) - \partial_k d^m\hat{g}^{kn} - \partial_k d^n\hat{g}^{mk},\\
sZ^a_m &= d^n\partial_n Z^a_m + \partial_m d^n Z^a_n + Z^c\partial_m a' + \partial_m(Z^c a'),\\
sZ^b_{mI} &= d^n\partial_n Z^b_{mI} + \partial_m d^n Z^b_{nI},\\
sZ^c &= d^n\partial_n Z^c,\\
sZ^d_{mn} &= 0,\\
s\hat{Z} &= \partial_n(d^n\hat{Z}) - \varepsilon^{mn}\partial_m(c'a')\partial_n a'.
\end{aligned}
\tag{3.16}
$$

The action (3.15) is invariant under the nilpotent BRST transformations (3.16).

Using equations of motion for the fields $Z^a_m(x)$, $Z^b_{mI}(x)$, $Z^c(x)$, $Z^d_{mn}(x)$, $\hat{Z}(x)$, $\hat{A}^m(x)$, $\hat{B}^{mI}(x)$, $\hat{C}(x)$ and $\hat{g}^{mn}(x)$, thus imposing gauge fixing conditions, we fix fields as

$$
\begin{aligned}
&\hat{A}^m = \hat{B}^{mI} = \hat{C} = 0, \qquad \hat{g}^{mn} = \eta^{mn},\\
&Z^a_m = Z^b_{mI} = Z^c = 0, \qquad Z^d_{mn} = V_{mn} - \frac{1}{2}\eta_{mn}\eta^{kl}V_{kl}, \qquad \hat{Z} = -\frac{1}{4}\eta^{mn}V_{mn},
\end{aligned}
\tag{3.17}
$$

where we denote

$$
\begin{aligned}
V_{mn} = &\frac{1}{2}\partial_m\xi^I\partial_n\xi_I + \partial_m\bar{\phi}^I\partial_n\phi_I + \partial_m\bar{f}\partial_n f\\
&+ \hat{a}^k\varepsilon_{km}\partial_n a' + \hat{b}^k_I\varepsilon_{km}\partial_n b'^I + \hat{c}^k\varepsilon_{km}\partial_n c' - \bar{d}_{mk}\partial_n d^k + \frac{1}{2}d^k\partial_k\bar{d}_{mn}\\
&+ (m \leftrightarrow n).
\end{aligned}
\tag{3.18}
$$

Then, we finally obtain the following gauge-fixed action,

$$
\begin{aligned}
S_{\text{gauge-fixed}} = \int \mathrm{d}^2x \Big\{ & -\frac{1}{2}\eta^{mn}\partial_m\xi^I\partial_n\xi_I - \eta^{mn}\partial_m\bar{\phi}^I\partial_n\phi_I - \eta^{mn}\partial_m\bar{f}\partial_n f \\
& - \hat{a}^m(\partial_m a + \varepsilon_m{}^n\partial_n a') - \hat{b}_I^m(\partial_m b^I + \varepsilon_m{}^n\partial_n b'^I) \\
& - \hat{c}^m(\partial_m c + \varepsilon_m{}^n\partial_n c') + \eta^{mn}\bar{d}_{mk}\partial_n d^k \\
& - 2a\hat{b}_I^m\partial_m\xi^I + \varepsilon_{mn}\hat{b}_I^m\hat{c}^n\phi^I + \frac{1}{2}(f + aa')\varepsilon_{mn}\hat{b}_I^m\hat{b}^{nI} \Big\}.
\end{aligned} \tag{3.19}
$$

The action (3.19) is invariant under the following on-shell nilpotent BRST transformation, in which the antifields and the auxiliary fields are eliminated,

$$
\begin{aligned}
s\xi^I &= d^n\partial_n\xi^I + a'\phi^I, \\
s\phi^I &= d^n\partial_n\phi^I, \\
s\bar{\phi}^I &= d^n\partial_n\bar{\phi}^I + b'^I - a'\xi^I, \\
sf &= d^n\partial_n f, \\
s\bar{f} &= d^n\partial_n\bar{f} + c', \\
sa &= d^n\partial_n a, \\
sa' &= d^n\partial_n a', \\
sb^I &= d^n\partial_n b^I + (f - aa')\phi^I, \\
sb'^I &= d^n\partial_n b'^I, \\
sc &= d^n\partial_n c + \frac{1}{2}\phi^I\phi_I, \\
sc' &= d^n\partial_n c', \\
sd^m &= d^n\partial_n d^m, \\
s\hat{a}^m &= \partial_n(d^n\hat{a}^m) - \partial_n d^m\hat{a}^n + \varepsilon^{mn}(\phi_I\partial_n\xi^I - \partial_n\phi_I\xi^I) - a'\hat{b}_I^m\phi^I \\
&\quad -(\varepsilon^{mn}c - \eta^{mn}c')\partial_n a' - \varepsilon^{mn}\partial_n(ca' + c'a), \\
s\hat{b}_I^m &= \partial_n(d^n\hat{b}_I^m) - \partial_n d^m\hat{b}_I^n + \varepsilon^{mn}\partial_n\phi_I, \\
s\hat{c}^m &= \partial_n(d^n\hat{c}^m) - \partial_n d^m\hat{c}^n + (\varepsilon^{mn}a - \eta^{mn}a')\partial_n a' + \varepsilon^{mn}\partial_n f, \\
s\bar{d}_{mn} &= V_{mn} - \frac{1}{2}\eta_{mn}\eta^{kl}V_{kl}.
\end{aligned} \tag{3.20}
$$

We can check the following relations for the nilpotency of the BRST transformation,

$$
\begin{aligned}
s^2\hat{c}^m &= -2\left(\frac{\delta_{\mathrm{L}}S}{\delta\bar{d}_{mn}}\right)a'\partial_n a', \\
s^2\bar{d}_{mn} &= -\left(\frac{\delta_{\mathrm{L}}S}{\delta\hat{c}^m}\right)a'\partial_n a' - \left(\frac{\delta_{\mathrm{L}}S}{\delta\hat{c}^n}\right)a'\partial_m a' + \eta_{mn}\eta^{kl}\left(\frac{\delta_{\mathrm{L}}S}{\delta\hat{c}^k}\right)a'\partial_l a'
\end{aligned}
$$

$$
+\eta_{mk}\eta_{nl}\left(\frac{\delta_{\rm L}S}{\delta\bar{d}_{kl}}\right)\eta^{pq}V_{pq},
$$

$$
s^2(\text{others})=0.
$$

Now we present a perturbative analysis of the gauge-fixed action. We would like to investigate the BRST Ward identities at the quantum level. Then, we find out that nonlocal anomalous terms obtained from one-loop calculations vanish by imposing a condition, which determines the critical dimension for this string model. For the explicit calculation it is convenient to introduce light-cone notations on the world-sheet†. Then, the gauge-fixed action (3.19) is expressed with these notations

$$
\begin{aligned}
S_{\text{gauge-fixed}}=\int \mathrm{d}^2x\,\Big\{&\partial_+\xi^I\partial_-\xi_I+2\partial_+\bar{\phi}^I\partial_-\phi_I+2\partial_+\bar{f}\,\partial_-f\\
&+\hat{a}_+\partial_-a_++\hat{a}_-\partial_+a_-+\hat{b}_{+I}\partial_-b^I_++\hat{b}_{-I}\partial_+b^I_-\\
&+\hat{c}_+\partial_-c_++\hat{c}_-\partial_+c_--\bar{d}_{++}\partial_-d^+-\bar{d}_{--}\partial_+d^-\\
&+(a_++a_-)(\,\hat{b}_{+I}\partial_-\xi^I+\hat{b}_{-I}\partial_+\xi^I\,)\\
&+\phi^I\hat{b}_{+I}\hat{c}_--\phi^I\hat{b}_{-I}\hat{c}_++\left(f+\frac{1}{2}a_+a_-\right)\hat{b}_{+I}\hat{b}^I_-\Big\},
\end{aligned}
\tag{3.21}
$$

where we denote $a_\pm(x)\equiv a(x)\mp a'(x)$, $b^I_\pm(x)\equiv b^I(x)\mp b'^I(x)$ and $c_\pm(x)\equiv c(x)\mp c'(x)$. Propagators are derived by taking inverses of bilinear parts in the action (3.21),

$$
\begin{aligned}
\langle\xi^I(x)\xi^J(y)\rangle_0&=\langle\bar{\phi}^I(x)\phi^J(y)\rangle_0\\
&=\int\frac{\mathrm{d}^2p}{i(2\pi)^2}\frac{1}{p^2+i\epsilon}\,e^{-ip(x-y)}\eta^{IJ},\\
\langle\bar{f}(x)f(y)\rangle_0&=\int\frac{\mathrm{d}^2p}{i(2\pi)^2}\frac{1}{p^2+i\epsilon}\,e^{-ip(x-y)},\\
\langle\hat{a}_\pm(x)a_\pm(y)\rangle_0&=\langle\hat{c}_\pm(x)c_\pm(y)\rangle_0=-\langle\bar{d}_{\pm\pm}(x)d^\pm(y)\rangle_0\\
&=\int\frac{\mathrm{d}^2p}{i(2\pi)^2}\frac{2ip^\mp}{p^2+i\epsilon}\,e^{-ip(x-y)},\\
\langle\hat{b}^I_\pm(x)b^J_\pm(y)\rangle_0&=\int\frac{\mathrm{d}^2p}{i(2\pi)^2}\frac{2ip^\mp}{p^2+i\epsilon}\,e^{-ip(x-y)}\eta^{IJ}.
\end{aligned}
$$

Now let us consider the following two-point function,

$$
A(p)_{++}\equiv\int\frac{\mathrm{d}^2x}{i(2\pi)^2}\langle V_{++}(x)V_{++}(0)\rangle\,e^{ipx}.
\tag{3.22}
$$

Here we mention that the two-point function (3.22) should vanish from the BRST symmetry $V_{++}(x)=s\bar{d}_{++}(x)$. Estimating all contributions arising from pairs (ξ^I,ξ_I), $(\bar{\phi}^I,\phi_I)$,

†Our convention of the light-cone coordinates on the world-sheet is $x^\pm=\frac{1}{\sqrt{2}}(x^0\pm x^1)$. The metric tensor and the Levi-Civitá symbol are given by $\eta_{++}=\eta_{--}=0$, $\eta_{+-}=\eta_{-+}=-1$ and $\varepsilon_{+-}=-\varepsilon_{-+}=-1$, respectively.

