Gauge Symmetry Breaking through Hosotani Mechanism in Softly Broken Supersymmetric QCD

Kazunori Takenaga *

School of Theoretical Physics,
Dublin Institute for Advanced Studies, 10 Burlington Road, Dublin 4, Ireland

Abstract

Gauge symmetry breaking through the Hosotani mechanism (the dynamics of nonintegrable phases) in softly broken supersymmetric QCD with N_F^{fd} flavors is studied. For N = even, there is a single $SU(N)$ symmetric vacuum state, while for N = odd, there is a doubly degenerate $SU(N)$ symmetric vacuum state in the model. We also study the generalized supersymmetric QCD by adding N_F^{adj} numbers of massless adjoint matter. The gauge symmetry breaking pattern such as $SU(3) \to SU(2) \times U(1)$ is possible for appropriate choices of the matter contents and values of supersymmetry breaking parameter. The massless state of the adjoint Higgs scalar is also discussed in the models.

DIAS-STP-02-04
May 2002

*email: takenaga@synge.stp.dias.ie

1 Introduction

Gauge symmetry breaking through the Hosotani mechanism [1, 2] (the dynamics of nonintegrable phases) is one of the remarkable phenomena in physics with extra dimensions. Component gauge fields for compactified directions, which are dynamical degrees of freedom and cannot be gauged away, can develop vacuum expectation values, and the gauge symmetry is broken dynamically. The existence of the zero mode for the component gauge field is crucial for the mechanism. Quantum effects shift the zero mode to induce the gauge symmetry breaking, reflecting the topology of the extra dimension.

The vacuum expectation values, which are nothing but the constant background fields, are also related with the eigenvalues (phases) of the Wilson line integrals along the compactified direction, and the gauge symmetry breaking corresponds to the nontrivial Wilson line integral. One can discuss the gauge symmetry breaking patterns of the theory by studying the effective potential for the phases [2].

Since the pioneering work by Hosotani [1], the dynamics of the nonintegrable phases has been studied in various models [2]–[7], namely, nonsupersymmetric gauge models. It has been known that the gauge symmetry breaking patterns depend on matter contents, i.e., the number, representation under the gauge group and boundary condition of matter.

In this paper, following author's works [8, 9], we study the gauge symmetry breaking patterns in supersymmetric $SU(N)$ gauge theory with N_F^{fd} numbers of massless fundamental matter (supersymmetric QCD) defined on $M^3 \otimes S^1$. Here M^3, S^1 are three-dimensional Minkowski space-time and a circle, respectively. And we also study the generalized supersymmetric QCD (supersymmetric QCD with massless adjoint matter).

The dynamics of the nonintegrable phases determines the vacuum structure of the theory. If we, however, introduce the matter multiplets, the vacuum expectation values of the squark fields in the multiplets also become the order parameters for gauge symmetry breaking. We assume that the gauge coupling constant g is small and ignore $O(g^2)$ contributions to the effective potential. In this approximation, there exist flat directions of the potential parametrized by the vacuum expectation values of the squark field. In order to concentrate on the dynamics of the nonintegrable phases, we take the trivial "point" on the flat direction, where all the vacuum expectation values of the squark fields vanish.

If the theory has supersymmetry, one cannot discuss the dynamical breaking of gauge symmetry based on perturbation theory because the perturbative effective potential for the nonintegrable phases vanishes due to the supersymmetry. One must break the supersymmetry in order to obtain nonvanishing effective potential [1]. We resort to the Scherk-

[1] This is not the case where the gauge charge such as the gauged $U(1)_R$ in supergravity models distinguishes bosons and fermions in a supermultiplet. In this case supersymmetry is broken spontaneously by the Hosotani mechanism [10].

Schwarz mechanism [11, 12], which is a natural candidate to break supersymmetry softly in this setup [13].

In the softly broken supersymmetric Yang-Mills theory, the $SU(N)$ gauge symmetry is not broken through the Hosotani mechanism. There are N vacuum states in the model, and the vacuum has Z_N symmetry. By adding N_F^{fd} sets of massless fundamental matter multiplet, the model describes the softly broken supersymmetric QCD with N_F^{fd} flavors. We find that in the case $N =$ even, there is a single $SU(N)$ symmetric vacuum state, while in the case $N =$ odd, there is a doubly degenerate $SU(N)$ symmetric vacuum state in the model. The degenerate two vacua is related each other by the symmetry transformations of the effective potential. Unlike the case of the softly broken supersymmetric Yang-Mills theory, there is no Z_2 symmetry for the degenerate vacuum because of the fundamental matter in the model. The vacuum configurations do not depend on the values of N_F^{fd} and supersymmetry breaking parameter.

We also discuss the mass of the adjoint Higgs scalar. The scalar is originally the component gauge field for the S^1 direction and behaves as adjoint Higgs at low energies. It acquires mass through the quantum correction in the extra dimension, and the mass is obtained by evaluating the second derivative of the effective potential at the minimum. The adjoint Higgs scalar is always massive in the softly broken supersymmetric QCD.

In the generalized supersymmetric QCD, we find that the partial gauge symmetry breaking such as $SU(2) \times U(1)$, which may be important for GUT symmetry breaking, is possible for appropriate choices of the matter contents and values of the supersymmetry breaking parameter. This gauge symmetry breaking pattern is not realized until one considers both of the massless adjoint and fundamental matter multiplets. We also find the massless state of the adjoint Higgs scalar within our approximation for the aforementioned gauge symmetry breaking pattern in the model.

In the next section we present the effective potentials for the nonintegrable phases of the models we study in this paper. We first discuss the gauge symmetry breaking patterns in the softly broken supersymmetric Yang-Mills theory in the section 3. Then, we proceed to the softly broken supersymmetric QCD, and the gauge symmetry breaking patterns are determined. The massless adjoint Higgs scalar is also discussed in the models. In section 4 we consider the generalized supersymmetric QCD. We are, especially, interested in the gauge symmetry breaking pattern such as $SU(3) \to SU(2) \times U(1)$ and the massless state of the adjoint Higgs scalar. The final section is devoted to conclusions and discussion.

2 Effective potential for nonintegrable phases

In this section we present the effective potentials for the nonintegrable phases of our models. We first consider the $SU(N)$ supersymmetric Yang-Mills theory for the latter convenience. The on-shell Lagrangian is given by

$$\mathcal{L} = \mathrm{tr}\left[-\frac{1}{2}F_{\hat{\mu}\hat{\nu}}F^{\hat{\mu}\hat{\nu}} - i\lambda\sigma^{\hat{\mu}}D_{\hat{\mu}}\bar{\lambda} + iD_{\hat{\mu}}\lambda\sigma^{\hat{\mu}}\bar{\lambda}\right]. \tag{1}$$

The coordinates of M^3 and S^1 are denoted by x^μ and y, respectively. $x^{\hat{\mu}}$ stands for (x^μ, y). We impose the boundary condition associated with the $U(1)_R$ symmetry on the gaugino field [8, 13],

$$\lambda(x^\mu, y+L) = \mathrm{e}^{i\beta}\lambda(x^\mu, y), \tag{2}$$

where L is the length of the circumference of S^1. The nontrivial phase β breaks supersymmetry softly. The gauge field $A_{\hat{\mu}}$ satisfies the periodic boundary condition.

