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Quantization of Neveu-Schwarz-Ramond Superstring Model in 10+2-dimensional Spacetime

Takuya Tsukioka* and Yoshiyuki Watabiki†

*School of Theoretical Physics, Dublin Institute for Advanced Studies,
10 Burlington Road, Dublin 4, Ireland
tsukioka@synge.stp.dias.ie

and

†Department of Physics, Tokyo Institute of Technology,
Oh-okayama, Meguro, Tokyo 152-8551, Japan
watabiki@th.phys.titech.ac.jp

Abstract

We construct a Neveu-Schwarz-Ramond superstring model which is invariant under supersymmetric $\mathrm{U}(1)_{\mathrm{V}}\times\mathrm{U}(1)_{\mathrm{A}}$ gauge transformations as well as the super-general coordinate, the super local Lorentz and the super-Weyl transformations on the string world-sheet. We quantize the superstring model by covariant BRST formulation *á la* Batalin and Vilkovisky and noncovariant light-cone gauge formulation. Upon the quantizations the model turns out to be formulated consistently in 10+2-dimensional background spacetime involving two time dimensions.

1 Introduction

It is the purpose of this paper to cast some further light upon constructions of theories involving two or more time dimensions. We propose an explicit Lagrangian description to the end from the viewpoint of string theory. It might be a clue for understanding the origin of time and spacetime itself to consider the physics in which two or more time coordinates are introduced.

From the point of view of the string unification, the relations between string theories in various dimensions have been studied and it was also conjectured that all of these string theories were regarded as different phases of an underlying theory in higher-dimensional spacetime. Meanwhile, the idea of extra time dimensions, which might be hidden dimensions, was suggested and studied. In this context, several unitary theories formulated in spacetime with extra time coordinates were investigated from various viewpoints [1–13], such as super p-brane scanning [1], $N = 2$ heterotic string theories [2], the perspective for F-theory [3, 4], two-time physics [5–8], 12-dimensional super Yang-Mills and supergravity theories [9], super (2,2)-brane [10] and superalgebraic analysis [11].

The study in this paper is focused on a model which is constructed in spacetime involving two time dimensions, although our idea for introducing extra time dimensions might be applied to formulate other theories involving more than two time dimensions. In particular, we would like to investigate a superstring model which is consistently formulated in 10+2-dimensional background spacetime. Our approach might make some connections to other models [1–13] from more fundamental and unified point of view.

Some years ago, one of the authors (Y.W.) had proposed a model which has a $\mathrm{U(1)_V} \times \mathrm{U(1)_A}$ gauge symmetry in two-dimensional spacetime and also applied the idea to string theories [14, 15]. The striking feature of these models is that extra negative norm states appear besides usual ones and these are removed by the quantization procedure as the same as string theories. This fact suggests two time coordinates might be introduced in the background spacetime. For the $\mathrm{U(1)_V}\times\mathrm{U(1)_A}$ bosonic string model, we explicitly carried out the quantization by covariant BRST and noncovariant light-cone gauge formulations and showed the critical dimension was 26+2 including two time dimensions [16].

This paper is a continuation of our work [16]. We wish to introduce supersymmetry into our previous $\mathrm{U(1)_V} \times \mathrm{U(1)_A}$ bosonic string model. The extensions are considered by two ways *i.e.* by introducing the supersymmetry on the string world-sheet (Neveu-Schwarz-Ramond model) and on the background spacetime (Green-Schwarz model). In this paper we focus our attentions on the $U(1)_V \times U(1)_A$ NSR superstring model. We propose an explicit Lagrangian description of the supersymmetric model by using the superspace formulation [17] and study the quantization. A subject for the Green-Schwarz superstring model based on our framework will be discussed in an additional work elsewhere [18].

The $U(1)_V \times U(1)_A$ superstring model is constructed as gauge field theory on two-dimensional world-sheet. Although the similar models were investigated in refs. [6, 12], an advantage of the formulation of our model is its manifest covariant expression in the background spacetime by using the $U(1)_V \times U(1)_A$ gauge symmetry, so that we can easily carry out the quantization with preserving the covariance. An obtained gauge-fixed action might be useful for the perturbation theory. That is the $U(1)_V \times U(1)_A$ gauge symmetry is essential in our model. In the formulation, the generalized Chern-Simons action [19] proposed by Kawamoto and one of the authors (Y.W.) as a new type of topological action plays an important key role. In fact, the generalized Chern-Simons action is introduced for the action to be covariant.

As we mentioned in the previous paper [16], there are two remarks for the quantization. These are also inherited to our superstring model. Firstly the action has a reducible symmetry which originally arises from gauge structures of the generalized Chern-Simons action [20]. Secondly the gauge algebra is open. In the covariant BRST quantization of the system including reducible and open gauge symmetry, we need to use the formulation developed by Batalin and Vilkovisky [21]. By adopting this method we explicitly show the covariant quantization is successfully carried out in the Lagrangian formulation.

In order to treat the dynamics of our model more directly, we also quantize the same model in noncovariant light-cone gauges. The suitable noncovariant gauge conditions can be imposed by residual symmetries of the supersymmetric $U(1)_V \times U(1)_A$ gauge symmetry and we can then solve all of the gauge constraints explicitly. We can also confirm that the existence of two time coordinates is not in conflict with the unitarity of the theory, since these are required by our "gauge" symmetry.

As an important feature of quantum string models, one can argue the critical dimension of the background spacetime [22–24]. In usual superstring theories, the critical dimension is 9+1 [25]. For our superstring model, the critical dimension turns out to be 10+2. We obtain this result directly from both the BRST and the noncovariant light-cone gauge formulations.

This paper is organized as follows: The brief review of the $U(1)_V \times U(1)_A$ bosonic string model is provided in Section 2. Then, we introduce the $\mathrm{U}(1)_{\mathrm{V}}\times\mathrm{U}(1)_{\mathrm{A}}$ superstring model involving $N=1$ supersymmetry on the world-sheet in Section 3. In this section, symmetries and semiclassical aspects of the $\mathrm{U}(1)_{\mathrm{V}}\times\mathrm{U}(1)_{\mathrm{A}}$ superstring model are explained. The covariant quantization for the model based on the Lagrangian formulation is presented in Section 4. In this section we investigate perturbative aspects of the quantized model and determine the critical dimension of our $\mathrm{U}(1)_{\mathrm{V}}\times\mathrm{U}(1)_{\mathrm{A}}$ superstring model. In Section 5, the quantization under noncovariant light-cone gauge fixing conditions is carried out. We then study the symmetry of the background spacetime and obtain the same critical dimension by direct computation of the full quantum Poincaré algebra. We also present a mass-shell relation of the model and discuss low energy quantum states. Conclusions and discussions are given in the final section. Appendixes A, B and C contain our notational conventions.

2 $\mathrm{U}(1)_{\mathrm{V}}\times\mathrm{U}(1)_{\mathrm{A}}$ bosonic string model

The $\mathrm{U}(1)_{\mathrm{V}}\times\mathrm{U}(1)_{\mathrm{A}}$ bosonic string model [14–16] described by two-dimensional field theory consists of scalar fields $\xi^I(x)$, $\phi^I(x)$ and $\bar{\phi}^I(x)$, gauge fields $A_m(x)$, $B^I_m(x)$ and $\tilde{C}(x)$ and the metric $g_{mn}(x)$. We shall consider closed string theories throughout this paper. The D scalar fields $\xi^I(x)$ are considered to be string coordinates in D-dimensional flat background spacetime with the metric:

$$\eta_{IJ} = \eta^{IJ} = \begin{cases} -1 & (I=J=0) \\ 1 & (I=J=i,\ \ i=1,\,2,\,\ldots,\,D-3) \\ -1 & (I=J=\hat{0}) \\ 1 & (I=J=\hat{1}) \\ 0 & (\text{otherwise}) \end{cases} \tag{2.1}$$

The indices I and J run through $0,\,1,\,2,\,\ldots,\,D-3,\,\hat{0},\,\hat{1}$. As we will explain, the unitarity of the theory requires two negative signatures to the background metric η_{IJ} (2.1), because the $\mathrm{U}(1)_{\mathrm{A}}$ gauge transformation as well as the general coordinate transformations removes a negative norm state.

The covariant action of the bosonic $\mathrm{U}(1)_{\mathrm{V}}\times\mathrm{U}(1)_{\mathrm{A}}$ string model [16] is

$$\begin{aligned} S = \int \mathrm{d}^2x\,\sqrt{-g}\,\Big\{ &-\frac{1}{2}g^{mn}\partial_m\xi^I\partial_n\xi_I - g^{mn}\partial_m\bar{\phi}^I\partial_n\phi_I \\ &+ \tilde{A}^m\phi_I\partial_m\xi^I + \tilde{B}^{mI}\partial_m\phi_I - \frac{1}{2}\tilde{C}\phi^I\phi_I\Big\}, \end{aligned} \tag{2.2}$$

where we denote

$$\sqrt{-g}\tilde{A}^m = \varepsilon^{mn}A_n, \quad \sqrt{-g}\tilde{B}^{mI} = \varepsilon^{mn}B^I_n, \quad \sqrt{-g}\tilde{C} = \frac{1}{2}\varepsilon^{mn}C_{mn},$$

and $g(x) = \det g_{mn}(x)$. The last two terms in (2.2) arise from the generalized Chern-Simons action [19] formulated in two-dimensional spacetime. The action (2.2) is invariant under the following gauge transformations including $\mathrm{U(1)_V \times U(1)_A}$ gauge transformations [16],

$$\begin{aligned}
\delta\xi^I &= v'\phi^I,\\
\delta\tilde{A}^m &= \frac{\varepsilon^{mn}}{\sqrt{-g}}\partial_n v + g^{mn}\partial_n v',\\
\delta\phi^I &= 0,\\
\delta\bar{\phi}^I &= u'^I,\\
\delta\tilde{B}^{mI} &= -v\frac{\varepsilon^{mn}}{\sqrt{-g}}\partial_n\xi^I + v'g^{mn}\partial_n\xi^I + \frac{\varepsilon^{mn}}{\sqrt{-g}}\partial_n u^I + g^{mn}\partial_n u'^I - \tilde{w}^m\phi^I,\\
\delta\tilde{C} &= \partial_m v'\tilde{A}^m - v'\nabla_m\tilde{A}^m + \nabla_m\tilde{w}^m,\\
\delta g_{mn} &= 0.
\end{aligned} \tag{2.3}$$

The parameters $\big(v(x), u^I(x)\big)$ and $\big(v'(x), u'^I(x)\big)$ correspond to the vector U(1) transformations "$\mathrm{U(1)_V}$" and the axial vector U(1) transformations "$\mathrm{U(1)_A}$", respectively. Although the scalar field $\bar{\phi}^I(x)$ might be gauged away by using the gauge degree of freedom for $u'^I(x)$, we leave this gauge degree of freedom in order to keep the $\mathrm{U(1)_V \times U(1)_A}$ gauge structure. The gauge transformations corresponding to the gauge parameters $u^I(x)$ and $\tilde{w}^m(x) = \varepsilon^{mn}w_n(x)/\sqrt{-g(x)}$ originally come from the generalized Chern-Simons theory [19]. The action (2.2) is also invariant under the general coordinate and the Weyl transformations

$$\begin{aligned}
\delta\xi^I &= k^n\partial_n\xi^I,\\
\delta\tilde{A}^m &= k^n\partial_n\tilde{A}^m - \partial_n k^m\tilde{A}^n + 2s\tilde{A}^m,\\
\delta\phi^I &= k^n\partial_n\phi^I,\\
\delta\bar{\phi}^I &= k^n\partial_n\bar{\phi}^I,\\
\delta\tilde{B}^{mI} &= k^n\partial_n\tilde{B}^{mI} - \partial_n k^m\tilde{B}^{nI} + 2s\tilde{B}^{mI},\\
\delta\tilde{C} &= k^n\partial_n\tilde{C} + 2s\tilde{C},\\
\delta g_{mn} &= k^l\partial_l g_{mn} + \partial_m k^l g_{ln} + \partial_n k^l g_{ml} - 2s g_{mn},
\end{aligned} \tag{2.4}$$

where $k^n(x)$ is a parameter for the general coordinate transformation and $s(x)$ is a scaling parameter for the Weyl transformation. The transformations (2.3) and (2.4) are all local.

It is worth to mention about some algebraic structures of the symmetry. The first is a reducibility of the symmetry. The system is on-shell reducible because the gauge transformations (2.3) have on-shell invariance under the following transformations of the gauge parameters with a reducible parameter $w'(x)$,

$$\begin{aligned} \delta' u^I &= w'\phi^I, \\ \delta' \tilde{w}^m &= \frac{\varepsilon^{mn}}{\sqrt{-g}}\partial_n w'. \end{aligned} \tag{2.5}$$

Since the transformations (2.5) are not reducible anymore, the action (2.2) is called a first-stage reducible system. The second is that the gauge algebra is open. This means that the gauge algebra closes only when the equations of motion are satisfied. Actually, a direct calculation of the commutator of two gauge transformations on $\tilde{B}^{mI}(x)$ leads to

$$[\delta_1, \delta_2]\tilde{B}^{mI} = \cdots - (v_1' v_2 - v_2' v_1)\frac{\varepsilon^{mn}}{\sqrt{-g}}\partial_n \phi^I,$$

where the dots $(\cdots)$ contain terms of the usual "structure constants" of the gauge algebra.

In addition to the gauge symmetries (2.3) and (2.4), the action (2.2) is invariant under the following global transformations,

$$\begin{aligned} \delta\xi^I &= \omega^I{}_J \xi^J + a^I, \\ \delta\tilde{A}^m &= r\tilde{A}^m + \sum_{i=1}^{2g} \alpha_i h^{(i)m}, \\ \delta\phi^I &= -r\phi^I + \omega^I{}_J \phi^J, \\ \delta\bar{\phi}^I &= r\bar{\phi}^I + \omega^I{}_J \bar{\phi}^J, \\ \delta\tilde{B}^{mI} &= r\tilde{B}^{mI} + \omega^I{}_J \tilde{B}^{mJ} + \sum_{i=1}^{2g}(\beta_i^I + \alpha_i \xi^I) h^{(i)m}, \\ \delta\tilde{C} &= 2r\tilde{C}, \\ \delta g_{mn} &= 0, \end{aligned} \tag{2.6}$$

where the parameters $\omega_{IJ} = -\omega_{JI}$, a^I and r are global parameters for the D-dimensional Lorentz transformation, the translation and the scale transformation, respectively. The functions $h^{(i)m}(x)$ are harmonic functions which satisfy $\nabla_m h^{(i)m}(x) = \varepsilon^{mn}\nabla_m h_n^{(i)}(x) = 0$ ($i = 1, 2, \ldots, 2g$; g = genus of two-dimensional spacetime) and α_i and β_i^I are global parameters.

As we have shown in ref. [16], the critical dimension of our bosonic string model (2.2) is

$$D = 28. \tag{2.7}$$

This means the quantum $\mathrm{U(1)_V \times U(1)_A}$ bosonic string theory is consistently formulated in 26+2-dimensional spacetime involving two time coordinates. The observation was directly obtained from both BRST quantization based on Batalin-Fradkin-Vilkovisky formulation and non-covariant light-cone quantization [16]. It would be interesting to extend the model by introducing world-sheet supersymmetry as we will explain in this paper.

3 $\mathrm{U(1)_V \times U(1)_A}$ NSR superstring model

In this section we construct a $\mathrm{U(1)_V \times U(1)_A}$ string model which holds $N = 1$ supersymmetry on the world-sheet *i.e.* Neveu-Schwarz-Ramond (NSR) type superstring model. In order to formulate supersymmetric theory, we use the $(1,1)$ type superspace with coordinates $z^M = (x^m,\ \theta^\mu)$, $(m = 0,1;\ \mu = 1,2)$ where θ^μ are fermionic coordinates [17]. The geometry of the superspace is given in Appendix B. Field variables of two-dimensional supergravity are a vielbein $E_M{}^A(z)$ and a connection $\Omega_M(z)$. In particular, we impose kinematic constraints on torsion components which are also explained in Appendix B.

We begin by introducing superfields $\Xi^I(z)$, $\Phi^I(z)$, $\bar\Phi^I(z)$, $\tilde\Psi^\alpha(z) \equiv (\bar\sigma\Psi(z))^\alpha$, $\tilde\Pi^{\alpha I}(z) \equiv (\bar\sigma\Pi^I(z))^\alpha$ and $\tilde\Lambda(z) \equiv -\frac{1}{2}(\bar\sigma)^{\alpha\beta}\Lambda_{\alpha\beta}(z)$, instead of the fields $\xi^I(x)$, $\phi^I(x)$, $\bar\phi^I(x)$, $\tilde A^m(x)$, $\tilde B^{mI}(x)$ and $\tilde C(x)$, respectively. The superfields $\Xi^I(z)$, $\Phi^I(z)$, $\bar\Phi^I(z)$ and $\tilde\Lambda(z)$ are bosonic scalar superfields, while $\tilde\Psi^\alpha(z)$ and $\tilde\Pi^{\alpha I}(z)$ are fermionic spinor superfields on the world-sheet. A covariant action for the $\mathrm{U(1)_V \times U(1)_A}$ NSR superstring model is then given by the similar form to the bosonic string action (2.2),

$$S = \int \mathrm{d}^2x\mathrm{d}^2\theta E\Big\{-\frac{1}{2}\mathcal{D}^\alpha\Xi^I\mathcal{D}_\alpha\Xi_I - \mathcal{D}^\alpha\bar\Phi^I\mathcal{D}_\alpha\Phi_I + \tilde\Psi^\alpha\Phi_I\mathcal{D}_\alpha\Xi^I + \tilde\Pi^{\alpha I}\mathcal{D}_\alpha\Phi_I - \frac{1}{2}\tilde\Lambda\Phi^I\Phi_I\Big\}. \tag{3.1}$$

The last two terms in (3.1) are constructed from the supersymmetric generalized Chern-Simons action [15].

The action (3.1) is invariant under the following local supersymmetric $\mathrm{U(1)_V \times U(1)_A}$ transformations,

$$\begin{aligned}
\delta\Xi^I &= V'\Phi^I,\\
\delta\tilde\Psi_\alpha &= (\bar\sigma\mathcal{D})_\alpha V + \mathcal{D}_\alpha V',\\
\delta\Phi^I &= 0,\\
\delta\bar\Phi^I &= U'^I, \qquad\qquad (3.2)\\
\delta\tilde\Pi^I_\alpha &= -V(\bar\sigma\mathcal{D})_\alpha\Xi^I + V'\mathcal{D}_\alpha\Xi^I + (\bar\sigma\mathcal{D})_\alpha U^I + \mathcal{D}_\alpha U'^I - \tilde W_\alpha\Phi^I,
\end{aligned}$$

$$\delta\tilde{\Lambda} = \mathcal{D}^\alpha V'\tilde{\Psi}_\alpha - V'\mathcal{D}^\alpha\tilde{\Psi}_\alpha + \mathcal{D}^\alpha\tilde{W}_\alpha,$$
$$\delta E_M{}^A = \delta\Omega_M = 0,$$

where the gauge parameters $V(z)$, $V'(z)$, $U^I(z)$ and $U'^I(z)$ are bosonic scalar superfields and $\tilde{W}_\alpha(z)$ is a fermionic spinor superfield. The parameters $\big(V(z), U^I(z)\big)$ and $\big(V'(z), U'^I(z)\big)$ correspond to the supersymmetric versions of the vector U(1) transformations "$\mathrm{U(1)_V}$" and the axial vector U(1) transformations "$\mathrm{U(1)_A}$", respectively. Again, we leave the gauge degree of freedom for the parameter $U'^I(z)$ as well as we did in the previous section. If this gauge degree of freedom is gauged away, the model turns to be the same one which we have discussed in the previous work [15]. The parameters $U^I(z)$ and $\tilde{W}_\alpha(z) \equiv (\bar{\sigma}W(z))_\alpha$ are related with the symmetries of the supersymmetric generalized Chern-Simons action [15].

The action (3.1) is also invariant under the super-general coordinate, the super local Lorentz and the super-Weyl scaling transformations,

$$\begin{aligned}
\delta\Xi^I &= K^N\partial_N\Xi^I, \\
\delta\tilde{\Psi}_\alpha &= K^N\partial_N\tilde{\Psi}_\alpha - \frac{1}{2}L(\bar{\sigma})_\alpha{}^\beta\tilde{\Psi}_\beta + \frac{1}{2}S\tilde{\Psi}_\alpha, \\
\delta\Phi^I &= K^N\partial_N\Phi^I, \\
\delta\bar{\Phi}^I &= K^N\partial_N\bar{\Phi}^I, \\
\delta\tilde{\Pi}^I_\alpha &= K^N\partial_N\tilde{\Pi}^I_\alpha - \frac{1}{2}L(\bar{\sigma})_\alpha{}^\beta\tilde{\Pi}^I_\beta + \frac{1}{2}S\tilde{\Pi}^I_\alpha, \\
\delta\tilde{\Lambda} &= K^N\partial_N\tilde{\Lambda} + S\tilde{\Lambda}, \\
\delta E_M{}^a &= K^N\partial_N E_M{}^a + \partial_M K^N E_N{}^a + E_M{}^b L\varepsilon_b{}^a - SE_M{}^a, \\
\delta E_M{}^\alpha &= K^N\partial_N E_M{}^\alpha + \partial_M K^N E_N{}^\alpha + \frac{1}{2}E_M{}^\beta L(\bar{\sigma})_\beta{}^\alpha - \frac{1}{2}SE_M{}^\alpha + \frac{i}{2}E_M{}^a(\sigma_a)^{\alpha\beta}\mathcal{D}_\beta S, \\
\delta\Omega_M &= K^N\partial_N\Omega_M + \partial_M K^N\Omega_N + \partial_M L + E_M{}^a\varepsilon_a{}^b\mathcal{D}_b S + E_M{}^\alpha(\bar{\sigma})_\alpha{}^\beta\mathcal{D}_\beta S,
\end{aligned} \tag{3.3}$$

where the superfields $K^N(z)$, $L(z)$ and $S(z)$ are gauge parameters for the super-general coordinate, the super local Lorentz and the super-Weyl scaling transformations, respectively.

Some algebraic structures of the gauge symmetry which we mentioned in the bosonic string model are also inherited to the superstring model. The gauge transformations (3.2) have on-shell invariance under the following transformations of the gauge parameters with a reducible scalar superfield parameter $W'(z)$,

$$\begin{aligned}
\delta' U^I &= W'\Phi^I, \\
\delta'\tilde{W}_\alpha &= (\bar{\sigma})_\alpha{}^\beta\mathcal{D}_\beta W'.
\end{aligned} \tag{3.4}$$

The gauge algebra is also open in the superstring model. Actually, a direct calculation of the commutator of two gauge transformations on $\tilde{\Pi}^I_\alpha(z)$ leads to

$$[\delta_1, \delta_2]\tilde{\Pi}^I_\alpha = \cdots - (V_1'V_2 - V_2'V_1)(\bar{\sigma})_\alpha{}^\beta \mathcal{D}_\beta \Phi^I,$$

where the dots $(\cdots)$ contain terms of the usual "structure constants" of the gauge algebra. From the points of view of these structures of the gauge symmetry we may adopt the Batalin-Vilkovisky formulation [21] which allows us to deal with reducible and open gauge symmetries to obtain covariant gauge-fixed theories. The on-shell reducibility is the characteristic feature of the gauge symmetry for the generalized Chern-Simons action and the quantization of such a system has been discussed in the previous works [20].

In addition to these gauge symmetries (3.2) and (3.3), the action (3.1) is invariant under the following global transformations,

$$\begin{aligned}
\delta\Xi^I &= \omega^I{}_J \Xi^J + a^I, \\
\delta\tilde{\Psi}_\alpha &= r\tilde{\Psi}_\alpha + \sum_{i=1}^{4g} \alpha_i H^{(i)}_\alpha, \\
\delta\Phi^I &= -r\Phi^I + \omega^I{}_J \Phi^J, \\
\delta\bar{\Phi}^I &= r\bar{\Phi}^I + \omega^I{}_J \bar{\Phi}^J, \\
\delta\tilde{\Pi}^I_\alpha &= r\tilde{\Pi}^I_\alpha + \omega^I{}_J \tilde{\Pi}^J_\alpha + \sum_{i=1}^{4g} (\beta^I_i + \alpha_i \Xi^I) H^{(i)}_\alpha, \\
\delta\tilde{\Lambda} &= 2r\tilde{\Lambda}, \\
\delta E_M{}^A &= \delta\Omega_M = 0,
\end{aligned} \tag{3.5}$$

where $\omega_{IJ} = -\omega_{JI}$, a^I, r, α_i and β^I_i are all constant parameters. Poincaré symmetry is ISO$(D-2,2)$ as the same as that of the bosonic model in the previous section. The functions $H^{(i)}_\alpha(z)$ result in harmonic functions on two-dimensional superspace which satisfy $\mathcal{D}H^{(i)} = \mathcal{D}\bar{\sigma}H^{(i)} = 0$ ($i = 1, 2, \ldots, 4g$; g = genus of two-dimensional spacetime).