$(\hat{a}_+, a_+)$, $(\hat{b}_{+I}, b^I_+)$, $(\hat{c}_+, c_+)$, $(\bar{d}_{++}, d^+)$ and $(\bar{f}, f)$ we can obtain the following result up to one-loop order,

$$\begin{aligned} A(p)_{++} &= \frac{1}{48\pi^3}\Big(D + 2D - 2 - 2D - 2 - 26 + 2\Big)\frac{(p^-)^3}{p^+} \\ &= \frac{D-28}{48\pi^3}\frac{(p^-)^3}{p^+}. \end{aligned} \tag{3.23}$$

In a similar way we obtain

$$A(p)_{--} = \frac{D-28}{48\pi^3}\frac{(p^+)^3}{p^-}. \tag{3.24}$$

Next we evaluate the other type of the two-point functions

$$\begin{aligned} A(p)_{+-} &\equiv \int \frac{\mathrm{d}^2 x}{i(2\pi)^2}\langle V_{++}(x)V_{--}(0)\rangle\, e^{ipx} \\ &= -\frac{D-8}{8\pi^2}\int \frac{\mathrm{d}k^+\mathrm{d}k^-}{i(2\pi)^2}\frac{k^+k^-}{k^+k^- + i\epsilon}\frac{(p-k)^+(p-k)^-}{(p-k)^+(p-k)^- + i\epsilon}. \end{aligned}$$

This two-point function is actually quadratically divergent. This divergent, however, will be absorbed adding a suitable local counter term to the action. We conclude then that the BRST anomaly vanishes if and only if

$$D = 28. \tag{3.25}$$

4 Covariant quantization in the Hamiltonian formulation

In this section we carry out the quantization of the classical action (2.12) in the covariant Hamiltonian formulation given by Batalin, Fradkin and Vilkovisky. We present that the gauge-fixed action and the BRST transformation obtained in this formulation coincide with the result of the Lagrangian formulation if we make a proper choice of a gauge-fermion and a suitable identification of ghosts and ghost momenta. We also obtain the BRST charge in this formulation.

First of all we decompose the world-sheet metric $g_{mn}(x)$ by using the following convenient parameterization [17],

$$g_{mn} = \begin{pmatrix} -N^2\gamma + N_1^2\gamma & N_1\gamma \\ N_1\gamma & \gamma \end{pmatrix}, \tag{4.1}$$

where $N(x)$ and $N_1(x)$ are the rescaled lapse and the shift function, respectively. Under these parameterization the factor $\gamma(x)$ decouples from Weyl invariant theory.

According to the ordinary Dirac's procedure, we introduce the following canonical momenta defined by $P_{\Phi^A}(x) \equiv \delta_{\rm L} S/\delta(\partial_0 \Phi^A(x))$ corresponding to fields $\Phi^A(x)$,

$$\begin{aligned} P^I_\xi &= \frac{1}{N}\partial_0 \xi^I - \frac{N_1}{N}\partial_1 \xi^I - A_1 \phi^I, \\ P^I_\phi &= \frac{1}{N}\partial_0 \bar{\phi}^I - \frac{N_1}{N}\partial_1 \bar{\phi}^I - B^I_1, \\ P^I_{\bar{\phi}} &= \frac{1}{N}\partial_0 \phi^I - \frac{N_1}{N}\partial_1 \phi^I, \end{aligned} \tag{4.2}$$

and

$$P_N = P_{N_1} = P_{A_m} = P^I_{B_m} = P_{C_{01}} = 0. \tag{4.3}$$

The relations (4.3) give primary constraints. A consistency check of these primary constraints yields a set of secondary constraints

$$\frac{1}{2}\Big(P^I_\xi P_{\xi I} + \partial_1 \xi^I \partial_1 \xi_I\Big) = 0, \tag{4.4}$$

$$P^I_\xi \partial_1 \xi_I = 0, \tag{4.5}$$

$$\phi_I \partial_1 \xi^I = 0, \tag{4.6}$$

$$\phi_I P^I_\xi = 0, \tag{4.7}$$

$$\partial_1 \phi^I = 0, \tag{4.8}$$

$$P^I_{\bar{\phi}} = 0, \tag{4.9}$$

$$\frac{1}{2}\phi^I \phi_I = 0, \tag{4.10}$$

and these conditions give no other relations. The constraints (4.4) and (4.5) correspond to the Virasoro constraints. We can easily show that the set of these constraints (4.3) and (4.4)-(4.10) is first-class. Introducing Lagrange multiplier fields $\lambda_i(x)$ corresponding to primary constraints (4.3), a total Hamiltonian is given by

$$\begin{aligned} H = \int {\rm d}x^1 \Big\{ & N\Big(\frac{1}{2}(P^I_\xi + A_1\phi^I)(P_{\xi I} + A_1 \phi_I) + \frac{1}{2}\partial_1\xi^I\partial_1\xi_I \\ & \qquad + (P^I_\phi + B^I_1)P_{\bar{\phi} I} + \partial_1\bar{\phi}^I\partial_1\phi_I\Big) \\ & + N_1\Big((P^I_\xi + A_1\phi^I)\partial_1\xi_I + (P^I_\phi + B^I_1)\partial_1\phi_I + P^I_{\bar{\phi}}\partial_1\bar{\phi}_I\Big) \\ & - A_0\phi_I\partial_1\xi^I - B^I_0\partial_1\phi_I - \frac{1}{2}C_{01}\phi^I\phi_I \\ & + \lambda_N P_N + \lambda_{N_1}P_{N_1} + \lambda_{A_m}P_{A_m} + \lambda_{B^I_m}P^I_{B_m} + \lambda_{C_{01}}P_{C_{01}}\Big\}. \end{aligned} \tag{4.11}$$

The total Hamiltonian (4.11) is weakly vanishing on the constraint surface defined by (4.3) and (4.4)-(4.10). The gauge transformations of the canonical momenta defined by (4.2) are given by

$$\begin{aligned}
\delta P^I_\xi &= \partial_1\Big(k^0 N\partial_1\xi^I + (k^1+k^0N_1)P^I_\xi\Big) \\
&\quad - \partial_1 v\phi^I - \partial_1\Big(k^0(A_0 - N_1A_1)\phi^I\Big) + v' P^I_{\bar\phi}, \\
\delta P^I_\phi &= - P^I_\xi\Big(v' + k^0 N A_1\Big) + \phi^I\Big(w_1 - A_1 v' - k^0 N A_1^2 + k^0 C_{01}\Big) \\
&\quad - \partial_1\Big(u^I - k^0 N\partial_1\bar\phi^I - (k^1+k^0N_1)P^I_\phi + k^0(B^I_0 - N_1 B^I_1)\Big) \\
&\quad + \Big(v + k^0(A_0 - N_1A_1)\Big)\partial_1\xi^I, \\
\delta P^I_{\bar\phi} &= \partial_1\Big(k^0 N\partial_1\phi^I + (k^1+k^0N_1)P^I_{\bar\phi}\Big).
\end{aligned}$$

In the construction of Batalin-Fradkin-Vilkovisky formulation [15] a phase space is extended so as to contain ghosts $\eta^A(x)$ and their canonically conjugate ghost momenta $\mathcal{P}_A(x)$ corresponding to constraints $G_{\eta^A}(x)$. Then a nilpotent BRST transformation is constructed and a physical phase space is defined as its cohomology which is a set of gauge invariant functions on the constraint surface. The role of the ghost momenta is to exclude functions vanishing on the constraint surface from the cohomology and gauge invariant functions are removed from the cohomology because of the action of the BRST transformation for the ghost.

First of all we separate the variables into dynamical and non-dynamical ones. By adopting gauge conditions $N(x) = 1$, $N_1(x) = 0$, $A_m(x) = 0$, $B^I_m(x) = 0$ and $C_{01}(x) = -\hat{C}(x) = -\hat{C}_0$ (const.), we have a set of dynamical phase space variables $\big(\xi^I(x), P^J_\xi(x)\big)$, $\big(\phi^I(x), P^J_\phi(x)\big)$ and $\big(\bar\phi^I(x), P^J_{\bar\phi}(x)\big)$ with the first-class constraints (4.4)-(4.10).

Here we rearrange the first-class constraints (4.4)-(4.10) into the following forms,

$$\begin{aligned}
G_{\eta_0} &= \frac{1}{2}\Big(P^I_\xi P_{\xi I} + \partial_1\xi^I\partial_1\xi_I\Big) + P^I_{\bar\phi}P_{\phi I} + \partial_1\bar\phi^I\partial_1\phi_I, \\
G_{\eta_1} &= P^I_\xi\partial_1\xi_I + P^I_{\bar\phi}\partial_1\bar\phi_I + P^I_\phi\partial_1\phi_I, \\
G_\eta &= \phi_I\partial_1\xi^I, \\
G_{\eta'} &= \phi_I P^I_\xi, \\
G_{\eta^I} &= \partial_1\phi^I, \\
G_{\eta'^I} &= P^I_{\bar\phi}, \\
G_{\bar\eta} &= \frac{1}{2}\phi^I\phi_I,
\end{aligned} \tag{4.12}$$

and introduce corresponding canonically conjugate pairs of ghosts and ghost momenta $\big(\eta_0(x), \mathcal{P}_0(x)\big)$, $\big(\eta_1(x), \mathcal{P}_1(x)\big)$, $\big(\eta(x), \mathcal{P}(x)\big)$, $\big(\eta'(x), \mathcal{P}'(x)\big)$, $\big(\eta^I(x), \mathcal{P}^J(x)\big)$, $\big(\eta'^I(x), \mathcal{P}'^J(x)\big)$ and $\big(\bar\eta(x), \bar{\mathcal{P}}(x)\big)$. Though the rearrangement of the constraints is not inevitable, it turns out that to choose these combinations of the constraints is the simplest way to lead to the gauge-fixed action (3.19) in the covariant Hamiltonian formulation.

As we explained in the previous section, the model has the reducible symmetry. Indeed the constraints $G_{\eta^I}(x)$ and $G_{\bar{\eta}}(x)$ are not independent due to the following relation,

$$G_{\eta''} \equiv \partial_1 G_{\bar{\eta}} - \phi_I G_{\eta^I} = 0. \tag{4.13}$$

Therefore it is necessary to introduce one more Grassmann even ghost $\eta''(x)$ and its momentum $\mathcal{P}''(x)$ corresponding to this reducibility condition.