Let us parametrize the vacuum expectation value of the component gauge field A_y for the S^1 direction as

$$\langle A_y\rangle \equiv \frac{1}{gL}\langle\Phi\rangle = \frac{1}{gL}\mathrm{diag}(\theta_1, \theta_2, \cdots, \theta_N) \qquad \text{with} \qquad \sum_{i=1}^{N}\theta_i = 0 \tag{3}$$

where g denotes the gauge coupling constant and θ_i is module of 2π. The phase θ_i called the nonintegrable phase is related with the eigenvalue of the Wilson line integral,

$$W_c \equiv \mathcal{P}\exp\left(-ig\oint_{S^1} dy\langle A_y\rangle\right) = \mathrm{diag}\left(\mathrm{e}^{-i\theta_1}, \mathrm{e}^{-i\theta_2}, \cdots, \mathrm{e}^{-i\theta_N}\right). \tag{4}$$

The residual gauge symmetry is generated by the generators of $SU(N)$ commuting with W_c [2].

By expanding the fields around the constant background (3) and integrating out the fluctuating fields up to the quadratic terms, one obtains the effective potential for the nonintegrable phases. Following the standard technique given in the papers [1, 2], we obtain the potential for the softly broken supersymmetric Yang-Mills theory [8],

$$V_{SYM}(\theta) = \frac{-2}{\pi^2 L^4}\sum_{n=1}^{\infty}\sum_{i,j=1}^{N}\frac{1}{n^4}\left(\cos[n(\theta_i - \theta_j)] - \cos[n(\theta_i - \theta_j - \beta)]\right), \tag{5}$$

where n stands for the Kaluza-Klein mode for the S^1 direction. The nontrivial phase β appears in the second term coming from the gaugino contribution to the constant background (3) and gives the nonvanishing effective potential. The number 2 in Eq. (5) counts the on-shell degrees of freedom of the gauge boson $(D-2)$ and gaugino $(2^{[D/2]}/2)$, which are equal in four dimensions. One can discuss how the gauge symmetry is broken through the dynamics of the nonintegrable phases in this model by finding the absolute minima of the effective potential.

Let us introduce N_F^{fd} sets of fundamental massless matter multiplet denoted by $Q(\bar{Q})$ belonging to the (anti)fundamental representation under $SU(N)$. The physical fields in $Q(\bar{Q})$ are quark $q(\bar{q})$ and squark $\phi_q(\bar{\phi}_q)$. We impose the boundary conditions associated with the $U(1)_R$ symmetry on the squark fields [8][2],

$$\phi_q(x^\mu, y+L) = \mathrm{e}^{i\beta}\phi_q(x^\mu, y), \quad \bar{\phi}_q(x^\mu, y+L) = \mathrm{e}^{i\beta}\bar{\phi}_q(x^\mu, y), \tag{6}$$

where we have suppressed the flavor index for the squark. The nontrivial phase β breaks supersymmetry softly and gives the nonvanishing effective potential. It has been pointed out that the phase is common to all flavors, so that the supersymmetry breaking terms in three dimensions are flavor blind [8, 13].

As noted in the introduction, we focus on the gauge symmetry breaking through the Hosotani mechanism and set all the vacuum expectation values of the squark fields vanishing. In order to evaluate the effective potential for the phases, one needs the mass operators for Q and $\bar{Q}$, which actually give the mass terms for the (s)quarks in three dimensions after compactifications. Since the matter multiplet $Q(\bar{Q})$ belongs to the (anti)fundamental representation under $SU(N)$ and the squark fields have the nontrivial phase β, the mass operator for ϕ_q and that for $\bar{\phi}_q$ have different forms [3]. On the other hand, the quark fields have no nontrivial phase, so that both of q and $\bar{q}$ give the same mass operators [4].

One can read the mass operators in the covariant derivatives for the squark fields,

$$(\partial_{\hat{\mu}}\phi_q^\dagger + ig\phi_q^\dagger A_{\hat{\mu}})(\partial_{\hat{\mu}}\phi_q - igA_{\hat{\mu}}\phi_q), \qquad (\partial_{\hat{\mu}}\bar{\phi}_q + ig\bar{\phi}_q A_{\hat{\mu}})(\partial_{\hat{\mu}}\bar{\phi}_q^\dagger - igA_{\hat{\mu}}\bar{\phi}_q^\dagger). \tag{7}$$

They are obtained as

$$\left(D_3^{\phi_q}\right)^2 = -\sum_{n=-\infty}^{\infty}\sum_{i=1}^{N}\left(\frac{2\pi}{L}\right)^2\left(n - \frac{\theta_i - \beta}{2\pi}\right)^2 \quad \text{for} \quad \phi_q, \tag{8}$$

$$\left(D_3^{\bar{\phi}_q}\right)^2 = -\sum_{n=-\infty}^{\infty}\sum_{i=1}^{N}\left(\frac{2\pi}{L}\right)^2\left(n - \frac{-\theta_i - \beta}{2\pi}\right)^2 \quad \text{for} \quad \bar{\phi}_q. \tag{9}$$

Here n stands for the Kaluza-Klein mode for the S^1 direction. That the prescription $\theta_i \to -\theta_i$ in Eq. (8) gives the Eq. (9) shows the field $\bar{\phi}_q$ belongs to the antifundamental representation under $SU(N)$. We see that ϕ_q and $\bar{\phi}_q$ contribute to the effective potential in a different manner [5].

Following again the standard prescription, we obtain the effective potential for the phases coming from the fundamental massless matter multiplets,

$$V_{matter}^{fd}(\theta) = \frac{2N_F^{fd}}{\pi^2 L^4}\sum_{n=1}^{\infty}\sum_{i=1}^{N}\frac{1}{n^4}\left((\cos(n\theta_i) - \cos[n(\theta_i - \beta)]) + (\cos(n\theta_i) - \cos[n(\theta_i + \beta)])\right)$$

[2]These boundary conditions are defined by the assignments of $U(1)_R$ charge on the fields based on the invariance of the action under the $U(1)_R$ transformation in the presence of the mass term $m\bar{Q}Q$. The discussion on the effective potential of the nonintegrable phases in this paper corresponds to the massless limit.

[3]This point has been overlooked in the previous paper [8].

[4]This is also clear from the fact that q and $\bar{q}$ forms a Dirac spinor satisfying the periodic boundary condition.

[5]The gauge group $SU(2)$ is an exceptional case as we will see in the section 4.

$$= \frac{2N_F^{fd}}{\pi^2 L^4} \sum_{n=1}^{\infty} \sum_{i=1}^{N} \frac{1}{n^4} (2\cos(n\theta_i) - \cos[n(\theta_i - \beta)] - \cos[n(\theta_i + \beta)]) \tag{10}$$

where the first term in Eq. (10) arises from the quarks $q, \bar{q}$, and the second and third terms come from ϕ_q and $\bar{\phi}_q$, respectively. By putting Eqs. (5) and (10) together, we obtain the effective potential for the softly broken supersymmetric QCD with N_F^{fd} numbers of the massless fundamental matter,

$$\begin{aligned} V_{SQCD}(\theta) &= V_{SYM}(\theta) + V_{matter}^{fd}(\theta) \\ &= \frac{-2}{\pi^2 L^4} \sum_{n=1}^{\infty} \sum_{i,j=1}^{N} \frac{1}{n^4} (\cos[n(\theta_i - \theta_j)] - \cos[n(\theta_i - \theta_j - \beta)]) \\ &+ \frac{2N_F^{fd}}{\pi^2 L^4} \sum_{n=1}^{\infty} \sum_{i=1}^{N} \frac{1}{n^4} (2\cos(n\theta_i) - \cos[n(\theta_i - \beta)] - \cos[n(\theta_i + \beta)]). \end{aligned} \tag{11}$$

For completeness, let us present the effective potential for the phases coming from N_F^{adj} numbers of the massless adjoint matter multiplet denoted by $Q^{adj}(q^{adj}, \phi_q^{adj})$. We impose the boundary condition associated with the $U(1)_R$ symmetry on the squark field in the same manner with Eq. (6),

$$\phi_q^{adj}(x^\mu, y + L) = e^{i\beta} \phi_q^{adj}(x^\mu, y). \tag{12}$$

The potential is given by [9]

$$V_{matter}^{adj}(\theta) = \frac{2N_F^{adj}}{\pi^2 L^4} \sum_{n=1}^{\infty} \sum_{i,j=1}^{N} \frac{1}{n^4} (\cos[n(\theta_i - \theta_j)] - \cos[n(\theta_i - \theta_j - \beta)]). \tag{13}$$

As a general remark, the phase θ_i gives no physical effects at least classically, but the effect is essential at the quantum level. It should be emphasized that these effective potentials (5), (10), (13) arise from taking into account the quantum correction in the extra dimension.