Now we introduce component fields for the superfields. In two-dimensional supergravity, we impose Wess-Zumino gauge for the vielbein $E_M{}^A(z)$ and the connection $\Omega_M(z)$ whose explicit forms are given in Appendix B. The other superfields are expressed as

$$\begin{aligned}
\Xi^I &= \xi^I + i(\theta\lambda^I) + \frac{i}{2}(\theta\theta)F^I, \\
\tilde{\Psi}_\alpha &= i\hat{\psi}_\alpha + i\theta_\alpha X' + i(\bar{\sigma}\theta)_\alpha X + i(\sigma^m\theta)_\alpha \tilde{A}_m + (\theta\theta)\psi_\alpha, \\
\Phi^I &= \phi^I + i(\theta\kappa^I) + \frac{i}{2}(\theta\theta)G^I, \\
\bar{\Phi}^I &= \bar{\phi}^I + i(\theta\bar{\kappa}^I) + \frac{i}{2}(\theta\theta)\bar{G}^I,
\end{aligned} \tag{3.6}$$

$$\tilde{\Pi}^I_\alpha = i\hat{\rho}^I_\alpha + i\theta_\alpha Y'^I + i(\bar{\sigma}\theta)_\alpha Y^I + i(\sigma^m\theta)_\alpha \tilde{B}^I_m + (\theta\theta)\rho^I_\alpha,$$
$$\tilde{\Lambda} = -2i\Big(H + i(\theta\pi) + \frac{i}{2}(\theta\theta)\tilde{C}\Big).$$

The $\mathrm{U(1)_V}\times\mathrm{U(1)_A}$ gauge parameters are also expressed as

$$\begin{aligned}
V &= v + i(\theta\mu) + \frac{i}{2}(\theta\theta)M,\\
V' &= v' + i(\theta\mu') + \frac{i}{2}(\theta\theta)M',\\
U^I &= u^I + i(\theta\nu^I) + \frac{i}{2}(\theta\theta)N^I,\\
U'^I &= u'^I + i(\theta\nu'^I) + \frac{i}{2}(\theta\theta)N'^I,\\
\tilde{W}_\alpha &= i\hat{\tau}_\alpha + i\theta_\alpha f' + i(\bar{\sigma}\theta)_\alpha f + i(\sigma^m\theta)_\alpha \tilde{w}_m + (\theta\theta)\tau_\alpha.
\end{aligned} \tag{3.7}$$

Before presenting the component expression of the classical action (3.1), we would like to clarify the gauge structure of physical component fields in the $\mathrm{U(1)_V}\times\mathrm{U(1)_A}$ superstring model. In terms of the component fields, the gauge transformations (3.2) are written down as

$$\begin{aligned}
\delta\xi^I &= v'\phi^I,\\
\delta\lambda^I_\alpha &= v'\kappa^I_\alpha + \mu'_\alpha\phi^I,\\
\delta F^I &= v'G^I - i(\mu'\kappa^I) + M'\phi^I,\\
\delta\hat{\psi}_\alpha &= (\bar{\sigma}\mu)_\alpha + \mu'_\alpha,\\
\delta X' &= M',\\
\delta X &= M,\\
\delta\tilde{A}_m &= \frac{1}{e}g_{mn}\varepsilon^{nl}\Big(\partial_l v - \frac{i}{2}(\mu\chi_l)\Big) + \partial_m v' - \frac{i}{2}(\mu'\chi_m),\\
\delta\psi_\alpha &= -\frac{1}{4}\Big((\partial_m v\bar{\sigma} + \partial_m v')\sigma^n\sigma^m\chi_n\Big)_\alpha + \frac{1}{2}(\bar{\sigma}\sigma^m\hat{\nabla}_m\mu)_\alpha + \frac{1}{2}(\sigma^m\hat{\nabla}_m\mu')_\alpha\\
&\quad + \frac{i}{8}\Big(\Big((\mu\chi_m)\bar{\sigma} + (\mu'\chi_m)\Big)\sigma^n\sigma^m\chi_n\Big)_\alpha - \frac{1}{4}\Big((M\bar{\sigma} + M')\sigma^m\chi_m\Big)_\alpha,\\
\delta\bar{\phi}^I &= u'^I,\\
\delta\bar{\kappa}^I_\alpha &= \nu'^I_\alpha,\\
\delta\bar{G}^I &= N'^I,\\
\delta\hat{\rho}^I_\alpha &= -\Big((v\bar{\sigma} - v')\lambda^I\Big)_\alpha + (\bar{\sigma}\nu^I)_\alpha + \nu'^I_\alpha - \hat{\tau}_\alpha\phi^I,\\
\delta Y'^I &= v'F^I + \frac{i}{2}\Big((\mu\bar{\sigma} - \mu')\lambda^I\Big) + N'^I + \frac{i}{2}(\hat{\tau}\kappa^I) - f'\phi^I,\\
\delta Y^I &= -vF^I + \frac{i}{2}\Big((\mu - \mu'\bar{\sigma})\lambda^I\Big) + N^I - \frac{i}{2}(\hat{\tau}\bar{\sigma}\kappa^I) - f\phi^I,
\end{aligned}$$

$$
\begin{aligned}
\delta \tilde{B}^I_m &= -\frac{v}{e} g_{mn}\varepsilon^{nl}\Big(\partial_l \xi^I - \frac{i}{2}(\chi_l \lambda^I)\Big) + v'\Big(\partial_m \xi^I - \frac{i}{2}(\chi_m \lambda^I)\Big) - \frac{i}{2}\Big((\mu\bar{\sigma} + \mu')\sigma_m \lambda^I\Big) \\
&\quad + \frac{1}{e} g_{mn}\varepsilon^{nl}\Big(\partial_l u^I - \frac{i}{2}(\nu^I \chi_l)\Big) + \partial_m u'^I - \frac{i}{2}(\nu'^I \chi_m) - \frac{i}{2}(\hat{\tau}\sigma_m \kappa^I) - \tilde{w}_m \phi^I, \qquad (3.8) \\
\delta \rho^I_\alpha &= -\frac{1}{2}\Big((v\bar{\sigma} - v')\sigma^m \hat{\nabla}_m \lambda^I\Big)_\alpha + \frac{1}{2}\Big((M\bar{\sigma} - M')\lambda^I\Big)_\alpha \\
&\quad + \frac{1}{4}\Big((v\bar{\sigma} - v')\sigma^n \sigma^m \chi_n + 2\sigma^m(\bar{\sigma}\mu + \mu')\Big)_\alpha \Big(\partial_m \xi^I - \frac{i}{2}(\chi_m \lambda^I)\Big) \\
&\quad + \frac{1}{4}\Big((v\bar{\sigma} - v')\sigma^m \chi_m - 2(\bar{\sigma}\mu - \mu')\Big)_\alpha F^I \\
&\quad - \frac{1}{4}\Big((\partial_m u^I \bar{\sigma} + \partial_m u'^I)\sigma^n \sigma^m \chi_n\Big)_\alpha + \frac{1}{2}(\bar{\sigma}\sigma^m \hat{\nabla}_m \nu^I)_\alpha + \frac{1}{2}(\sigma^m \hat{\nabla}_m \nu'^I)_\alpha \\
&\quad - \frac{1}{4}\Big((N^I \bar{\sigma} + N'^I)\sigma^m \chi_m\Big)_\alpha + \frac{i}{8}\Big(\big((\nu^I \chi_m)\bar{\sigma} + (\nu'^I \chi_m)\big)\sigma^n \sigma^m \chi_n\Big)_\alpha \\
&\quad + \frac{1}{2}\hat{\tau}_\alpha G^I - \frac{1}{2}\Big((f\bar{\sigma} + f' + \tilde{w}_m \sigma^m)\kappa^I\Big)_\alpha - \tau_\alpha \phi^I, \\
\delta H &= -v'X' - \frac{i}{2}(\mu'\hat{\psi}) + f', \\
\delta \pi_\alpha &= -v'\psi_\alpha - \frac{1}{2}v'(\sigma^m \hat{\nabla}_m \hat{\psi})_\alpha + \frac{1}{4}v'\Big(\sigma^m(X' + X\bar{\sigma} + \tilde{A}_n \sigma^n)\chi_m\Big)_\alpha \\
&\quad - \frac{1}{2}\Big((3X' - X\bar{\sigma} - \tilde{A}_m \sigma^m)\mu'\Big)_\alpha - \frac{1}{2}\Bigg(\Big(M' - \Big(\partial_m v' - \frac{i}{2}(\mu'\chi_m)\Big)\sigma^m\Big)\hat{\psi}\Bigg)_\alpha \\
&\quad + \frac{1}{2}(\sigma^m \hat{\nabla}_m \hat{\tau})_\alpha - \frac{1}{4}\Big(\sigma^m(f\bar{\sigma} + f' + \tilde{w}_n \sigma^n)\chi_m\Big)_\alpha + \tau_\alpha, \\
\delta \tilde{C} &= \Big(\partial_m v' - \frac{i}{2}(\mu'\chi_m)\Big)\Big(\tilde{A}^m - \frac{i}{4}(\hat{\psi}\sigma^n \sigma^m \chi_n)\Big) \\
&\quad - v'\Big(e^{am}\hat{\nabla}_m \tilde{A}_a - \frac{i}{4}(\chi_m \sigma^m \chi_n)\tilde{A}^n - \frac{i}{4}(\chi_m \sigma^n \sigma^m \hat{\nabla}_n \hat{\psi}) + \frac{i}{2}(\psi\sigma^m \chi_m) \\
&\qquad - \frac{i}{2e}\varepsilon^{mn}(\hat{\psi}\bar{\sigma}\hat{\nabla}_m \chi_n) + \frac{i}{8}(\chi_m \sigma^n \sigma^m \chi_n)X'\Big) \\
&\quad + 2i(\mu'\psi) + \frac{i}{2}(\hat{\psi}\sigma^m \hat{\nabla}_m \mu') + \frac{i}{2}(\mu'\sigma^m \hat{\nabla}_m \hat{\psi}) - \frac{i}{4}\Big(\mu'\sigma^m(X' + X\bar{\sigma} + \tilde{A}_n \sigma^n)\chi_m\Big) \\
&\quad - 2M'\Big(X' + \frac{i}{8}(\hat{\psi}\sigma^m \chi_m)\Big) - \frac{i}{4}(\chi_m \sigma^n \sigma^m \hat{\nabla}_n \hat{\tau}) - \frac{i}{2e}\varepsilon^{mn}(\hat{\tau}\bar{\sigma}\hat{\nabla}_m \chi_n) \\
&\quad + \frac{i}{8}f'(\chi_m \sigma^n \sigma^m \chi_n) + e^{am}\hat{\nabla}_m \tilde{w}_a - \frac{i}{4}\tilde{w}^m(\chi_n \sigma^n \chi_m) + \frac{i}{2}(\tau\sigma^m \chi_m), \\
\delta \phi^I &= \delta \kappa^I_\alpha = \delta G^I = 0.
\end{aligned}
$$

By using the trivial gauge degrees of freedom for the parameters $\mu_\alpha(x)$, $M'(x)$, $M(x)$, $\nu'^I_\alpha(x)$, $\nu^I_\alpha(x)$, $N'^I(x)$, $N^I(x)$, $f'(x)$ and $\tau_\alpha(x)$ in the gauge transformations (3.8), we can impose gauge fixing conditions*

$$
\hat{\psi}_\alpha = X' = X = \bar{\kappa}^I_\alpha = \hat{\rho}^I_\alpha = Y'^I = Y^I = H = \pi_\alpha = 0. \qquad (3.9)
$$

*As the $U(1)_V \times U(1)_A$ gauge symmetries are essential in our string models, we would like to keep these symmetries without gauging away the field $\bar{\phi}^I(x)$.

In order to compensate the Wess-Zumino gauge (B.17) in two-dimensional supergravity within the gauge (3.9), the $\mathrm{U}(1)_{\mathrm{V}}\times\mathrm{U}(1)_{\mathrm{A}}$ gauge parameters should be field dependent. From the gauge transformations (3.8) and the corresponding super-general coordinate, super local Lorentz and super-Weyl transformations (B.30a) and (B.30b) for the fields $\hat{\psi}_\alpha(x)$, $X'(x)$, $X(x)$, $\bar{\kappa}^I_\alpha(x)$, $\hat{\rho}^I_\alpha(x)$, $Y'^I(x)$, $Y^I(x)$, $H(x)$ and $\pi_\alpha(x)$, the gauge parameters allowed within the Wess-Zumino gauge (B.17) and the gauge (3.9) take the following forms,

$$\begin{aligned}
\mu_\alpha &= -(\bar{\sigma}\mu')_\alpha - (\bar{\sigma}\sigma^m\zeta)_\alpha\tilde{A}_m,\\
M' &= -i(\zeta\psi) + \frac{i}{4}(\zeta\sigma^m\sigma^n\chi_m)\tilde{A}_n,\\
M &= -i(\zeta\bar{\sigma}\psi) - \frac{i}{4}(\zeta\bar{\sigma}\sigma^m\sigma^n\chi_m)\tilde{A}_n,\\
\nu'^I_\alpha &= -(\sigma^m\zeta)_\alpha\partial_m\bar{\phi}^I - \zeta_\alpha\bar{G}^I,\\
\nu^I_\alpha &= \Big((v-v'\bar{\sigma})\lambda^I\Big)_\alpha - (\bar{\sigma}\nu'^I)_\alpha + (\bar{\sigma}\hat{\tau})_\alpha\phi^I - (\bar{\sigma}\sigma^m\zeta)_\alpha\tilde{B}^I_m,\\
N'^I &= -v'F^I - \frac{i}{2}(\hat{\tau}\kappa^I) + \frac{i}{2}(\zeta\sigma^m\lambda^I)\tilde{A}_m - i(\zeta\rho^I) + \frac{i}{4}(\zeta\sigma^m\sigma^n\chi_m)\tilde{B}^I_n,\\
N^I &= vF^I + \frac{i}{2}(\hat{\tau}\bar{\sigma}\kappa^I) + f\phi^I + \frac{i}{2}(\zeta\sigma^m\bar{\sigma}\lambda^I)\tilde{A}_m - i(\zeta\bar{\sigma}\rho^I) - \frac{i}{4}(\zeta\bar{\sigma}\sigma^m\sigma^n\chi_m)\tilde{B}^I_n,\\
f' &= 0,\\
\tau_\alpha &= v'\Big(\psi - \frac{1}{4}\tilde{A}_m\sigma^n\sigma^m\chi_n\Big)_\alpha - \frac{1}{2}(\sigma^m\mu')_\alpha\tilde{A}_m - \frac{1}{2}(\sigma^m\hat{\nabla}_m\hat{\tau})_\alpha\\
&\quad+\frac{1}{4}\Big(\sigma^m(f\bar{\sigma}+\tilde{w}_n\sigma^n)\chi_m\Big)_\alpha - \zeta_\alpha\tilde{C}.
\end{aligned} \tag{3.10}$$

The classical action for the $\mathrm{U}(1)_{\mathrm{V}}\times\mathrm{U}(1)_{\mathrm{A}}$ superstring model (3.1) is then expressed by

$$\begin{aligned}
S = \int d^2x e\Big\{ &-\frac{1}{2}g^{mn}\partial_m\xi^I\partial_n\xi_I - \frac{i}{2}(\lambda^I\sigma^m\partial_m\lambda_I)\\
&+\frac{i}{2}(\lambda_I\sigma^m\sigma^n\chi_m)\partial_n\xi^I - \frac{1}{16}(\chi_m\sigma^n\sigma^m\chi_n)(\lambda^I\lambda_I) + \frac{1}{2}F^IF_I\\
&-g^{mn}\partial_m\bar{\phi}^I\partial_n\phi_I + \bar{G}^IG_I\\
&+\Big(\tilde{A}^m\partial_m\xi^I + i(\psi\lambda^I)\Big)\phi_I + \tilde{B}^{mI}\partial_m\phi_I + i(\rho^I\kappa_I) - \frac{1}{2}\tilde{C}\phi^I\phi_I\Big\},
\end{aligned} \tag{3.11}$$

where we redefine some of the fields as follows,

$$\begin{aligned}
\psi_\alpha - \frac{1}{4}\tilde{A}_m(\sigma^n\sigma^m\chi_n)_\alpha &\to \psi_\alpha,\\
\rho^I_\alpha + \frac{1}{2}\partial_m\bar{\phi}^I(\sigma^n\sigma^m\chi_n)_\alpha - \frac{1}{2}\tilde{A}_m(\sigma^m\lambda^I)_\alpha - \frac{1}{4}\tilde{B}^I_m(\sigma^n\sigma^m\chi_n)_\alpha &\to \rho^I_\alpha.
\end{aligned} \tag{3.12}$$

Under the field redefinitions (3.12), the gauge transformations within the Wess-Zumino gauge (B.17) and the gauge (3.9) are given by

$$\delta\xi^I = v'\phi^I + k^n\partial_n\xi^I + i(\zeta\lambda^I),$$

$$
\begin{aligned}
\delta\lambda^I_\alpha &= v'\kappa^I_\alpha + \mu'_\alpha\phi^I + k^n\partial_n\lambda^I_\alpha + (\sigma^m\zeta)_\alpha\Big(\partial_m\xi^I - \frac{i}{2}(\chi_m\lambda^I)\Big) + \zeta_\alpha F^I - \frac{1}{2}l(\bar\sigma\lambda^I)_\alpha + \frac{1}{2}s\lambda^I_\alpha,\\
\delta F^I &= v'G^I - i(\mu'\kappa^I) + k^n\partial_n F^I - i(\zeta\psi)\phi^I - \frac{i}{2}(\zeta\sigma^m\sigma^n\chi_m)\partial_n\xi^I\\
&\quad + i(\zeta\sigma^m\nabla_m\lambda^I) + \frac{1}{8}(\zeta\lambda^I)(\chi_m\sigma^n\sigma^m\chi_n) - \frac{i}{2}(\zeta\sigma^m\chi_m)F^I + sF^I,\\
\delta\tilde A^m &= \frac{1}{e}\varepsilon^{mn}\partial_n v + g^{mn}\partial_n v' - \frac{i}{2}(\mu'\sigma^n\sigma^m\chi_n)\\
&\quad + k^n\partial_n\tilde A^m - \partial_n k^m\tilde A^n + i(\zeta\sigma^m\psi) - i(\zeta\sigma^n\chi_n)\tilde A^m + 2s\tilde A^m,\\
\delta\psi_\alpha &= -\frac{1}{2}\partial_m v'(\sigma^n\sigma^m\chi_n)_\alpha + (\sigma^m\nabla_m\mu')_\alpha - \frac{i}{8}\mu'_\alpha(\chi_m\sigma^n\sigma^m\chi_n)\\
&\quad + k^n\partial_n\psi_\alpha + \zeta_\alpha\nabla_m\tilde A^m - i(\zeta\sigma^m\chi_m)\psi_\alpha - \frac{i}{2}(\zeta\sigma^m\psi)\chi_{m\alpha} - \frac{1}{2}l(\bar\sigma\psi)_\alpha + \frac{3}{2}s\psi_\alpha,\\
\delta\phi^I &= k^n\partial_n\phi^I + i(\zeta\kappa^I),\\
\delta\kappa^I_\alpha &= k^n\partial_n\kappa^I_\alpha + (\sigma^m\zeta)_\alpha\Big(\partial_m\phi^I - \frac{i}{2}(\chi_m\kappa^I)\Big) + \zeta_\alpha G^I - \frac{1}{2}l(\bar\sigma\kappa^I)_\alpha + \frac{1}{2}s\kappa^I_\alpha,\\
\delta G^I &= k^n\partial_n G^I - \frac{i}{2}(\zeta\sigma^m\sigma^n\chi_m)\partial_n\phi^I\\
&\quad + i(\zeta\sigma^m\nabla_m\kappa^I) + \frac{1}{8}(\zeta\kappa^I)(\chi_m\sigma^n\sigma^m\chi_n) - \frac{i}{2}(\zeta\sigma^m\chi_m)G^I + sG^I,\\
\delta\bar\phi^I &= u'^I + k^n\partial_n\bar\phi^I, \qquad\qquad (3.13)\\
\delta\bar G^I &= -v'F^I - \frac{i}{2}(\hat\tau\kappa^I) + k^n\partial_n\bar G^I - i(\zeta\rho^I) - \frac{i}{2}(\zeta\sigma^m\chi_m)\bar G^I + s\bar G^I,\\
\delta\tilde B^{mI} &= -\frac{v}{e}\varepsilon^{mn}\partial_n\xi^I + v'g^{mn}\Big(\partial_n\xi^I - \frac{i}{2}(\chi_l\sigma_n\sigma^l\lambda^I)\Big) - i(\mu'\sigma^m\lambda^I)\\
&\quad + \frac{1}{e}\varepsilon^{mn}\partial_n u^I + g^{mn}\partial_n u'^I - \frac{i}{2}(\hat\tau\sigma^m\kappa^I) - \tilde w^m\phi^I + k^n\partial_n\tilde B^{mI} - \partial_n k^m\tilde B^{nI}\\
&\quad + i(\zeta\lambda^I)\tilde A^m - i(\zeta\sigma^m\sigma^n\sigma^l\chi_n)\partial_l\bar\phi^I + \frac{i}{2}(\zeta\sigma^n\sigma^m\chi_n)\bar G^I\\
&\quad - i(\zeta\sigma^n\chi_n)\tilde B^{mI} + i(\zeta\sigma^m\rho^I) + 2s\tilde B^{mI},\\
\delta\rho^I_\alpha &= v'\Big(\sigma^m\nabla_m\lambda^I - \frac{1}{2}\partial_m\xi^I\sigma^n\sigma^m\chi_n - \frac{i}{8}(\chi_m\sigma^n\sigma^m\chi_n)\lambda^I - \phi^I\psi - \frac{1}{2}\tilde A^m\sigma_m\kappa^I\Big)_\alpha + \mu'_\alpha F^I\\
&\quad + \frac{1}{2}\Big((G^I - \partial_m\phi^I\sigma^m)\hat\tau\Big)_\alpha - \frac{i}{4}(\hat\tau\sigma^m\sigma^n\chi_m)(\sigma_n\kappa^I)_\alpha - \frac{1}{2}f(\bar\sigma\kappa^I)_\alpha - \frac{1}{2}\tilde w^m(\sigma_m\kappa^I)_\alpha\\
&\quad + k^n\partial_n\rho^I_\alpha - \zeta_\alpha\Big(g^{mn}\nabla_m\partial_n\phi^I + \tilde A^m\partial_m\xi^I + i(\psi\lambda^I) - \nabla_m\tilde B^{mI} - \tilde C\phi^I\Big)\\
&\quad - (\sigma^m\nabla_m\zeta)_\alpha\bar G^I - (\sigma^m\zeta)_\alpha\partial_m\bar G^I + \frac{i}{8}\zeta_\alpha(\chi_m\sigma^n\sigma^m\chi_n)\bar G^I\\
&\quad - i(\zeta\sigma^m\chi_m)\rho^I_\alpha - \frac{i}{2}(\zeta\sigma^m\rho^I)\chi_{m\alpha} - \frac{1}{2}l(\bar\sigma\rho^I)_\alpha + \frac{3}{2}s\rho^I_\alpha,\\
\delta\tilde C &= \partial_m v'\tilde A^m - v'\nabla_m\tilde A^m + 2i(\mu'\psi) + \nabla_m\tilde w^m + k^n\partial_n\tilde C - i(\zeta\sigma^m\chi_m)\tilde C + 2s\tilde C,
\end{aligned}
$$

where we use the covariant derivative ∇_m for the torsion free connection $\omega_m(x)$ defined via (A.12), (A.13), (A.16) and (A.20) and we also redefine the gauge parameter $\tilde w^m(x)$ as

$$
\tilde w^m - \frac{i}{2e}\varepsilon^{mn}(\hat\tau\bar\sigma\chi_n) \to \tilde w^m. \qquad (3.14)
$$

Let us consider the reducible symmetry. By introducing the reducible parameter $W'(z)$ as

$$W' = w' + i(\theta\check{w}') + \frac{i}{2}(\theta\theta)\bar{w}', \tag{3.15}$$

the reducible transformations (3.4) are expressed in terms of the component fields

$$\begin{aligned} \delta' u^I &= w'\phi^I, \\ \delta'\hat{\tau}_\alpha &= (\bar{\sigma}\check{w}')_\alpha, \\ \delta' f &= \bar{w}', \\ \delta'\tilde{w}^m &= \frac{1}{e}\varepsilon^{mn}\partial_n w', \end{aligned} \tag{3.16}$$

where we use the redefinition of the gauge parameter (3.14). One can easily check the reducible transformation (3.4) is also consistent with the Wess-Zumino gauge.