After the step by step construction according to the systematic procedure [22], we obtain the following BRST transformations in the extended phase space,

$$\begin{aligned}
s\xi^I &= -P_\xi^I \eta_0 - \partial_1 \xi^I \eta_1 - \phi^I \eta' - \mathcal{P}^I \eta_0 \eta + \mathcal{P}'^I \eta_0 \eta', \\
sP_\xi^I &= -\partial_1(\partial_1 \xi^I \eta_0) - \partial_1(P_\xi^I \eta_1) - \partial_1(\phi^I \eta) + \partial_1(\mathcal{P}'^I \eta_0 \eta) - \partial_1(\mathcal{P}^I \eta_0 \eta'), \\
s\phi^I &= -P_{\bar{\phi}}^I \eta_0 - \partial_1 \phi^I \eta_1, \\
sP_\phi^I &= -\partial_1(\partial_1 \bar{\phi}^I \eta_0) - \partial_1(P_\phi^I \eta_1) + P_\xi^I \eta' + \partial_1 \xi^I \eta + \phi^I \bar{\eta} - \partial_1 \eta^I - \mathcal{P}'^I \eta_0 \bar{\eta} - \mathcal{P}^I \eta'', \\
s\bar{\phi}^I &= -P_\phi^I \eta_0 - \partial_1 \bar{\phi}^I \eta_1 - \eta'^I, \\
sP_{\bar{\phi}}^I &= -\partial_1(\partial_1 \phi^I \eta_0) - \partial_1(P_{\bar{\phi}}^I \eta_1), \\
s\eta_0 &= -\eta_0 \partial_1 \eta_1 - \eta_1 \partial_1 \eta_0, \\
s\mathcal{P}_0 &= -\frac{1}{2} P_\xi^I P_{\xi I} - \frac{1}{2} \partial_1 \xi^I \partial_1 \xi_I - P_{\bar{\phi}}^I P_{\phi I} - \partial_1 \bar{\phi}^I \partial_1 \phi_I \\
&\quad + P_\xi^I \mathcal{P}_I \eta - P_\xi^I \mathcal{P}'_I \eta' - \partial_1 \xi^I \mathcal{P}'_I \eta + \partial_1 \xi^I \mathcal{P}_I \eta' - \phi^I \mathcal{P}'_I \bar{\eta} \\
&\quad + \mathcal{P}_0 \partial_1 \eta_1 + \partial_1(\mathcal{P}_0 \eta_1) + \mathcal{P}_1 \partial_1 \eta_0 + \partial_1(\mathcal{P}_1 \eta_0) + \mathcal{P}' \partial_1 \eta + \mathcal{P} \partial_1 \eta' + \mathcal{P}'^I \partial_1 \eta_I + \mathcal{P}^I \partial_1 \eta'_I \\
&\quad - \mathcal{P}^I \mathcal{P}'_I \eta'' - \mathcal{P}'' \eta \partial_1 \eta - \mathcal{P}'' \eta' \partial_1 \eta' + \mathcal{P}'' \bar{\eta} \partial_1 \eta_0 + \partial_1(\mathcal{P}'' \bar{\eta} \eta_0), \\
s\eta_1 &= -\eta_0 \partial_1 \eta_0 - \eta_1 \partial_1 \eta_1, \\
s\mathcal{P}_1 &= -P_\xi^I \partial_1 \xi_I - P_\phi^I \partial_1 \phi_I - P_{\bar{\phi}}^I \partial_1 \bar{\phi}_I \\
&\quad + \mathcal{P}_0 \partial_1 \eta_0 + \partial_1(\mathcal{P}_0 \eta_0) + \mathcal{P}_1 \partial_1 \eta_1 + \partial_1(\mathcal{P}_1 \eta_1) \\
&\quad + \mathcal{P} \partial_1 \eta + \mathcal{P}' \partial_1 \eta' + \mathcal{P}^I \partial_1 \eta_I + \mathcal{P}'^I \partial_1 \eta'_I - \partial_1 \bar{\mathcal{P}} \bar{\eta} - \mathcal{P}'' \partial_1 \eta'', \\
s\eta &= -\eta_0 \partial_1 \eta' - \eta_1 \partial_1 \eta, \\
s\mathcal{P} &= -\phi_I \partial_1 \xi^I - P_\xi^I \mathcal{P}_I \eta_0 + \partial_1 \xi^I \mathcal{P}'_I \eta_0 \\
&\quad + \bar{\mathcal{P}} \partial_1 \eta' + \partial_1(\bar{\mathcal{P}} \eta') + \partial_1(\mathcal{P}' \eta_0) + \partial_1(\mathcal{P} \eta_1) + \mathcal{P}'' \eta_0 \partial_1 \eta + \partial_1(\mathcal{P}'' \eta_0 \eta), \\
s\eta' &= -\eta_0 \partial_1 \eta - \eta_1 \partial_1 \eta', \\
s\mathcal{P}' &= -\phi_I P_\xi^I + P_\xi^I \mathcal{P}'_I \eta_0 - \partial_1 \xi^I \mathcal{P}_I \eta_0
\end{aligned} \tag{4.14}$$

$$
\begin{aligned}
&+\bar{\mathcal{P}}\partial_1\eta+\partial_1(\bar{\mathcal{P}}\eta)+\partial_1(\mathcal{P}\eta_0)+\partial_1(\mathcal{P}'\eta_1)+\mathcal{P}''\eta_0\partial_1\eta'+\partial_1(\mathcal{P}''\eta_0\eta'),\\
s\eta^I &= -\eta_0\partial_1\eta'^I-\eta_1\partial_1\eta^I-\partial_1\xi^I\eta_0\eta'-P_\xi^I\eta_0\eta-\mathcal{P}'^I\eta_0\eta''+\phi^I\eta'',\\
s\mathcal{P}^I &= -\partial_1\phi^I+\partial_1(\mathcal{P}'^I\eta_0)+\partial_1(\mathcal{P}^I\eta_1),\\
s\eta'^I &= -\eta_0\partial_1\eta^I-\eta_1\partial_1\eta'^I+\phi^I\eta_0\bar{\eta}+P_\xi^I\eta_0\eta'+\partial_1\xi^I\eta_0\eta+\mathcal{P}^I\eta_0\eta'',\\
s\mathcal{P}'^I &= -P_{\bar{\phi}}^I+\partial_1(\mathcal{P}^I\eta_0)+\partial_1(\mathcal{P}'^I\eta_1),\\
s\bar{\eta} &= -\partial_1(\eta_1\bar{\eta})-\eta\partial_1\eta'-\eta'\partial_1\eta+\partial_1\eta'',\\
s\bar{\mathcal{P}} &= -\frac{1}{2}\phi^I\phi_I+\phi^I\mathcal{P}'_I\eta_0+\partial_1\bar{\mathcal{P}}\eta_1-\mathcal{P}''\eta_0\partial_1\eta_0,\\
s\eta'' &= -\eta_1\partial_1\eta''-\eta_0\eta\partial_1\eta-\eta_0\eta'\partial_1\eta'-\bar{\eta}\eta_0\partial_1\eta_0,\\
s\mathcal{P}'' &= \partial_1\bar{\mathcal{P}}-\phi_I\mathcal{P}^I-\partial_1(\mathcal{P}''\eta_1)+\mathcal{P}^I\mathcal{P}'_I\eta_0.
\end{aligned}
$$

By using the generalized Poisson brackets, a nilpotent BRST charge $\Omega_{\rm min}$, which realizes the BRST transformations $sX\equiv\{\Omega_{\rm min},X\}$ for any canonical variables X, is defined by

$$
\begin{aligned}
\Omega_{\rm min}=\int \mathrm{d}x^1\Big\{&\eta_0\Big(\frac{1}{2}\Big(P_\xi^IP_{\xi I}+\partial_1\xi^I\partial_1\xi_I\Big)+P_{\bar{\phi}}^IP_{\phi I}+\partial_1\bar{\phi}^I\partial_1\bar{\phi}_I\Big)\\
&+\eta_1\Big(P_\xi^I\partial_1\xi_I+P_{\bar{\phi}}^I\partial_1\bar{\phi}_I+P_\phi^I\partial_1\phi_I\Big)\\
&+\eta\phi_I\partial_1\xi^I+\eta'\phi_IP_\xi^I+\eta^I\partial_1\phi_I+\eta'_IP_{\bar{\phi}}^I+\frac{1}{2}\bar{\eta}\phi^I\phi_I\\
&+\eta''\Big(\partial_1\bar{\mathcal{P}}-\phi_I\mathcal{P}^I\Big)\\
&+P_\xi^I\mathcal{P}_I\eta_0\eta-P_\xi^I\mathcal{P}'_I\eta_0\eta'+\xi^I\partial_1(\mathcal{P}'_I\eta_0\eta)-\xi^I\partial_1(\mathcal{P}_I\eta_0\eta')-\phi^I\mathcal{P}'_I\eta_0\bar{\eta}\\
&+\mathcal{P}_0\Big(\eta_0\partial_1\eta_1+\eta_1\partial_1\eta_0\Big)+\mathcal{P}_1\Big(\eta_0\partial_1\eta_0+\eta_1\partial_1\eta_1\Big)\\
&+\mathcal{P}\Big(\eta_0\partial_1\eta'+\eta_1\partial_1\eta\Big)+\mathcal{P}'\Big(\eta_0\partial_1\eta+\eta_1\partial_1\eta'\Big)\\
&+\mathcal{P}_I\Big(\eta_0\partial_1\eta'^I+\eta_1\partial_1\eta^I\Big)+\mathcal{P}'_I\Big(\eta_0\partial_1\eta^I+\eta_1\partial_1\eta'^I\Big)\\
&+\bar{\mathcal{P}}\Big(\eta'\partial_1\eta+\eta\partial_1\eta'+\partial_1(\eta_1\bar{\eta})\Big)+\mathcal{P}^I\mathcal{P}'_I\eta_0\eta''\\
&+\mathcal{P}''\Big(\eta_1\partial_1\eta''+\eta_0\eta\partial_1\eta+\eta_0\eta'\partial_1\eta'+\bar{\eta}\eta_0\partial_1\eta_0\Big)\Big\}.
\end{aligned}
\tag{4.15}
$$

In order to fix the gauge, we extend the phase space further and introduce sets of canonical variables $\big(\bar{\lambda}(x),\lambda(x)\big)$ and $\big(\bar{\rho}(x),\rho(x)\big)$. Their statics are bosonic for $\big(\bar{\lambda}(x),\lambda(x)\big)$ and fermionic for $\big(\bar{\rho}(x),\rho(x)\big)$ and the canonical structures are defined by

$$
\begin{aligned}
\{\bar{\lambda},\lambda\}&=-\{\lambda,\bar{\lambda}\}=1,\\
\{\bar{\rho},\rho\}&=\{\rho,\bar{\rho}\}=-1.
\end{aligned}
\tag{4.16}
$$

BRST transformations are also extended to these variables as

$$
\begin{aligned}
s\bar{\lambda} &= \rho, \qquad & s\rho &= 0, \\
s\bar{\rho} &= \lambda, \qquad & s\lambda &= 0.
\end{aligned}
\tag{4.17}
$$