3 Supersymmetric QCD with N_F^{fd} flavors

In this section we discuss the gauge symmetry breaking through the Hosotani mechanism based on the obtained effective potentials in the previous section. Before doing it, let us mention about the vacuum structure of the model, which is peculiar to softly broken supersymmetric gauge theories.

Strictly speaking, the dynamics of nonintegrable phases itself does not give the whole information on the vacuum structure of softly broken supersymmetric gauge theories. This is because, as noted in the introduction, the vacuum expectation values of the squark fields $\langle\phi_q\rangle, \langle\bar{\phi}_q\rangle \in \mathbf{C}$ are also the order parameters for gauge symmetry breaking. If one wishes to study the entire vacuum structure, one should take into account the order parameters in addition to the nonintegrable phases. This means that one has to include the tree-level potential and one-loop corrections to the vacuum expectation values of the squark fields as well.

The tree-level potential, which arises from the covariant derivative and the quartic couplings for the squark field, is given by [6]

$$\begin{aligned}
V_{tree} &= g^2\left(\langle\phi_q^\dagger\rangle\langle A_y\rangle^2\langle\phi_q\rangle + \langle\bar{\phi}_q\rangle\langle A_y\rangle^2\langle\bar{\phi}_q^\dagger\rangle\right) + g^2\left(\langle\phi_q^\dagger\rangle T^a\langle\phi_q\rangle - \langle\bar{\phi}_q\rangle T^a\langle\bar{\phi}_q^\dagger\rangle\right)^2 \\
&= \frac{1}{L^2}\sum_{i=1}^{N}\theta_i^2\left(|\langle\phi_{qi}\rangle|^2 + \left|\langle\bar{\phi}_q^i\rangle\right|^2\right) + g^2\left(\langle\phi_q^\dagger\rangle T^a\langle\phi_q\rangle - \langle\bar{\phi}_q\rangle T^a\langle\bar{\phi}_q^\dagger\rangle\right)^2, \qquad (14)
\end{aligned}$$

where we have used Eq. (3) and $T^a(a=1,\cdots,N^2-1)$ stands for the generator of $SU(N)$. Let us note that the interactions between $\langle\phi_q\rangle, \langle\bar{\phi}_q\rangle$ and θ_i are $O(1)$, while the self-interactions among the squarks are of order g^2. And the one-loop correction to the vacuum expectation values of the squark fields, which is not written explicitly, is also of order g^2.

If the gauge coupling g is very small, then, one may ignore the $O(g^2)$ terms, so that the term which does not have the gauge coupling dependence becomes dominant contribution to the vacuum structure of the theory. In this approximation, the total effective potential is given by

$$V(\theta, \langle\phi_q\rangle, \langle\bar{\phi}_q\rangle) = \frac{1}{L^2}\sum_{i=1}^{N}\theta_i^2\left(|\langle\phi_{qi}\rangle|^2 + \left|\langle\bar{\phi}_q^i\rangle\right|^2\right) + V_{SQCD}(\theta), \qquad (15)$$

where $V_{SQCD}(\theta)$ is given by Eq. (11). The relevant interaction to generate the effective potential (15) is only the gauge interaction, which is $O(1)$. That is why the total effective potential does not have the dependence on the gauge coupling.

The first term in Eq. (15), which stands for the tree-level potential, is positive semi-definite. The configuration that minimizes it is given by $\langle\phi_{qi}\rangle = \langle\bar{\phi}_q^i\rangle = 0$ for nonzero values of $\theta_i(i=1,\cdots,N)$. In fact, as we will see soon, the nonzero values of θ_i are the case where the absolute minima of $V_{SQCD}(\theta)$ is realized. As a result, the tree-level potential does not affect the vacuum structure of the model in this approximation. Therefore, the vacuum structure is determined by the dynamics of the nonintegrable phases alone in this model.

3.1 Gauge symmetry breaking via Hosotani mechanism

Let us now consider the effective potential $V_{SQCD}(\theta)$ in order to study the dynamics of the nonintegrable phases, i.e., gauge symmetry breaking through the Hosotani mechanism. Our strategy to find the vacuum configuration of the potential is to minimize $V_{SYM}(\theta)$

[6] Since the tree-level potential in the model is not the Higgs type potential, we do not expect the phase structures depending on the size of S^1 such as the ones studied in Ref. [14].

and $V_{matter}(\theta)$ separately, and we take the common configuration for both of them, which actually gives the absolute minima of the potential $V_{SQCD}(\theta)$.

Since $V_{SYM}(\theta)$ can be recasted as [9]

$$V_{SYM}(\theta) = \frac{-2}{\pi^2 L^4} \sum_{n=1}^{\infty} \sum_{i,j=1}^{N} \frac{1}{n^4}[1-\cos(n\beta)]\left(N + 2 \sum_{1\leq i<j\leq N} \cos[n(\theta_i - \theta_j)]\right), \tag{16}$$

it is easy to see that this potential is minimized at $\theta_i - \theta_j = 0$. Taking account of $\sum_{i=1}^{N} \theta_i = 0$ and a module of 2π of θ_i, we obtain

$$\theta_i(i = 1, \cdots, N) = \frac{2\pi}{N} m, \qquad m = 0, \cdots, N-1. \tag{17}$$

This means $e^{i\theta_i} = e^{2\pi i m/N}$, so that the Wilson line integral just corresponds to an element of the center of $SU(N)$, and it commutes with all the generators of $SU(N)$. Hence, the gauge symmetry is not broken in this model. This is the same result with the case of the nonsupersymmetric Yang-Mills theory [1].

It is important to note that there are N vacuum states corresponding to the values of m. The N vacua are physically equivalent because, for example, the mass spectra on the vacua are exactly the same with each other. The fields A_μ, λ stay in massless on the vacuum configuration (17). The vacuum has Z_N symmetry. A way of looking at the Z_N symmetry is to consider the gauge transformation (regular, nonperiodic) defined by

$$U^{(m)}(y) = \exp \frac{2\pi i y}{L} \begin{pmatrix} \frac{m}{N} & & & \\ & \frac{m}{N} & & \\ & & \ddots & \\ & & & -\frac{(N-1)m}{N} \end{pmatrix}. \tag{18}$$

This transformation does not change the boundary conditions of the fields $A_{\hat{\mu}}, \lambda$ because they belong to the adjoint representation under $SU(N)$. It is easy to see that the N vacuum states are related each other by this transformation.