Now we are going to an on-shell formulation of the model by eliminating the "auxiliary" fields $F^I(x)$, $G^I(x)$, $\bar{G}^I(x)$, $\kappa^I_\alpha(x)$ and $\rho^I_\alpha(x)$. All of the gauge transformations for these auxiliary fields are proportional to the equations of motion, so that we can eliminate these fields within the on-shell formulation. Then, the action (3.11) becomes the following form,

$$\begin{aligned} S = \int \mathrm{d}^2 x e\Big\{ &-\frac{1}{2}g^{mn}\partial_m\xi^I\partial_n\xi_I - \frac{i}{2}(\lambda^I\sigma^m\partial_m\lambda_I) \\ &+\frac{i}{2}(\lambda_I\sigma^m\sigma^n\chi_m)\partial_n\xi^I - \frac{1}{16}(\chi_m\sigma^n\sigma^m\chi_n)(\lambda^I\lambda_I) \\ &-g^{mn}\partial_m\bar{\phi}^I\partial_n\phi_I + \Big(\tilde{A}^m\partial_m\xi^I + i(\psi\lambda^I)\Big)\phi_I + \tilde{B}^{mI}\partial_m\phi_I - \frac{1}{2}\tilde{C}\phi^I\phi_I\Big\}, \end{aligned} \tag{3.17}$$

and the gauge transformations are given by

$$\begin{aligned} \delta\xi^I &= v'\phi^I + k^n\partial_n\xi^I + i(\zeta\lambda^I), \\ \delta\lambda^I_\alpha &= \mu'_\alpha\phi^I + k^n\partial_n\lambda^I_\alpha + (\sigma^m\zeta)_\alpha\Big(\partial_m\xi^I - \frac{i}{2}(\chi_m\lambda^I)\Big) - \frac{1}{2}l(\bar{\sigma}\lambda^I)_\alpha + \frac{1}{2}s\lambda^I_\alpha, \\ \delta\tilde{A}^m &= \frac{1}{e}\varepsilon^{mn}\partial_n v + g^{mn}\partial_n v' - \frac{i}{2}(\mu'\sigma^n\sigma^m\chi_n) \\ &\quad +k^n\partial_n\tilde{A}^m - \partial_n k^m\tilde{A}^n + i(\zeta\sigma^m\psi) - i(\zeta\sigma^n\chi_n)\tilde{A}^m + 2s\tilde{A}^m, \\ \delta\psi_\alpha &= -\frac{1}{2}\partial_m v'(\sigma^n\sigma^m\chi_n)_\alpha + (\sigma^m\nabla_m\mu')_\alpha - \frac{i}{8}\mu'_\alpha(\chi_m\sigma^n\sigma^m\chi_n) \\ &\quad +k^n\partial_n\psi_\alpha + \zeta_\alpha\nabla_m\tilde{A}^m - i(\zeta\sigma^m\chi_m)\psi_\alpha - \frac{i}{2}(\zeta\sigma^m\psi)\chi_{m\alpha} - \frac{1}{2}l(\bar{\sigma}\psi)_\alpha + \frac{3}{2}s\psi_\alpha, \\ \delta\phi^I &= k^n\partial_n\phi^I, \\ \delta\bar{\phi}^I &= u'^I + k^n\partial_n\bar{\phi}^I, \\ \delta\tilde{B}^{mI} &= -\frac{v}{e}\varepsilon^{mn}\partial_n\xi^I + v'g^{mn}\Big(\partial_n\xi^I - \frac{i}{2}(\chi_l\sigma_n\sigma^l\lambda^I)\Big) - i(\mu'\sigma^m\lambda^I) \end{aligned} \tag{3.18}$$

$$
\begin{aligned}
&+\frac{1}{e}\varepsilon^{mn}\partial_n u^I + g^{mn}\partial_n u'^I - \tilde{w}^m\phi^I + k^n\partial_n\tilde{B}^{mI} - \partial_n k^m\tilde{B}^{nI} \\
&+i(\zeta\lambda^I)\tilde{A}^m - i(\zeta\sigma^m\sigma^n\sigma^l\chi_n)\partial_l\bar{\phi}^I - i(\zeta\sigma^n\chi_n)\tilde{B}^{mI} + 2s\tilde{B}^{mI}, \\
\delta\tilde{C} &= \partial_m v'\tilde{A}^m - v'\nabla_m\tilde{A}^m + 2i(\mu'\psi) + \nabla_m\tilde{w}^m + k^n\partial_n\tilde{C} - i(\zeta\sigma^m\chi_m)\tilde{C} + 2s\tilde{C}, \\
\delta e_m{}^a &= k^n\partial_n e_m{}^a + \partial_m k^n e_n{}^a + i(\zeta\sigma^a\chi_m) + l e_m{}^b\varepsilon_b{}^a - s e_m{}^a, \\
\delta\chi_{m\alpha} &= k^n\partial_n\chi_{m\alpha} + \partial_m k^n\chi_{n\alpha} + 2(\nabla_m\zeta)_\alpha - \frac{i}{2}(\chi_m\bar{\sigma}\sigma^l\chi_l)(\bar{\sigma}\zeta)_\alpha \\
&-\frac{1}{2}l(\bar{\sigma}\chi_m)_\alpha - \frac{1}{2}s\chi_{m\alpha} - (\sigma_m\check{s})_\alpha.
\end{aligned}
$$

Since the gauge parameters $\hat{\tau}_\alpha(x)$ and $f(x)$ disappear from the gauge transformation (3.18), the reducible transformations with which we would like to work are

$$
\begin{aligned}
\delta' u^I &= w'\phi^I, \\
\delta'\tilde{w}^m &= \frac{1}{e}\varepsilon^{mn}\partial_n w'.
\end{aligned} \tag{3.19}
$$

In addition to the gauge symmetry, the action (3.17) is also invariant under the following global transformations,

$$
\begin{aligned}
\delta\xi^I &= \omega^I{}_J\xi^J + a^I, \\
\delta\lambda^I_\alpha &= \omega^I{}_J\lambda^J_\alpha, \\
\delta\tilde{A}^m &= r\tilde{A}^m + \sum_{i=1}^{2g}\alpha_i h^{(i)m}, \\
\delta\psi_\alpha &= r\psi_\alpha, \\
\delta\phi^I &= -r\phi^I + \omega^I{}_J\phi^J, \\
\delta\bar{\phi}^I &= r\bar{\phi}^I + \omega^I{}_J\bar{\phi}^J, \\
\delta\tilde{B}^{mI} &= r\tilde{B}^{mI} + \omega^I{}_J\tilde{B}^{mJ} + \sum_{i=1}^{2g}(\beta^I_i + \alpha_i\xi^I)h^{(i)m}, \\
\delta\tilde{C} &= 2r\tilde{C}, \\
\delta e_m{}^a &= \delta\chi_{m\alpha} = 0.
\end{aligned} \tag{3.20}
$$

We take the classical action (3.17), the gauge transformation (3.18) and the reducibility condition (3.19) as the starting point for the quantization we will discuss in this paper. The superpartners of the fields $\phi^I(x)$, $\tilde{B}^{mI}(x)$ and $\tilde{C}(x)$ in the generalized Chern-Simons action disappear in the action (3.17), since the supersymmetry transformations of these fields are trivial.

Before getting into the quantization of the model, it is worth to mention semiclassical aspects of the action (3.17), by eliminating gauge fields through their equations of motion.

Indeed, this manipulation might be helpful to understand the heart of the model. The field $\bar{\phi}^I(x)$ can be gauged away by using the gauge degree of freedom for the parameter $u'^I(x)$. Equations of motion for the fields $\tilde{B}^{mI}(x)$ and $\tilde{C}(x)$ give gauge constraints

$$\begin{aligned} \partial_m \phi^I &= 0, \\ \phi^I \phi_I &= 0. \end{aligned} \tag{3.21}$$

It is possible to find nontrivial solutions for these constraints if the background spacetime metric includes two time-like signatures. In the light-cone notation†, one of the interesting solutions which is naturally related with the usual superstring action is $\phi^{\hat{-}}(x) = \phi^\mu(x) = 0$ and $\phi^{\hat{+}}(x) = \text{const.}$. After substituting this solution into the action (3.17), the action becomes

$$\begin{aligned} S = \int \mathrm{d}^2 x e \Big\{ &-\frac{1}{2} g^{mn} \partial_m \xi^I \partial_n \xi_I - \frac{i}{2} (\lambda^I \sigma^m \partial_m \lambda_I) \\ &+\frac{i}{2} (\lambda_I \sigma^m \sigma^n \chi_m) \partial_n \xi^I - \frac{1}{16} (\chi_m \sigma^n \sigma^m \chi_n)(\lambda^I \lambda_I) \\ &-\Big(\tilde{A}^m \partial_m \xi^{\hat{-}} + i(\psi \lambda^{\hat{-}}) \Big) \phi^{\hat{+}} \Big\}. \end{aligned} \tag{3.22}$$

In the action (3.22), relations $\partial_m \xi^{\hat{-}}(x) = 0$ and $\lambda_\alpha^{\hat{-}}(x) = 0$ are given by the equation of motion for $\tilde{A}^m(x)$ and $\psi^\alpha(x)$, respectively. Then, the final form of the action becomes the usual NSR superstring action

$$\begin{aligned} S = \int \mathrm{d}^2 x e \Big\{ &-\frac{1}{2} g^{mn} \partial_m \xi^\mu \partial_n \xi_\mu - \frac{i}{2} (\lambda^\mu \sigma^m \partial_m \lambda_\mu) \\ &+\frac{i}{2} (\lambda_\mu \sigma^m \sigma^n \chi_m) \partial_n \xi^\mu - \frac{1}{16} (\chi_m \sigma^n \sigma^m \chi_n)(\lambda^\mu \lambda_\mu) \Big\}. \end{aligned} \tag{3.23}$$

Thus, the superstring action (3.23) is regarded as a gauge-fixed version of the action (3.17). The scalar field $\phi^I(x)$ plays an important role for the covariant formulation of the $\mathrm{U}(1)_\mathrm{V} \times \mathrm{U}(1)_\mathrm{A}$ superstring model in the background spacetime which involves two time coordinates. From this manipulation it is suggested that the critical dimension of the background spacetime is defined as $D - 3 = 9$, *i.e.* $D = 12$. However, the dimension D should be determined in the quantum analysis as we will investigate in this paper. We would also like to emphasize that the quantization will be carried out with preserving D-dimensional covariance.

† We use a convention of the light-cone coordinates for the background spacetime as $x^I = (x^\mu, x^{\hat{+}}, x^{\hat{-}})$ where $x^{\hat{\pm}} = \frac{1}{\sqrt{2}}(x^{\hat{0}} \pm x^{\hat{1}})$ and the index μ runs through $0, 1, \ldots, D-3$.

4 Covariant quantization in the Lagrangian formulation

In this section we consider the covariant quantization of the action. As we explained in the previous section, the action has first-stage reducible and open gauge symmetries. In order to quantize the action we adopt the field-antifield formulation *á la* Batalin-Vilkovisky [21].

In the construction of Batalin-Vilkovisky formulation [26,27], ghost and ghost for ghost fields according to the reducibility condition and corresponding each antifields are introduced. The Grassmann parities of the antifields are opposite to those of the corresponding fields. If a field has ghost number n, its antifield has ghost number $-n-1$. We denote a set of fields and their antifields

$$\Phi^A(x) = \Big(\varphi^i(x), \mathcal{C}_0^{a_0}(x), \mathcal{C}_1^{a_1}(x), \ldots, \mathcal{C}_N^{a_N}(x)\Big),$$
$$\Phi^*_A(x) = \Big(\varphi^*_i(x), \mathcal{C}^*_{0,a_0}(x), \mathcal{C}^*_{1,a_1}(x), \ldots, \mathcal{C}^*_{N,a_N}(x)\Big),$$

respectively. The fields $\varphi^i(x)$ are classical fields, on the other hand, the fields $\mathcal{C}_n^{a_n}(x)$ [$n = 0$, $1, \ldots, N$] are ghost and ghost for ghost fields corresponding to N-th reducible conditions. The classical fields $\varphi^i(x)$ and the ghost fields $\mathcal{C}_n^{a_n}(x)$ have the ghost number 0 and $n+1$, respectively. Then a minimal action $S_{\text{min}}(\Phi, \Phi^*)$ is defined by solving the following master equation,

$$\Big(S_{\text{min}}(\Phi, \Phi^*), S_{\text{min}}(\Phi, \Phi^*)\Big) = 0, \tag{4.1}$$

with the boundary conditions

$$S_{\text{min}}(\Phi, \Phi^*)\Big|_{\Phi^*=0} = S_{\text{classical}}(\varphi), \tag{4.2a}$$
$$\frac{\delta_{\text{L}}\delta_{\text{R}} S_{\text{min}}(\Phi, \Phi^*)}{\delta\mathcal{C}_n^{a_n}\delta\mathcal{C}^*_{n-1,a_{n-1}}}\Bigg|_{\Phi^*=0} = R^{a_{n-1}}_{n,a_n}(\Phi), \qquad (n = 0, 1, \ldots, N). \tag{4.2b}$$

Here the antibracket is defined by

$$(X, Y) \equiv \frac{\delta_{\text{R}} X}{\delta\Phi^*_A}\frac{\delta_{\text{L}} Y}{\delta\Phi^A} - \frac{\delta_{\text{R}} X}{\delta\Phi^A}\frac{\delta_{\text{L}} Y}{\delta\Phi^*_A}. \tag{4.3}$$

In this notation, $\mathcal{C}^*_{-1,a_{-1}}(x) \equiv \varphi^*_i(x)$ are the antifields of the classical fields $\varphi^i(x)$. The terms $R^{a_{-1}}_{0,a_0}(\Phi)$ and $R^{a_{n-1}}_{n,a_n}(\Phi)$ represent the gauge transformations and the n-th reducibility transformations, respectively. The master equation is solved order by order with respect to the ghost number. The BRST transformations of fields and antifields are given by

$$s\Phi^A = \Big(S_{\text{min}}, \Phi^A\Big), \qquad s\Phi^*_A = \Big(S_{\text{min}}, \Phi^*_A\Big). \tag{4.4}$$

Eqs. (4.1) and (4.4) assure that the BRST transformation is nilpotent and the minimal action is invariant under the BRST transformation*.

Now we apply the Batalin-Vilkovisky formulation to the quantization of our model. First of all, we take the action (3.17) as the classical action $S_{\rm classical}$. The algebra of the gauge transformations (3.18) is given by

$$
\begin{aligned}
&[\ \delta(v_1),\ \delta(v_2')\ ] = \delta\Big(\tilde{w}^m \equiv -\frac{\varepsilon^{mn}}{e} v_1 \partial_n v_2'\Big),\\
&[\ \delta(v_1),\ \delta(k_2)\ ] = \delta\Big(v \equiv k_2^n \partial_n v_1\Big),\\
&[\ \delta(v_1),\ \delta(\zeta_2)\ ] = \delta\Big(u^I \equiv i v_1 (\zeta_2 \lambda^I)\Big),\\
&[\ \delta(v_1'),\ \delta(v_2')\ ] = \delta\Big(\tilde{w}^m \equiv v_1' g^{mn} \partial_n v_2' - v_2' g^{mn} \partial_n v_1'\Big),\\
&[\ \delta(v_1'),\ \delta(\mu_2')\ ] = \delta\Big(\tilde{w}^m \equiv -\frac{i}{2} v_1' (\mu_2' \sigma^l \sigma^m \chi_l)\Big),\\
&[\ \delta(v_1'),\ \delta(k_2)\ ] = \delta\Big(v' \equiv k_2^n \partial_n v_1'\Big),\\
&[\ \delta(v_1'),\ \delta(\zeta_2)\ ] = \delta\Big(\mu_\alpha' \equiv (\sigma^m \zeta_2)_\alpha \partial_m v_1'\Big) + \delta\Big(u^I \equiv -i v_1' (\zeta_2 \bar{\sigma} \lambda^I)\Big) + \delta\Big(\tilde{w}^m \equiv i v_1' (\zeta_2 \sigma^m \psi)\Big),\\
&[\ \delta(\mu_1'),\ \delta(\mu_2')\ ] = \delta\Big(\tilde{w}^m \equiv -2i (\mu_1' \sigma^m \mu_2')\Big),\\
&[\ \delta(\mu_1'),\ \delta(k_2)\ ] = \delta\Big(\mu_\alpha' \equiv k_2^n \partial_n \mu_{1\alpha}'\Big),\\
&[\ \delta(\mu_1'),\ \delta(\zeta_2)\ ] = \delta\Big(v \equiv i (\mu_1' \bar{\sigma} \zeta_2)\Big) + \delta\Big(v' \equiv i (\mu_1' \zeta_2)\Big)\\
&\qquad\qquad\qquad\qquad + \delta\Big(\mu_\alpha' \equiv -\frac{i}{2} (\sigma^m \zeta_2)_\alpha (\mu_1' \chi_m)\Big) + \delta\Big(\tilde{w}^m \equiv -i (\mu_1' \zeta_2) \tilde{A}^m\Big),\\
&[\ \delta(\mu_1'),\ \delta(l_2)\ ] = \delta\Big(\mu_\alpha' \equiv -\frac{1}{2} (\bar{\sigma} \mu_1')_\alpha l_2\Big),\\
&[\ \delta(\mu_1),\ \delta(s_2)\ ] = \delta\Big(\mu_\alpha' \equiv \frac{1}{2} \mu_{1\alpha}' s_2\Big),\\
&[\ \delta(u_1^I),\ \delta(k_2)\ ] = \delta\Big(u^I \equiv k_2^n \partial_n u_1^I\Big),\\
&[\ \delta(u_1'^I),\ \delta(k_2)\ ] = \delta\Big(u'^I \equiv k_2^n \partial_n u_1'^I\Big),\\
&[\ \delta(\tilde{w}_1),\ \delta(k_2)\ ] = \delta\Big(\tilde{w}^m \equiv k_2^n \partial_n \tilde{w}_1^m - \partial_n k_2^m \tilde{w}_1^n\Big),\\
&[\ \delta(\tilde{w}_1),\ \delta(\zeta_2)\ ] = \delta\Big(\tilde{w}^m \equiv -i \tilde{w}_1^m (\zeta_2 \sigma^n \chi_n)\Big),\\
&[\ \delta(\tilde{w}_1),\ \delta(s_2)\ ] = \delta\Big(\tilde{w}^m \equiv 2 \tilde{w}_1^m s_2\Big), \qquad\qquad (4.5)\\
&[\ \delta(k_1),\ \delta(k_2)\ ] = \delta\Big(k^n \equiv k_2^l \partial_l k_1^n - k_1^l \partial_l k_2^n\Big),\\
&[\ \delta(k_1),\ \delta(\zeta_2)\ ] = \delta\Big(\zeta_\alpha \equiv -k_1^n \partial_n \zeta_{2\alpha}\Big),\\
&[\ \delta(k_1),\ \delta(l_2)\ ] = \delta\Big(l \equiv -k_1^n \partial_n l_2\Big),\\
&[\ \delta(k_1),\ \delta(s_2)\ ] = \delta\Big(s \equiv -k_1^n \partial_n s_2\Big),
\end{aligned}
$$

*Our convention for the Leipnitz rule of the BRST operation is given by $s(XY) = (sX)Y + (-)^{|X|} X(sY)$, where $|X|$ is a Grassmann parity of the field X.

$$[\ \delta(k_1),\ \delta(\check{s}_2)\ ] = \delta\big(\check{s}_\alpha \equiv -k_1^n\partial_n\check{s}_{2\alpha}\big),$$

$$\begin{aligned}
[\ \delta(\zeta_1),\ \delta(\zeta_2)\ ] = {} & \delta\Big(v \equiv 2ie\varepsilon_{mn}(\zeta_1\sigma^m\zeta_2)\tilde{A}^n\Big) + \delta\Big(\mu'_\alpha \equiv i(\zeta_1\sigma^m\zeta_2)(\sigma_m\psi)_\alpha + i(\zeta_1\bar{\sigma}\zeta_2)(\bar{\sigma}\psi)_\alpha\Big) \\
& +\delta\Big(u^I \equiv 2ie\varepsilon_{mn}(\zeta_1\sigma^m\zeta_2)\tilde{B}^{nI} - 2i\frac{\varepsilon^{mn}}{e}(\zeta_1\sigma_m\zeta_2)\partial_n\bar{\phi}^I\Big) \\
& +\delta\Big(u'^I \equiv 2i(\zeta_1\sigma^m\zeta_2)\partial_m\bar{\phi}^I\Big) + \delta\Big(\tilde{w}^m \equiv 2i(\zeta_1\sigma^m\zeta_2)\tilde{C}\Big) \\
& +\delta\Big(k^n \equiv -2i(\zeta_1\sigma^n\zeta_2)\Big) + \delta\Big(\zeta_\alpha \equiv i(\zeta_1\sigma^m\zeta_2)\chi_{m\alpha}\Big) \\
& +\delta\Big(l \equiv 2i(\zeta_1\sigma^m\zeta_2)\Big(\omega_m - \frac{i}{2}(\chi_m\bar{\sigma}\sigma^n\chi_n)\Big)\Big) \\
& +\delta\Bigg(\check{s}_\alpha \equiv i\Big(\Big((\zeta_1\sigma^n\zeta_2)\sigma_n + (\zeta_1\bar{\sigma}\zeta_2)\bar{\sigma}\Big)\bar{\sigma}\frac{\varepsilon^{pq}}{e}\Big(\nabla_p\chi_q - \frac{i}{4}(\chi_p\bar{\sigma}\sigma^l\chi_l)\bar{\sigma}\chi_q\Big)\Big)_\alpha\Bigg),
\end{aligned}$$

$$[\ \delta(\zeta_1),\ \delta(l_2)\ ] = \delta\Big(\zeta_\alpha \equiv -\frac{1}{2}(\bar{\sigma}\zeta_1)_\alpha l_2\Big),$$

$$[\ \delta(\zeta_1),\ \delta(s_2)\ ] = \delta\Big(\zeta_\alpha \equiv -\frac{1}{2}\zeta_{1\alpha}s_2\Big) + \delta\Big(\check{s}_\alpha \equiv -(\sigma^m\zeta_1)_\alpha\partial_m s_2\Big),$$

$$[\ \delta(\zeta_1),\ \delta(\check{s}_2)\ ] = \delta\Big(l \equiv i(\zeta_1\bar{\sigma}\check{s}_2)\Big) + \delta\Big(s \equiv -i(\zeta_1\check{s}_2)\Big) + \delta\Big(\check{s}_\alpha \equiv \frac{i}{2}(\sigma^m\zeta_1)_\alpha(\chi_m\check{s}_2)\Big),$$

$$[\ \delta(l_1),\ \delta(\check{s}_2)\ ] = \delta\Big(\check{s}_\alpha \equiv \frac{1}{2}(\bar{\sigma}\check{s}_2)_\alpha l_1\Big),$$

$$[\ \delta(s_1),\ \delta(\check{s}_2)\ ] = \delta\Big(\check{s}_\alpha \equiv -\frac{1}{2}\check{s}_{2\alpha}s_1\Big),$$

$$(\text{others}) = 0.$$

In the above gauge algebra, some commutation relations are closed within on-shell,

$$\begin{aligned}
[\ \delta(v_1),\ \delta(v'_2)\ ]\tilde{B}^{mI} &= \delta(\tilde{w})\tilde{B}^{mI} + v_1v'_2\frac{\varepsilon^{mn}}{e^2}\frac{\delta S}{\delta\tilde{B}^n_I}, \\
[\ \delta(v'_1),\ \delta(\zeta_2)\ ]\lambda^I_\alpha &= \delta(\mu')\lambda^I_\alpha + (\sigma^m\zeta_2)_\alpha v'_1\frac{1}{e}\frac{\delta S}{\delta\tilde{B}^m_I}, \\
[\ \delta(v'_1),\ \delta(\zeta_2)\ ]\tilde{B}^{mI} &= \delta(\mu')\tilde{B}^{mI} + \delta(u^I)\tilde{B}^{mI} + \delta(\tilde{w})\tilde{B}^{mI} + v'_1\Big(\zeta_2\sigma^m\frac{1}{e}\frac{\delta_L S}{\delta\lambda_I}\Big), \\
[\ \delta(\zeta_1),\ \delta(\zeta_2)\ ]\lambda^I_\alpha &= \delta(\mu')\lambda^I_\alpha + \delta(k)\lambda^I_\alpha + \delta(\zeta)\lambda^I_\alpha + \delta(l)\lambda^I_\alpha \\
&\quad -(\zeta_1\sigma^m\zeta_2)\Big(\sigma_m\frac{1}{e}\frac{\delta_L S}{\delta\lambda_I}\Big)_\alpha - (\zeta_1\bar{\sigma}\zeta_2)\Big(\bar{\sigma}\frac{1}{e}\frac{\delta_L S}{\delta\lambda_I}\Big)_\alpha, \\
[\ \delta(\zeta_1),\ \delta(\zeta_2)\ ]\phi^I &= \delta(k)\phi^I + 2i(\zeta_1\sigma^m\zeta_2)\frac{1}{e}\frac{\delta S}{\delta\tilde{B}^{mI}}, \\
[\ \delta(\zeta_1),\ \delta(\zeta_2)\ ]\tilde{B}^{mI} &= \delta(v)\tilde{B}^{mI} + \delta(\mu')\tilde{B}^{mI} + \delta(u^I)\tilde{B}^{mI} + \delta(u'^I)\tilde{B}^{mI} + \delta(\tilde{w})\tilde{B}^{mI} \\
&\quad +\delta(k)\tilde{B}^{mI} + \delta(\zeta)\tilde{B}^{mI} - 2i(\zeta_1\sigma^m\zeta_2)\frac{1}{e}\frac{\delta S}{\delta\phi_I}.
\end{aligned}$$

We are now ready to quantize the model in the Batalin-Vilkovisky formulation.