The corresponding extended BRST charge is given by

$$
\Omega = \Omega_{\mathrm{min}} + \Omega_{\mathrm{nonmin}}, \tag{4.18}
$$

$$
\Omega_{\mathrm{nonmin}} = -\int \mathrm{d}x^1\, \lambda\rho.
$$

Now the gauge-fixed action is obtained by a Legendre transformation from the Hamiltonian in the extended phase space,

$$
\begin{aligned}
S_{\text{gauge-fixed}} = \int \mathrm{d}x^0 \Big\{ \int \mathrm{d}x^1 \Big(& \partial_0\xi^I P_{\xi I} + \partial_0\bar{\phi}^I P_{\bar{\phi} I} + \partial_0\phi^I P_{\phi I} \\
& + \partial_0\eta_0\mathcal{P}_0 + \partial_0\eta_1\mathcal{P}_1 + \partial_0\eta\mathcal{P} + \partial_0\eta'\mathcal{P}' \\
& + \partial_0\eta^I\mathcal{P}_I + \partial_0\eta'^I\mathcal{P}'_I + \partial_0\bar{\eta}\bar{\mathcal{P}} + \partial_0\eta''\mathcal{P}'' \\
& + \partial_0\bar{\rho}\rho + \partial_0\bar{\lambda}\lambda \Big) - H_K \Big\},
\end{aligned}
\tag{4.19}
$$

where H_K is a gauge-fixed Hamiltonian expressed by using a gauge-fixing fermion K,

$$
H_K = \{\Omega, K\}. \tag{4.20}
$$

The gauge-fixed Hamiltonian H_K consists of gauge-fixing terms and ghost parts only since the total Hamiltonian of the system has vanished. There is no systematic way to find K so as to yield a covariant expression. Here, however, we can use the result in the Lagrangian formulation as a clue. Actually we would like to show that the two formulations give an equivalent result. We have found that the following gauge-fixing fermion K works as desired

$$
K = \int \mathrm{d}x^1 \Big(-\mathcal{P}_0 + \bar{\eta}\partial_1\bar{\lambda} + \bar{\rho}\mathcal{P}'' \Big). \tag{4.21}
$$

By integrating out the momentum variables $P^I_\xi(x)$, $P^I_\phi(x)$, $P^I_{\bar{\phi}}(x)$, $\mathcal{P}''(x)$ and $\lambda(x)$ with this gauge-fixing fermion, we obtain the following relations,

$$
\begin{aligned}
P^I_\xi &= \partial_0\xi^I + \mathcal{P}^I\eta - \mathcal{P}'^I\eta', \\
P^I_{\bar{\phi}} &= \partial_0\phi^I, \\
P^I_\phi &= \partial_0\bar{\phi}^I, \\
\lambda &= \partial_0\eta'' - \eta_1\partial_1\bar{\rho} + \bar{\eta}\partial_1\eta_0 - \eta\partial_1\eta - \eta'\partial_1\eta', \\
\mathcal{P}'' &= \partial_0\bar{\lambda}.
\end{aligned}
\tag{4.22}
$$

Then, the gauge-fixed action becomes

$$
\begin{aligned}
S_{\text{gauge-fixed}} = \int \mathrm{d}^2x \Big\{ & \frac{1}{2}\partial_0\xi^I\partial_0\xi_I - \frac{1}{2}\partial_1\xi^I\partial_1\xi_I \\
& + \partial_0\bar{\phi}^I\partial_0\phi_I - \partial_1\bar{\phi}^I\partial_1\phi_I \\
& - \mathcal{P}_0\partial_0\eta_0 + \mathcal{P}_1\partial_1\eta_0 - \mathcal{P}_1\partial_0\eta_1 + \mathcal{P}_0\partial_1\eta_1 \\
& - \mathcal{P}\partial_0\eta + \mathcal{P}'\partial_1\eta - \mathcal{P}'\partial_0\eta' + \mathcal{P}\partial_1\eta' \\
& - \mathcal{P}_I\partial_0\eta^I + \mathcal{P}'_I\partial_1\eta^I - \mathcal{P}'_I\partial_0\eta'^I + \mathcal{P}_I\partial_1\eta'^I \\
& - \bar{\mathcal{P}}\partial_0\bar{\eta} + \rho\partial_1\bar{\eta} - \rho\partial_0\bar{\rho} + \bar{\rho}\partial_1\bar{\mathcal{P}} \\
& + \partial_0\bar{\lambda}\partial_0\eta'' - \partial_1\bar{\lambda}\partial_1\eta'' \\
& + \partial_0\xi^I\mathcal{P}_I\eta - \partial_1\xi^I\mathcal{P}'_I\eta - \partial_0\xi^I\mathcal{P}'_I\eta' + \partial_1\xi^I\mathcal{P}_I\eta' - \phi^I\mathcal{P}'_I\bar{\eta} + \phi^I\mathcal{P}_I\bar{\rho} \\
& + \partial_1\bar{\lambda}\eta\partial_1\eta' + \partial_1\bar{\lambda}\eta'\partial_1\eta - \partial_1\bar{\lambda}\partial_1(\bar{\eta}\eta_1) \\
& - \partial_0\bar{\lambda}\eta_1\partial_1\bar{\rho} + \partial_0\bar{\lambda}\bar{\eta}\partial_1\eta_0 - \partial_0\bar{\lambda}\eta\partial_1\eta - \partial_0\bar{\lambda}\eta'\partial_1\eta' \\
& - \mathcal{P}^I\mathcal{P}'_I\eta_0\bar{\rho} + \mathcal{P}^I\mathcal{P}'_I\eta\eta' - \mathcal{P}^I\mathcal{P}'_I\eta'' \Big\}.
\end{aligned} \tag{4.23}
$$

If we redefine the field variables as:

$$
\begin{aligned}
&\eta_0 \to -d^0, && \mathcal{P}_0 \to -\bar{d}_{00} - \hat{c}^0\partial_1\bar{f} + \hat{c}^1\partial_0\bar{f}, \\
&\eta_1 \to -d^1, && \mathcal{P}_1 \to -\bar{d}_{01} + \hat{c}^1\partial_1\bar{f}, \\
&\eta \to a, && \mathcal{P} \to \hat{a}^0 - \partial_1(\bar{f}a) + \bar{f}\partial_0 a' + \hat{b}^0_I\xi^I, \\
&\eta' \to -a', && \mathcal{P}' \to -\hat{a}^1 + \partial_1(\bar{f}a') - \bar{f}\partial_0 a - \hat{b}^1_I\xi^I, \\
&\eta^I \to b^I - a\xi^I, && \mathcal{P}^I \to \hat{b}^{0I}, \\
&\eta'^I \to -b'^I + a'\xi^I, && \mathcal{P}'^I \to -\hat{b}^{1I}, \\
&\bar{\eta} \to -\hat{c}^0, && \bar{\mathcal{P}} \to -c + d^0\partial_1\bar{f}, \\
&\eta'' \to f + d^0\hat{c}^1, && \bar{\lambda} \to \bar{f}, \\
&\rho \to c' + d^0\partial_0\bar{f} + d^1\partial_1\bar{f}, && \bar{\rho} \to \hat{c}^1,
\end{aligned}
$$

the action (4.23) and the BRST transformations (4.14) completely coincide with the gauge-fixed action (3.19) and the on-shell BRST transformations (3.20) in the Lagrangian formulation. After these manipulations we also obtain the final form of the BRST charge (4.18),

$$
\begin{aligned}
\Omega = \int \mathrm{d}x^1 \Big\{ -d^0 \Big(& \frac{1}{2}\partial_0\xi^I\partial_0\xi_I + \frac{1}{2}\partial_1\xi^I\partial_1\xi_I + \partial_0\bar{\phi}^I\partial_0\phi_I + \partial_1\bar{\phi}^I\partial_1\phi_I \\
& - \hat{a}^1\partial_0 a' + \hat{a}^0\partial_1 a' - \hat{b}^1_I\partial_0 b'^I + \hat{b}^0_I\partial_1 b'^I - \hat{c}^1\partial_0 c' + \hat{c}^0\partial_1 c'
\end{aligned}
$$

$$
\begin{aligned}
&\quad +\partial_0\bar{f}\partial_0 f+\partial_1\bar{f}\partial_1 f\Big)\\
&-d^1\Big(\partial_0\xi^I\partial_1\xi_I+\partial_0\bar{\phi}^I\partial_1\phi_I+\partial_1\bar{\phi}^I\partial_0\phi_I\\
&\quad +\hat{a}^0\partial_0 a'-\hat{a}^1\partial_1 a'+\hat{b}^0_I\partial_0 b'^I-\hat{b}^1_I\partial_1 b'^I+\hat{c}^0\partial_0 c'-\hat{c}^1\partial_1 c'\\
&\quad +\partial_0\bar{f}\partial_1 f+\partial_1\bar{f}\partial_0 f\Big)\\
&-\bar{d}_{0n}d^m\partial_m d^n\\
&-a'(\phi_I\partial_0\xi^I-\partial_0\phi_I\xi^I)+a(\phi_I\partial_1\xi^I-\partial_1\phi_I\xi^I)\\
&-b'_I\partial_0\phi^I+b_I\partial_1\phi^I-\frac{1}{2}\hat{c}^0\phi^I\phi_I\\
&-c'(\partial_0 f+a\partial_0 a'+a'\partial_0 a)+c(\partial_1 f+a\partial_1 a'+a'\partial_1 a)-(f+aa')\hat{b}^0_I\phi^I\Big\}.
\end{aligned}
$$

5 Noncovariant quantization in the light-cone gauge formulation

In this section we investigate the dynamics of the model defined by the constraints (4.3) and (4.4)-(4.10) and Hamiltonian (4.11) in noncovariant gauge and obtain the same result of the critical dimension as in the covariant quantization*. In addition, we present a mass-shell relation of the model and give low energy quantum states. According to imposing the noncovariant gauge fixing conditions, we explicitly solve the constraints to some of the variables from the equations of motion.

We begin by considering conditions for the scalar field $\phi^I(\tau,\sigma)$. It is convenient to introduce Fourier mode expansions of the canonical pair $\big(\phi^I(\tau,\sigma),P^J_\phi(\tau,\sigma)\big)$,

$$
\begin{aligned}
\phi^I(\tau,\sigma)&=\phi^I(\tau)+\frac{1}{\sqrt{2\pi}}\sum_{m\neq 0}\phi^I_m(\tau)e^{im\sigma},\\
P^I_\phi(\tau,\sigma)&=\frac{p^I_\phi(\tau)}{2\pi}+\frac{1}{\sqrt{2\pi}}\sum_{m\neq 0}p^I_{\phi m}(\tau)e^{im\sigma}.
\end{aligned}
\tag{5.1}
$$

Poisson brackets are defined by

$$
\begin{aligned}
\{\phi^I(\tau),p^J_\phi(\tau)\}&=\eta^{IJ},\\
\{\phi^I_m(\tau),p^J_{\phi n}(\tau)\}&=\eta^{IJ}\delta_{m+n},\\
\text{otherwise}&=0.
\end{aligned}
\tag{5.2}
$$

In terms of the Fourier modes, the constraint (4.8) is equivalent to $\phi^I_m(\tau)=0$. We will later adopt a gauge fixing condition for this constraint. On the other hand, the equation of

*In this section, we use conventions of the world-sheet coordinates as $x^0\equiv\tau$ and $x^1\equiv\sigma$. We also parameterize the spatial coordinate as $0\leq\sigma\leq 2\pi$ and impose the periodical boundary conditions on any fields $\Phi^A(\tau,\sigma)$ as $\Phi^A(\tau,\sigma)=\Phi^A(\tau,\sigma+2\pi)$.

motion for $\phi^I(\tau,\sigma)$ on the constraint surface is $\partial_\tau\phi^I(\tau,\sigma)=0$. Together with the constraint $\phi^I_m(\tau)=0$, we then set the configuration of the scalar field as $\phi^I(\tau,\sigma)=\phi^I(\tau)=\phi^I(=\text{const.})$.