Let us next consider the potential $V_{matter}^{fd}(\theta)$ given by Eq. (10) and find the configuration that minimizes it. This is interesting in its own light because, as we will see later, this potential corresponds to the case of the generalized supersymmetric QCD with $N_F^{adj} = 1$. The potential is recasted as

$$\begin{aligned} V_{matter}^{fd}(\theta) &= \frac{2N_F^{fd}}{\pi^2 L^4} \sum_{n=1}^{\infty} \sum_{i=1}^{N} \frac{1}{n^4}(2\cos(n\theta_i) - \cos[n(\theta_i - \beta)] - \cos[n(\theta_i + \beta)]) \\ &= \frac{4N_F^{fd}}{\pi^2 L^4} \sum_{n=1}^{\infty} \sum_{i=1}^{N} \frac{1}{n^4}[1-\cos(n\beta)]\cos(n\theta_i). \end{aligned} \tag{19}$$

We see that the nontrivial phase β does not affect the location of the absolute minima of the potential. In finding the minimum, let us note that the potential is invariant under [7]

$$\beta \to 2\pi - \beta. \tag{20}$$

[7]The potential is also invariant under $\beta \to \beta + 2\pi i k, k \in \mathbf{Z}$. This corresponds to $\lambda \to e^{2\pi i k}\lambda$.

This invariance means that the potential is symmetric under the reflection with respect to $\beta = \pi$ for fixed θ_i. The region given by $0 < \beta \leq \pi$ is enough to study the potential. Moreover, the potential also posessess the invariance under

$$\theta_i \to 2\pi - \theta_i, \qquad i = 1, \cdots, N. \tag{21}$$

The maximal symmetry of $V^{fd}_{matter}(\theta)$ is given by the transformations with Eqs. (20) and (21).

Taking into account Eqs. (20) and (21), we see that the region given by $\theta_i - \beta \geq 0$ is enough to study the potential. Thanks to this, one does not need the classification depending on the sign of $\theta_i - \beta$ when one uses the formula,

$$\sum_{n=1}^{\infty} \frac{1}{n^4} \cos(nx) = \frac{-1}{48} x^2 (x - 2\pi)^2 + \frac{\pi^4}{90} \qquad (0 \leq x \leq 2\pi). \tag{22}$$

Noting an expression obtained by applying the formula (22),

$$\sum_{n=1}^{\infty} \frac{1}{n^4} (2\cos(n\theta) - \cos[n(\theta - \beta)] - \cos[n(\theta + \beta)]]) = \frac{\beta^2}{24} (6\theta^2 - 12\pi\theta + \beta^2 + 4\pi^2), \tag{23}$$

we have

$$V^{fd}_{matter} = \frac{2N_F^{fd}}{\pi^2 L^4} \frac{\beta^2}{24} \left(\sum_{i=1}^{N-1} \left(6\theta_i^2 - 12\pi\theta_i + \beta^2 + 4\pi^2\right) + 6\left(\sum_{i=1}^{N-1} \theta_i\right)^2 - 12\pi \sum_{i=1}^{N-1} \theta_i + \beta^2 + 4\pi^2 \right). \tag{24}$$

The extremum condition $\partial V_{matter}/\partial \theta_k (k = 1, \cdots, N-1) = 0$ yields

$$\theta_k + (\theta_1 + \cdots + \theta_{N-1}) = 0 \pmod{2\pi}, \qquad k = 1, \cdots, N-1. \tag{25}$$

The solution to Eq. (25) is obtained as $\theta_k = 2\pi q/N$ $(q = 0, \cdots, N-1)$. Since $\theta_N = -\sum_{k=1}^{N-1} \theta_k = 2\pi q/N$, we finally have $\theta_i (i = 1, \cdots, N) = 2\pi q/N$.

Unlike the case of the softly broken supersymmetric Yang-Mills theory, the effective potential has different energies for different values of q in the present case. The minimum of the function (23) is achieved at $\theta = \pi$. If all the θ_i's can take this value, the potential $V^{fd}_{matter}(\theta)$ is obviously minimized at $\theta_i = \pi$ $(i = 1, \cdots, N)$. In fact, this is the case when $N =$ even and corresponds to $q_{even} = N/2$. For $N =$ odd, the value which is as close as possible to π gives the lowest energy of the potential. It is given by $q^{(1)}_{odd} = (N-1)/2$, i.e., $\theta^{(1)}_i = (N-1)\pi/N$. The potential is invariant under Eq. (21), so that the configuration with $q^{(2)}_{odd} = (N+1)/2$ corresponding to $\theta^{(2)}_i = (N+1)\pi/N (= 2\pi - \theta^{(1)}_i)$ gives the same energy with that for $q^{(1)}_{odd} = (N-1)/2$ and also becomes a vacuum configuration [8].

The two vacuum configurations $\theta^{(1)}_i, \theta^{(2)}_i$ are not distinct. In order to see it, let us consider the mass spectra for ϕ_q on the vacua $\theta^{(1)}_i, \theta^{(2)}_i$. They are given by $(n - (\theta^{(1)}_i - \beta)/2\pi)^2$ and $(n - (\theta^{(2)}_i - \beta)/2\pi))^2$ from Eq. (8). The former is reduced to the latter by

[8] Note that the physical region of $\theta_i (i = 1, \cdots, N)$ is $0 \leq \theta_i \leq 2\pi$.

the transformations with Eqs. (20) and (21) and vice versa. Since they are the symmetry transformation of the effective potential, both of the mass spectra are physically identical each other.

The vacuum configuration for the case N = odd is a doubly degenerate. There is, however, no Z_2 symmetry for the vacuum configurations in the present case because the model contains the massless matter multiplet belonging to the (anti)fundamental representation under $SU(N)$. The gauge transformation with Eq. (18) changes the boundary condition of the field in the multiplet. In fact, we see that

$$\phi'_q(y+L) = \mathrm{e}^{i(\beta+\frac{2\pi}{N})}\phi'_q(y), \tag{26}$$

where $\phi'_q = U^{(m=1)}(y)\phi_q$.

We have obtained the vacuum configuration which minimizes $V^{fd}_{matter}(\theta)$ as

$$\theta_i(i=1,\cdots,N) = \begin{cases} \pi & \cdots N = \text{even}, \\ \frac{N-1}{N}\pi, \quad (\text{or } \frac{N+1}{N}\pi) & \cdots N = \text{odd}. \end{cases} \tag{27}$$

As we have noticed before, they do not depend on N_F^{fd} and the supersymmetry breaking parameter β by the Scherk-Schwarz mechanism. The vacuum configurations respect the $SU(N)$ gauge symmetry and are the parts of the center of $SU(N)$.

We are ready to find the common configuration between Eqs. (17) and (27), which gives the absolute minima of the effective potential (11). It is given by Eq. (27) obviously. We conclude that for N = even, there is a single vacuum state, while for N = odd, there is a doubly degenerate vacuum state in the softly broken supersymmetric QCD with N_F^{fd} flavors.

Here we confirm the discussion on the tree-level potential at the beginning of this section. As we have studied above, the configuration that minimizes the effective potential (11) is given by the nonzero values of $\theta_i(i=1,\cdots,N)$, so that only the vanishing vacuum expectation values of the squark fields minimize the total potential (15).