The classical fields $\varphi^i(x)$ consist of $\xi^I(x)$, $\lambda^I_\alpha(x)$, $\tilde{A}^m(x)$, $\psi_\alpha(x)$, $\phi^I(x)$, $\bar{\phi}^I(x)$, $\tilde{B}^{mI}(x)$, $\tilde{C}(x)$, $e_m{}^a(x)$ and $\chi_{m\alpha}(x)$. Here we introduce ghost fields $a(x)$, $a'(x)$, $\alpha'_\alpha(x)$, $b^I(x)$, $b'^I(x)$, $\tilde{c}^m(x)$, $d^m(x)$, $\gamma_\alpha(x)$, $c_L(x)$, $c_W(x)$ and $\check{c}_{S\alpha}(x)$ corresponding to the gauge parameters $v(x)$,

$v'(x)$, $\mu'_\alpha(x)$, $u^I(x)$, $u'^I(x)$, $\tilde{w}^m(x)$, $k^m(x)$, $\zeta_\alpha(x)$, $l(x)$, $s(x)$ and $\check{s}_\alpha(x)$ and a ghost for ghost field $f(x)$ to the reducible parameter $w'(x)$. The ghost fields $a(x)$, $a'(x)$, $b^I(x)$, $b'^I(x)$, $\tilde{c}^m(x)$, $d^m(x)$, $c_L(x)$ and $c_W(x)$ are fermionic, whereas the ghost fields $\alpha'_\alpha(x)$, $\gamma_\alpha(x)$ and $\check{c}_{S\alpha}(x)$ and the ghost for ghost field $f(x)$ are bosonic. Since the $\mathrm{U(1)_V}\times\mathrm{U(1)_A}$ superstring model is a first-stage reducible system, the boundary conditions (4.2b) with $n = 0$, 1 correspond to the gauge transformations (3.18) and the reducibility conditions (3.19), respectively. Then, we can solve the master equation perturbatively in the order of antifields,

$$
\begin{aligned}
S_{\min} = S_{\text{classical}} \\
+\int \mathrm{d}^2x \Big\{ & -\xi^*_I\Big(a'\phi^I + d^n\partial_n\xi^I + i(\gamma\lambda^I)\Big) \\
& +(\lambda^*_I\alpha')\phi^I + d^n(\lambda^*_I\partial_n\lambda^I) + (\lambda^*_I\sigma^m\gamma)\Big(\partial_m\xi^I - \frac{i}{2}(\chi_m\lambda^I)\Big) \\
& -\frac{1}{2}c_L(\lambda^*_I\bar{\sigma}\lambda^I) + \frac{1}{2}c_W(\lambda^*_I\lambda^I) - \frac{1}{e}(\lambda^*_I\sigma^m\gamma)a'\tilde{B}^{*I}_m \\
& +\frac{1}{4e}(\lambda^*_I\sigma^m\lambda^{*I})(\gamma\sigma_m\gamma) + \frac{1}{4e}(\lambda^*_I\bar{\sigma}\lambda^{*I})(\gamma\bar{\sigma}\gamma) \\
& -\tilde{A}^*_m\Big(\frac{\varepsilon^{mn}}{e}\partial_n a + g^{mn}\partial_n a' - \frac{i}{2}(\alpha'\sigma^n\sigma^m\chi_n) + d^n\partial_n\tilde{A}^m - \partial_n d^m\tilde{A}^n \\
& \qquad +i(\gamma\sigma^m\psi) - i(\gamma\sigma^n\chi_n)\tilde{A}^m + 2c_W\tilde{A}^m\Big) \\
& -\frac{1}{2}\partial_m a'(\psi^*\sigma^n\sigma^m\chi_n) + (\psi^*\sigma^m\nabla_m\alpha') - \frac{i}{8}(\psi^*\alpha')(\chi_m\sigma^n\sigma^m\chi_n) \\
& +d^n(\psi^*\partial_n\psi) + (\psi^*\gamma)\nabla_m\tilde{A}^m - i(\gamma\sigma^m\chi_m)(\psi^*\psi) - \frac{i}{2}(\gamma\sigma^m\psi)(\psi^*\chi_m) \\
& -\frac{1}{2}c_L(\psi^*\bar{\sigma}\psi) + \frac{3}{2}c_W(\psi^*\psi) \\
& -\phi^*_I\Big(d^n\partial_n\phi^I - \frac{i}{e}(\gamma\sigma^m\gamma)\tilde{B}^{*I}_m\Big) - \bar{\phi}^*_I\Big(b'^I + d^n\partial_n\bar{\phi}^I\Big) \\
& -\tilde{B}^*_{mI}\Big(-\frac{a}{e}\varepsilon^{mn}\partial_n\xi^I + a'g^{mn}\Big(\partial_n\xi^I - \frac{i}{2}(\chi_l\sigma_n\sigma^l\lambda^I)\Big) - i(\alpha'\sigma^m\lambda^I) \\
& \qquad +\frac{1}{e}\varepsilon^{mn}\partial_n b^I + g^{mn}\partial_n b'^I - \tilde{c}^m\phi^I + d^n\partial_n\tilde{B}^{mI} - \partial_n d^m\tilde{B}^{nI} \\
& \qquad +i(\gamma\lambda^I)\tilde{A}^m - i(\gamma\sigma^m\sigma^n\sigma^l\chi_n)\partial_l\bar{\phi}^I - i(\gamma\sigma^n\chi_n)\tilde{B}^{mI} + 2c_W\tilde{B}^{mI} \\
& \qquad +\frac{1}{2e^2}aa'\varepsilon^{mn}\tilde{B}^{*I}_n - \frac{1}{e}a(\gamma\sigma^m\lambda^{*I}) + \frac{i}{e}(\gamma\sigma^m\gamma)\phi^{*I} + \frac{1}{2e^2}\varepsilon^{mn}\tilde{B}^{*I}_n f\Big) \\
& -\tilde{C}^*\Big(\partial_m a'\tilde{A}^m - a'\nabla_m\tilde{A}^m + 2i(\alpha'\psi) + \nabla_m\tilde{c}^m + d^n\partial_n\tilde{C} \\
& \qquad -i(\gamma\sigma^m\chi_m)\tilde{C} + 2c_W\tilde{C}\Big) \\
& -e^{*m}_a\Big(d^n\partial_n e_m{}^a + \partial_m d^n e_n{}^a + i(\gamma\sigma^a\chi_m) + c_L e_m{}^b\varepsilon_b{}^a - c_W e_m{}^a\Big) \\
& +d^n(\chi^{*m}\partial_n\chi_m) + \partial_m d^n(\chi^{*m}\chi_n) + 2(\chi^{*m}\nabla_m\gamma) - \frac{i}{2}(\chi_m\bar{\sigma}\sigma^l\chi_l)(\chi^{*m}\bar{\sigma}\gamma) \\
& -\frac{1}{2}c_L(\chi^{*m}\bar{\sigma}\chi_m) - \frac{1}{2}c_W(\chi^{*m}\chi_m) - (\chi^{*m}\sigma_m\check{c}_S)
\end{aligned}
$$

$$
\begin{aligned}
&+a^*\Big(d^n\partial_n a + i(\gamma\bar\sigma\alpha') + ie\varepsilon_{mn}(\gamma\sigma^m\gamma)\tilde A^n\Big) + a'^*\Big(d^n\partial_n a' - i(\gamma\alpha')\Big)\\
&+d^n(\alpha'^*\partial_n\alpha') + (\alpha'^*\sigma^m\gamma)\partial_m a' - \frac{i}{2}(\alpha'^*\sigma^m\gamma)(\alpha'\chi_m)\\
&-\frac{i}{2}(\gamma\sigma^m\gamma)(\alpha'^*\sigma_m\psi) - \frac{i}{2}(\gamma\bar\sigma\gamma)(\alpha'^*\bar\sigma\psi) - \frac{1}{2}c_L(\alpha'^*\bar\sigma\alpha') + \frac{1}{2}c_W(\alpha'^*\alpha')\\
&+b_I^*\Big(d^n\partial_n b^I + i(\gamma\lambda^I)a - i(\gamma\bar\sigma\lambda^I)a' + ie\varepsilon_{mn}(\gamma\sigma^m\gamma)\tilde B^{nI}\\
&\qquad -i\frac{\varepsilon^{mn}}{e}(\gamma\sigma_m\gamma)\partial_n\bar\phi^I + \phi^I f\Big)\\
&+b_I'^*\Big(d^n\partial_n b'^I + i(\gamma\sigma^m\gamma)\partial_m\bar\phi^I\Big)\\
&+\tilde c_m^*\Big(\frac{\varepsilon^{mn}}{e}a\partial_n a' - g^{mn}a'\partial_n a' - \frac{i}{2}(\alpha'\sigma^n\sigma^m\chi_n)a' - i(\alpha'\sigma^m\alpha')\\
&\qquad +d^n\partial_n\tilde c^m - \partial_n d^m\tilde c^n + i(\gamma\sigma^m\psi)a' + i(\gamma\alpha')\tilde A^m - i(\gamma\sigma^n\chi_n)\tilde c^m\\
&\qquad +i(\gamma\sigma^m\gamma)\tilde C + 2c_W\tilde c^m + \frac{\varepsilon^{mn}}{e}\partial_n f\Big)\\
&+d_m^*\Big(d^n\partial_n d^m - i(\gamma\sigma^m\gamma)\Big)\\
&+d^n(\gamma^*\partial_n\gamma) - \frac{i}{2}(\gamma\sigma^m\gamma)(\gamma^*\chi_m) - \frac{1}{2}c_L(\gamma^*\bar\sigma\gamma) - \frac{1}{2}c_W(\gamma^*\gamma)\\
&+c_L^*\Big(d^n\partial_n c_L + i(\gamma\sigma^m\gamma)\Big(\omega_m - \frac{i}{2}(\chi_m\bar\sigma\sigma^n\chi_n)\Big) + i(\check c_S\bar\sigma\gamma)\Big)\\
&+c_W^*\Big(d^n\partial_n c_W + i(\check c_S\gamma)\Big)\\
&+d^n(\check c_S^*\partial_n\check c_S) - \frac{i}{2}(\gamma\sigma^m\gamma)\Big(\check c_S^*\sigma_m\bar\sigma\frac{\varepsilon^{pq}}{e}\Big(\nabla_p\chi_q - \frac{i}{4}(\chi_p\bar\sigma\sigma^l\chi_l)\bar\sigma\chi_q\Big)\Big)\\
&-\frac{i}{2}(\gamma\bar\sigma\gamma)\Big(\check c_S^*\frac{\varepsilon^{pq}}{e}\Big(\nabla_p\chi_q - \frac{i}{4}(\chi_p\bar\sigma\sigma^l\chi_l)\bar\sigma\chi_q\Big)\Big) - \partial_m c_W(\check c_S^*\sigma^m\gamma)\\
&-\frac{i}{2}(\check c_S\chi_m)(\check c_S^*\sigma^m\gamma) + \frac{1}{2}(\check c_S^*\bar\sigma\check c_S)c_L - \frac{1}{2}(\check c_S^*\check c_S)c_W\\
&-f^*\Big(d^n\partial_n f - i(\gamma\alpha')a + i(\gamma\bar\sigma\alpha')a' + ie\varepsilon_{mn}(\gamma\sigma^m\gamma)\tilde c^n\Big)\Big\}. \qquad (4.6)
\end{aligned}
$$

The gauge degrees of freedom are fixed by introducing a nonminimal action which must be added to the minimal one and choosing a suitable gauge-fixing fermion. By using the gauge parameters for the general coordinate, Weyl, local supersymmetry and super-Weyl transformations, we here choose super-orthonormal gauge conditions $e_m{}^a(x) = \delta_m{}^a$ for the zweibein field and $\chi_{m\alpha}(x) = 0$ for the gravitino field. In the same way for the bosonic model [16], the $\mathrm{U}(1)_\mathrm{V}\times\mathrm{U}(1)_\mathrm{A}$ gauge parameters $\big(v(x), v'(x), \alpha_i\big)$, $\big(u^I(x), u'^I(x), \beta_i^I\big)$ and $\tilde w^m$ allow to choose gauges $\tilde A^m(x) = \tilde B^{mI}(x) = 0$ and $\tilde C(x) = \tilde C_0$, where $\tilde C_0$ is a constant parameter. We also impose a gauge condition $\partial_m\Big(eg^{mn}e\varepsilon_{nk}\tilde c^k(x)\Big) = 0$ to fix the residual gauge degrees of freedom from the reducibility condition. In addition to these, we fix a gauge $\psi_\alpha(x) = 0$ by using the gauge parameter $\mu'_\alpha(x)$ in this supersymmetric model. In order to adopt all of these gauge fixing conditions, we introduce the nonminimal action

$S_{\rm nonmin}$,

$$S_{\rm nonmin} = \int {\rm d}^2x \Big\{ \varepsilon^{mn}\hat{a}^*_m Z^{(a)}_n + \varepsilon^{mn}\hat{b}^{*I}_m Z^{(b)}_{nI} + c^* Z^{(c)} - (\bar{\alpha}^* \check{Z}^{(\alpha)}) + \bar{d}^{*}_m{}^{a} Z^m{}_a - (\beta^*_m \check{Z}^m) - \bar{f}^* c' \Big\}, \qquad (4.7)$$

and the gauge-fixing fermion Ψ,

$$\Psi = \int {\rm d}^2x \Big\{ e\varepsilon_{mn}\hat{a}^m \tilde{A}^n + e\varepsilon_{mn}\hat{b}^m_I \tilde{B}^{nI} + ec(\tilde{C} - \tilde{C}_0) + ie(\bar{\alpha}\psi) - e\bar{d}^m{}_a(e_m{}^a - \delta_m{}^a) + \frac{e}{2}(\beta^m \chi_m) + \bar{f}\partial_m(eg^{mn}e\varepsilon_{nk}\tilde{c}^k) \Big\}. \qquad (4.8)$$

The antighost fields $\hat{a}^m(x)$, $\hat{b}^m_I(x)$, $c(x)$, $\bar{d}^m{}_a(x)$ and $c'(x)$ are fermionic, while $\bar{\alpha}_\alpha(x)$ and $\beta^m{}_\alpha(x)$ are bosonic. The auxiliary fields $Z^{(a)}_m(x)$, $Z^{(b)}_{mI}(x)$, $Z^{(c)}(x)$, $Z^m{}_a(x)$ and $\bar{f}(x)$ are bosonic, whereas $\check{Z}^{(\alpha)}_\alpha(x)$ and $\check{Z}^m{}_\alpha(x)$ are fermionic†.

The BRST transformations of the field $\Phi^A(x)$ and the antifields $\Phi^*_A(x)$ are now given by

$$s\Phi^A = \Big(S_{\rm min} + S_{\rm nonmin}, \Phi^A\Big), \qquad s\Phi^*_A = \Big(S_{\rm min} + S_{\rm nonmin}, \Phi^*_A\Big). \qquad (4.9)$$

Then, the BRST transformations of the fields $\Phi^A(x)$ are

$$\begin{aligned}
s\xi^I &= a'\phi^I + d^n\partial_n\xi^I + i(\gamma\lambda^I), \\
s\lambda^I_\alpha &= \alpha'_\alpha\phi^I + d^n\partial_n\lambda^I_\alpha + (\sigma^m\gamma)_\alpha\Big(\partial_m\xi^I - \frac{i}{2}(\chi_m\lambda^I)\Big) - \frac{1}{2}c_L(\bar{\sigma}\lambda^I)_\alpha + \frac{1}{2}c_W\lambda^I_\alpha \\
&\quad -\frac{1}{e}(\sigma^m\gamma)_\alpha a'\tilde{B}^{*I}_m + \frac{1}{2e}(\sigma^m\lambda^{*I})_\alpha(\gamma\sigma_m\gamma) + \frac{1}{2e}(\bar{\sigma}\lambda^{*I})_\alpha(\gamma\bar{\sigma}\gamma), \\
s\tilde{A}^m &= \frac{\varepsilon^{mn}}{e}\partial_n a + g^{mn}\partial_n a' - \frac{i}{2}(\alpha'\sigma^n\sigma^m\chi_n) \\
&\quad +d^n\partial_n\tilde{A}^m - \partial_n d^m\tilde{A}^n + i(\gamma\sigma^m\psi) - i(\gamma\sigma^n\chi_n)\tilde{A}^m + 2c_W\tilde{A}^m, \\
s\psi_\alpha &= -\frac{1}{2}\partial_m a'(\sigma^n\sigma^m\chi_n)_\alpha + (\sigma^m\nabla_m\alpha')_\alpha - \frac{i}{8}\alpha'_\alpha(\chi_m\sigma^n\sigma^m\chi_n) \\
&\quad +d^n\partial_n\psi_\alpha + \gamma_\alpha\nabla_m\tilde{A}^m - i(\gamma\sigma^m\chi_m)\psi_\alpha - \frac{i}{2}(\gamma\sigma^m\psi)\chi_{m\alpha} - \frac{1}{2}c_L(\bar{\sigma}\psi)_\alpha + \frac{3}{2}c_W\psi_\alpha, \\
s\phi^I &= d^n\partial_n\phi^I - \frac{i}{e}(\gamma\sigma^m\gamma)\tilde{B}^{*I}_m, \\
s\bar{\phi}^I &= b'^I + d^n\partial_n\bar{\phi}^I, \\
s\tilde{B}^{mI} &= -\frac{a}{e}\varepsilon^{mn}\partial_n\xi^I + a'g^{mn}\Big(\partial_n\xi^I - \frac{i}{2}(\chi_l\sigma_n\sigma^l\lambda^I)\Big) - i(\alpha'\sigma^m\lambda^I) \\
&\quad +\frac{1}{e}\varepsilon^{mn}\partial_n b^I + g^{mn}\partial_n b'^I - \tilde{c}^m\phi^I + d^n\partial_n\tilde{B}^{mI} - \partial_n d^m\tilde{B}^{nI}
\end{aligned}$$

†The auxiliary fields $Z^{(a)}_m(x)$, $Z^{(b)}_{mI}(x)$ and $Z^{(c)}(x)$ are equivalent to $Z^a_m(x)$, $Z^b_{mI}(x)$ and $Z^c(x)$ in our previous paper [16], respectively. In order to avoid confusing the attached indices a, b and c for $Z^a_m(x)$, $Z^b_{mI}(x)$ and $Z^c(x)$ with local Lorentz indices, we have changed the notations.

$$
\begin{aligned}
&+i(\gamma\lambda^I)\tilde{A}^m - i(\gamma\sigma^m\sigma^n\sigma^l\chi_n)\partial_l\bar{\phi}^I - i(\gamma\sigma^n\chi_n)\tilde{B}^{mI} + 2c_W\tilde{B}^{mI} \\
&+\frac{1}{e^2}aa'\varepsilon^{mn}\tilde{B}^{*I}_n - \frac{1}{e}a(\gamma\sigma^m\lambda^{*I}) + \frac{i}{e}(\gamma\sigma^m\gamma)\phi^{*I} + \frac{1}{e^2}\varepsilon^{mn}\tilde{B}^{*I}_n f,
\end{aligned}
$$

$$
\begin{aligned}
s\tilde{C} &= \partial_m a'\tilde{A}^m - a'\nabla_m\tilde{A}^m + 2i(\alpha'\psi) + \nabla_m\tilde{c}^m + d^n\partial_n\tilde{C} - i(\gamma\sigma^m\chi_m)\tilde{C} + 2c_W\tilde{C}, \\
se_m{}^a &= d^n\partial_n e_m{}^a + \partial_m d^n e_n{}^a + i(\gamma\sigma^a\chi_m) + c_L e_m{}^b\varepsilon_b{}^a - c_W e_m{}^a, \qquad (4.10a) \\
s\chi_{m\alpha} &= d^n\partial_n\chi_{m\alpha} + \partial_m d^n\chi_{n\alpha} + 2(\nabla_m\gamma)_\alpha - \frac{i}{2}(\chi_m\bar{\sigma}\sigma^l\chi_l)(\bar{\sigma}\gamma)_\alpha \\
&\quad -\frac{1}{2}c_L(\bar{\sigma}\chi_m)_\alpha - \frac{1}{2}c_W\chi_{m\alpha} - (\sigma_m\check{c}_S)_\alpha, \\
sa &= d^n\partial_n a + i(\gamma\bar{\sigma}\alpha') + ie\varepsilon_{mn}(\gamma\sigma^m\gamma)\tilde{A}^n, \\
sa' &= d^n\partial_n a' - i(\gamma\alpha'), \\
s\alpha'_\alpha &= d^n\partial_n\alpha'_\alpha - (\sigma^m\gamma)_\alpha\partial_m a' + \frac{i}{2}(\sigma^m\gamma)_\alpha(\alpha'\chi_m) + \frac{i}{2}(\gamma\sigma^m\gamma)(\sigma_m\psi)_\alpha + \frac{i}{2}(\gamma\bar{\sigma}\gamma)(\bar{\sigma}\psi)_\alpha \\
&\quad -\frac{1}{2}c_L(\bar{\sigma}\alpha')_\alpha + \frac{1}{2}c_W\alpha'_\alpha, \\
sb^I &= d^n\partial_n b^I + i(\gamma\lambda^I)a - i(\gamma\bar{\sigma}\lambda^I)a' + ie\varepsilon_{mn}(\gamma\sigma^m\gamma)\tilde{B}^{nI} - i\frac{\varepsilon^{mn}}{e}(\gamma\sigma_m\gamma)\partial_n\bar{\phi}^I + \phi^I f, \\
sb'^I &= d^n\partial_n b'^I + i(\gamma\sigma^m\gamma)\partial_m\bar{\phi}^I, \\
s\tilde{c}^m &= \frac{\varepsilon^{mn}}{e}a\partial_n a' - g^{mn}a'\partial_n a' - \frac{i}{2}(\alpha'\sigma^n\sigma^m\chi_n)a' - i(\alpha'\sigma^m\alpha') + d^n\partial_n\tilde{c}^m - \partial_n d^m\tilde{c}^n \\
&\quad +i(\gamma\sigma^m\psi)a' + i(\gamma\alpha')\tilde{A}^m - i(\gamma\sigma^n\chi_n)\tilde{c}^m + i(\gamma\sigma^m\gamma)\tilde{C} + 2c_W\tilde{c}^m + \frac{\varepsilon^{mn}}{e}\partial_n f, \\
sd^m &= d^n\partial_n d^m - i(\gamma\sigma^m\gamma), \\
s\gamma_\alpha &= d^n\partial_n\gamma_\alpha + \frac{i}{2}(\gamma\sigma^m\gamma)\chi_{m\alpha} - \frac{1}{2}c_L(\bar{\sigma}\gamma)_\alpha - \frac{1}{2}c_W\gamma_\alpha, \\
sc_L &= d^n\partial_n c_L + i(\gamma\sigma^m\gamma)\Big(\omega_m - \frac{i}{2}(\chi_m\bar{\sigma}\sigma^n\chi_n)\Big) + i(\check{c}_S\bar{\sigma}\gamma), \\
sc_W &= d^n\partial_n c_W + i(\check{c}_S\gamma), \\
s\check{c}_{S\alpha} &= d^n\partial_n\check{c}_{S\alpha} + \frac{i}{2}\Big(\Big((\gamma\sigma^m\gamma)\sigma_m + (\gamma\bar{\sigma}\gamma)\bar{\sigma}\Big)\bar{\sigma}\frac{\varepsilon^{pq}}{e}\Big(\nabla_p\chi_q - \frac{i}{4}(\chi_p\bar{\sigma}\sigma^l\chi_l)\bar{\sigma}\chi_q\Big)\Big)_\alpha \\
&\quad -\partial_m c_W(\sigma^m\gamma)_\alpha - \frac{i}{2}(\check{c}_S\chi_m)(\sigma^m\gamma)_\alpha - \frac{1}{2}(\bar{\sigma}\check{c}_S)_\alpha c_L + \frac{1}{2}\check{c}_{S\alpha}c_W, \\
sf &= d^n\partial_n f - i(\gamma\alpha')a + i(\gamma\bar{\sigma}\alpha')a' + ie\varepsilon_{mn}(\gamma\sigma^m\gamma)\tilde{c}^n,
\end{aligned}
$$

and

$$
\begin{aligned}
s\hat{a}^m &= \varepsilon^{mn}Z^{(a)}_n, & sZ^{(a)}_m &= 0, \\
s\hat{b}^m_I &= \varepsilon^{mn}Z^{(b)}_{nI}, & sZ^{(b)}_{mI} &= 0, \\
sc &= Z^{(c)}, & sZ^{(c)} &= 0, \\
s\bar{\alpha}_\alpha &= \check{Z}^{(\alpha)}_\alpha, & s\check{Z}^{(\alpha)}_\alpha &= 0, \qquad (4.10b) \\
s\bar{f} &= c', & sc' &= 0, \\
s\bar{d}^m{}_a &= Z^m{}_a, & sZ^m{}_a &= 0,
\end{aligned}
$$

$$s\beta^m{}_\alpha = \check{Z}^m{}_\alpha, \qquad\qquad s\check{Z}^m{}_\alpha = 0.$$

Now, let us construct a gauge-fixed action. The antifields are eliminated by using the gauge-fixing fermion (4.8) via equations $\Phi^*_A(x) = \delta_{\rm L}\Psi/\delta\Phi^A(x)$. In order to specify the physical degrees of freedom of the two-dimensional supergravity sector, it might be useful to decompose the antighost fields $\bar{d}^m{}_a(x)$ and $\beta^m{}_\alpha(x)$ as follows,

$$\bar{d}^m{}_a = e^n{}_a \bar{d}^m{}_n + \frac{\varepsilon^{mn}}{e} e_{na}\bar{c}_L + e^m{}_a \bar{c}_W, \tag{4.11a}$$

$$\beta^m{}_\alpha = \bar{\beta}^m{}_\alpha + (\sigma^m\beta)_\alpha, \tag{4.11b}$$

where we define

$$\begin{aligned}
\bar{d}^m{}_n &\equiv \frac{1}{2}\Big(\bar{d}^m{}_a e_n{}^a + \bar{d}^l{}_a e^{ma} g_{ln} - \delta^m{}_n \bar{d}^l{}_a e_l{}^a\Big),\\
\bar{c}_L &\equiv \frac{1}{2}\varepsilon^{ab}\bar{d}^m{}_a e_{mb},\\
\bar{c}_W &\equiv \frac{1}{2}\bar{d}^m{}_a e_m{}^a,\\
\bar{\beta}^m{}_\alpha &\equiv \frac{1}{2}(\sigma_n\sigma^m\beta^n)_\alpha,\\
\beta_\alpha &\equiv \frac{1}{2}(\sigma_m\beta^m)_\alpha.
\end{aligned}$$