As we did in the previous section, by using the gauge parameters $k^n(\tau,\sigma)$ for the general coordinate transformations we first adopt gauge fixing conditions for the constraints $P_N(\tau,\sigma)=0$ and $P_{N_1}(\tau,\sigma)=0$ as the orthonormal gauge $N(\tau,\sigma)=1$ and $N_1(\tau,\sigma)=0$. The $\mathrm{U(1)_V\times U(1)_A}$ gauge parameters $v(\tau,\sigma)$, $v'(\tau,\sigma)$ and the global parameters α_i can fix to be $A_m(\tau,\sigma)=0$ corresponding to the constraints $P_{A_m}(\tau,\sigma)=0$. However, the system still has residual symmetries concerned with these gauge parameters $k^n(\tau,\sigma)$, $v(\tau,\sigma)$ and $v'(\tau,\sigma)$. Taking these symmetries into account, we can adopt the following gauge fixing conditions on "two" light-cone coordinates† of the background spacetime within the gauge $N(\tau,\sigma)=1$, $N_1(\tau,\sigma)=0$ and $A_m(\tau,\sigma)=0$,

$$\begin{aligned}\xi^+(\tau,\sigma)&=\frac{p^+}{2\pi}\tau, &\qquad P_\xi^+(\tau,\sigma)&=\frac{p^+}{2\pi},\\ \xi^{\hat+}(\tau,\sigma)&=\frac{p^{\hat+}}{2\pi}\tau, &\qquad P_\xi^{\hat+}(\tau,\sigma)&=\frac{p^{\hat+}}{2\pi},\end{aligned}\tag{5.3}$$

where p^+ and $p^{\hat+}$ are light-cone components of the center of mass momenta. Therefore we can eliminate "two" unphysical components of the coordinates of the background spacetime. Indeed the gauge fixing conditions (5.3) correspond to ones for the first-class constraints (4.4)-(4.7).

In order to show how these conditions (5.3) are accomplished, we use Fourier mode expansions of the canonical pair $\big(\xi^I(\tau,\sigma),P_\xi^J(\tau,\sigma)\big)$. Under the gauge $N(\tau,\sigma)=1$, $N_1(\tau,\sigma)=0$ and $A_m(\tau,\sigma)=0$, the equations of motion for $\xi^I(\tau,\sigma)$ and $P^I_\xi(\tau,\sigma)$ turn to be free wave equations and their solutions are

$$\begin{aligned}\xi^I(\tau,\sigma)&=x^I+\frac{p^I}{2\pi}\tau+\frac{i}{2\sqrt{\pi}}\sum_{m\neq0}\frac{1}{m}\Big(\alpha^I_m e^{-im(\tau-\sigma)}+\tilde\alpha^I_m e^{-im(\tau+\sigma)}\Big),\\ P^I_\xi(\tau,\sigma)&=\frac{p^I}{2\pi}+\frac{1}{2\sqrt{\pi}}\sum_{m\neq0}\Big(\alpha^I_m e^{-im(\tau-\sigma)}+\tilde\alpha^I_m e^{-im(\tau+\sigma)}\Big),\end{aligned}\tag{5.4}$$

and Poisson brackets are given by

$$\begin{aligned}&\{x^I,p^J\}=\eta^{IJ},\\ &\{\alpha^I_m,\alpha^J_n\}=\{\tilde\alpha^I_m,\tilde\alpha^J_n\}=-im\eta^{IJ}\delta_{m+n},\\ &\text{otherwise}=0.\end{aligned}\tag{5.5}$$

†From the definition of the metric (2.1), we denote the light-cone coordinates of the background spacetime as $x^I=(x^+,x^-,x^i,x^{\hat+},x^{\hat-})$, where $x^\pm\equiv\frac{1}{\sqrt2}(x^0\pm x^{D-3})$ and $x^{\hat\pm}\equiv\frac{1}{\sqrt2}(x^{\hat0}\pm x^{\hat1})$ and the index i runs through $1, 2, \ldots, D-4$.

In terms of the Fourier modes, the constraints (4.4)-(4.7) are equivalent to

$$L_m = \tilde{L}_m = 0, \tag{5.6a}$$

$$\phi_I \alpha^I_m = \phi_I \tilde{\alpha}^I_m = 0, \tag{5.6b}$$

where we define the Virasoro generators as

$$L_m \equiv \frac{1}{2}\sum_k \alpha^I_{m-k}\alpha_{Ik}, \qquad \tilde{L}_m \equiv \frac{1}{2}\sum_k \tilde{\alpha}^I_{m-k}\tilde{\alpha}_{Ik},$$

and we denote $\alpha^I_0 = \tilde{\alpha}^I_0 \equiv p^I/(2\sqrt{\pi})$. The gauge fixing conditions (5.3) are equivalent to

$$\begin{aligned} &x^+ = x^{\hat{+}} = 0, \\ &\alpha^+_m = \alpha^{\hat{+}}_m = \tilde{\alpha}^+_m = \tilde{\alpha}^{\hat{+}}_m = 0, \qquad (m \neq 0). \end{aligned} \tag{5.7}$$

Now let us explain the procedure to obtain the gauge fixing conditions (5.7). Within the orthonormal gauge we can perform changes of the background spacetime coordinates with the gauge parameters $k^n(\tau,\sigma)$ provided that conditions $\partial_\tau k^\tau(\tau,\sigma) = \partial_\sigma k^\sigma(\tau,\sigma)$ and $\partial_\tau k^\sigma(\tau,\sigma) = \partial_\sigma k^\tau(\tau,\sigma)$ are satisfied. Here we take the following parameterizations of $k^n(\tau,\sigma)$ which satisfy these conditions,

$$k^+(\tau,\sigma) \equiv \frac{1}{\sqrt{2}}(k^\tau + k^\sigma) = \frac{1}{\sqrt{2}}\sum_m \tilde{k}_m e^{-im(\tau+\sigma)},$$

$$k^-(\tau,\sigma) \equiv \frac{1}{\sqrt{2}}(k^\tau - k^\sigma) = \frac{1}{\sqrt{2}}\sum_m k_m e^{-im(\tau-\sigma)}.$$

In addition to these, the $\mathrm{U}(1)_\mathrm{V} \times \mathrm{U}(1)_\mathrm{A}$ gauge parameters $v(\tau,\sigma)$ and $v'(\tau,\sigma)$ can be also used to perform changes of the coordinates within the gauge $A_m(\tau,\sigma) = 0$ provided that conditions $\partial_\tau v'(\tau,\sigma) = -\partial_\sigma v(\tau,\sigma)$ and $\partial_\tau v(\tau,\sigma) = -\partial_\sigma v'(\tau,\sigma)$ are satisfied. We take the following parameterizations of $v(\tau,\sigma)$ and $v'(\tau,\sigma)$ to realize these conditions,

$$v(\tau,\sigma) = v + \frac{i}{2\sqrt{\pi}}\sum_{m\neq 0}\frac{1}{m}\Big(v_m e^{-im(\tau-\sigma)} - \tilde{v}_m e^{-im(\tau+\sigma)}\Big),$$

$$v'(\tau,\sigma) = v' + \frac{i}{2\sqrt{\pi}}\sum_{m\neq 0}\frac{1}{m}\Big(v_m e^{-im(\tau-\sigma)} + \tilde{v}_m e^{-im(\tau+\sigma)}\Big).$$

The gauge transformations corresponding to these parameters are consistent with the equations of motion for $\xi^I(\tau,\sigma)$ and $P^I_\xi(\tau,\sigma)$. Because, in terms of the Fourier modes, the gauge transformations are given by

$$\begin{aligned} &\delta x^I = \frac{1}{2\sqrt{\pi}}\sum_m k_m \alpha^I_{-m} + \frac{1}{2\sqrt{\pi}}\sum_m \tilde{k}_m \tilde{\alpha}^I_{-m} + v'\phi^I, \\ &\delta p^I = 0, \\ &\delta\alpha^I_m = -im\sum_n k_{m-n}\alpha^I_n + v_m\phi^I, \qquad (m\neq 0), \\ &\delta\tilde{\alpha}^I_m = -im\sum_n \tilde{k}_{m-n}\tilde{\alpha}^I_n + \tilde{v}_m\phi^I, \qquad (m\neq 0). \end{aligned} \tag{5.8}$$

It is worth to mention that these gauge transformations are the same ones in usual string theories, except for the gauge transformations corresponding to the parameters v', v_m and $\tilde{v}_m$. However, we would like to emphasize that these gauge transformations can be disappear on the following components,

$$
\begin{aligned}
\delta(\phi^{\hat{+}} x^{+} - \phi^{+} x^{\hat{+}}) &= \frac{1}{2\sqrt{\pi}} \sum_m \Big(\phi^{\hat{+}} (k_m \alpha^{+}_{-m} + \tilde{k}_m \tilde{\alpha}^{+}_{-m}) - \phi^{+} (k_m \alpha^{\hat{+}}_{-m} + \tilde{k}_m \tilde{\alpha}^{\hat{+}}_{-m}) \Big), \\
\delta(\phi^{\hat{+}} \alpha^{+}_m - \phi^{+} \alpha^{\hat{+}}_m) &= -im \sum_n k_{m-n} (\phi^{\hat{+}} \alpha^{+}_n - \phi^{+} \alpha^{\hat{+}}_n), \qquad (m \neq 0), \\
\delta(\phi^{\hat{+}} \tilde{\alpha}^{+}_m - \phi^{+} \tilde{\alpha}^{\hat{+}}_m) &= -im \sum_n \tilde{k}_{m-n} (\phi^{\hat{+}} \tilde{\alpha}^{+}_n - \phi^{+} \tilde{\alpha}^{\hat{+}}_n), \qquad (m \neq 0).
\end{aligned}
$$

By using the gauge degrees of freedom for k_m and $\tilde{k}_m$, which is the same manipulation to realize the light-cone gauge fixing condition in usual string theories, we can adopt gauge conditions

$$
\begin{aligned}
&\phi^{\hat{+}} x^{+} - \phi^{+} x^{\hat{+}} = 0, \\
&\phi^{\hat{+}} \alpha^{+}_m - \phi^{+} \alpha^{\hat{+}}_m = 0, \qquad (m \neq 0), \\
&\phi^{\hat{+}} \tilde{\alpha}^{+}_m - \phi^{+} \tilde{\alpha}^{\hat{+}}_m = 0, \qquad (m \neq 0),
\end{aligned}
\tag{5.9}
$$

if the following condition is satisfied,

$$
\phi^{\hat{+}} p^{+} - \phi^{+} p^{\hat{+}} \neq 0. \tag{5.10}
$$

Next we use the gauge degrees of freedom for v', v_m and $\tilde{v}_m$ in (5.8). To keep the condition (5.10) both of the scalar fields $\phi^{\hat{+}}$ and ϕ^{+} can not be vanish simultaneously. If $\phi^{\hat{+}} \neq 0$, we can adopt the following gauge fixing conditions of the $\hat{+}$ component,

$$
\begin{aligned}
&x^{\hat{+}} = 0, \\
&\alpha^{\hat{+}}_m = \tilde{\alpha}^{\hat{+}}_m = 0, \qquad (m \neq 0),
\end{aligned}
\tag{5.11}
$$

without spoiling the gauge fixing conditions (5.9). From (5.9) and (5.11) we can then obtain the gauge fixing conditions (5.7). In the similar way, we also conclude the same gauge fixing conditions (5.7), in the case $\phi^{+} \neq 0$. Therefore without the loss of the generality we choose the case $\phi^{\hat{+}} \neq 0$ throughout the rest of this paper.