Let us now study the mass of the adjoint Higgs scalar. The scalar is originally the component gauge field for the S^1 direction and behaves as adjoint Higgs at low energies. It acquires mass through the quantum correction in the extra dimension. The mass is obtained by the second derivative of the effective potential (11) at the minimum,

$$\frac{\partial^2 V_{SQCD}}{\partial\theta_i\partial\theta_j} = \frac{C_H^{SQCD}}{\pi^2 L^4} M_{ij}, \qquad C_H^{SQCD} \equiv \beta^2\left(N + N_F^{fd}\right), \tag{28}$$

where the matrix M_{ij} is given by

$$M_{ij} \equiv \begin{pmatrix} 2 & 1 & \cdots & \cdots & 1 \\ 1 & 2 & & & \vdots \\ \vdots & & \ddots & & \vdots \\ \vdots & & & \ddots & \vdots \\ 1 & \cdots & \cdots & \cdots & 2 \end{pmatrix}. \tag{29}$$

All the (off-)diagonal elements of the matrix are 2(1). As studied in Ref. [9], this matrix is easily diagonalized, and the mass is obtained as

$$m_{\Phi}^2 = \frac{g^2 C_H^{SQCD}}{\pi^2 L^2}\frac{N}{2}. \tag{30}$$

The mass of the adjoint Higgs scalar is $SU(N)$ invariant, reflecting the $SU(N)$-symmetric vacuum configuration of the model. It is easy to see that there is no possibility to have $C_H^{SQCD} = 0$, so that the adjoint Higgs scalar is always massive and cannot be massless.

4 Supersymmetric QCD with massless adjoint matter

In this section we proceed to study the generalized version of supersymmetric QCD by introducing N_F^{adj} numbers of massless adjoint matter multiplet. Let us first discuss the tree-level potential within our approximation in this model.

If we add the massless adjoint matter, the tree-level potential becomes, ignoring the $O(g^2)$ terms and the flavor index [9],

$$V_{tree} = \frac{1}{L^2}\sum_{i=1}^{N}\theta_i^2\left(|\langle\phi_{qi}\rangle|^2 + \left|\langle\bar{\phi}_q^i\rangle\right|^2\right) + \frac{2}{L^2}\mathrm{tr}\left|\left[\langle\Phi\rangle,\ \langle\phi_q^{adj}\rangle\right]\right|^2. \tag{31}$$

The second term comes from the covariant derivative of the squark field in the adjoint representation under $SU(N)$. The total effective potential is, then, given by

$$V_{total} = \frac{1}{L^2}\sum_{i=1}^{N}\theta_i^2\left(|\langle\phi_{qi}\rangle|^2 + \left|\langle\bar{\phi}_q^i\rangle\right|^2\right) + \frac{2}{L^2}\mathrm{tr}\left|\left[\langle\Phi\rangle,\ \langle\phi_q^{adj}\rangle\right]\right|^2 + V_{GSQCD}(\theta). \tag{32}$$

$V_{GSQCD}(\theta)$ is given by

$$\begin{aligned}
V_{GSQCD}(\theta) &\equiv V_{SQCD}(\theta) + V_{matter}^{adj}(\theta)\\
&= \frac{2N_F^{adj}-2}{\pi^2 L^4}\sum_{n=1}^{\infty}\sum_{i,j=1}^{N}\frac{1}{n^4}(\cos[n(\theta_i-\theta_j)] - \cos[n(\theta_i-\theta_j-\beta)])\\
&+ \frac{2N_F^{fd}}{\pi^2 L^4}\sum_{n=1}^{\infty}\sum_{i=1}^{N}\frac{1}{n^4}(2\cos(n\theta_i) - \cos[n(\theta_i-\beta)] - \cos[n(\theta_i+\beta)]),
\end{aligned} \tag{33}$$

where the first term in Eq. (33) stands for the contributions from the supersymmetric Yang-Mills theory and N_F^{adj} numbers of the massless adjoint matter [9].

Let us note that one cannot rotate $\langle\phi_q^{adj}\rangle$ into a diagonal form by utilizing the $SU(N)$ degrees of freedom because we have already used them to parametrize $\langle A_y\rangle$ as the diagonal form given by Eq. (3). The first and second terms in Eq. (32) are positive semi-definite. In order to minimize the second term in Eq. (32), $\langle\phi_q^{adj}\rangle$ have only a diagonal form. Then, it

[9] We have ignored the terms coming from the trilinear coupling of the chiral superfields, $\bar{Q}Q^{adj}Q$ by assuming that the coupling are of order g, hence $O(g^2)$ in the potential.

commutes with $\langle\Phi\rangle$ for any values of θ_i and yields the vanishing second term $[\langle\Phi\rangle,\ \langle\phi_q^{adj}\rangle] = 0$. Therefore, $\langle\phi_q^{adj}\rangle$ is undetermined in this approximation and parametrizes the flat direction of the potential.

In addition to $\langle\phi_q^{adj}\rangle$, the vacuum expectation values of ϕ_q and $\bar{\phi}_q$ can also parametrize the flat direction of the potential. If all the θ_i's take nonzero values, $\langle\phi_q\rangle = \langle\bar{\phi}_q\rangle = 0$ gives the vanishing first term in Eq. (32). In this case, there is no flat direction of the potential parametrized by $\langle\phi_q\rangle$ and $\langle\bar{\phi}_q\rangle$. This was the situation in the softly broken supersymmetric QCD. If some of θ_i's, however, take the values of zero, say, $\theta_k \neq 0 (k = 1, \cdots, l < N-1)$, the corresponding $\langle\phi_{qk}\rangle$ and $\langle\bar{\phi}_q^k\rangle$ can take arbitrary values in keeping the vanishing first term and parametrize the flat direction of the potential. In our approximation ignoring the $O(g^2)$ terms, the effective potential has the flat direction in general.

In this paper, we are interested in the dynamics of the nonintegrable phases, or one can say that we study the gauge symmetry breaking in this model at the trivial "point", where all the vacuum expectation values of the squark fields $\phi_q, \bar{\phi}_q, \phi_q^{adj}$ vanish. We ignore the tree-level potential, first and second terms in Eq. (32) and focus on the effective potential $V_{GSQCD}(\theta)$ only.

Here we notice that the effective potential $V_{GSQCD}(\theta)$ is reduced to $V_{matter}^{fd}(\theta)$ for $N_F^{adj} = 1$. The contributions from the vector multiplet $(A_{\hat{\mu}}, \lambda)$ and the massless adjoint multiplet (q^{adj}, ϕ_q^{adj}) to the constant background (3) cancel each other. This is because in four dimensions the two massless multiplets form $\mathcal{N} = 2$ supersymmetry to have the $SU(2)_R$ symmetry, so that we still have $\mathcal{N} = 1$ supersymmetry for the two multiplets even though we imposed the boundary conditions (2), (12) associated with the $U(1)_R$ symmetry [8]. As we have already studied in the previous section, the vacuum configuration for this special case is given by Eq. (27) from the potential $V_{matter}^{fd}(\theta)$ alone. The $SU(N)$ gauge symmetry is not broken for any values of N_F^{fd} and β. In order to avoid the cancellation, one needs to impose the boundary condition associated with the $SU(2)_R$ symmetry in addition to $U(1)_R$.

4.1 $SU(2)$ case

The effective potential (33) seems to have a simple form. It is, however, hard to study the vacuum configuration of the potential fully analytically. As we will show in the next subsection, the location of the minima of the potential changes according to the values of the phase β. The only exceptional case is the $SU(2)$ gauge group. The effective potential for the case of $SU(2)$ becomes

$$\begin{aligned} V_{GQSCD}(\theta) &= \frac{2N_F^{adj} - 2}{\pi^2 L^4} \sum_{n=1}^{\infty} \frac{1}{n^4} \Big(2(1 - \cos(n\beta)) \\ &+ 2\cos(2n\theta) - \cos[n(2\theta - \beta)] - \cos[n(2\theta + \beta)] \Big) \end{aligned}$$

$$+\quad \frac{2\times(2N_F^{fd})}{\pi^2L^4}\sum_{n=1}^{\infty}\frac{1}{n^4}(2\cos(n\theta)-\cos[n(\theta-\beta)]-\cos[n(\theta+\beta)]). \quad (34)$$

Let us note that the contributions from ϕ_q and $\bar{\phi}_q$ to the potential (34) have the same forms. This is because $\mathbf{2}$ and $\bar{\mathbf{2}}$ of $SU(2)$ are equivalent. The $SU(2)$ gauge group is special in this sense.