The field $\bar{d}^m{}_n(x)$ is symmetric $\varepsilon^n{}_m\bar{d}^m{}_n(x) = 0$ and traceless $\delta^n{}_m\bar{d}^m{}_n(x) = 0$, and the field $\bar{\beta}^m{}_\alpha(x)$ is σ-traceless $(\sigma_m)_\alpha{}^\beta\bar{\beta}^m{}_\beta(x) = 0$. Then, the gauge-fixed action is given by

$$\begin{aligned}
S_{\rm gauge-fixed} &= S_{\rm min} + S_{\rm nonmin}\Big|_{\Phi^* = \frac{\delta\Psi}{\delta\Phi}}\\
&= \int {\rm d}^2x \Big\{ -\frac{1}{2}eg^{mn}\partial_m\xi^I\partial_n\xi_I - \frac{i}{2}e(\lambda^I\sigma^m\partial_m\lambda_I) - eg^{mn}\partial_m\bar{\phi}^I\partial_n\phi_I\\
&\qquad - \hat{a}^m\Big(\partial_m a + e\varepsilon_{mk}g^{kn}\partial_n a'\Big) - \hat{b}^m_I\Big(\partial_m b^I + e\varepsilon_{mk}g^{kn}\partial_n b'^I\Big)\\
&\qquad - \hat{c}^m\Big(\partial_m c + e\varepsilon_{mk}g^{kn}\partial_n c'\Big) + ie(\bar{\alpha}\sigma^m\partial_m\alpha') - eg^{mn}\partial_m\bar{f}\partial_n f\\
&\qquad + e\bar{d}^m{}_n\partial_m d^n + e(\bar{\beta}^m\partial_m\gamma)\\
&\qquad - 2a\hat{b}^m_I\partial_m\xi^I + \varepsilon_{mn}\hat{b}^m_I\hat{c}^n\phi^I\\
&\qquad + \frac{1}{2}(f + aa')\varepsilon_{mn}\hat{b}^m_I\hat{b}^{nI} + ie\varepsilon_{mn}\hat{b}^m_I(\alpha'\sigma^n\lambda^I)\\
&\qquad - e\tilde{A}^m Z^{(a)}_m - e\tilde{B}^{mI}Z^{(b)}_{mI} + e\tilde{C}Z^{(c)} + ie(\psi\check{Z}^{(\alpha)})\\
&\qquad - 2e\bar{c}_L c_L - 2e\bar{c}_W c_W - e(\beta\check{c}_S)\\
&\qquad - e(e_m{}^a - \delta_m{}^a)Z^m{}_a + \frac{e}{2}(\chi_m\check{Z}^m)\Big\},
\end{aligned} \tag{4.12}$$

where we redefine some of the fields as follows,

$$Z^{(a)}_m - \phi_I(\partial_m\xi^I) - \varepsilon_{mn}d^k\partial_k\hat{a}^n - \partial_m d^k\varepsilon_{kn}\hat{a}^n + c\partial_m a' + \partial_m(ca')$$

$$-ie\varepsilon_{mk}g^{kn}\partial_n\bar{f}(\gamma\alpha') - i\varepsilon_{mn}\hat{b}_I^n(\gamma\lambda^I) + i\partial_m(\bar{\alpha}\gamma) \to Z_m^{(a)},$$
$$Z_{mI}^{(b)} - \partial_m\phi_I - \varepsilon_{mn}d^k\partial_k\hat{b}_I^n - \partial_m d^k\varepsilon_{kn}\hat{b}_I^n \to Z_{mI}^{(b)},$$
$$Z^{(c)} - \frac{1}{2}\phi^I\phi_I - d^n\partial_n c + ie\varepsilon_{mk}g^{kn}\partial_n\bar{f}(\gamma\sigma^m\gamma) \to Z^{(c)},$$
$$\begin{aligned}\check{Z}_\alpha^{(\alpha)} + \phi_I\lambda_\alpha^I - d^n\partial_n\bar{\alpha}_\alpha - 2c\alpha'_\alpha + e\varepsilon_{mk}g^{kn}\partial_n\bar{f}a'(\sigma^m\gamma)_\alpha - \varepsilon_{mn}\hat{a}^n(\sigma^m\gamma)_\alpha \\ -\frac{1}{4}\partial_m d^n(\sigma^m\sigma_n\bar{\alpha})_\alpha - \frac{i}{2}(\bar{\alpha}\chi_m)(\sigma^m\gamma)_\alpha \\ +\frac{1}{2}\Big(c_L - \frac{1}{2}\frac{\varepsilon^{mk}}{e}g_{kn}\partial_m d^n\Big)(\bar{\sigma}\bar{\alpha})_\alpha - \frac{1}{2}\Big(c_W - \frac{1}{2}\partial_n d^n\Big)\bar{\alpha}_\alpha \to \check{Z}_\alpha^{(\alpha)},\end{aligned}$$
$$\begin{aligned}Z^m{}_a - \frac{i}{2}\frac{\varepsilon^{mn}}{e}\partial_n\Big(e_a{}^k\Big(\alpha'\sigma_k\bar{\sigma}(\bar{\alpha} - 2\bar{f}\bar{\sigma}\alpha')\Big)\Big) + \partial_n(\bar{d}^m{}_a d^n) - i\bar{d}^m{}_a(\gamma\sigma^n\chi_n) \\ +2\bar{d}^m{}_a\Big(c_W - \frac{1}{2}\partial_n d^n\Big) - \frac{1}{2}\frac{\varepsilon^{mn}}{e}\partial_n\Big(e_{la}(\beta^l\bar{\sigma}\gamma)\Big) \to Z^m{}_a,\end{aligned}$$
$$\begin{aligned}\check{Z}^m{}_\alpha + i(\sigma^n\sigma^m\lambda_I)_\alpha\partial_n\xi^I - \frac{1}{8}(\sigma^n\sigma^m\chi_n)_\alpha(\lambda^I\lambda_I) + ia'\varepsilon_{nk}\hat{b}_I^k(\sigma^n\sigma^m\lambda^I)_\alpha \\ +2i\varepsilon_{nk}\hat{b}_I^k\partial_l\bar{\phi}^I(\sigma^l\sigma^m\sigma^n\gamma)_\alpha + ie\varepsilon_{nk}g^{kl}\partial_l\bar{f}a'(\sigma^n\sigma^m\alpha')_\alpha + 2i\varepsilon_{nk}g^{kl}\partial_l\bar{f}\hat{c}^n(\sigma^m\gamma)_\alpha \\ -2i\varepsilon_{nk}g^{km}\partial_l\bar{f}\hat{c}^n(\sigma^l\gamma)_\alpha + 2i\varepsilon_{nk}g^{ml}\partial_l\bar{f}\hat{c}^k(\sigma^n\gamma)_\alpha - i\varepsilon_{nk}\hat{a}^k(\sigma^n\sigma^m\alpha')_\alpha \\ -i\partial_n a'(\sigma^n\sigma^m\bar{\alpha})_\alpha + \frac{1}{4}(\bar{\alpha}\alpha')(\sigma^n\sigma^m\chi_n)_\alpha - 2i(\sigma^a\gamma)_\alpha\bar{d}^m{}_a - d^n\partial_n\beta^m{}_\alpha \\ +\partial_n d^m\beta^n{}_\alpha + i(\gamma\sigma^n\chi_n)\beta^m{}_\alpha - \frac{i}{2}(\bar{\sigma}\sigma^l\chi_l)_\alpha(\beta^m\bar{\sigma}\gamma) + \frac{1}{2}(\bar{\sigma}\beta^m)_\alpha c_L - \frac{5}{2}\beta^m{}_\alpha c_W \to \check{Z}^m{}_\alpha,\end{aligned}$$
$$\hat{a}^m - \varepsilon^{mn}\partial_n(\bar{f}a) + e\bar{f}g^{mn}\partial_n a' - \hat{b}_I^m\xi^I \to \hat{a}^m,$$
$$b^I + a\xi^I \to b^I,$$
$$b'^I + a'\xi^I \to b'^I,$$
$$c' - d^n\partial_n\bar{f} \to c',$$
$$\bar{\alpha}_\alpha - 2\bar{f}(\bar{\sigma}\alpha')_\alpha \to \bar{\alpha}_\alpha,$$
$$c_L - \frac{1}{2}\frac{\varepsilon^{mn}}{e}g_{nl}\partial_m d^l \to c_L,$$
$$c_W - \frac{1}{2}\partial_m d^m \to c_W,$$
$$\check{c}_{S\alpha} - (\sigma^m\partial_m\gamma)_\alpha \to \check{c}_{S\alpha},$$

and we denote $\hat{c}^m(x) \equiv e\tilde{c}^m(x)$. It should be noted that we remove a BRST exact term

$$-e\tilde{C}_0\Big(Z^{(c)} - c\Big(d^n\partial_n e + i(\gamma\sigma^n\chi_n) - 2\Big(c_W - \frac{1}{2}\partial_n d^n\Big)\Big)\Big) = -s(e\tilde{C}_0 c),$$

from the above action.

Using equations of motion of the gauge-fixed action (4.12), thus imposing the gauge fixing conditions, we consistently fix the fields as

$$\tilde{A}^m = \tilde{B}^{mI} = \tilde{C} = \psi_\alpha = c_L = c_W = \check{c}_{S\alpha} = \chi_{m\alpha} = 0,$$

$$
\begin{aligned}
&Z_m^{(a)} = Z_{mI}^{(b)} = Z^{(c)} = \check{Z}_\alpha^{(\alpha)} = \bar{c}_L = \bar{c}_W = \beta_\alpha = \check{Z}^m{}_\alpha = 0,\\
&e_m{}^a = \delta_m{}^a,\\
&Z^m{}_a = -\frac{1}{2}\delta^m{}_a\eta^{kl}\partial_k\xi^I\partial_l\xi_I + \delta^n{}_a\eta^{mk}\partial_k\xi^I\partial_n\xi_I\\
&\qquad -\frac{i}{2}\delta^m{}_a(\lambda^I\sigma^n\partial_n\lambda_I) + \frac{i}{2}\delta^n{}_a(\lambda^I\sigma^m\partial_n\lambda_I)\\
&\qquad -\delta^m{}_a\eta^{kl}\partial_k\bar{\phi}^I\partial_l\phi_I + \delta^n{}_a\eta^{mk}\partial_k\bar{\phi}^I\partial_n\phi_I + \delta^n{}_a\eta^{mk}\partial_n\bar{\phi}^I\partial_k\phi_I\\
&\qquad -\delta^m{}_a\hat{a}^n\varepsilon_n{}^l\partial_l a' + \delta^n{}_a\hat{a}^k\varepsilon_k{}^m\partial_n a' + \delta^n{}_a\hat{a}^k\varepsilon_{kn}\eta^{ml}\partial_l a'\\
&\qquad -\delta^m{}_a\hat{b}_I^n\varepsilon_n{}^l\partial_l b'^I + \delta^n{}_a\hat{b}_I^k\varepsilon_k{}^m\partial_n b'^I + \delta^n{}_a\hat{b}_I^k\varepsilon_{kn}\eta^{ml}\partial_l b'^I\\
&\qquad -\delta^m{}_a\hat{c}^n\varepsilon_n{}^l\partial_l c' + \delta^n{}_a\hat{c}^k\varepsilon_k{}^m\partial_n c' + \delta^n{}_a\hat{c}^k\varepsilon_{kn}\eta^{ml}\partial_l c'\\
&\qquad +i\delta^m{}_a(\bar{\alpha}\sigma^n\partial_n\alpha') - i\delta^n{}_a(\bar{\alpha}\sigma^m\partial_n\alpha')\\
&\qquad -\delta^m{}_a\eta^{kl}\partial_k\bar{f}\partial_l f + \delta^n{}_a\eta^{mk}\partial_k\bar{f}\partial_n f + \delta^n{}_a\eta^{mk}\partial_n\bar{f}\partial_k f\\
&\qquad +\delta^m{}_a\bar{d}^k{}_n\partial_k d^n + \delta^n{}_a\bar{d}^k{}_n\partial_k d^m - \delta^n{}_a\bar{d}^m{}_k\partial_n d^k\\
&\qquad +\delta^m{}_a(\bar{\beta}^n\partial_n\gamma) - \delta^n{}_a(\bar{\beta}^m\partial_n\gamma)\\
&\qquad +i\delta^m{}_a\varepsilon_{kl}\hat{b}_I^k(\alpha'\sigma^l\lambda^I) + i\delta^n{}_a\varepsilon_{nk}\hat{b}_I^k(\alpha'\sigma^m\lambda^I).
\end{aligned}
\tag{4.13}
$$

Then, we finally obtain the gauge-fixed action

$$
\begin{aligned}
S_{\text{gauge-fixed}} = \int \mathrm{d}^2x\Big\{ &-\frac{1}{2}\eta^{mn}\partial_m\xi^I\partial_n\xi_I - \frac{i}{2}(\lambda^I\sigma^m\partial_m\lambda_I) - \eta^{mn}\partial_m\bar{\phi}^I\partial_n\phi_I\\
&-\hat{a}^m\Big(\partial_m a + \varepsilon_m{}^n\partial_n a'\Big) - \hat{b}_I^m\Big(\partial_m b^I + \varepsilon_m{}^n\partial_n b'^I\Big)\\
&-\hat{c}^m\Big(\partial_m c + \varepsilon_m{}^n\partial_n c'\Big) + i(\bar{\alpha}\sigma^m\partial_m\alpha') - \eta^{mn}\partial_m\bar{f}\partial_n f\\
&+\eta^{mn}\bar{d}_{mk}\partial_n d^k + (\bar{\beta}^m\partial_m\gamma)\\
&-2a\hat{b}_I^m\partial_m\xi^I + \varepsilon_{mn}\hat{b}_I^m\hat{c}^n\phi^I\\
&+\frac{1}{2}(f + aa')\varepsilon_{mn}\hat{b}_I^m\hat{b}^{nI} + i\varepsilon_{mn}\hat{b}_I^m(\alpha'\sigma^n\lambda^I)\Big\}.
\end{aligned}
\tag{4.14}
$$

The action (4.14) is invariant under the following on-shell nilpotent BRST transformations which are obtained from (4.10a) and (4.10b) by eliminating the antifields and the auxiliary fields,

$$
\begin{aligned}
s\xi^I &= a'\phi^I + d^n\partial_n\xi^I + i(\gamma\lambda^I),\\
s\lambda_\alpha^I &= \alpha'_\alpha\phi^I + d^n\partial_n\lambda_\alpha^I + (\sigma^m\gamma)_\alpha\partial_m\xi^I + \frac{1}{4}\partial_m d_n(\sigma^m\sigma^n\lambda^I)_\alpha - \varepsilon_{mn}a'\hat{b}^{mI}(\sigma^n\gamma)_\alpha,\\
s\phi^I &= d^n\partial_n\phi^I + i(\gamma\sigma^m\gamma)\varepsilon_{mn}\hat{b}^{nI},\\
s\bar{\phi}^I &= b'^I - a'\xi^I + d^n\partial_n\bar{\phi}^I,\\
sa &= d^n\partial_n a + i(\gamma\bar{\sigma}\alpha'),
\end{aligned}
$$

$$
\begin{aligned}
sa' &= d^n\partial_n a' - i(\gamma\alpha'),\\
s\alpha'_\alpha &= d^n\partial_n\alpha'_\alpha - (\sigma^m\gamma)_\alpha\partial_m a' + \frac{1}{4}(\sigma^m\sigma^n\alpha')_\alpha\partial_m d_n,\\
sb^I &= (f - aa')\phi^I + d^n\partial_n b^I + 2i(\gamma\lambda^I)a - i(\gamma\bar\sigma\lambda^I)a' - i\varepsilon^{mn}(\gamma\sigma_m\gamma)\partial_n\bar\phi^I + i(\gamma\bar\sigma\alpha')\xi^I,\\
sb'^I &= d^n\partial_n b'^I + i(\gamma\lambda^I)a' + i(\gamma\sigma^m\gamma)\partial_m\bar\phi^I - i(\gamma\alpha')\xi^I,\\
s\hat c^m &= (\varepsilon^{mn}a - \eta^{mn}a')\partial_n a' - i(\alpha'\sigma^m\alpha') + \varepsilon^{mn}\partial_n f + \partial_n(d^n\hat c^m) - \partial_n d^m\hat c^n,\\
sd^m &= d^n\partial_n d^m - i(\gamma\sigma^m\gamma),\\
s\gamma_\alpha &= d^n\partial_n\gamma_\alpha - \frac{1}{4}(\sigma^n\sigma^m\gamma)_\alpha\partial_m d_n, \qquad (4.15)\\
sf &= d^n\partial_n f - i(\gamma\alpha')a + i(\gamma\bar\sigma\alpha')a' + i\varepsilon_{mn}(\gamma\sigma^m\gamma)\hat c^n,\\
s\hat a^m &= \varepsilon^{mn}(\phi_I\partial_n\xi^I - \partial_n\phi_I\xi^I) - a'\hat b_I^m\phi^I - (\varepsilon^{mn}c - \eta^{mn}c')\partial_n a' - \varepsilon^{mn}\partial_n(ca' + c'a)\\
&\quad + \partial_n(d^n\hat a^m) - \partial_n d^m\hat a^n + i\varepsilon^{mn}\partial_n(\gamma\bar\alpha) + 2i\hat b_I^m(\gamma\lambda^I) + i\partial_n\bar f(\gamma\sigma^n\sigma^m\alpha'),\\
s\hat b_I^m &= \varepsilon^{mn}\partial_n\phi_I + \partial_n(d^n\hat b_I^m) - \partial_n d^m\hat b_I^n,\\
sc &= \frac{1}{2}\phi^I\phi_I + d^n\partial_n c + i\varepsilon^{mn}\partial_m\bar f(\gamma\sigma_n\gamma),\\
s\bar\alpha_\alpha &= 2\Big((c - c'\bar\sigma)\alpha'\Big)_\alpha - \phi_I\lambda^I_\alpha + d^n\partial_n\bar\alpha_\alpha + \frac{1}{4}\partial_m d_n(\sigma^m\sigma^n\bar\alpha)_\alpha - \varepsilon_{mn}(\hat a^m + \hat b_I^m\xi^I)(\sigma^n\gamma)_\alpha\\
&\quad + (\varepsilon^{mn}a' + \eta^{mn}a)\partial_m\bar f(\sigma_n\gamma)_\alpha,\\
s\bar f &= c' + d^n\partial_n\bar f,\\
sc' &= d^n\partial_n c' + i(\gamma\sigma^m\gamma)\partial_m\bar f,\\
s\bar d_{mn} &= V_{mn} - \frac{1}{2}\eta_{mn}(\eta^{kl}V_{kl}),\\
s\bar\beta_{m\alpha} &= J_{m\alpha},
\end{aligned}
$$

where we denote

$$
\begin{aligned}
V_{mn} &\equiv \frac{1}{2}\partial_m\xi^I\partial_n\xi_I + \frac{i}{4}(\lambda^I\sigma_m\partial_n\lambda_I) + \partial_m\bar\phi^I\partial_n\phi_I\\
&\quad + \hat a^k\varepsilon_{km}\partial_n a' + \hat b_I^k\varepsilon_{km}\partial_n b'^I + \hat c^k\varepsilon_{km}\partial_n c' - \frac{i}{4}(\bar\alpha\sigma_m\partial_n\alpha') + \frac{i}{4}(\alpha'\sigma_m\partial_n\bar\alpha)\\
&\quad + \partial_m\bar f\partial_n f - \bar d_{mk}\partial_n d^k + \frac{1}{2}d^k\partial_k\bar d_{mn} - \frac{3}{4}(\bar\beta_m\partial_n\gamma) + \frac{1}{4}(\gamma\partial_m\bar\beta_n) + \frac{i}{2}\varepsilon_{mk}\hat b_I^k(\alpha'\sigma_n\lambda^I)\\
&\quad + (m \leftrightarrow n), \qquad (4.16a)\\
J_{m\alpha} &\equiv -i\Big(\partial_n\xi^I + \varepsilon_{nk}a'\hat b^{kI}\Big)(\sigma^n\sigma_m\lambda_I)_\alpha - 2i\Big(\varepsilon_{ln}\hat b_I^n\partial_k\bar\phi^I + \varepsilon_{ln}\hat c^n\partial_k\bar f\Big)(\sigma^k\sigma_m\sigma^l\gamma)_\alpha\\
&\quad - i\Big(\varepsilon_k{}^n(\hat a^k + \hat b_I^k\xi^I) - (\varepsilon^{kn}a' + \eta^{kn}a)\partial_k\bar f\Big)(\sigma_n\sigma_m\alpha')_\alpha + i\partial_n a'(\sigma^n\sigma_m\bar\alpha)_\alpha\\
&\quad + 2i\bar d_{mn}(\sigma^n\gamma)_\alpha + \frac{3}{2}\partial_m d^n\bar\beta_{n\alpha} + d^n\partial_n\bar\beta_{m\alpha}. \qquad (4.16b)
\end{aligned}
$$

We list below the statistics and the ghost numbers of each fields for the convenience:

fields	statistics	ghost numbers
$\bar{f}$	bosonic	-2
$\hat{a}^m,\ \hat{b}^m,\ c,\ c',\ \bar{d}_{mn}$	fermionic	-1
$\bar{\alpha}_\alpha,\ \bar{\beta}^m{}_\alpha$	bosonic	
λ^I_α	fermionic	0
$\xi^I,\ \phi^I,\ \bar{\phi}^I$	bosonic	
$a,\ a',\ b^I,\ b'^I,\ \hat{c}^m,\ d^m$	fermionic	1
$\alpha'_\alpha,\ \gamma_\alpha$	bosonic	
f	bosonic	2

The ghost fields $\big(a(x),\ a'(x),\ \hat{a}^m(x),\ \alpha'_\alpha(x),\ \bar{\alpha}_\alpha(x)\big)$, $\big(b^I(x),\ b'^I(x),\ \hat{c}^m(x),\ \hat{b}^m(x),\ c(x),\ c'(x),\ f(x),\ \bar{f}(x)\big)$ and $\big(d^m(x),\ \bar{d}_{mn}(x),\ \gamma_\alpha(x),\ \bar{\beta}^m{}_\alpha(x)\big)$ come from the symmetries of the $\mathrm{U(1)_V \times U(1)_A}$, of the generalized Chern-Simons action and of the supergravity, respectively.

We now present a perturbative analysis of the gauge-fixed action (4.14) and investigate BRST Ward identities at the quantum level. Then, we find out that nonlocal anomalous terms obtained from loop calculations vanish by imposing a condition which determines the critical dimension for this superstring model. For the explicit calculation it might be convenient to introduce light-cone notations on the world-sheet[‡]. The gauge-fixed action (4.14) is then expressed with these notations

$$\begin{aligned} S_{\text{gauge-fixed}} = \int \mathrm{d}^2x \Big\{ & \partial_+\xi^I\partial_-\xi_I + \frac{i}{\sqrt{2}}\lambda^I_+\partial_-\lambda_{I+} + \frac{i}{\sqrt{2}}\lambda^I_-\partial_+\lambda_{I-} + 2\partial_+\bar{\phi}^I\partial_-\phi_I \\ & +\hat{a}_+\partial_-a_+ + \hat{a}_-\partial_+a_- + \hat{b}_{+I}\partial_-b^I_+ + \hat{b}_{-I}\partial_+b^I_- \\ & +\hat{c}_+\partial_-c_+ + \hat{c}_-\partial_+c_- - i\sqrt{2}\bar{\alpha}_+\partial_-\alpha'_+ - i\sqrt{2}\bar{\alpha}_-\partial_+\alpha'_- + 2\partial_+\bar{f}\partial_-f \\ & -\bar{d}_{++}\partial_-d^+ - \bar{d}_{--}\partial_+d^- - \bar{\beta}_+\partial_-\gamma_+ + \bar{\beta}_-\partial_+\gamma_- \\ & +(a_+ + a_-)(\hat{b}_{+I}\partial_-\xi^I + \hat{b}_{-I}\partial_+\xi^I) + \hat{b}_{+I}\hat{c}_-\phi^I - \hat{b}_{-I}\hat{c}_+\phi^I \\ & +\Big(f + \frac{1}{2}a_+a_-\Big)\hat{b}_{+I}\hat{b}^I_- + i\sqrt{2}\hat{b}_{+I}\alpha'_-\lambda^I_- - i\sqrt{2}\hat{b}_{-I}\alpha'_+\lambda^I_+ \Big\}, \end{aligned} \qquad (4.17)$$

where we denote

$$\lambda^I_+ \equiv \lambda^I_1, \qquad \lambda^I_- \equiv \lambda^I_2,$$

[‡] Our convention of the light-cone coordinates on the world-sheet is $x^\pm = \frac{1}{\sqrt{2}}(x^0 \pm x^1)$. The metric tensor η_{mn} and Levi-Civitá symbol ε_{mn} are given by $\eta_{++} = \eta_{--} = 0$, $\eta_{+-} = \eta_{-+} = -1$ and $\varepsilon_{+-} = -\varepsilon_{-+} = -1$, respectively.

$$\alpha'_+ \equiv \alpha'_1, \qquad \alpha'_- \equiv \alpha'_2, \qquad \bar{\alpha}_+ \equiv \bar{\alpha}_1, \qquad \bar{\alpha}_- \equiv \bar{\alpha}_2,$$
$$\gamma_+ \equiv \gamma_2, \qquad \gamma_- \equiv \gamma_1, \qquad \bar{\beta}_+ \equiv \bar{\beta}_{+\alpha=1}, \qquad \bar{\beta}_- \equiv \bar{\beta}_{-\alpha=2},$$
$$a_\pm \equiv a \mp a',$$
$$b^I_\pm \equiv b^I \mp b'^I,$$
$$c_\pm \equiv c \mp c'.$$

Propagators are derived by taking inverses of bilinear parts in the action (4.17),

$$\begin{aligned}
\langle \xi^I(x)\xi^J(y)\rangle_0 &= \langle \bar{\phi}^I(x)\phi^J(y)\rangle_0 \\
&= \int \frac{\mathrm{d}^2p}{i(2\pi)^2}\ \frac{1}{p^2+i\epsilon}\ e^{-ip(x-y)}\eta^{IJ}, \\
\langle \lambda^I_\pm(x)\lambda^J_\pm(y)\rangle_0 &= \int \frac{\mathrm{d}^2p}{i(2\pi)^2}\ \frac{-\sqrt{2}p^\mp}{p^2+i\epsilon}\ e^{-ip(x-y)}\eta^{IJ}, \\
\langle \hat{a}_\pm(x)a_\pm(y)\rangle_0 &= \langle \hat{c}_\pm(x)c_\pm(y)\rangle_0 = -\langle \bar{d}_{\pm\pm}(x)d^\pm(y)\rangle_0 \\
&= \int \frac{\mathrm{d}^2p}{i(2\pi)^2}\ \frac{-2ip^\mp}{p^2+i\epsilon}\ e^{-ip(x-y)}, \\
\langle \hat{b}^I_\pm(x)b^I_\pm(y)\rangle_0 &= \int \frac{\mathrm{d}^2p}{i(2\pi)^2}\ \frac{-2ip^\mp}{p^2+i\epsilon}\ e^{-ip(x-y)}\eta^{IJ}, \\
\langle \bar{\alpha}_\pm(x)\alpha'_\pm(y)\rangle_0 &= \int \frac{\mathrm{d}^2p}{i(2\pi)^2}\ \frac{\sqrt{2}p^\mp}{p^2+i\epsilon}\ e^{-ip(x-y)}, \\
\langle \bar{\beta}_\pm(x)\gamma_\pm(y)\rangle_0 &= \int \frac{\mathrm{d}^2p}{i(2\pi)^2}\ \frac{\mp 2ip^\mp}{p^2+i\epsilon}\ e^{-ip(x-y)}, \\
\langle \bar{f}(x)f(y)\rangle_0 &= \int \frac{\mathrm{d}^2p}{i(2\pi)^2}\ \frac{1}{p^2+i\epsilon}\ e^{-ip(x-y)}.
\end{aligned}$$

Now let us consider the following two-point functions,

$$A(p)_{++} \equiv \int \frac{\mathrm{d}^2x}{i(2\pi)^2}\ \langle V_{++}(x)V_{++}(0)\rangle\ e^{ipx}, \tag{4.18a}$$

$$B(p)_{++} \equiv \int \frac{\mathrm{d}^2x}{i(2\pi)^2}\ \langle J_+(x)J_+(0)\rangle\ e^{ipx}, \tag{4.18b}$$

where we denote $J_+(x) \equiv J_{+\alpha=1}(x)$. Here we mention that the two-point functions (4.18a) and (4.18b) should vanish from the point of view of the BRST symmetries $V_{++}(x) = s\bar{d}_{++}(x)$ and $J_+(x) = s\bar{\beta}_+(x)$. By estimating all of the contributions arising from (ξ^I, ξ_I), $(\lambda^I_+, \lambda_{I+})$, $(\bar{\phi}^I, \phi_I)$, $(\hat{a}_+, a_+)$, $(\hat{b}_{+I}, b^I_+)$, $(\hat{c}_+, c_+)$, $(\bar{\alpha}_+, \alpha'_+)$, $(\bar{f}, f)$, $(\bar{d}_{++}, d^+)$ and $(\bar{\beta}_+, \gamma_+)$, we can obtain the following result for the two-point function (4.18a) up to one-loop order,

$$\begin{aligned}
A(p)_{++} &= \frac{1}{48\pi^3}\Big(D + \frac{1}{2}D + 2D - 2 - 2D - 2 - 1 + 2 - 26 + 11\Big)\frac{(p^-)^3}{p^+} \\
&= \frac{D-12}{32\pi^3}\frac{(p^-)^3}{p^+}.
\end{aligned} \tag{4.19a}$$

Furthermore we can obtain the following result for (4.18b),

$$B(p)_{++} = \frac{1}{4\sqrt{2}\pi^3}\Big(D-2-10\Big)\frac{(p^-)^2}{p^+}$$
$$= \frac{D-12}{4\sqrt{2}\pi^3}\frac{(p^-)^2}{p^+}. \tag{4.19b}$$

In a similar way we can find

$$A(p)_{--} \equiv \int \frac{\mathrm{d}^2 x}{i(2\pi)^2}\,\langle V_{--}(x)V_{--}(0)\rangle\, e^{ipx} = \frac{D-12}{32\pi^3}\frac{(p^+)^3}{p^-}, \tag{4.20a}$$
$$B(p)_{--} \equiv \int \frac{\mathrm{d}^2 x}{i(2\pi)^2}\,\langle J_{-}(x)J_{-}(0)\rangle\, e^{ipx} = \frac{D-12}{4\sqrt{2}\pi^3}\frac{(p^+)^2}{p^-}, \tag{4.20b}$$

where $J_-(x) \equiv J_{-\alpha=2}(x)$. Although we need to check the other two-point functions, *i.e.* $A(p)_{+-}$ and $B(p)_{+-}$, these two-point functions are actually divergent. However, this divergence can be absorbed adding suitable local counter terms to the action. Therefore, we conclude that the BRST anomalies vanish if and only if

$$D = 12. \tag{4.21}$$

5 Quantization in the light-cone gauge formulation

In this section we carry out the quantization of the classical action (3.17) in the light-cone gauge and derive the same critical dimension of the model in the covariant gauge. In addition, we mention a mass-shell relation of the model.