We next adopt the gauge fixing condition $C_{\tau\sigma}(\tau, \sigma) = -\hat{C}(\tau, \sigma) = -\hat{C}_0$ (const.) with respect to constraints $P_{C_{\tau\sigma}}(\tau, \sigma) = 0$, by using the gauge parameter $w_m(\tau, \sigma)$. Within this gauge we also have a residual gauge symmetry corresponding to the gauge parameter $w_m (= \text{const.})$ which will be used later.

Using the remaining gauge parameters $u^I(\tau,\sigma)$ and $u'^I(\tau,\sigma)$, we can further impose gauge fixing conditions for the constraints $P^I_{\bar\phi}(\tau,\sigma)=0$, $P^I_{B_m}(\tau,\sigma)=0$ and $\partial_\sigma\phi^I(\tau,\sigma)=0$. In order to specify the gauge fixing condition it is also convenient to introduce Fourier mode expansions. We list below these for the canonical pairs $\left(\bar\phi^I(\tau,\sigma), P^J_{\bar\phi}(\tau,\sigma)\right)$ and $\left(B^I_m(\tau,\sigma), P^J_{B_m}(\tau,\sigma)\right)$ and their Poisson brackets:

- $(\bar\phi^I, P^J_{\bar\phi})$ sector:

$$\begin{aligned}\bar\phi^I(\tau,\sigma) &= \bar\phi^I(\tau)+\frac{1}{\sqrt{2\pi}}\sum_{m\neq 0}\bar\phi^I_m(\tau)e^{im\sigma},\\ P^I_{\bar\phi}(\tau,\sigma) &= \frac{p^I_{\bar\phi}(\tau)}{2\pi}+\frac{1}{\sqrt{2\pi}}\sum_{m\neq 0}p^I_{\bar\phi m}(\tau)e^{im\sigma},\end{aligned}\tag{5.12}$$

  and Poisson brackets,

$$\begin{aligned}\{\bar\phi^I(\tau), p^J_{\bar\phi}(\tau)\} &= \eta^{IJ},\\ \{\bar\phi^I_m(\tau), p^J_{\bar\phi n}(\tau)\} &= \eta^{IJ}\delta_{m+n},\\ \text{otherwise} &= 0.\end{aligned}\tag{5.13}$$

- $(B^I_\tau, P^J_{B_\tau})$ sector:

$$\begin{aligned}B^I_\tau(\tau,\sigma) &= B^I_\tau(\tau)+\frac{1}{\sqrt{2\pi}}\sum_{m\neq 0}B^I_{\tau m}(\tau)e^{im\sigma},\\ P^I_{B_\tau}(\tau,\sigma) &= \frac{p^I_{B_\tau}(\tau)}{2\pi}+\frac{1}{\sqrt{2\pi}}\sum_{m\neq 0}p^I_{B_\tau m}(\tau)e^{im\sigma},\end{aligned}\tag{5.14}$$

  and Poisson brackets,

$$\begin{aligned}\{B^I_\tau(\tau), p^J_{B_\tau}(\tau)\} &= \eta^{IJ},\\ \{B^I_{\tau m}(\tau), p^J_{B_\tau n}(\tau)\} &= \eta^{IJ}\delta_{m+n},\\ \text{otherwise} &= 0,\end{aligned}\tag{5.15}$$

  and the similar relations for $(B^I_\sigma, P^J_{B_\sigma})$.

Then, equations of motion for $\bar\phi^I(\tau,\sigma)$ and $P^I_\phi(\tau,\sigma)$,

$$\begin{aligned}\partial_\tau\bar\phi^I(\tau,\sigma) &= P^I_\phi(\tau,\sigma)+B^I_\sigma(\tau,\sigma),\\ \partial_\tau P^I_\phi(\tau,\sigma) &= \partial^2_\sigma\bar\phi^I(\tau,\sigma)-\partial_\sigma B^I_\tau(\tau,\sigma)-\hat C_0\phi^I,\end{aligned}\tag{5.16}$$

are expressed by the Fourier modes,

$$\partial_\tau\bar\phi^I(\tau) = \frac{p^I_\phi(\tau)}{2\pi}+B^I_\sigma(\tau),\tag{5.17a}$$

$$\partial_\tau \bar{\phi}^I_m(\tau) = p^I_{\phi m}(\tau) + B^I_{\sigma m}(\tau), \tag{5.17b}$$

$$\partial_\tau p^I_\phi(\tau) = -2\pi \hat{C}_0 \phi^I, \tag{5.17c}$$

$$\partial_\tau p^I_{\phi m}(\tau) = -m^2 \bar{\phi}^I_m(\tau) - imB^I_{\tau m}(\tau). \tag{5.17d}$$

The equation of motion (5.17c) for the non-oscillator mode of $P^I_\phi(\tau, \sigma)$ can be solved as

$$p^I_\phi(\tau) = p^I_\phi - 2\pi \hat{C}_0 \phi^I \tau, \tag{5.18}$$

where p^I_ϕ is a zero-mode and Poisson bracket is defined by

$$\{\phi^I, p^J_\phi\} = \eta^{IJ}. \tag{5.19}$$

On the Fourier components, the constraints $P^I_{\bar{\phi}}(\tau, \sigma) = 0$, $P^I_{B_m}(\tau, \sigma) = 0$ and $\partial_\sigma \phi^I(\tau, \sigma) = 0$ are equivalent to

$$\begin{aligned} p^I_{\bar{\phi}}(\tau) &= p^I_{\bar{\phi} m}(\tau) = 0, \\ p^I_{B_\tau}(\tau) &= p^I_{B_\tau m}(\tau) = 0, \\ p^I_{B_\sigma}(\tau) &= p^I_{B_\sigma m}(\tau) = 0, \\ \phi^I_m(\tau) &= 0. \end{aligned} \tag{5.20}$$

Now we impose gauge fixing conditions corresponding to the constraints (5.20). The gauge fixing conditions are determined so as to be compatible with the equations of motion (5.17a)-(5.17d). By making the gauge transformations

$$\begin{aligned} \delta \bar{\phi}^I &= u'^I, \\ \delta B^I_\tau &= \partial_\tau u^I + \partial_\sigma u'^I, \\ \delta B^I_\sigma &= \partial_\tau u'^I + \partial_\sigma u^I, \\ \delta P^I_\phi &= -\partial_\sigma u^I, \end{aligned}$$

with gauge parameters

$$\begin{aligned} u^I(\tau, \sigma) &= -\int^\tau \mathrm{d}\tau' B^I_\tau(\tau') - \frac{i}{\sqrt{2\pi}} \sum_{m \neq 0} \frac{1}{m} p^I_{\phi m}(\tau) e^{im\sigma}, \\ u'^I(\tau, \sigma) &= -\bar{\phi}^I(\tau) - \frac{1}{\sqrt{2\pi}} \sum_{m \neq 0} \bar{\phi}^I_m(\tau) e^{im\sigma}, \end{aligned}$$

and the equations of motion (5.17a)-(5.17d), we obtain the following gauge fixing conditions,

$$
\begin{aligned}
\bar{\phi}^I(\tau) &= \bar{\phi}^I_m(\tau) = 0, \\
B^I_\tau(\tau) &= B^I_{\tau m}(\tau) = 0, \\
B^I_\sigma(\tau) = -\frac{p^I_\phi(\tau)}{2\pi}, \qquad B^I_{\sigma m}(\tau) &= 0, \\
p^I_{\phi m}(\tau) &= 0.
\end{aligned}
\tag{5.21}
$$

Finally we consider the constraint

$$
\frac{1}{2}\phi^I \phi_I = 0. \tag{5.22}
$$

As we explained, the model has still residual gauge symmetry $w_\sigma (=$ const.) within the gauge $C_{\tau\sigma}(\tau, \sigma) = -\hat{C}_0$. Using this symmetry

$$
\delta P^I_\phi = \phi^I w_\sigma,
$$

we can make the one of the zero-mode components of $P^I_\phi(\tau, \sigma)$ to be vanish. By taking the case $\phi^{\hat{+}} \neq 0$ and choosing the gauge parameter w_σ as

$$
w_\sigma = -\frac{p^{\hat{+}}_\phi}{2\pi\phi^{\hat{+}}},
$$

we impose a gauge fixing condition

$$
p^{\hat{+}}_\phi = 0. \tag{5.23}
$$

We shall here summarize the correspondence between the constraints (5.6a), (5.6b), (5.20) and (5.22) and the gauge fixing conditions (5.7), (5.21) and (5.23) obtained from the above manipulation within the gauge $N(\tau, \sigma) = 1$, $N_1(\tau, \sigma) = 0$, $A_m(\tau, \sigma) = 0$ and $C_{\tau\sigma}(\tau, \sigma) = -\hat{C}_0$:

constraints	**gauge fixing conditions**
$L_0 + \tilde{L}_0 = 0,$	$x^+ = 0,$
$L_m = \tilde{L}_m = 0,$	$\alpha^+_m = \tilde{\alpha}^+_m = 0, \quad (m \neq 0),$
$\phi_I p^I = 0,$	$x^{\hat{+}} = 0,$
$\phi_I \alpha^I_m = \phi_I \tilde{\alpha}^I_m = 0,$	$\alpha^{\hat{+}}_m = \tilde{\alpha}^{\hat{+}}_m = 0, \quad (m \neq 0),$
$p^I_{\bar{\phi}}(\tau) = p^I_{\bar{\phi} m}(\tau) = 0,$	$\bar{\phi}^I(\tau) = \bar{\phi}^I_m(\tau) = 0,$
$p^I_{B_\tau}(\tau) = p^I_{B_\tau m}(\tau) = 0,$	$B^I_\tau(\tau) = B^I_{\tau m}(\tau) = 0,$
$p^I_{B_\sigma}(\tau) = p^I_{B_\sigma m}(\tau) = 0,$	$B^I_\sigma(\tau) = -\frac{p^I_\phi(\tau)}{2\pi}, \qquad B^I_{\sigma m}(\tau) = 0,$
$\phi^I_m(\tau) = 0,$	$p^I_{\phi m}(\tau) = 0,$
$\frac{1}{2}\phi^I \phi_I = 0,$	$p^{\hat{+}}_\phi = 0.$

Under these gauge fixing conditions, the dynamics of the model is described by the zero-modes and the oscillator modes of the transverse string coordinates $\xi^i(\tau,\sigma)$, the zero-modes of light-cone coordinates $\xi^{\hat{\pm}}(\tau,\sigma)$ and $\xi^{\pm}(\tau,\sigma)$ and the zero-modes of the fields $\phi^I(\tau,\sigma)$ and $P_\phi^I(\tau,\sigma)\big(=-B_\sigma^I(\tau,\sigma)\big)$.