The potential (34) happens to be invariant under Eqs. (20) and (21), so that the region given by $\theta-\beta\geq 0$ is enough to study the potential, and we can apply the formula (22) to the effective potential. We obtain that

$$\begin{aligned} V_{GQSCD}(\theta) &= \frac{2N_F^{adj}-2}{\pi^2L^4}\frac{\beta^2}{24}\Big((\beta-2\pi)^2+24\theta^2-24\pi\theta+\beta^2+4\pi^2\Big) \\ &+ \frac{2\times(2N_F^{fd})}{\pi^2L^4}\frac{\beta^2}{24}\Big(6\theta^2-12\pi\theta+\beta^2+4\pi^2\Big). \end{aligned} \quad (35)$$

By solving the extremum condition $\partial V_{QSCD}/\partial\theta=0$, we have

$$\theta^{(1)}=\frac{N_F^{fd}+N_F^{adj}-1}{N_F^{fd}+2(N_F^{adj}-1)}\pi. \quad (36)$$

The other solution, which is obtained by taking into account the invariance of the potential under Eq. (21),

$$\theta^{(2)}=2\pi-\theta^{(1)}=\frac{N_F^{fd}+3(N_F^{adj}-1)}{N_F^{fd}+2(N_F^{adj}-1)}\pi \quad (37)$$

is not distinct from the solution $\theta^{(1)}$. The squark mass spectra on the solutions are identical each other due to Eq. (20) (and/or Eq. (21)). There is a doubly degenerate vacuum state. The vacuum configuration breaks the $SU(2)$ gauge symmetry to $U(1)$ spontaneously.

The second derivative of the effective potential at the minimum gives the mass of the adjoint Higgs scalar as we have stated in the section 3. We find that

$$m_\Phi^2\equiv(gL)^2\frac{\partial^2V_{GSQCD}}{\partial\theta^2}=\frac{2g^2\beta^2}{\pi^2L^2}\Big(2(N_F^{adj}-1)+N_F^{fd}\Big). \quad (38)$$

No massless state of the adjoint Higgs scalar appears except for $(N_F^{adj},N_F^{fd})=(1,0)$, whose flavor number corresponds to the aforementioned $\mathcal{N}=2$ supersymmetry in four dimensions.

4.2 $SU(3)$ case

Let us next consider the $SU(3)$ gauge group. Even in this case, we find interesting physics such as the partial gauge symmetry breaking and massless adjoint Higgs scalar, which is never observed in the models studied in Ref. [9] and the previous section.

In order to see that the vacuum configuration changes according to the values of β by the Scherk-Schwarz mechanism, we first assume that β is very small, but nonzero. After finding the vacuum configuration for the small values of β, we next study the vacuum configuration for $\beta = \pi$. The potential (33) for the case of $SU(3)$ is still invariant under Eq. (20), so that $0 < \beta \le \pi$ is relevant. Then, we compare the configurations for the two cases.

We may apply the formula (22) to the potential (33) for the small values of β. We obtain that

$$\begin{aligned} V_{GQSCD}(\theta) &= \frac{2N_F^{adj}-2}{\pi^2 L^4}\beta^2\left[\frac{N}{48}(\beta-2\pi)^2+\frac{N(N-1)}{48}(\beta^2+4\pi^2)\right. \\ &+ \left.\frac{N}{2}\left(\sum_{i=1}^{N-1}\theta_i^2+\sum_{1\le i<j\le N-1}\theta_i\theta_j\right)-\pi\sum_{i=1}^{N-1}(N-i)\theta_i\right] \\ &+ \frac{4N_F^{fd}}{\pi^2 L^4}\frac{\beta^2}{48}\left[12\left(\sum_{i=1}^{N-1}\theta_i^2+\left(\sum_{i=1}^{N-1}\theta_i\right)^2\right)-48\pi\sum_{i=1}^{N-1}\theta_i+N(\beta^2+4\pi^2)\right], \end{aligned} \tag{39}$$

where we have used the result obtained in Ref. [9] for the first term in Eq. (33). The extremum condition $\partial V_{GSQCD}/\partial\theta_k (k=1,\cdots,N-1)=0$ yields

$$\frac{1}{2\pi}\left(N(N_F^{adj}-1)+N_F^{fd}\right)(\theta_k+(\theta_1+\cdots+\theta_{N-1})) = N_F^{fd}+(N_F^{adj}-1)(N-k). \tag{40}$$

This is written in the form, denoting $d \equiv N(N_F^{adj}-1)+N_F^{fd}$,

$$\frac{d}{2\pi}\begin{pmatrix} 2 & 1 & \cdots & \cdots & 1 \\ 1 & 2 & & & \vdots \\ \vdots & & \ddots & & \vdots \\ \vdots & & & \ddots & \vdots \\ 1 & \cdots & \cdots & \cdots & 2 \end{pmatrix}\begin{pmatrix} \theta_1 \\ \theta_2 \\ \theta_3 \\ \vdots \\ \theta_{N-2} \\ \theta_{N-1} \end{pmatrix} = N_F^{fd}\begin{pmatrix} 1 \\ 1 \\ 1 \\ \vdots \\ 1 \\ 1 \end{pmatrix} + (N_F^{adj}-1)\begin{pmatrix} N-1 \\ N-2 \\ N-3 \\ \vdots \\ 2 \\ 1 \end{pmatrix}, \tag{41}$$

where the matrix of the left-hand side in Eq. (41) is the same with the one in Eq. (29). The inverse of the matrix is given by

$$\frac{1}{N}\begin{pmatrix} N-1 & -1 & \cdots & \cdots & -1 \\ -1 & N-1 & & & \vdots \\ \vdots & & \ddots & & \vdots \\ \vdots & & & \ddots & \vdots \\ -1 & \cdots & \cdots & \cdots & N-1 \end{pmatrix}. \tag{42}$$

All the (off-)diagonal elements of the matrix are $N-1(-1)$. The solution to Eq. (40) is, then, found to be

$$\theta_k = \frac{N_F^{fd}}{d}\frac{2\pi}{N}+\frac{(N_F^{adj}-1)}{d}\pi\big(N-(2k-1)\big), \qquad k=1,\cdots,N-1 \tag{43}$$

with

$$\theta_N = -\sum_{k=1}^{N-1}\theta_k = \frac{-2\pi}{d}\frac{N-1}{N}\left(N_F^{fd}+\frac{N}{2}(N_F^{adj}-1)\right). \tag{44}$$