In the canonical formulation it might be useful to introduce the following new variables for the inverse zweibein fields $e_a{}^m(x)$,

$$e_\pm{}^m \equiv \frac{e}{2}\Big(e_0{}^m \pm e_1{}^m\Big). \tag{5.1}$$

According to the ordinary Dirac's procedure*, we introduce canonical momenta defined by $P_{\Phi^A}(x) \equiv \delta_{\mathrm{L}} S/\delta(\partial_0 \Phi^A(x))$ corresponding to fields $\Phi^A(x)^\dagger$,

$$\begin{aligned} P_{\xi_I} &= -e\Big(g^{0m}\partial_m \xi_I - \tilde{A}^0\phi_I - \frac{i}{2}(\lambda_I\sigma^m\sigma^a\chi_m)e_a{}^0\Big), \\ P_{\phi_I} &= -e\Big(g^{0m}\partial_m\bar{\phi}_I - \tilde{B}^0_I\Big), \\ P_{\bar{\phi}_I} &= -eg^{0m}\partial_m\phi_I, \end{aligned} \tag{5.2}$$

*Our convention of the generalized Poisson bracket is given in Appendix C. We take the independent variables $\Phi^A(x)$ as $\xi^I(x)$, $\lambda^I_\alpha(x)$, $\phi^I(x)$, $\bar{\phi}^I(x)$, $A_m(x)$, $B^I_m(x)$, $C_{01}(x)$, $\psi_\alpha(x)$, $e_\pm{}^m(x)$ and $\chi_{m\alpha}(x)$.

†For the spinor fields, we denote their two components as $\lambda^I_+(x) \equiv \lambda^I_1(x)$ and $\lambda^I_-(x) \equiv \lambda^I_2(x)$. The other spinor fields also obey the same conventions.

and

$$P_{\lambda I\pm} = \frac{\delta_{\rm L} S}{\delta(\partial_0 \lambda^I_\pm)} = -ie_\mp{}^0 \lambda_{I\pm}, \quad \Rightarrow \quad \theta_{I\pm} \equiv P_{\lambda I\pm} + ie_\mp{}^0\lambda_{I\pm} = 0, \tag{5.3a}$$

$$P_A{}^m = P_{B}{}^m_I = P_{C_{01}} = P_{\psi_\pm} = P_{e_\pm m} = P_{\chi_\pm}^m = 0. \tag{5.3b}$$

The Poisson brackets are defined by

$$\begin{aligned}
\{\xi^I, P_{\xi J}\} &= \{\phi^I, P_{\phi J}\} = \{\bar\phi^I, P_{\bar\phi J}\} = \delta^I_J, \\
\{\lambda^I_\pm, P_{\lambda J\pm}\} &= \{P_{\lambda J\pm}, \lambda^I_\pm\} = -\delta^I_J, \\
\{A_m, P_A{}^n\} &= \delta^n_m, \\
\{B^I_m, P_{B}{}^n_J\} &= \delta^n_m \delta^I_J, \\
\{C_{01}, P_{C_{01}}\} &= 1, \\
\{\psi_\pm, P_{\psi_\pm}\} &= \{P_{\psi_\pm}, \psi_\pm\} = -1, \\
\{e_\pm{}^m, P_{e_\pm n}\} &= \delta^m_n, \\
\{\chi_{m\pm}, P_{\chi_\pm}^n\} &= \{P_{\chi_\pm}^n, \chi_{m\pm}\} = -\delta^n_m.
\end{aligned} \tag{5.4}$$

The relations (5.3a) and (5.3b) give primary constraints. By introducing Lagrange multiplier fields $\rho_i(x)$ for primary constraints $\varphi^i(x)$, canonical Hamiltonian can be written in terms of the phase space variables as

$$\begin{aligned}
H = &\int \mathrm{d}x^1 \Big\{ \partial_0 \Phi^A P_{\Phi^A} - \mathcal{L} + \rho_i \varphi^i \Big\} \\
= &\int \mathrm{d}x^1 \Big\{ -\frac{e_-{}^1}{e_-{}^0} \Big(\frac{1}{4} \Big(P_\xi{}^I + A_1\phi^I + \partial_1\xi^I\Big)\Big(P_{\xi I} + A_1\phi_I + \partial_1\xi_I\Big) - P_{\lambda I+}\partial_1\lambda^I_+ \\
&\qquad + \Big(P_{\bar\phi}{}^I + \partial_1\phi^I\Big)\Big(P_{\phi I} + B_{1I} + \partial_1\bar\phi_I\Big) - \frac{i}{2}\Big(P_\xi{}^I + A_1\phi^I + \partial_1\xi^I\Big)\lambda_{I+}\chi_{1-}\Big) \\
&+ \frac{e_+{}^1}{e_+{}^0}\Big(\frac{1}{4}\Big(P_\xi{}^I + A_1\phi^I - \partial_1\xi^I\Big)\Big(P_{\xi I} + A_1\phi_I - \partial_1\xi_I\Big) + P_{\lambda I-}\partial_1\lambda^I_- \\
&\qquad + \Big(P_{\bar\phi}{}^I - \partial_1\phi^I\Big)\Big(P_{\phi I} + B_{1I} - \partial_1\bar\phi_I\Big) - \frac{i}{2}\Big(P_\xi{}^I + A_1\phi^I - \partial_1\xi^I\Big)\lambda_{I-}\chi_{1+}\Big) \\
&- A_0\phi_I\partial_1\xi^I - B^I_0\partial_1\phi_I - \frac{1}{2}C_{01}\phi^I\phi_I - 2i\Big(e_+{}^0e_-{}^1 - e_-{}^0e_+{}^1\Big)\phi_I\Big(\psi_-\lambda^I_+ - \psi_+\lambda^I_-\Big) \\
&+ \frac{i}{2}\Big(P_\xi{}^I + A_1\phi^I + \partial_1\xi^I\Big)\lambda_{I+}\chi_{0-} - \frac{i}{2}\Big(P_\xi{}^I + A_1\phi^I - \partial_1\xi^I\Big)\lambda_{I-}\chi_{0+} \\
&- \theta_{I+}\rho_\lambda{}^I_+ - \theta_{I-}\rho_\lambda{}^I_- \\
&+ \rho_{A m}P_A{}^m + \rho_{B}{}^I_m P_{B}{}^m_I + \rho_{C_{01}}P_{C_{01}} + \rho_{\psi+}P_{\psi+} + \rho_{\psi-}P_{\psi-} \\
&+ \rho_{e_+}{}^m P_{e_+ m} + \rho_{e_-}{}^m P_{e_- m} + \rho_{\chi_+ m}P_{\chi_+}^m + \rho_{\chi_- m}P_{\chi_-}^m \Big\},
\end{aligned} \tag{5.5}$$

where we redefine the multiplier $\rho_{\lambda}{}^{I}_{\pm}(x)$ for the primary constraint (5.3a) as

$$\rho_{\lambda}{}^{I}_{\pm} + \partial_0 \lambda^I_{\pm} + \frac{e_{\mp}{}^1}{e_{\mp}{}^0}\partial_1 \lambda^I_{\pm} \to \rho_{\lambda}{}^{I}_{\pm}.$$

A consistency check of the primary constraints (5.3a) and (5.3b) yields a set of secondary constraints

$$\frac{1}{4}\Big(P_{\xi}{}^{I} \pm \partial_1 \xi^I\Big)\Big(P_{\xi I} \pm \partial_1 \xi_I\Big) \mp P_{\lambda I\pm}\partial_1 \lambda^I_{\pm} = 0, \quad (5.6a)$$

$$\Big(P_{\xi}{}^{I} \pm \partial_1 \xi^I\Big)\lambda_{I\pm} = 0, \quad (5.6b)$$

$$\phi_I \lambda^I_{\pm} = 0, \quad (5.6c)$$

$$\phi_I \partial_1 \xi^I = 0, \quad (5.6d)$$

$$\phi_I P_{\xi}{}^{I} = 0, \quad (5.6e)$$

$$\partial_1 \phi^I = 0, \quad (5.6f)$$

$$P_{\bar{\phi}}{}^{I} = 0, \quad (5.6g)$$

$$\frac{1}{2}\phi^I \phi_I = 0, \quad (5.6h)$$

and these secondary constraints give no other relations[‡].

Let us specify the algebraic structure of the constraints. The constraints (5.3a) have non-vanishing Poisson brackets with each other

$$\begin{aligned} \{\theta^I_{\pm}, \theta^J_{\pm}\} &= -2ie_{\mp}{}^0 \eta^{IJ}, \\ \{\theta^I_{\pm}, \theta^J_{\mp}\} &= 0, \end{aligned} \quad (5.7)$$

and are therefore second class. Similarly, the constraints $P_{e_{\pm 0}}(x) = 0$ and (5.6a)-(5.6c) have non-vanishing Poisson brackets with the constraints (5.3a) and hence are also second class. However, we can take the following linear combinations for these constraints,

$$\begin{aligned} &P_{e_{\pm 0}} = 0 \\ &\quad \to\ P_{e_{\pm 0}} + \frac{1}{2e_{\pm}{}^0}\theta_{I\mp}\lambda^I_{\mp} = 0, && (5.8a) \\ &\frac{1}{4}\Big(P_{\xi}{}^{I} \pm \partial_1 \xi^I\Big)\Big(P_{\xi I} \pm \partial_1 \xi_I\Big) \mp P_{\lambda I\pm}\partial_1 \lambda^I_{\pm} = 0 \\ &\quad \to\ \frac{1}{4}\Big(P_{\xi}{}^{I} \pm \partial_1 \xi^I\Big)\Big(P_{\xi I} \pm \partial_1 \xi_I\Big) \mp \Big(P_{\lambda I\pm} - \frac{1}{2}\theta_{I\pm}\Big)\partial_1\Big(\lambda^I_{\pm} + \frac{i}{2e_{\mp}{}^0}\theta^I_{\pm}\Big) = 0, && (5.8b) \\ &\Big(P_{\xi}{}^{I} \pm \partial_1 \xi^I\Big)\lambda_{I\pm} = 0 \end{aligned}$$

‡The consistency checks for some of the constraints determine the multipliers as functionals of the canonical variables. However, these explicit forms are not important because these contributions to the canonical Hamiltonian might be ignored if we introduce the Dirac brackets.

$$\rightarrow \ \Big(P_\xi{}^I \pm \partial_1 \xi^I\Big)\Big(\lambda_{I\pm} + \frac{i}{2e_\mp{}^0}\theta_{I\pm}\Big) = 0, \tag{5.8c}$$

$$\phi_I \lambda^I_\pm = 0$$

$$\rightarrow \ \phi_I\Big(\lambda^I_\pm + \frac{i}{2e_\mp{}^0}\theta^I_\pm\Big) = 0, \tag{5.8d}$$

so that the above constraints (5.8a)-(5.8d) turn out to have vanishing Poisson brackets with the constraints (5.3a). Then, we can separate all of the constraints into second class (5.3a) and first class (5.3b), (5.8a)-(5.8d) and (5.6d)-(5.6h). For the second class constraints, we introduce the following Dirac brackets instead of the Poisson brackets,

$$\begin{aligned} \{P_{e_\mp{}^0}, \lambda^I_\pm\} &= \frac{\lambda^I_\pm}{2e_\mp{}^0}, \\ \{\lambda^I_\pm, \lambda^J_\pm\} &= -\frac{i}{2e_\mp{}^0}\eta^{IJ}, \end{aligned} \tag{5.9}$$

and we set the second class constraints (5.3a) as identities. Then, the first class constraints (5.8a)-(5.8d) are simply replaced to the original ones.

Now we investigate the dynamics of the model defined by the canonical Hamiltonian (5.5) with the first class constraints (5.3b) and (5.6a)-(5.6h). Imposing noncovariant gauge fixing conditions, we explicitly solve the constraints to some of the variables from the equations of motion[§].

We begin by considering conditions for the scalar field $\phi^I(\tau,\sigma)$. We find it convenient to introduce Fourier mode expansions of the canonical pair $\Big(\phi^I(\tau,\sigma), P_{\phi J}(\tau,\sigma)\Big)$,

$$\begin{aligned} \phi^I(\tau,\sigma) &= \phi^I(\tau) + \frac{1}{\sqrt{2\pi}}\sum_{m\neq 0}\phi^I_m(\tau)e^{im\sigma}, \\ P_{\phi I}(\tau,\sigma) &= \frac{p_{\phi I}(\tau)}{2\pi} + \frac{1}{\sqrt{2\pi}}\sum_{m\neq 0}p_{\phi Im}(\tau)e^{im\sigma}. \end{aligned} \tag{5.10}$$

Then, Poisson brackets are written by

$$\begin{gathered} \{\phi^I(\tau), p_{\phi J}(\tau)\} = \delta^I_J, \\ \{\phi^I_m(\tau), p_{\phi Jn}(\tau)\} = \delta^I_J\delta_{m+n}, \\ \text{otherwise} = 0. \end{gathered} \tag{5.11}$$

In terms of the Fourier modes, the constraint (5.6f) is equivalent to $\phi^I_m(\tau) = 0$. On the other hand, the equation of motion for $\phi^I(\tau,\sigma)$ on the constraint surface is $\partial_\tau\phi^I(\tau,\sigma) = 0$.

[§]Hereafter we use the conventions of the world-sheet coordinates as $x^0 \equiv \tau$ and $x^1 \equiv \sigma$. We also parameterize the spatial coordinate as $0 \leq \sigma \leq 2\pi$.

Together with the constraint $\phi^I_m(\tau)=0$, we then set the configuration of the scalar field as $\phi^I(\tau,\sigma)=\phi^I(\tau)=\phi^I(=\text{const.})$.

We first impose orthonormal gauge fixing conditions $e_a{}^m(\tau,\sigma)=\delta_a{}^m$ for the constraints $P_{e_{\pm m}}(\tau,\sigma)=0$, by using the gauge parameters $k^n(\tau,\sigma)$, $l(\tau,\sigma)$ and $s(\tau,\sigma)$ for the general coordinate, the local Lorentz and the Weyl scaling transformations, respectively. In addition, we can also adopt $\chi_{m\pm}(\tau,\sigma)=0$ as a gauge fixing condition for the constraint $P_{\chi_\pm}^{\ m}(\tau,\sigma)=0$, by using the gauge parameters $\zeta_\pm(\tau,\sigma)$ and $\check{s}_\pm(\tau,\sigma)$ for the local supersymmetry and the super-Weyl scaling transformations, respectively. The bosonic $\mathrm{U(1)_V\times U(1)_A}$ gauge parameters $v(\tau,\sigma)$, $v'(\tau,\sigma)$ and the global parameter α_i can fix to be $A_m(\tau,\sigma)=0$ for the constraint $P_A{}^m(\tau,\sigma)=0$, while the fermionic gauge parameters $\mu'_\pm(\tau,\sigma)$ can fix to be $\psi_\pm(\tau,\sigma)=0$ for the constraint $P_{\psi_\pm}(\tau,\sigma)=0$. However, this is not the end of the story. The system still has residual symmetries concerned with these gauge parameters [16]. As we will explain below, taking these symmetries into account, we can adopt the following gauge fixing conditions on "two" light-cone coordinates[¶] of the background spacetime within the gauge $e_a{}^m(\tau,\sigma)=\delta_a{}^m$ and $\chi_{m\pm}(\tau,\sigma)=A_m(\tau,\sigma)=\psi_\pm(\tau,\sigma)=0$,

$$\begin{aligned}
\xi^+(\tau,\sigma) &= \frac{p^+}{2\pi}\tau, & P_\xi^+(\tau,\sigma) &= \frac{p^+}{2\pi}, \\
\xi^{\hat+}(\tau,\sigma) &= \frac{p^{\hat+}}{2\pi}\tau, & P_\xi^{\hat+}(\tau,\sigma) &= \frac{p^{\hat+}}{2\pi}, \\
\lambda^+_\pm(\tau,\sigma) &= 0, \\
\lambda^{\hat+}_\pm(\tau,\sigma) &= 0,
\end{aligned} \tag{5.12}$$

where p^+ and $p^{\hat+}$ are light-cone components of the center of mass momenta. Therefore we can eliminate "two" unphysical components of the coordinates of the background spacetime and their superpartners. Indeed the gauge fixing conditions (5.12) correspond to ones for the first class constraints (5.6a)-(5.6e).

In order to show how these conditions (5.12) are accomplished, it might be useful to introduce Fourier mode expansions. In the gauge $e_a{}^m(\tau,\sigma)=\delta_a{}^m$ and $\chi_{m\pm}(\tau,\sigma)=A_m(\tau,\sigma)=\psi_\pm(\tau,\sigma)=0$, the dynamics of the coordinates $\xi^I(\tau,\sigma)$ and $\lambda^I_\pm(\tau,\sigma)$ turns out to be given by free wave equations and free Dirac equations with some constraints. For

[¶] From the definition of the metric (2.1), we denote the light-cone coordinates of the background spacetime as $x^I=(x^+,x^-,x^i,x^{\hat+},x^{\hat-})$, where $x^\pm\equiv\frac{1}{\sqrt2}(x^0\pm x^{D-3})$ and $x^{\hat\pm}\equiv\frac{1}{\sqrt2}(x^{\hat0}\pm x^{\hat1})$ and the index i runs through 1, 2, ..., $D-4$.

the bosonic coordinates, the solutions of the equation of motion are

$$\begin{aligned} \xi^I(\tau,\sigma) &= x^I + \frac{p^I}{2\pi}\tau + \frac{i}{2\sqrt{\pi}}\sum_{m\neq 0}\frac{1}{m}\Big(\alpha^I_m e^{-im(\tau-\sigma)} + \tilde{\alpha}^I_m e^{-im(\tau+\sigma)}\Big), \\ P_\xi{}^I(\tau,\sigma) &= \frac{p^I}{2\pi} + \frac{1}{2\sqrt{\pi}}\sum_{m\neq 0}\Big(\alpha^I_m e^{-im(\tau-\sigma)} + \tilde{\alpha}^I_m e^{-im(\tau+\sigma)}\Big), \end{aligned} \tag{5.13}$$

and Poisson brackets are given by

$$\begin{aligned} &\{x^I, p^J\} = \eta^{IJ}, \\ &\{\alpha^I_m, \alpha^J_n\} = \{\tilde{\alpha}^I_m, \tilde{\alpha}^J_n\} = -im\eta^{IJ}\delta_{m+n}, \\ &\text{otherwise} = 0. \end{aligned} \tag{5.14}$$

For the fermionic coordinates, the solutions depend on their boundary conditions *i.e.* periodic (Ramond model) and antiperiodic (Neveu-Schwarz model),

$$\begin{aligned} \lambda^I_-(\tau,\sigma) &= \frac{1}{\sqrt{2\pi}}\sum_{r\in\mathbf{Z}+a} b^I_r e^{-ir(\tau-\sigma)}, \\ \lambda^I_+(\tau,\sigma) &= \frac{1}{\sqrt{2\pi}}\sum_{r\in\mathbf{Z}+a} \tilde{b}^I_r e^{-ir(\tau+\sigma)}, \end{aligned} \tag{5.15}$$

where $a = 0$ for Ramond and $a = 1/2$ for Neveu-Schwarz model, respectively. Their Poisson brackets are given by

$$\begin{aligned} \{b^I_r, b^J_s\} &= \{\tilde{b}^I_r, \tilde{b}^J_s\} = -i\eta^{IJ}\delta_{r+s}, \\ \{b^I_r, \tilde{b}^J_s\} &= 0. \end{aligned} \tag{5.16}$$

In terms of the Fourier modes, the constraints (5.6a)-(5.6e) are equivalent to

$$\begin{aligned} &L_m = L^{(\alpha)}_m + L^{(b)}_m = 0, \\ &\tilde{L}_m = \tilde{L}^{(\alpha)}_m + \tilde{L}^{(b)}_m = 0, \\ &G_r = \tilde{G}_r = 0, \\ &\phi_I \alpha^I_m = \phi_I \tilde{\alpha}^I_m = 0, \\ &\phi_I b^I_r = \phi_I \tilde{b}^I_r = 0. \end{aligned} \tag{5.17}$$

In the above eqs., we define the super-Virasoro generators as

$$\begin{aligned} L^{(\alpha)}_m &\equiv \frac{1}{2}\sum_n \alpha^I_{-n}\alpha_{I\,m+n}, & \tilde{L}^{(\alpha)}_m &\equiv \frac{1}{2}\sum_n \tilde{\alpha}^I_{-n}\tilde{\alpha}_{I\,m+n}, \\ L^{(b)}_m &\equiv \frac{1}{2}\sum_{r\in\mathbf{Z}+a}\Big(r+\frac{m}{2}\Big) b^I_{-r} b_{I\,m+r}, & \tilde{L}^{(b)}_m &\equiv \frac{1}{2}\sum_{r\in\mathbf{Z}+a}\Big(r+\frac{m}{2}\Big) \tilde{b}^I_{-r} \tilde{b}_{I\,m+r}, \\ G_r &\equiv \sum_n \alpha^I_{-n} b_{I\,r+n}, & \tilde{G}_r &\equiv \sum_n \tilde{\alpha}^I_{-n} \tilde{b}_{I\,r+n}, \end{aligned}$$

where we denote $\alpha_0^I = \tilde{\alpha}_0^I \equiv p^I/(2\sqrt{\pi})$. The gauge fixing conditions (5.12) are equivalent to

$$x^+ = x^{\hat{+}} = 0,$$
$$\alpha_m^+ = \alpha_m^{\hat{+}} = \tilde{\alpha}_m^+ = \tilde{\alpha}_m^{\hat{+}} = 0, \quad (m \neq 0), \tag{5.18a}$$

and

$$b_r^+ = b_r^{\hat{+}} = \tilde{b}_r^+ = \tilde{b}_r^{\hat{+}} = 0. \tag{5.18b}$$

Now let us explain the procedure to get the gauge fixing conditions (5.18a) and (5.18b). For the fermionic sectors, within the super-orthonormal gauge, we can change the fermionic coordinates $\lambda_\pm^I(\tau,\sigma)$ with the gauge parameters $\zeta_\pm(\tau,\sigma)$ provided that conditions $\partial_\tau\zeta_+(\tau,\sigma) = -\partial_\sigma\zeta_+(\tau,\sigma)$ and $\partial_\tau\zeta_-(\tau,\sigma) = \partial_\sigma\zeta_-(\tau,\sigma)$ are satisfied. Here we take the following forms which realize these conditions,

$$\zeta_+(\tau,\sigma) = \frac{1}{\sqrt{2\pi}} \sum_{r\in\mathbf{Z}+a} \zeta_r e^{-ir(\tau-\sigma)},$$
$$\zeta_-(\tau,\sigma) = \frac{1}{\sqrt{2\pi}} \sum_{r\in\mathbf{Z}+a} \tilde{\zeta}_r e^{-ir(\tau+\sigma)}.$$

In analogy to the bosonic $\mathrm{U(1)_V \times U(1)_A}$ string case [16], the fermionic $\mathrm{U(1)_V \times U(1)_A}$ gauge parameters $\mu'_\pm(\tau,\sigma)$ can be also used to change the fermionic coordinates within the gauge $\psi_\pm(\tau,\sigma) = 0$ provided that conditions $\partial_\tau\mu'_+(\tau,\sigma) = \partial_\sigma\mu'_+(\tau,\sigma)$ and $\partial_\tau\mu'_-(\tau,\sigma) = -\partial_\sigma\mu'_-(\tau,\sigma)$ are satisfied. We take the following forms for $\mu'_\pm(\tau,\sigma)$ to realize these conditions,

$$\mu'_-(\tau,\sigma) = \frac{1}{\sqrt{2\pi}} \sum_{r\in\mathbf{Z}+a} \mu'_r e^{-ir(\tau-\sigma)},$$
$$\mu'_+(\tau,\sigma) = \frac{1}{\sqrt{2\pi}} \sum_{r\in\mathbf{Z}+a} \tilde{\mu}'_r e^{-ir(\tau+\sigma)}.$$

The gauge transformations corresponding to these parameters are consistent with the equations of motion for the fermionic coordinates $\lambda_\pm^I(\tau,\sigma)$, because, in terms of the Fourier modes, the gauge transformations are given by

$$\delta b_r^I = \frac{1}{\sqrt{\pi}} \sum_n \zeta_{r-n}\alpha_n^I + \mu'_r\phi^I,$$
$$\delta \tilde{b}_r^I = -\frac{1}{\sqrt{\pi}} \sum_n \tilde{\zeta}_{r-n}\tilde{\alpha}_n^I + \tilde{\mu}'_r\phi^I. \tag{5.19}$$

It is worth to mention that these gauge transformations are the same ones in usual string theories, except for the gauge transformations corresponding to the parameters μ'_r and $\tilde{\mu}'_r$.