In fact these gauge conditions completely fix the gauge degrees of freedom and these are consistent with the equations of motion. As the constraints are quadratic in the Fourier modes, we can solve the constraints directly and the dependent variables are expressed in terms of the independent variables. Here are the independent canonical variables

$$
\begin{aligned}
\{x^-,p^+\} &= \{x^{\hat{-}},p^{\hat{+}}\} = -1,\\
\{x^i,p^j\} &= \delta^{ij},\\
\{\alpha^i_m,\alpha^j_n\} &= \{\tilde{\alpha}^i_m,\tilde{\alpha}^j_n\} = -im\delta^{ij}\delta_{m+n},\\
\{\phi^+,p_\phi^-\} &= \{\phi^-,p_\phi^+\} = \{\phi^{\hat{+}},p_\phi^{\hat{-}}\} = -1,\\
\{\phi^i,p_\phi^j\} &= \delta^{ij},
\end{aligned}
\tag{5.24}
$$

and the remaining non-vanishing dependent variables are

$$
\begin{aligned}
p^- &= \frac{-1}{\phi^{\hat{+}}p^+-\phi^+p^{\hat{+}}}\Big\{\frac{p^{\hat{+}}p^{\hat{+}}}{\phi^{\hat{+}}}\big(\phi^+\phi^- - \tfrac{1}{2}\phi^i\phi_i\big)\\
&\qquad\qquad -p^{\hat{+}}\big(\phi^-p^+-\phi^ip_i\big)-2\pi\phi^{\hat{+}}\big(L_0^{\mathrm{tr}}+\tilde{L}_0^{\mathrm{tr}}\big)\Big\},\\
\alpha_m^- &= \frac{-1}{\phi^{\hat{+}}p^+-\phi^+p^{\hat{+}}}\big(p^{\hat{+}}\phi^i\alpha_{mi}-2\sqrt{\pi}\phi^{\hat{+}}L_m^{\mathrm{tr}}\big), \qquad (m\neq 0),\\
\tilde{\alpha}_m^- &= \frac{-1}{\phi^{\hat{+}}p^+-\phi^+p^{\hat{+}}}\big(p^{\hat{+}}\phi^i\tilde{\alpha}_{mi}-2\sqrt{\pi}\phi^{\hat{+}}\tilde{L}_m^{\mathrm{tr}}\big), \qquad (m\neq 0),\\
p^{\hat{-}} &= \frac{1}{\phi^{\hat{+}}p^+-\phi^+p^{\hat{+}}}\Big\{\frac{p^{\hat{+}}p^{+}}{\phi^{\hat{+}}}\big(\phi^+\phi^- - \tfrac{1}{2}\phi^i\phi_i\big)\\
&\qquad\qquad -p^{+}\big(\phi^-p^+-\phi^ip_i\big)-2\pi\phi^{+}\big(L_0^{\mathrm{tr}}+\tilde{L}_0^{\mathrm{tr}}\big)\Big\},\\
\alpha_m^{\hat{-}} &= \frac{1}{\phi^{\hat{+}}p^+-\phi^+p^{\hat{+}}}\big(p^{+}\phi^i\alpha_{mi}-2\sqrt{\pi}\phi^{+}L_m^{\mathrm{tr}}\big), \qquad (m\neq 0),\\
\tilde{\alpha}_m^{\hat{-}} &= \frac{1}{\phi^{\hat{+}}p^+-\phi^+p^{\hat{+}}}\big(p^{+}\phi^i\tilde{\alpha}_{mi}-2\sqrt{\pi}\phi^{+}\tilde{L}_m^{\mathrm{tr}}\big), \qquad (m\neq 0),\\
\phi^{\hat{-}} &= -\frac{1}{\phi^{\hat{+}}}\big(\phi^+\phi^- - \tfrac{1}{2}\phi^i\phi_i\big),
\end{aligned}
\tag{5.25}
$$

where the transverse parts of the Virasoro generators L_m^{tr} and $\tilde{L}_m^{\mathrm{tr}}$ are defined by

$$
L_m^{\mathrm{tr}} \equiv \frac{1}{2}\sum_k \alpha^i_{m-k}\alpha_{ik}, \qquad \tilde{L}_m^{\mathrm{tr}} \equiv \frac{1}{2}\sum_k \tilde{\alpha}^i_{m-k}\tilde{\alpha}_{ik}.
$$

Now let us investigate the symmetry of the D-dimensional background spacetime. The translation and the Lorentz transformation generators derived from the classical action (2.12) are given by

$$
\begin{aligned}
P^I &\equiv \int_0^{2\pi} \mathrm{d}\sigma\, P_\xi^I \\
&= p^I, && (5.26\text{a}) \\
M^{IJ} &\equiv \int_0^{2\pi} \mathrm{d}\sigma \left(\xi^I P_\xi^J + \phi^I P_\phi^J + \bar{\phi}^I P_{\bar{\phi}}^J + B_\tau^I P_{B_\tau}^J + B_\sigma^I P_{B_\sigma}^J - (I \leftrightarrow J)\right) \\
&= x^I p^J - \frac{i}{2} \sum_{m \neq 0} \frac{1}{m}\left(\alpha_{-m}^I \alpha_m^J + \tilde{\alpha}_{-m}^I \tilde{\alpha}_m^J\right) + \phi^I p_\phi^J - (I \leftrightarrow J). && (5.26\text{b})
\end{aligned}
$$

Using the independent canonical variables (5.24), the Poincaré algebra ISO$(D-2,2)$ is satisfied,

$$
\begin{aligned}
\{P^I, P^J\} &= 0, \\
\{M^{IJ}, P^K\} &= \eta^{IK} P^J - \eta^{JK} P^I, \\
\{M^{IJ}, M^{KL}\} &= \eta^{IK} M^{JL} - \eta^{JK} M^{IL} - \eta^{IL} M^{JK} + \eta^{JL} M^{IK},
\end{aligned}
\tag{5.27}
$$

if the level matching condition $L_0^{\mathrm{tr}} = \tilde{L}_0^{\mathrm{tr}}$ is imposed. Conversely, the gauge fixing procedure we considered is the way to preserve the full D-dimensional Poincaré symmetry.

According to the ordinary string theories in the light-cone gauge, we have to examine Poincaré algebra (5.27) in the quantum theory [21]. The checking of the Poincaré algebra is again straightforward, except for commutation relations $[M^{i-}, M^{j-}]$, $[M^{i\hat{-}}, M^{j\hat{-}}]$, and $[M^{i-}, M^{j\hat{-}}]$. After lengthy computation, we can obtain the following results,

$$
\begin{aligned}
[M^{i-}, M^{j-}] &= \frac{4\pi {\phi^{\hat{+}}}^2}{(\phi^{\hat{+}} p^+ - \phi^+ p^{\hat{+}})^2} A^{ij}, \\
[M^{i\hat{-}}, M^{j\hat{-}}] &= \frac{4\pi {\phi^+}^2}{(\phi^{\hat{+}} p^+ - \phi^+ p^{\hat{+}})^2} A^{ij}, \\
[M^{i-}, M^{j\hat{-}}] &= i\delta^{ij} M^{-\hat{-}} - \frac{4\pi \phi^{\hat{+}} \phi^+}{(\phi^{\hat{+}} p^+ - \phi^+ p^{\hat{+}})^2} A^{ij}.
\end{aligned}
\tag{5.28}
$$

An anomalous term A^{ij} is

$$
\begin{aligned}
A^{ij} = &-2\left(1 - \frac{D-4}{24}\right) \sum_{m=1}^{\infty} m\left(\alpha_{-m}^i \alpha_m^j + \tilde{\alpha}_{-m}^i \tilde{\alpha}_m^j - (i \leftrightarrow j)\right) \\
&+ \left(a_0 - \frac{D-4}{12}\right) \sum_{m=1}^{\infty} \frac{1}{m}\left(\alpha_{-m}^i \alpha_m^j + \tilde{\alpha}_{-m}^i \tilde{\alpha}_m^j - (i \leftrightarrow j)\right),
\end{aligned}
$$

where the constant a_0 denotes the ordering ambiguity of the sum $L_0^{\mathrm{tr}} + \tilde{L}_0^{\mathrm{tr}}$ in (5.25) by adopting the normal-ordering prescription. The anomalous term A^{ij} vanishes if and only if

$$D = 28, \quad a_0 = 2. \tag{5.29}$$

Then, the Poincaré algebra ISO(26, 2) is satisfied in the quantum theory.

A mass-shell relation of this string model is given by

$$\begin{aligned} m^2 &= -P^I P_I \\ &= 4\pi\left(N + \tilde{N} - a_0\right), \end{aligned} \tag{5.30}$$

where level operators N and $\tilde{N}$ are defined by

$$N \equiv \sum_{m=1}^{\infty} \alpha^i_{-m}\alpha_{im}, \qquad \tilde{N} \equiv \sum_{m=1}^{\infty} \tilde{\alpha}^i_{-m}\tilde{\alpha}_{im}.$$

The level matching condition $L_0^{\mathrm{tr}} = \tilde{L}_0^{\mathrm{tr}}$ is then expressed as $N = \tilde{N}$. Therefore, this closed bosonic string model also involves tachyon in the ground state and graviton $g_{IJ}(\xi)$, two-form field $B_{IJ}(\xi)$ and dilaton $\phi(\xi)$ in the first excited massless state.