These solutions become

$$(\theta_1,\theta_2)=\left(\frac{2}{3}\pi,\ \ \frac{N_F^{fd}}{3(N_f^{adj}-1)+N_F^{fd}}\frac{2\pi}{3}\right). \tag{45}$$

for the $SU(3)$ gauge group, which is of our interest. Except for the case of $N_F^{adj}=1$, the configuration breaks $SU(3)$ to $U(1)\times U(1)$. Therefore, for the small values of β, the gauge symmetry is maximally broken, which still holds for the $SU(N)$ gauge group. As an example, the solutions for certain values of N_F^{adj} and N_F^{fd} are given by

$$\begin{aligned}
(\theta_1,\theta_2) &= \left(\frac{2}{3}\pi,\frac{1}{6}\pi\right)\cdots(N_F^{adj},N_F^{fd})=(2,1),\\
&= \left(\frac{2}{3}\pi,\frac{4}{15}\pi\right)\cdots(N_F^{adj},N_F^{fd})=(2,2),\\
&= \left(\frac{2}{3}\pi,\frac{1}{3}\pi\right)\cdots(N_F^{adj},N_F^{fd})=(2,3).
\end{aligned} \tag{46}$$

Let us next study the vacuum configuration at $\beta=\pi$. The possible gauge symmetry breaking patterns are [10]

$$SU(3)\to\begin{cases} SU(3) & \cdots\ \ (\theta_1,\theta_2)=(\frac{2}{3}\pi,\frac{2}{3}\pi),\\ SU(2)\times U(1) & \cdots\ \ (\theta_1,\theta_2)=(\pi,0)+\text{permutations},\\ U(1)\times U(1) & \cdots\ \ (\theta_1,\theta_2)=(\frac{2}{3}\pi,0)+\text{permutations}.\end{cases} \tag{47}$$

By studying the determinant of the Hessian,

$$H_{ij}\equiv\left.\frac{\partial^2 V_{GSQCD}}{\partial\theta_i\partial\theta_j}\right|_{\beta=\pi} \tag{48}$$

and comparing the potential energy for the given gauge symmetry breaking pattern (47), we know the position and stability of the global minima of the effective potential. And at the same time, as we will see later, the matrix gives the information on the mass of the adjoint Higgs scalar at $\beta=\pi$. Depending on the numbers of flavors N_F^{adj}, N_F^{fd}, the gauge symmetry breaking patters are different. We obtain [11]

$$\begin{aligned}
0<N_F^{fd}\leq\frac{3}{7}(N_F^{adj}-1) \quad &\cdots(\theta_1,\theta_2)=\left(\frac{2}{3}\pi,0\right)+\text{permutations},\\
(N_F^{adj}-1)<N_F^{fd}\leq 9(N_F^{adj}-1) \quad &\cdots(\theta_1,\theta_2)=(\pi,0)+\text{permutations},\\
9(N_F^{adj}-1)<N_F^{fd} \quad &\cdots(\theta_1,\theta_2)=\left(\frac{2}{3}\pi,\frac{2}{3}\pi\right).
\end{aligned} \tag{49}$$

The vacuum configuration at $\beta=\pi$ corresponding to our example (46) is given by $(\theta_1,\theta_2)=(\pi,0)$ and its permutations, for which the residual gauge symmetry is $SU(2)\times U(1)$. Therefore, we observe that the vacuum configuration changes according to the

[10] We have confirmed that the configurations given by $(\theta_1,\theta_2)=(0,0),(\pi/3,\pi/3)$ do not alter our discussions.

[11] The configuration for the region $3(N_F^{adj}-1)/7\leq N_F^{fd}<N_F^{adj}-1$ is not given by $(\theta_1,\theta_2)=(\pi,0)$, but is close to it and respects $U(1)\times U(1)$ symmetry.

values of the phase β. The configuration in Eq. (46) starts to change as β becomes large, keeping $U(1)\times U(1)$ symmetry, and arrive at $(\theta_1,\theta_2)=(\pi,0)$ at $\beta=\pi$, where $SU(2)\times U(1)$ symmetry is realized [12].

What is interesting is that the gauge symmetry breaking pattern $SU(3)\to SU(2)\times U(1)$ cannot be realized until one considers the softly broken suppersymmetric QCD with the massless adjoint matter. Actually, as we have studied in the previous section, the gauge symmetry breaking pattern in the softly broken supersymmetric QCD and Yang-Mills theory is $SU(N)\to SU(N)$ and that in the softly broken supersymmetric gauge theory with only the massless adjoint matter is $SU(N)\to U(1)^{N-1}$ [9]. This partial gauge symmetry breaking has been pointed out in the nonsupersymmetric gauge theory with both of the massless adjoint and fundamental matter [15].

If we change the number of the flavors, the vacuum configuration at $\beta=\pi$ also changes. For $(N_F^{adj},N_F^{fd})=(4,1)$, the vacuum configuration is given by $(\theta_1,\theta_2)=(2\pi/3,\pi/15)$ from Eq. (45) for the small values of β, while at $\beta=\pi$, taking account of Eq. (49), it is given by $(\theta_1,\theta_2)=(2\pi/3,0)$. The configuration at $\beta=\pi$ still respects $U(1)\times U(1)$ symmetry though the configurations themselves are different for the two cases.

The above observation implies that if N_F^{adj} increases, then, the first term in Eq. (33) dominates in the effective potential. The vacuum configuration tends to realize the maximal breaking of $SU(3)$. This is consistent with the result that the dynamics of the nonintegrable phases for the massless adjoint matter always results the maximal breaking of $SU(N)$, i.e., $U(1)^{N-1}$ [9]. If we, instead, increase N_F^{fd} for fixed N_F^{adj}, the vacuum configuration tends toward the original gauge symmetry. This is because the second term in Eq. (33) dominates in the effective potential for large number of N_F^{fd}, and the potential has the $SU(N)$ symmetric vacuum as we have studied in the section 3.

Let us finally discuss the massless state of the adjoint Higgs scalar. To this end, we study the determinant of the Hessian for the configuration $(\theta_1,\theta_2)=(\pi,0)$,

$$\det H\Big|_{\beta=\pi}=\Big(N_F^{fd}-(N_F^{adj}-1)\Big)\Big(9(N_F^{adj}-1)-N_F^{fd}\Big)\,. \qquad (50)$$

The determinant vanishes for the case $N_F^{fd}=N_F^{adj}-1$ or $N_F^{fd}=9(N_F^{adj}-1)$ except for the aforementioned $\mathcal{N}=2$ supersymmetry. The conditions are satisfied without any fine-tuning of the parameters as long as N_F^{adj} and N_F^{fd} are discrete numbers. In our example, $(N_F^{adj},N_F^{fd})=(2,1)$ satisfies the former condition. The vanishing determinant implies that the Hessian contains the massless mode, which is nothing but the massless adjoint Higgs scalar in our approximation [13]. The massless state of the adjoint Higgs scalar has

[12] The gauge symmetry breaking pattern becomes $SU(3)\to SU(2)$ for the configuration $(\theta_1,\theta_2)=(\pi,0)$ if we consider the nonzero values of $\langle\phi_{q2}\rangle,\langle\bar{\phi}_q^2\rangle$.

[13] This vanishing determinant is modified if we consider the nonzero values of the vacuum expectation values for the squark fields.

also been pointed out in the nonsupersymmetric gauge theories [15].

For comparison to the case of $\beta = \pi$, let us evaluate the second derivative of the effective potential (33) for the small values of β. The vacuum configuration in this case is given by Eq. (43) and breaks the $SU(N)$ gauge symmetry to $U(1)^{N-1}$ spontaneously. The second derivative is calculated, using (39), as

$$\frac{\partial^2 V_{GSQCD}}{\partial\theta_i\partial\theta_j} = \frac{C_H^{GSQCD}}{\pi^2 L^4} M_{ij} \qquad C_H^{GSQCD} \equiv \beta^2 \left(N(N_F^{adj}-1) + N_F^{fd}\right), \tag{51}$$

where M_{ij} is given by Eq. (29). The matrix does not have the zero eigenvalue, and the coefficient C_H^{GSQCD} never vanishes except for the aforementioned $\mathcal{N}=2$ supersymmetry. Therefore, the adjoint Higgs scalar for the small values of β is always massive and cannot be massless.