However, we would like to emphasize that these gauge transformations can be disappear on the following combinations,

$$\delta(\phi^{\hat{+}} b_r^{+} - \phi^{+} b_r^{\hat{+}}) = \frac{1}{\sqrt{\pi}} \sum_n \zeta_{r-n} (\phi^{\hat{+}} \alpha_n^{+} - \phi^{+} \alpha_n^{\hat{+}}),$$
$$\delta(\phi^{\hat{+}} \tilde{b}_r^{+} - \phi^{+} \tilde{b}_r^{\hat{+}}) = -\frac{1}{\sqrt{\pi}} \sum_n \tilde{\zeta}_{r-n} (\phi^{\hat{+}} \tilde{\alpha}_n^{+} - \phi^{+} \tilde{\alpha}_n^{\hat{+}}).$$

In analogy to taking the light-cone gauge in usual string theories, by using the gauge degrees of freedom for ζ_r and $\tilde{\zeta}_r$, we can recursively adopt gauge conditions

$$\begin{aligned} \phi^{\hat{+}} b_r^{+} - \phi^{+} b_r^{\hat{+}} &= 0, \\ \phi^{\hat{+}} \tilde{b}_r^{+} - \phi^{+} \tilde{b}_r^{\hat{+}} &= 0, \end{aligned} \tag{5.20}$$

if the following condition is satisfied,

$$\phi^{\hat{+}} p^{+} - \phi^{+} p^{\hat{+}} \neq 0. \tag{5.21}$$

Next we use the gauge degrees of freedom for μ_r' and $\tilde{\mu}_r'$ in (5.19). In order to keep the condition (5.21) one cannot vanish both of the scalar fields $\phi^{\hat{+}}$ and ϕ^{+} simultaneously. If $\phi^{\hat{+}} \neq 0$, we can adopt the following gauge fixing conditions of the $\hat{+}$ component,

$$b_r^{\hat{+}} = \tilde{b}_r^{\hat{+}} = 0, \tag{5.22}$$

without spoiling the gauge fixing conditions (5.20). From (5.20) and (5.22) we can then arrive at the gauge fixing conditions (5.18b). In the similar way, we also conclude the same gauge fixing conditions (5.18b), in the case $\phi^{+} \neq 0$. Therefore we may assume the case $\phi^{\hat{+}} \neq 0$ throughout this paper without loss of generality. For the bosonic sectors, the procedure to obtain the light-cone gauge (5.18a) within the gauge (5.18b) is essentially as same as in the previous work [16], and therefore does not need to be repeated here.

The gauge fixing procedures for the remaining constraints $P_{B_I^m}(\tau,\sigma) = P_{C_{01}}(\tau,\sigma) = 0$ and (5.6f)-(5.6h) are also as same as in the bosonic $\mathrm{U(1)_V \times U(1)_A}$ string model. As we explicitly showed in the paper [16], after imposing suitable gauge fixing conditions, the dynamics for the remaining phase space variables is completely determined, so that it is described by the zero-modes of the fields $\phi^I(\tau,\sigma)$ and $P_\phi{}^I(\tau,\sigma)\Big(= -B_\sigma^I(\tau,\sigma)\Big)$,

$$\begin{aligned} \phi^I(\tau,\sigma) &= \phi^I, \\ P_\phi{}^I(\tau,\sigma) &= \frac{p_\phi{}^I}{2\pi} - \hat{C}_0 \phi^I \tau, \end{aligned} \tag{5.23}$$

with second class constraints

$$\frac{1}{2}\phi^I\phi_I = 0, \quad {p_\phi}^{\hat{+}} = 0, \tag{5.24}$$

where the constant $\hat{C}_0$ arises from a gauge fixing condition $C_{\tau\sigma}(\tau,\sigma) = -\hat{C}_0$. In the two-time physics, these zero-modes are regarded as canonical particle modes [6,8].

Now let us summarize the correspondence between the constraints and the gauge fixing conditions[||]:

constraints	gauge fixing conditions
$L_0 + \tilde{L}_0 = 0,$	$x^+ = 0,$
$L_m = \tilde{L}_m = 0,$	$\alpha^+_m = \tilde{\alpha}^+_m = 0, \quad (m \neq 0),$
$G_r = \tilde{G}_r = 0,$	$b^+_r = \tilde{b}^+_r = 0,$
$\phi_I p^I = 0,$	$x^{\hat{+}} = 0,$
$\phi_I \alpha^I_m = \phi_I \tilde{\alpha}^I_m = 0,$	$\alpha^{\hat{+}}_m = \tilde{\alpha}^{\hat{+}}_m = 0, \quad (m \neq 0),$
$\phi_I b^I_r = \phi_I \tilde{b}^I_r = 0,$	$b^{\hat{+}}_r = \tilde{b}^{\hat{+}}_r = 0,$
$\frac{1}{2}\phi^I\phi_I = 0,$	${p_\phi}^{\hat{+}} = 0.$

We are now ready to discuss the dynamics of the system.

As the constraints are quadratic in the Fourier modes, we can solve these directly and the dependent variables are expressed in terms of the independent variables. Here are the non-vanishing Poisson brackets of the independent canonical variables

$$\begin{aligned}
\{x^-, p^+\} &= \{x^{\hat{-}}, p^{\hat{+}}\} = -1, \\
\{x^i, p^j\} &= \delta^{ij}, \\
\{\alpha^i_m, \alpha^j_n\} &= \{\tilde{\alpha}^i_m, \tilde{\alpha}^j_n\} = -im\delta^{ij}\delta_{m+n}, \\
\{b^i_r, b^j_s\} &= \{\tilde{b}^i_r, \tilde{b}^j_s\} = -i\delta^{ij}\delta_{r+s}, \\
\{\phi^+, {p_\phi}^-\} &= \{\phi^-, {p_\phi}^+\} = \{\phi^{\hat{+}}, {p_\phi}^{\hat{-}}\} = -1, \\
\{\phi^i, {p_\phi}^j\} &= \delta^{ij},
\end{aligned} \tag{5.25}$$

and the remaining non-vanishing dependent variables are written down as

$$p^- = \frac{-1}{\phi^{\hat{+}}p^+ - \phi^+ p^{\hat{+}}}\left(\frac{p^{\hat{+}}p^{\hat{+}}}{\phi^{\hat{+}}}\left(\phi^+\phi^- - \frac{1}{2}\phi^i\phi_i\right) - p^{\hat{+}}\left(\phi^- p^+ - \phi^i p_i\right) - 2\pi\phi^{\hat{+}}\left(L_0^{\mathrm{tr}} + \tilde{L}_0^{\mathrm{tr}}\right)\right),$$

[||] As in usual closed string theories, a constraint $L_0 - \tilde{L}_0 = 0$ leads residual gauge invariance *i.e.* the translation along the world-sheet coordinate σ. This constraint results in the level matching condition for physical states in quantum theories.

$$
\begin{aligned}
\alpha_m^{-} &= \frac{-1}{\phi^{\hat +}p^{+} - \phi^{+}p^{\hat +}}\Big(p^{\hat +}\phi^{i}\alpha_{im} - 2\sqrt{\pi}\phi^{\hat +}L_m^{\rm tr}\Big), \qquad (m \neq 0),\\
\tilde\alpha_m^{-} &= \frac{-1}{\phi^{\hat +}p^{+} - \phi^{+}p^{\hat +}}\Big(p^{\hat +}\phi^{i}\tilde\alpha_{im} - 2\sqrt{\pi}\phi^{\hat +}\tilde L_m^{\rm tr}\Big), \qquad (m \neq 0),\\
p^{\hat -} &= \frac{1}{\phi^{\hat +}p^{+} - \phi^{+}p^{\hat +}}\bigg(\frac{p^{\hat +}p^{+}}{\phi^{\hat +}}\Big(\phi^{+}\phi^{-} - \frac{1}{2}\phi^{i}\phi_{i}\Big) - p^{+}\Big(\phi^{-}p^{+} - \phi^{i}p_{i}\Big) - 2\pi\phi^{+}\Big(L_0^{\rm tr} + \tilde L_0^{\rm tr}\Big)\bigg),\\
\alpha_m^{\hat -} &= \frac{1}{\phi^{\hat +}p^{+} - \phi^{+}p^{\hat +}}\Big(p^{+}\phi^{i}\alpha_{im} - 2\sqrt{\pi}\phi^{+}L_m^{\rm tr}\Big), \qquad (m \neq 0),\\
\tilde\alpha_m^{\hat -} &= \frac{1}{\phi^{\hat +}p^{+} - \phi^{+}p^{\hat +}}\Big(p^{+}\phi^{i}\tilde\alpha_{im} - 2\sqrt{\pi}\phi^{+}\tilde L_m^{\rm tr}\Big), \qquad (m \neq 0), \qquad\qquad (5.26)\\
b_r^{-} &= \frac{-1}{\phi^{\hat +}p^{+} - \phi^{+}p^{\hat +}}\Big(p^{\hat +}\phi^{i}b_{ir} - 2\sqrt{\pi}\phi^{\hat +}G_r^{\rm tr}\Big),\\
\tilde b_r^{-} &= \frac{-1}{\phi^{\hat +}p^{+} - \phi^{+}p^{\hat +}}\Big(p^{\hat +}\phi^{i}\tilde b_{ir} - 2\sqrt{\pi}\phi^{\hat +}\tilde G_r^{\rm tr}\Big),\\
b_r^{\hat -} &= \frac{1}{\phi^{\hat +}p^{+} - \phi^{+}p^{\hat +}}\Big(p^{+}\phi^{i}b_{ir} - 2\sqrt{\pi}\phi^{+}G_r^{\rm tr}\Big),\\
\tilde b_r^{\hat -} &= \frac{1}{\phi^{\hat +}p^{+} - \phi^{+}p^{\hat +}}\Big(p^{+}\phi^{i}\tilde b_{ir} - 2\sqrt{\pi}\phi^{+}\tilde G_r^{\rm tr}\Big),\\
\phi^{\hat -} &= -\frac{1}{\phi^{\hat +}}\Big(\phi^{+}\phi^{-} - \frac{1}{2}\phi^{i}\phi_{i}\Big),
\end{aligned}
$$

where transverse parts of the super-Virasoro generators $L_m^{\rm tr}(= L_m^{(\alpha){\rm tr}} + L_m^{(b){\rm tr}})$, $\tilde L_m^{\rm tr}(= \tilde L_m^{(\alpha){\rm tr}} + \tilde L_m^{(b){\rm tr}})$, $G_r^{\rm tr}$ and $\tilde G_r^{\rm tr}$ are defined by

$$
\begin{aligned}
L_m^{(\alpha){\rm tr}} &\equiv \frac{1}{2}\sum_{n}\alpha^{i}_{-n}\alpha_{im+n}, & \tilde L_m^{(\alpha){\rm tr}} &\equiv \frac{1}{2}\sum_{n}\tilde\alpha^{i}_{-n}\tilde\alpha_{im+n},\\
L_m^{(b){\rm tr}} &\equiv \frac{1}{2}\sum_{r\in {\bf Z}+a}\Big(r + \frac{m}{2}\Big)b^{i}_{-r}b_{im+r}, & \tilde L_m^{(b){\rm tr}} &\equiv \frac{1}{2}\sum_{r\in {\bf Z}+a}\Big(r + \frac{m}{2}\Big)\tilde b^{i}_{-r}\tilde b_{im+r},\\
G_r^{\rm tr} &\equiv \sum_{n}\alpha^{i}_{-n}b_{ir+n}, & \tilde G_r^{\rm tr} &\equiv \sum_{n}\tilde\alpha^{i}_{-n}\tilde b_{ir+n}.
\end{aligned}
$$

Now let us investigate the symmetry of the D-dimensional background spacetime. The generators for the translation and the Lorentz transformation are derived from the classical action (3.17). In terms of the Fourier modes, these are

$$
P^{I} = p^{I}, \qquad (5.27a)
$$

$$
\begin{aligned}
M^{IJ} = x^{I}p^{J} &- \frac{i}{2}\sum_{m\neq 0}\frac{1}{m}\Big(\alpha^{I}_{-m}\alpha^{J}_{m} + \tilde\alpha^{I}_{-m}\tilde\alpha^{J}_{m}\Big) - \frac{i}{2}\sum_{r\in {\bf Z}+a}\Big(b^{I}_{-r}b^{J}_{r} + \tilde b^{I}_{-r}\tilde b^{J}_{r}\Big) + \phi^{I}{p_\phi}^{J}\\
&-(I \leftrightarrow J). \qquad (5.27b)
\end{aligned}
$$

Using the independent canonical variables, the Poincaré algebra ISO$(D-2,2)$ is satisfied,

$$
\{P^{I}, P^{J}\} = 0,
$$

$$
\begin{aligned}
\{M^{IJ}, P^K\} &= \eta^{IK} P^J - \eta^{JK} P^I, \\
\{M^{IJ}, M^{KL}\} &= \eta^{IK} M^{JL} - \eta^{JK} M^{IL} - \eta^{IL} M^{JK} + \eta^{JL} M^{IK},
\end{aligned}
\tag{5.28}
$$

if a constraint $L_0^{\rm tr} = \tilde{L}_0^{\rm tr}$ is imposed. The gauge fixing procedure we considered is the way to preserve the full D-dimensional Poincaré symmetry.

According to the ordinary string theories in the light-cone gauge, we have to examine Poincaré algebra (5.28) in the quantum theory [24]. The checking of the Poincaré algebra is again straightforward, except for commutation relations $[M^{i-}, M^{j-}]$, $[M^{i\hat{-}}, M^{j\hat{-}}]$, $[M^{i-}, M^{j\hat{-}}]$, $[M^{i-}, M^{-\hat{-}}]$ and $[M^{i\hat{-}}, M^{-\hat{-}}]$. The explicit forms of these Lorentz generators are given in Appendix D. After lengthy computation, we can obtain the following results,

$$
\begin{aligned}
[M^{i-}, M^{j-}] &= \frac{4\pi {\phi^{\hat{+}}}^2}{(\phi^{\hat{+}} p^+ - \phi^+ p^{\hat{+}})^2} A^{ij}, \\
[M^{i\hat{-}}, M^{j\hat{-}}] &= \frac{4\pi {\phi^{+}}^2}{(\phi^{\hat{+}} p^+ - \phi^+ p^{\hat{+}})^2} A^{ij}, \\
[M^{i-}, M^{j\hat{-}}] &= i\delta^{ij} M^{-\hat{-}} - \frac{4\pi \phi^{\hat{+}} \phi^+}{(\phi^{\hat{+}} p^+ - \phi^+ p^{\hat{+}})^2} A^{ij}, \\
[M^{i-}, M^{-\hat{-}}] &= \frac{4\pi \phi^{\hat{+}} \phi_j}{(\phi^{\hat{+}} p^+ - \phi^+ p^{\hat{+}})^2} A^{ij}, \\
[M^{i\hat{-}}, M^{-\hat{-}}] &= -\frac{4\pi \phi^{+} \phi_j}{(\phi^{\hat{+}} p^+ - \phi^+ p^{\hat{+}})^2} A^{ij}.
\end{aligned}
\tag{5.29}
$$

Anomalous terms A^{ij} are

$$
\begin{aligned}
A^{ij} = &-\left(\sum_{m=1}^{\infty} m\left(\alpha^i_{-m}\alpha^j_m + \tilde{\alpha}^i_{-m}\tilde{\alpha}^j_m\right) + 4\sum_{r=\frac{1}{2}}^{\infty} r^2\left(b^i_{-r}b^j_r + \tilde{b}^i_{-r}\tilde{b}^j_r\right)\right)\left(1 - \frac{D-4}{8}\right) \\
&+ \left(\sum_{m=1}^{\infty} \frac{1}{m}\alpha^i_{-m}\alpha^j_m + \sum_{r=\frac{1}{2}}^{\infty} b^i_{-r}b^j_r\right)\left(:L_0^{\rm tr}: - :\tilde{L}_0^{\rm tr}: + a_0 + \tilde{a}_0 - \frac{D-4}{8}\right) \\
&+ \left(\sum_{m=1}^{\infty} \frac{1}{m}\tilde{\alpha}^i_{-m}\tilde{\alpha}^j_m + \sum_{r=\frac{1}{2}}^{\infty} \tilde{b}^i_{-r}\tilde{b}^j_r\right)\left(:\tilde{L}_0^{\rm tr}: - :L_0^{\rm tr}: + \tilde{a}_0 + a_0 - \frac{D-4}{8}\right) \\
&-(i \leftrightarrow j),
\end{aligned}
\tag{5.30a}
$$

for (NS, NS) sector, which means both right and left movers satisfy the Neveu-Schwarz boundary conditions,

$$
\begin{aligned}
A^{ij} = &-\left(\sum_{m=1}^{\infty} m\left(\alpha^i_{-m}\alpha^j_m + \tilde{\alpha}^i_{-m}\tilde{\alpha}^j_m\right) + 4\sum_{r=1}^{\infty} r^2\left(b^i_{-r}b^j_r + \tilde{b}^i_{-r}\tilde{b}^j_r\right)\right)\left(1 - \frac{D-4}{8}\right) \\
&+ \left(\sum_{m=1}^{\infty} \frac{1}{m}\alpha^i_{-m}\alpha^j_m + \sum_{r=1}^{\infty} b^i_{-r}b^j_r + \frac{1}{2}b^i_0 b^j_0\right)\left(:L_0^{\rm tr}: - :\tilde{L}_0^{\rm tr}: + a_0 + \tilde{a}_0\right)
\end{aligned}
$$

$$
\begin{aligned}
&+\left(\sum_{m=1}^{\infty}\frac{1}{m}\tilde{\alpha}^i_{-m}\tilde{\alpha}^j_m+\sum_{r=1}^{\infty}\tilde{b}^i_{-r}\tilde{b}^j_r+\frac{1}{2}\tilde{b}^i_0\tilde{b}^j_0\right)\left(:\tilde{L}^{\rm tr}_0:-:L^{\rm tr}_0:+\tilde{a}_0+a_0\right)\\
&-(i\leftrightarrow j),
\end{aligned}
\tag{5.30b}
$$

for (R, R) sector and

$$
\begin{aligned}
A^{ij}=&-\left(\sum_{m=1}^{\infty}m\left(\alpha^i_{-m}\alpha^j_m+\tilde{\alpha}^i_{-m}\tilde{\alpha}^j_m\right)+4\sum_{r=\frac{1}{2}}^{\infty}r^2 b^i_{-r}b^j_r+4\sum_{r=1}^{\infty}r^2\tilde{b}^i_{-r}\tilde{b}^j_r\right)\left(1-\frac{D-4}{8}\right)\\
&+\left(\sum_{m=1}^{\infty}\frac{1}{m}\alpha^i_{-m}\alpha^j_m+\sum_{r=\frac{1}{2}}^{\infty}b^i_{-r}b^j_r\right)\left(:L^{\rm tr}_0:-:\tilde{L}^{\rm tr}_0:+a_0+\tilde{a}_0-\frac{D-4}{8}\right)\\
&+\left(\sum_{m=1}^{\infty}\frac{1}{m}\tilde{\alpha}^i_{-m}\tilde{\alpha}^j_m+\sum_{r=1}^{\infty}\tilde{b}^i_{-r}\tilde{b}^j_r+\frac{1}{2}\tilde{b}^i_0\tilde{b}^j_0\right)\left(:\tilde{L}^{\rm tr}_0:-:L^{\rm tr}_0:+\tilde{a}_0+a_0\right)\\
&-(i\leftrightarrow j),
\end{aligned}
\tag{5.30c}
$$

for (NS, R) sector. The constants a_0 and $\tilde{a}_0$ denote the ordering ambiguity of the operators $L^{\rm tr}_0$ and $\tilde{L}^{\rm tr}_0$ by adopting the normal-ordering prescription,

$$
L^{\rm tr}_0 \;\to\; :L^{\rm tr}_0:-a_0, \qquad \tilde{L}^{\rm tr}_0 \;\to\; :\tilde{L}^{\rm tr}_0:-\tilde{a}_0.
$$

The level matching condition for physical states is now expressed as

$$
:L^{\rm tr}_0:-a_0=:\tilde{L}^{\rm tr}_0:-\tilde{a}_0, \quad\leftrightarrow\quad N-a_0=\tilde{N}-\tilde{a}_0, \tag{5.31}
$$

where number operators N and $\tilde{N}$ are defined by

$$
N\equiv\sum_{m=1}^{\infty}\alpha^i_{-m}\alpha_{im}+\sum_{r\in\mathbf{Z}+a>0}^{\infty}rb^i_{-r}b_{ir}, \qquad \tilde{N}\equiv\sum_{m=1}^{\infty}\tilde{\alpha}^i_{-m}\tilde{\alpha}_{im}+\sum_{r\in\mathbf{Z}+a>0}^{\infty}r\tilde{b}^i_{-r}\tilde{b}_{ir}.
$$

Imposing the level matching condition (5.31), the anomalous terms A^{ij} vanish on physical states if and only if

$$
D=12, \qquad
\begin{aligned}
&a_0=\tilde{a}_0=\frac{1}{2}, && \big(\text{(NS, NS) sector}\big),\\
&a_0=\tilde{a}_0=0, && \big(\text{(R, R) sector}\big),\\
&a_0=\frac{1}{2},\quad \tilde{a}_0=0, && \big(\text{(NS, R) sector}\big).
\end{aligned}
\tag{5.32}
$$

Then, the Poincaré algebra ISO(10, 2) is satisfied in the quantum theory.

A mass-shell relation of this superstring model is given by

$$
\begin{aligned}
m^2&=-P^IP_I\\
&=4\pi\left(N+\tilde{N}-a_0-\tilde{a}_0\right).
\end{aligned}
\tag{5.33}
$$

As a common future of our string models [16] and the two-time physics [5], on-shell degrees of freedom are equivalent to usual string theories, because our extra spacetime coordinates are introduced by the "gauge" symmetries.

6 Conclusions and discussions

We have explicitly constructed the $\mathrm{U(1)_V \times U(1)_A}$ NSR superstring model by using the superfield formulation and discussed the quantization of the model. Even though the system had reducible and open gauge symmetries, we have shown that the covariant quantization has been successfully carried out in the Lagrangian formulation *á la* Batalin and Vilkovisky. Furthermore we have presented the noncovariant light-cone gauge formulation and investigated the symmetry of the background spacetime. With careful considerations of the residual $\mathrm{U(1)_V \times U(1)_A}$ gauge symmetries, we have specified the gauge fixing conditions corresponding to the first-class constraints. Under these suitable conditions, we have been able to clarify dynamical independent variables and solve the first-class constraints explicitly. Although manifest covariance has been lost, we have confirmed the full D-dimensional Poincaré algebra of the background spacetime by direct computation.

Since the quantizations of the model have been successfully carried out, we could argue the critical dimension of the superstring model. In our case, it turns out to be 10+2. This means the background spacetime involves two time coordinates. Conversely, the requirement of two negative signatures in the background metric is natural one due to the gauge invariance of our model. The critical dimension has been obtained from both the BRST Ward identity in the BRST formulation and the D-dimensional quantum Poincaré algebra in the noncovariant light-cone gauge formulation. Therefore, we have concluded a consistent quantum theory of our $\mathrm{U(1)_V \times U(1)_A}$ superstring model has been formulated in 10+2-dimensional background spacetime. We have also discussed the quantum states. Contributions toward the mass-shell relation from zero-modes of the scalar field $\phi^I(x)$ are completely canceled, so that our superstring model possesses the same spectra as usual string theories.