6 Conclusions and discussions

We have investigated the quantization of the $\mathrm{U}(1)_{\mathrm{V}} \times \mathrm{U}(1)_{\mathrm{A}}$ bosonic string model in two-dimensional quantum field theory. Even though the system has reducible and open gauge symmetries, we have shown that the covariant quantization has been successfully carried out in the Lagrangian formulation *á la* Batalin and Vilkovisky. In the covariant Batalin-Fradkin-Vilkovisky Hamiltonian formulation, we have considered the first-class constraints and the constraint algebra corresponding to the gauge symmetries and led to the same gauge-fixed action and BRST transformation as those of the Lagrangian formulation under the proper choice of the gauge-fermion and the identification of the fields. In addition we have obtained the BRST charge which generates the BRST transformations. Furthermore we have presented the noncovariant light-cone gauge formulation and investigated the symmetry of the background spacetime. With careful considerations of residual gauge symmetries, we have specified the gauge fixing conditions corresponding to the first-class constraints. Under these suitable conditions, we have been able to clarify dynamical independent variables and solve the first-class constraints explicitly. Although manifest covariance has been lost, we have confirmed the full D-dimensional Poincaré algebra of the background spacetime by direct computation.

Since the quantizations of the model have been successfully carried out, we can argue the critical dimension of the string model. In our case, it turns to be 26+2. This means the background spacetime involves two time-like coordinates. Conversely, the requirement of two negative signatures in the background metric is natural one due to the gauge invariance of our model. The critical dimension has been obtained from both the BRST Ward identity in the BRST formulation and the D-dimensional quantum Poincaré algebra in the noncovariant light-cone gauge formulation. Therefore, we have concluded a consistent quantum theory of our $\mathrm{U}(1)_{\mathrm{V}} \times \mathrm{U}(1)_{\mathrm{A}}$ string model has only been formulated in 26+2-dimensional background spacetime. We have also considered the quantum states from the mass-shell relation. Contributions toward the mass-shell relation from zero-modes of the scalar field $\phi^I(x)$ are completely canceled, so that our closed bosonic string model possesses the same spectra as usual string theories.

We propose the quantum $\mathrm{U}(1)_{\mathrm{V}}\times\mathrm{U}(1)_{\mathrm{A}}$ string model as a device to formulate the physics involving two time coordinates. In the formulation, the generalized Chern-Simons action has played an important role. From this viewpoint, it would be interesting to consider a low energy effective action which might be derived from our formulation of string theory. If we consider a background gauge field $A_I(\xi)$ which could be obtained from our open string or superstring model, it should have an additional gauge symmetry $\delta A_I(\xi) = \phi_I \Omega(\xi)$, where $\Omega(\xi)$ is a gauge parameter and ϕ_I is a constant null field, corresponding to the constraints (5.6b). Such a gauge symmetry has been discussed in the formulation of 10+2-dimensional supersymmetric Yang-Mills theory [3]. In this context, the generalized Chern-Simons action [12] which can be formulated in arbitrary dimensions is also supposed to be useful for constructing the low energy effective action.

The form of the classical action (2.12) suggests that this model should be more naturally defined in higher-dimensional field theories, namely, that membranes or p-branes are more fundamental than strings in our formulation. Actually, the action (2.12) is derived from a membrane action by adopting a compactification prescription. The $\mathrm{U}(1)_{\mathrm{V}} \times \mathrm{U}(1)_{\mathrm{A}}$ string model might be the first useful example which suggests higher-dimensional object like membrane or p-brane in the framework of perturbative field theory without using the concept of "string duality".

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Appendix A. Two-dimensional world-sheet

The two-dimensional spacetime coordinates are denoted by $x^m (= (\tau, \sigma))$. The two-dimensional flat metric η_{mn} and Levi-Civitá symbol ε_{mn} are given by

$$\eta_{mn} = \eta^{mn} = \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}, \qquad \varepsilon_{mn} = -\varepsilon^{mn} = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}.$$

In the curved two-dimensional spacetime, the metric is given by $g_{mn}(x)$ and the covariant derivative ∇_m operates to fields as

$$\nabla_m V_n = \partial_m V_n - \Gamma^l{}_{mn} V_l,$$
$$\nabla_m V^n = \partial_m V^n + \Gamma^n{}_{ml} V^l,$$

where $\Gamma^l{}_{mn}$ is the Christoffel symbol defined by $\Gamma^l{}_{mn} = \frac{1}{2} g^{lk}(\partial_m g_{kn} + \partial_n g_{mk} - \partial_k g_{mn})$.

The functional derivative with respect to a symmetric tensor $V^{mn}(x) = V^{nm}(x)$ is

$$\frac{\delta V^{mn}(x)}{\delta V^{kl}(y)} = (\delta_k^m \delta_l^n + \delta_l^m \delta_k^n)\, \delta(x-y).$$

Then, the antibracket (3.3) is explicitly written as

$$(X, Y) = \frac{\delta_{\rm R} X}{\delta \xi_I^*} \frac{\delta_{\rm L} Y}{\delta \xi^I} + \frac{\delta_{\rm R} X}{\delta \hat{A}_m^*} \frac{\delta_{\rm L} Y}{\delta \hat{A}^m} + \frac{1}{2} \frac{\delta_{\rm R} X}{\delta \hat{g}_{mn}^*} \frac{\delta_{\rm L} Y}{\delta \hat{g}^{mn}} + \ldots.$$

Appendix B. Generalized Poisson bracket

A generalized Poisson bracket [22] is defined by

$$\{F, G\} \equiv \left(\frac{\delta_{\rm L} F}{\delta \varphi^i} \frac{\delta_{\rm L} G}{\delta P_{\varphi^i}} - \frac{\delta_{\rm L} F}{\delta P_{\varphi^i}} \frac{\delta_{\rm L} G}{\delta \varphi^i} \right) + (-)^{|F|} \left(\frac{\delta_{\rm L} F}{\delta \theta^\alpha} \frac{\delta_{\rm L} G}{\delta P_{\theta^\alpha}} + \frac{\delta_{\rm L} F}{\delta P_{\theta^\alpha}} \frac{\delta_{\rm L} G}{\delta \theta^\alpha} \right),$$

where canonical variables φ^i and P_{φ^i} are bosonic, and θ^α and P_{θ^α} are fermionic. In the above definition the contraction of the indices contains the integration of space or spacetime and $|F|$ is the Grassmann parity of F. This generalized Poisson bracket will be replaced by the graded commutation relation multiplied by $-i$ upon quantization, as usual. The explicit forms of the basic Poisson brackets are given by

$$\{\varphi^i, P_{\varphi^j}\} = -\{P_{\varphi^j}, \varphi^i\} = \delta^i_j,$$
$$\{\theta^\alpha, P_{\theta^\beta}\} = \{P_{\theta^\beta}, \theta^\alpha\} = -\delta^\alpha_\beta.$$

The algebraic properties of the Poisson bracket are as follows:

$$\{F, G\} = -(-)^{|F||G|}\{G, F\},$$
$$\{F_1F_2, G\} = F_1\{F_2, G\} + (-)^{|G||F_2|}\{F_1, G\}F_2.$$

Appendix C. BRST formulation of $\mathrm{U}(1)_\mathrm{V} \times \mathrm{U}(1)_\mathrm{A}$ model without two-dimensional gravity

In this appendix we summarize the Lagrangian BRST quantization of $\mathrm{U}(1)_\mathrm{V} \times \mathrm{U}(1)_\mathrm{A}$ model *without* two-dimensional gravity. Since the quantization of this model is much simpler than that of the model coupled with two-dimensional gravity we have investigated throughout this paper, the following result might be helpful to understand the quantization of the $\mathrm{U}(1)_\mathrm{V} \times \mathrm{U}(1)_\mathrm{A}$ gauge structure.

The action of $\mathrm{U}(1)_\mathrm{V} \times \mathrm{U}(1)_\mathrm{A}$ model without two-dimensional gravity is

$$S = \int \mathrm{d}^2x \Big(-\frac{1}{2}\eta^{mn}\partial_m\xi^I\partial_n\xi_I - \eta^{mn}\partial_m\bar{\phi}^I\partial_n\phi_I + \tilde{A}^m\phi_I\partial_m\xi^I + \tilde{B}^{mI}\partial_m\phi_I - \frac{1}{2}\tilde{C}\phi^I\phi_I \Big), \tag{C.1}$$

which is invariant under the following local gauge transformation,

$$\begin{aligned}
\delta\xi^I &= v'\phi^I, \\
\delta\phi^I &= 0, \\
\delta\bar{\phi}^I &= u'^I, \\
\delta\tilde{A}^m &= \varepsilon^{mn}\partial_n v + \eta^{mn}\partial_n v', \\
\delta\tilde{B}^{mI} &= \varepsilon^{mn}\partial_n u^I + \eta^{mn}\partial_n u'^I - v\varepsilon^{mn}\partial_n\xi^I + v'\eta^{mn}\partial_n\xi^I - \tilde{w}^m\phi^I, \\
\delta\tilde{C} &= \partial_m\tilde{w}^m + \partial_m v'\tilde{A}^m - v'\partial_m\tilde{A}^m.
\end{aligned} \tag{C.2}$$

After performing the BRST formulation, one obtains the following gauge-fixed action

$$S_{\text{gauge-fixed}} = \int \mathrm{d}^2x \Big\{ -\frac{1}{2}\eta^{mn}\partial_m\xi^I\partial_n\xi_I - \eta^{mn}\partial_m\bar{\phi}^I\partial_n\phi_I - \eta^{mn}\partial_m\bar{f}\partial_n f$$

$$
\begin{aligned}
&- \hat{a}^m(\partial_m a + \varepsilon_m{}^n \partial_n a') - \hat{b}_I^m(\partial_m b^I + \varepsilon_m{}^n \partial_n b'^I) \\
&- \hat{c}^m(\partial_m c + \varepsilon_m{}^n \partial_n c') \\
&- 2a\hat{b}_I^m \partial_m \xi^I + \varepsilon_{mn}\hat{b}_I^m \hat{c}^n \phi^I + \frac{1}{2}(f + aa')\varepsilon_{mn}\hat{b}_I^m \hat{b}^{nI} \Big\}.
\end{aligned} \tag{C.3}
$$

The action (C.3) is invariant under the nilpotent BRST transformations

$$
\begin{aligned}
s\xi^I &= a'\phi^I, \\
s\phi^I &= 0, \\
s\bar{\phi}^I &= b'^I - a'\xi^I, \\
sf &= 0, \\
s\bar{f} &= c', \\
sa &= 0, \\
sa' &= 0, \\
sb^I &= (f - aa')\phi^I, \\
sb'^I &= 0, \\
sc &= \frac{1}{2}\phi^I \phi_I, \\
sc' &= 0, \\
s\hat{a}^m &= \varepsilon^{mn}(\phi_I \partial_n \xi^I - \partial_n \phi_I \xi^I) - a'\hat{b}_I^m \phi^I \\
&\quad - (\varepsilon^{mn} c - \eta^{mn} c')\partial_n a' - \varepsilon^{mn}\partial_n(ca' + c'a), \\
s\hat{b}_I^m &= \varepsilon^{mn}\partial_n \phi_I, \\
s\hat{c}^m &= (\varepsilon^{mn} a - \eta^{mn} a')\partial_n a' + \varepsilon^{mn}\partial_n f.
\end{aligned} \tag{C.4}
$$

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