5 Conclusions and discussion

We have studied the gauge symmetry breaking patterns through the Hosotani mechanism (the dynamics of the nonintegrable phases) in the supersymmetric QCD with N_F^{fd} numbers of the massless fundamental matter and its generalized version by introducing N_F^{adj} numbers of the massless adjoint matter. The supersymmetry is broken softly by the Scherk-Schwarz mechanism to give the nonvanishing effective potentials for the phases.

We have first studied the softly broken supersymmetric Yang-Mills theory. The $SU(N)$ gauge symmetry is not broken, and there are N vacuum states given by Eq. (17). The N vacua are physically equivalent, Z_N symmetric and are related each other by the gauge transformation with Eq. (18). The fields A_μ, λ stay in massless on the vacuum configuration.

By introducing N_F^{fd} sets of the massless fundamental matter multiplet, we have obtained the softly broken supersymmetric QCD with N_F^{fd} flavors. The $SU(N)$ gauge symmetry is not broken again in this model, but the vacuum configuration itself depend on the number of color N. For $N=$ even, there is a single vacuum state, while for $N=$ odd, there is a doubly degenerate vacuum state. The symmetry transformations with Eqs. (20) and (21) of the effective potential relate the degenerate two vacua. The Z_2 symmetry is broken by the massless matter multiplet belonging to the (anti)fundamental representation under $SU(N)$. The adjoint Higgs scalar is always massive in the two models except for the case of the accidental $\mathcal{N}=2$ supersymmetry in four dimensions.

We have also discussed the gauge symmetry breaking patterns in the generalized version of supersymmetric QCD (supersymmetric QCD with the massless adjoint matter). We have first studied the case of $SU(2)$ and found the vacuum configuration given by Eq. (36), which breaks the $SU(2)$ gauge symmetry to $U(1)$ spontaneously. There is no massless state of the adjoint Higgs scalar in this case.

In order to see how the gauge symmetry is broken through the Hosotani mechanism for higher rank gauge group, we have considered the $SU(3)$ gauge group and chosen the appropriate numbers of the flavors as a demonstration. The vacuum configuration changes according to the values of the supersymmetry breaking parameter β by the Scherk-Schwarz mechanism. We have explicitly shown that the vacuum configurations for small values of β and $\beta = \pi$ are given by the different configurations, which realize the different gauge symmetry breaking patterns. It is possible to have the gauge symmetry pattern such as $SU(3) \to SU(2) \times U(1)$ for the choice given by Eq. (46) at $\beta = \pi$. This symmetry breaking pattern is peculiar to the model and is never observed in the models studied in Ref. [9] and the section 3.

We have investigated the massless state of the adjoint Higgs scalar by studying the determinant of the Hessian (48) for the small values of β and $\beta = \pi$. We have shown that the massless adjoint Higgs scalar is impossible for the small values of β. At $\beta = \pi$, however, we have obtained the condition for the vanishing determinant of the Hessian without any fine-tuning, which implies the existence of the massless adjoint Higgs scalar in our approximation. And we have given the explicit example of the parameter choices for the massless state. It seems that in order to have the massless adjoint Higgs, the partial gauge symmetry breaking such as $SU(3) \to SU(2) \times U(1)$ is necessary. Hence, the massless state is specific feature to the generalized version of the softly broken supersymmetric QCD.

We have also discussed the tendency of gauge symmetry breaking pattern at $\beta = \pi$ by varying the number of the flavor in the generalized supersymmetric QCD. If the number of the massless adjoint matter N_F^{adj} increases for fixed number of the fundamental matter N_F^{fd}, the gauge symmetry breaking patterns tend toward the maximal breaking of the original gauge symmetry, say, $U(1) \times U(1)$ in our example. On the other hand, if N_F^{fd} increases for fixed N_F^{adj}, it does toward the vacuum configuration respecting the original gauge symmetry, $(\theta_1, \theta_2) = (\frac{2}{3}\pi, \frac{2}{3}\pi)$ in our example.

It may be interesting to ask what gauge symmetry pattern is realized if we consider the higher rank gauge group such as $SU(5)$ in the generalized supersymmetric QCD. Taking into account the lessons in this paper, one has to select carefully the numbers of flavors N_F^{adj}, N_F^{fd} in order to realize the partial gauge symmetry breaking such as $SU(5) \to SU(3) \times SU(2) \times U(1)$, which may be relevant to the mechanism of GUT symmetry breaking. We need further studies in order to determine the gauge symmetry breaking patterns for the higher rank gauge group in the model. This will be reported elsewhere.

We have assumed the gauge coupling g is very small and ignored the $O(g^2)$ terms in the effective potential. In this approximation there exists the flat direction of the potential parametrized by the vacuum expectation values of the squark fields, namely $\langle \phi_q^{adj} \rangle$. We have chosen the trivial "point" corresponding to the vanishing vacuum expectation values of them, and we have studied the gauge symmetry breaking patterns through the dynamics of the nonintegrable phases alone. In order to determine the whole vacuum structure, one needs to take into account the ignored $O(g^2)$ term including the tree-level potential and one-loop corrections to the vacuum expectation values of the squark fields.

We have implicitly assumed the mass term for the squarks, from which we have defined the boundary condition of the squark field associated with the $U(1)_R$ symmetry. We have taken the massless limit in order to study the gauge symmetry breaking patterns. It is expected that the nonvanishing mass term may also influence the gauge symmetry breaking [16]. It is important and interesting to study the effect of the mass term on the Hosotani mechanism.

Acknowledgements

The author would like to thank the Dublin Institute for Advanced Study for warm hospitality.

References

[1] Y. Hosotani, Phys. Lett. **B126**, 309 (1983).

[2] Y. Hosotani, Ann. Phys. (N.Y.) **190**, 233 (1989).

[3] A. T. Davies and A. McLachlan, Phys. Lett. **B200**, 305 (1988); Nucl. Phys. **B317**, 237 (1989).

[4] J. E. Hetrick and C. L. Ho, Phys. Rev. **D40**, 4085 (1989).

[5] A. Higuchi and L. Parker, Phys. Rev. **D37**, 2853 (1988).

[6] C. L. Ho and Y. Hosotani, Nucl. Phys. **B345**, 445 (1990).

[7] A. McLachlan, Nucl. Phys. **B338**, 188 (1990).

[8] K. Takenaga, Phys. Rev. **D58**, 026004 (1998); **61**, 129902(E) (2000).

[9] K. Takenaga, Phys. Rev. **D64**, 066001 (2001).

[10] G. V. Gersdorff, M. Quiros and A. Riotto, hep-th/0204041.

[11] J. Scherk and J.H. Schwarz, Phys. Lett. **B82**, 60 (1979).

[12] P. Fayet, Phys. Lett. **B159**, 121 (1985); Nucl. Phys. **B263**, 87 (1986).

[13] K. Takenaga, Phys. Lett. **B425**, 114 (1998).

[14] H. Hatanaka, K. Ohnishi, M. Sakamoto and K.Takenaga, hep-th/0111183, to appear in Prog. Theor. Phys. **107**, (2002) and in preparation.

[15] H. Hatanaka, Prog. Theor. Phys. **102**, 407 (1999).

[16] Y. Hosotani, Phys. Lett. **B129**, 193 (1983).