We propose our string models as devices to formulate the physics involving two time coordinates and to search for a fundamental theory with an underlying complex nature of spacetime which would be linked via dualities to M-theory, type II string theories and F-theory from higher-dimensional points of view. Our explicit Lagrangian formulation might be a clue to understand spacetime itself.

The classical actions and the gauge symmetries of our string models strongly suggest that these model should be more naturally defined in higher-dimensional field theories, namely, that membranes or p-branes are more fundamental than strings in our formulation. From the point of view of constraint algebras, the action might be derived from a membrane action by adopting a compactification prescription. In this case, Kaluza-Klein fields which arise from the compactification might correspond to the gauge fields in our present formulation.

One of the remarkable features of our higher-dimensional formulation is that models are allowed to have pairs of the extra time and space coordinates. Therefore, by applying our mechanism, one can construct a succession of supersymmetric models formulated in background spacetimes, involving some time coordinates, where Majorana-Weyl fermions can live consistently.

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Appendix A. Two-dimensional world-sheet

We denote the characters $a, b, c, \cdots$ and $m, n, l, \cdots$ as flat local Lorentz and curved spacetime indices, respectively.

The two-dimensional flat metric η_{ab} and Levi-Cività symbol ε_{ab} are given by

$$\eta_{ab} = \eta^{ab} = \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}, \qquad \varepsilon_{ab} = -\varepsilon^{ab} = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}. \tag{A.1}$$

As the two-dimensional Clifford algebra, the σ-matrices satisfy

$$\{\sigma^a, \sigma^b\} = 2\eta^{ab}. \tag{A.2}$$

Their explicit representations are

$$(\sigma^0)_\alpha{}^\beta = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}, \qquad (\sigma^1)_\alpha{}^\beta = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix},$$

and

$$(\bar{\sigma})_\alpha{}^\beta \equiv (\sigma^0\sigma^1)_\alpha{}^\beta = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}.$$

The spinor metric is given by

$$\eta_{\alpha\beta} = \eta^{\alpha\beta} = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}, \tag{A.3}$$

and the spinor indices are raised or lowered by using the metric $\eta^{\alpha\beta}$ and $\eta_{\alpha\beta}$*,

$$\theta^\alpha = \theta_\beta \eta^{\beta\alpha}, \qquad \theta_\alpha = \eta_{\alpha\beta}\theta^\beta. \tag{A.4}$$

A bilinear form of spinors θ and χ is defined by

$$(\theta \mathcal{M}_\sigma \chi) \equiv \theta^\alpha (\mathcal{M}_\sigma)_\alpha{}^\beta \chi_\beta,$$

where $\mathcal{M}_\sigma$ denotes any products of the σ-matrices. An integration of spinor coordinates is given by

$$\int \mathrm{d}^2\theta \equiv \frac{1}{2}\int \mathrm{d}\theta^2 \mathrm{d}\theta^1, \tag{A.5}$$

and its normalization is defined by

$$\int \mathrm{d}^2\theta(\theta\theta) = 1.$$

One can introduce an orthonormal basis "zweibein" one-form field of the local Lorentz flame for each cotangent space of two-dimensional spacetime

$$e^a = \mathrm{d}x^m e_m{}^a, \tag{A.6}$$

where the indexes $a(=0,1)$ and $m(=0,1)$ label the local Lorentz flame and curved spacetime, respectively. Orthonormality for the zweibein $e_m{}^a(x)$ implies

$$g^{mn} e_m{}^a e_n{}^b = \eta^{ab},$$

by using inverse metric $g^{mn}(x)$. One may assume the invertibility of the zweibein,

$$e_m{}^a e_a{}^n = \delta_m{}^n, \qquad e_a{}^m e_m{}^b = \delta_a{}^b.$$

In the curved spacetime, the metric $g_{mn}(x)$ and the Levi-Cività symbol ε^{mn} are given by

$$g_{mn} = e_m{}^a e_n{}^b \eta_{ab}, \qquad \varepsilon^{mn} = e e_a{}^m e_b{}^n \varepsilon^{ab}, \qquad \varepsilon_{mn} = \frac{1}{e} e_m{}^a e_n{}^b \varepsilon_{ab}, \tag{A.7}$$

where $e(x) \equiv \det e_m{}^a(x) = \sqrt{-g(x)}$. One might prefer to use the Levi-Cività tensors,

$$\mathcal{E}^{mn} \equiv \frac{1}{e}\varepsilon^{mn}, \qquad \mathcal{E}_{mn} \equiv e\varepsilon_{mn}.$$

The zweibein $e_m{}^a(x)$ allows to covert the local Lorentz indices to the spacetime indices and back,

$$A^a = e_m{}^a A^m, \qquad A^m = A^a e_a{}^m.$$

*One might prefer to regard the equation $\theta^\alpha = \theta_\beta \eta^{\beta\alpha}$ as the Dirac conjugate relation $\bar{\psi} = \psi^\dagger \sigma^0 = \psi^T C$, where the spinor ψ is a real (Majorana) spinor and the charge conjugation matrix is defined as $C \equiv \sigma^0$.

One can also define σ-matrices in curved spacetime $\sigma^m(x)$ as

$$\sigma^m \equiv \sigma^a e_a{}^m. \tag{A.8}$$

The zweibein obeys the following SO$(1,1)$ local Lorentz transformation

$$\delta e_m{}^a = l e_m{}^b \varepsilon_b{}^a, \tag{A.9}$$

where the symbol $\varepsilon_b{}^a$ is a generator of the vectorial representation for the local Lorentz group SO$(1,1)$ and the function $l(x)$ is a corresponding gauge parameter. In order to define a covariant derivative on the local Lorentz group, a spin connection one-form $\omega_a{}^b(x)$ is introduced as

$$\omega_a{}^b = \mathrm{d}x^m \omega_m \varepsilon_a{}^b = \omega \varepsilon_a{}^b, \tag{A.10}$$

and its local Lorentz transformation is defined by

$$\delta\omega_m = \partial_m l. \tag{A.11}$$

The connections allow us to define covariant derivatives acting on the local Lorentz indices as

$$\begin{aligned} \nabla_m \phi &= \partial_m \phi, \\ \nabla_m A_a &= \partial_m A_a + \omega_m \varepsilon_a{}^b A_b, \\ \nabla_m A^a &= \partial_m A^a - \omega_m A^b \varepsilon_b{}^a, \end{aligned} \tag{A.12}$$

whereas covariant derivatives acting on the ordinary curved spacetime indices are defined by

$$\begin{aligned} \nabla_m \phi &= \partial_m \phi, \\ \nabla_m A_n &= \partial_m A_n - \Gamma^l{}_{mn} A_l, \\ \nabla_m A^n &= \partial_m A^n + \Gamma^n{}_{ml} A^l, \end{aligned} \tag{A.13}$$

where the coefficient $\Gamma^l{}_{mn}(x)$ is usual Christoffel connection. From the equivalence for the covariant derivatives between local Lorentz flame and curved spacetime flame, one can obtain the following relation between spin connections and Christoffel connections,

$$\Gamma^l{}_{mn} = e_a{}^l(\partial_m e_n{}^a - \omega_m e_n{}^b \varepsilon_b{}^a), \tag{A.14}$$

or equivalently

$$\nabla_m e_n{}^a = 0. \tag{A.15}$$

In (A.15), we use the following definition for the covariant derivative for the field $A_m{}^a(x)$ involving "mixed" indices,

$$\nabla_m A_n{}^a \equiv \partial_m A_n{}^a - \Gamma^l{}_{mn} A_l{}^a - \omega_m A_n{}^b \varepsilon_b{}^a. \tag{A.16}$$

In two-dimensional case, the metric compatibility which corresponds to $\nabla_k g_{mn}(x) = 0$ is automatically satisfied by the following way

$$\begin{aligned}\nabla_m \eta_{ab} &= \partial_m \eta_{ab} + \omega_m \varepsilon_a{}^c \eta_{cb} + \omega_m \varepsilon_b{}^c \eta_{ac} \\ &= \omega_m \varepsilon_a{}^c \eta_{cb} + \omega_m \varepsilon_b{}^c \eta_{ac} \\ &= 0.\end{aligned}$$

The torsion two-form $T^a(x)$ and the curvature two-form $R_a{}^b(x)$ are then defined as

$$T^a = \mathrm{d}e^a - \omega e^b \varepsilon_b{}^a \equiv -\frac{1}{2} e^c e^b T_{bc}{}^a, \tag{A.17a}$$

$$R_a{}^b = \mathrm{d}\omega_a{}^b + \omega_a{}^c \omega_c{}^b \equiv -\frac{1}{2} e^d e^c R_{cda}{}^b. \tag{A.17b}$$

In the curved spacetime, the torsion components are given as

$$\begin{aligned}T_{mn}{}^l &\equiv e_a{}^l T_{mn}{}^a \\ &= e_a{}^l \Big(\partial_m e_n{}^a - \omega_m e_n{}^b \varepsilon_b{}^a - (m \leftrightarrow n)\Big) \\ &= \Gamma^l{}_{mn} - \Gamma^l{}_{nm}.\end{aligned}$$

If one impose the usual torsion free condition $T_{mn}{}^l(x) = 0$, the spin connections $\omega_m(x)$ are expressed in terms of the zweibein fields

$$\omega_m = \frac{1}{e} e_{am} \varepsilon^{nl} \partial_n e_l{}^a. \tag{A.18}$$

In addition to the above definition for the covariant derivative on the bosonic fields, one can also define covariant derivatives acting on spinor fields. The generator of the spinorial representation of the local Lorentz group SO(1,1) is given by $\frac{1}{2}(\bar{\sigma})_\alpha{}^\beta$. The local Lorentz transformations of spinor fields are then given by

$$\begin{aligned}\delta\psi_\alpha &= -\frac{1}{2} l(\bar{\sigma}\psi)_\alpha, \\ \delta\psi^\alpha &= -\frac{1}{2} l(\bar{\sigma}\psi)^\alpha.\end{aligned} \tag{A.19}$$

From these transformations, one can define the following covariant derivative acting on the spinor fields $\psi_\alpha(x)$ and $\psi^\alpha(x)$,

$$\begin{aligned}(\nabla_m\psi)_\alpha &= \partial_m\psi_\alpha + \frac{1}{2}\omega_m(\bar\sigma\psi)_\alpha, \\ (\nabla_m\psi)^\alpha &= \partial_m\psi^\alpha + \frac{1}{2}\omega_m(\bar\sigma\psi)^\alpha.\end{aligned} \tag{A.20}$$

One can easily check the metric compatibility for the spinor metric $\eta^{\alpha\beta}$ and $\eta_{\alpha\beta}$,

$$\nabla_m\eta_{\alpha\beta} = \nabla_m\eta^{\alpha\beta} = 0,$$

and the following covariant constant relations

$$\nabla_m\delta_\alpha{}^\beta = \nabla_m(\sigma^a)_\alpha{}^\beta = \nabla_m(\bar\sigma)_\alpha{}^\beta = 0.$$

Appendix B. Geometry of superspace

In order to construct supersymmetric theory on the two-dimensional world-sheet, we use the $(1,1)$ type superspace with coordinates $z^M = (x^m,\ \theta^\mu)$ $(m=0,1;\ \mu=1,2)$. The coordinates x^m and θ^μ are bosonic and fermionic, respectively,

$$z^M z^N = (-)^{|M||N|} z^N z^M, \tag{B.1}$$

where $|M|$ is a Grassmann parity of the coordinate z^M.

One can define a differential p-form in $(1,1)$ superspace as

$$\Phi(z) \equiv \frac{(-)^{p(p-1)/2}}{p!}\mathrm{d}z^{M_p}\cdots\mathrm{d}z^{M_1}\Phi_{M_1\cdots M_p}(z), \tag{B.2}$$

where the coefficient function $\Phi_{M_1\cdots M_p}(z)$ is "graded" antisymmetric in its indices, due to the "graded" anticommutativity of the differential forms,

$$\mathrm{d}z^M\mathrm{d}z^N = -(-)^{|M||N|}\mathrm{d}z^N\mathrm{d}z^M.$$

The exterior derivative for the p-form is defined by

$$\begin{aligned}\mathrm{d}\Phi(z) &\equiv \mathrm{d}z^L\partial_L\Big(\frac{(-)^{p(p-1)/2}}{p!}\mathrm{d}z^{M_p}\cdots\mathrm{d}z^{M_1}\Phi_{M_1\cdots M_p}(z)\Big) \\ &= \frac{(-)^{(p+1)p/2}}{p!}\mathrm{d}z^{M_p}\cdots\mathrm{d}z^{M_1}\mathrm{d}z^L\partial_L\Phi(z).\end{aligned}$$

Under the "usual" wedge product of p-form $\Phi(z)$ and q-form $\Psi(z)$,

$$\Phi(z)\Psi(z) \equiv \frac{(-)^{p(p-1)/2+q(q-1)/2}}{p!q!} \mathrm{d}z^{M_p} \cdots \mathrm{d}z^{M_1} \Phi_{M_1 \cdots M_p}(z) \mathrm{d}z^{N_q} \cdots \mathrm{d}z^{N_1} \Psi_{N_1 \cdots N_q}(z),$$

the Leipnitz rule for the product of p-form $\Phi(z)$ and q-form $\Psi(z)$ is given by

$$\mathrm{d}(\Phi(z)\Psi(z)) = \mathrm{d}\Phi(z)\Psi(z) + (-)^p \Phi(z)\mathrm{d}\Psi(z).$$

At each points on superspace, a local Lorentz frame is defined by introducing a vielbein one-form

$$E^A(z) = \mathrm{d}z^M E_M{}^A(z), \tag{B.3}$$

where the indices $A = (a, \alpha)$ $(a = 0, 1;\ \alpha = 1, 2)$ denote the local Lorentz flame. The vielbeins $E_M{}^A(z)$ are invertible superfields

$$E_M{}^A(z) E_A{}^N(z) = \delta_M{}^N, \qquad E_A{}^M(z) E_M{}^B(z) = \delta_A{}^B. \tag{B.4}$$

In the local Lorentz frame, the exterior derivative is written by

$$\mathrm{d} = \mathrm{d}z^M \partial_M = E^A \partial_A, \tag{B.5}$$

where we denote the derivative ∂_A as

$$\partial_A \equiv E_A{}^M \partial_M.$$

The "graded" Lie bracket is given by

$$[\partial_A, \partial_B\} = C_{AB}{}^C \partial_C, \tag{B.6}$$

where

$$C_{AB}{}^C \equiv (\partial_A E_B{}^N) E_N{}^C - (-)^{|A||B|} (\partial_B E_A{}^N) E_N{}^C.$$

The vielbeins obey the following local Lorentz transformations,

$$\delta E^A(z) = E^B(z) L(z) \varepsilon_B{}^A, \tag{B.7}$$

where the generator of SO$(1,1)$ local Lorentz group is defined by

$$\varepsilon_A{}^B = \begin{pmatrix} \varepsilon_a{}^b & 0 \\ 0 & \frac{1}{2}(\bar{\sigma})_\alpha{}^\beta \end{pmatrix},$$

and the superfield $L(z)$ is a gauge parameter. In order to define supercovariant derivative on SO(1,1) local Lorentz group, a connection one-form is introduced as

$$\Omega_A{}^B(z) = \mathrm{d}z^M \Omega_M(z)\varepsilon_A{}^B = \Omega(z)\varepsilon_A{}^B, \tag{B.8}$$

and its local Lorentz transformation is defined by

$$\delta\Omega_M(z) = \partial_M L(z). \tag{B.9}$$

The connections allow us to define supercovariant derivatives on scalar field $\Phi(z)$ and vector fields $\Psi^A(z)$ and $\Psi_A(z)$ as

$$\begin{aligned}
\mathcal{D}\Phi &= \mathrm{d}\Phi = E^C\partial_C\Phi, \\
\mathcal{D}\Psi_A &= \mathrm{d}\Psi_A + \Omega\varepsilon_A{}^B\Psi_B = E^C\Big(\partial_C\Psi_A + \Omega_C\varepsilon_A{}^B\Psi_B\Big) \equiv E^C\mathcal{D}_C\Psi_A, \\
\mathcal{D}\Psi^A &= \mathrm{d}\Psi^A - \Omega\Psi^B\varepsilon_B{}^A = E^C\Big(\partial_C\Psi^A - \Omega_C\Psi^B\varepsilon_B{}^A\Big) \equiv E^C\mathcal{D}_C\Psi^A,
\end{aligned} \tag{B.10}$$

where we denote $\Omega_A(z) \equiv E_A{}^N\Omega_N(z)$.

The torsion two-form $T^A(z)$ and the curvature two-form $R_A{}^B(z)$ are then defined as

$$T^A = \mathcal{D}E^A \equiv -\frac{1}{2}E^C E^B T_{BC}{}^A, \tag{B.11a}$$

$$R_A{}^B = \mathrm{d}\Omega_A{}^B + \Omega_A{}^C\Omega_C{}^B \equiv -\frac{1}{2}E^D E^C R_{CDA}{}^B. \tag{B.11b}$$

In the local Lorentz flame, the torsion and the curvature are explicitly given by

$$\begin{aligned}
T_{BC}{}^A &= -C_{BC}{}^A - \Omega_B\varepsilon_C{}^A + (-)^{|B||C|}\Omega_C\varepsilon_B{}^A, \\
R_{CDA}{}^B &= \Big(\partial_C\Omega_D - (-)^{|C||D|}\partial_D\Omega_C - C_{CD}{}^E\Omega_E\Big)\varepsilon_A{}^B \\
&\equiv F_{CD}\varepsilon_A{}^B.
\end{aligned}$$

The torsion and the curvature satisfy the Bianchi identities

$$\mathcal{D}T^A = -E^B R_B{}^A, \tag{B.12a}$$

$$\mathcal{D}R_A{}^B = 0. \tag{B.12b}$$

Because the tangent group SO(1,1) is Abelian, the identity (B.12b) becomes trivial relation. On the other hand, the identity (B.12a) becomes the following nontrivial one in the local Lorentz flame,

$$\begin{aligned}
&\mathcal{D}_A T_{BC}{}^D + (-)^{|A|(|B|+|C|)}\mathcal{D}_B T_{CA}{}^D + (-)^{|C|(|A|+|B|)}\mathcal{D}_C T_{AB}{}^D \\
&+T_{AB}{}^E T_{EC}{}^D + (-)^{|A|(|B|+|C|)}T_{BC}{}^E T_{EA}{}^D + (-)^{|C|(|A|+|B|)}T_{CA}{}^E T_{EB}{}^D \\
=& -R_{ABC}{}^D - (-)^{|A|(|B|+|C|)}R_{BCA}{}^D - (-)^{|C|(|A|+|B|)}R_{CAB}{}^D.
\end{aligned}$$

The dynamical variables of two-dimensional supergravity are the vielbein $E_M{}^A(z)$ and the connection $\Omega_M(z)$. The degrees of freedom of these superfields are $80 = (4 \times 4 \times 4) + (4 \times 4)$, because two bosonic fields and one Majorana spinor field are contained in one single superfield. In order to clarify true physical degrees of freedom, it may be useful to impose the following kinematic constraints on some of the torsion components $T_{AB}{}^C(z)$,

$$\begin{aligned} T_{\beta\gamma}{}^a &= T_{\gamma\beta}{}^a = 2i(\sigma^a)_{\beta\gamma}, \\ T_{bc}{}^a &= -T_{cb}{}^a = 0, \\ T_{\beta\gamma}{}^\alpha &= T_{\gamma\beta}{}^\alpha = 0. \end{aligned} \tag{B.13}$$

These constraints reduce the degrees of freedom to $24 = 80 - \big((2 \times 3 \times 4) + (2 \times 1 \times 4) + (2 \times 3 \times 4)\big)$. Other torsion and curvature components are determined from the above constraints by introducing one single scalar superfield $\hat{S}(z)$ [17]:

$$\begin{aligned} T_{\beta c}{}^a &= -T_{c\beta}{}^a = 0, \\ T_{b\gamma}{}^\alpha &= -T_{\gamma b}{}^\alpha = \frac{1}{4}(\sigma_b)_\gamma{}^\alpha \hat{S}, \\ T_{bc}{}^\alpha &= -T_{cb}{}^\alpha = -\frac{i}{4}\varepsilon_{bc}(\bar{\sigma})^{\alpha\beta}\mathcal{D}_\beta \hat{S}, \end{aligned} \tag{B.14}$$

and

$$\begin{aligned} F_{\alpha\beta} &= F_{\beta\alpha} = i(\bar{\sigma})_{\alpha\beta}\hat{S}, \\ F_{\alpha b} &= -F_{b\alpha} = \frac{1}{2}(\bar{\sigma}\sigma_b)_\alpha{}^\beta \mathcal{D}_\beta \hat{S}, \\ F_{ab} &= -F_{ba} = \frac{i}{4}\varepsilon_{ab}\mathcal{D}^\alpha\mathcal{D}_\alpha \hat{S} + \frac{1}{4}\varepsilon_{ab}\hat{S}^2. \end{aligned} \tag{B.15}$$

Under the super-general coordinate and the super local Lorentz transformations, the vielbeins $E_M{}^A(z)$ and the connections $\Omega_M(z)$ transform as

$$\delta E_M{}^A = K^N \partial_N E_M{}^A + \partial_M K^N E_N{}^A + E_M{}^B L \varepsilon_B{}^A, \tag{B.16a}$$

$$\delta \Omega_M = K^N \partial_N \Omega_M + \partial_M K^N \Omega_N + \partial_M L, \tag{B.16b}$$

where $K^N(z)$ and $L(z)$ are local parameters for the super-general coordinate and super local Lorentz transformations, respectively. If one denotes these gauge parameters as

$$\begin{aligned} K^n &= k^n_{(0)} + i\theta^\rho k^n_{(1)\rho} + \frac{i}{2}(\theta\theta)k^n_{(2)}, \\ K^\nu &= k^\nu_{(0)} + i\theta^\rho k^\nu_{(1)\rho} + \frac{i}{2}(\theta\theta)k^\nu_{(2)}, \\ L &= l_{(0)} + i\theta^\rho l_{(1)\rho} + \frac{i}{2}(\theta\theta)l_{(2)}, \end{aligned}$$

and use the degrees of freedom for the parameters $k^n_{(1)\rho}(x)$, $k^n_{(2)}(x)$, $k^\nu_{(1)\rho}(x)$ $k^\nu_{(2)}(x)$, $l_{(1)\rho}(x)$ and $l_{(2)}(x)$, one can impose the following Wess-Zumino gauge,

$$\begin{aligned} E_\mu{}^a &= i\theta^\rho E_{\rho\mu}{}^a + \mathcal{O}(\theta^2), \\ E_\mu{}^\alpha &= \delta_\mu{}^\alpha + i\theta^\rho E_{\rho\mu}{}^\alpha + \mathcal{O}(\theta^2), \\ \Omega_\mu &= i\theta^\rho \omega_{\rho\mu} + \mathcal{O}(\theta^2), \end{aligned} \tag{B.17}$$

where $E_{\rho\mu}{}^a(x) = E_{\mu\rho}{}^a(x)$, $E_{\rho\mu}{}^\alpha(x) = E_{\mu\rho}{}^\alpha(x)$ and $\omega_{\rho\mu}(x) = \omega_{\mu\rho}(x)$. Since the degrees of freedom of the Wess-Zumino gauge are $(2\times 2)+2+(2\times 2)+2+2+1 = 15$, the remaining degrees of freedom are $24-15=9$ and the remaining gauge degrees of freedom are $20-15=5$ which we should specify later. We identify these 9 degrees of freedom to the following field contents,

$$\begin{aligned} E_m{}^a|_{\theta=0} &\equiv e_m{}^a, && \text{(zweibein)}, \\ E_m{}^\alpha|_{\theta=0} &\equiv \frac{1}{2}\chi_m{}^\alpha, && \text{(Rarita-Schwinger field)}, \\ \hat{S}|_{\theta=0} &\equiv A, && \text{(auxiliary field)}. \end{aligned} \tag{B.18}$$

By using these independent variables $e_m{}^a(x)$, $\chi_m{}^\alpha(x)$ and $A(x)$, we can then determine all of the field variables:

- vielbeins:

$$\begin{aligned} E_m{}^a &= e_m{}^a + i(\theta\sigma^a\chi_m) + \frac{i}{4}(\theta\theta)e_m{}^a A, \\ E_m{}^\alpha &= \frac{1}{2}\chi_m{}^\alpha - \frac{1}{4}(\theta\sigma_m)^\alpha A - \frac{1}{2}(\theta\bar{\sigma})^\alpha\hat{\omega}_m - \frac{3}{16}i(\theta\theta)\chi_m{}^\alpha A + \frac{i}{8}(\theta\theta)(\check{v}\sigma_m)^\alpha, \\ E_\mu{}^a &= i\theta^\lambda(\sigma^a)_{\lambda\mu}, \\ E_\mu{}^\alpha &= \delta_\mu{}^\alpha - \frac{i}{8}(\theta\theta)\delta_\mu{}^\alpha A. \end{aligned} \tag{B.19}$$

- inverse vielbeins:

$$\begin{aligned} E_a{}^m &= e_a{}^m - \frac{i}{2}(\theta\sigma^m\chi_a) - \frac{1}{8}(\theta\theta)(\chi_a\sigma^n\sigma^m\chi_n), \\ E_\alpha{}^m &= -i\theta^\lambda(\sigma^m)_{\lambda\alpha} - \frac{1}{4}(\theta\theta)(\sigma^n\sigma^m\chi_n)_\alpha, \\ E_a{}^\mu &= -\frac{1}{2}\chi_a{}^\mu + \frac{i}{4}(\theta\sigma^n\chi_a)\chi_n{}^\mu + \frac{1}{4}(\theta\sigma_a)^\mu A + \frac{1}{2}(\theta\bar{\sigma})^\mu\hat{\omega}_a \\ &\quad + \frac{1}{16}(\theta\theta)(\chi_a\sigma^l\sigma^n\chi_l)\chi_n{}^\mu - \frac{i}{8}(\theta\theta)(\chi_a\sigma^n\bar{\sigma})^\mu\hat{\omega}_n - \frac{i}{8}(\theta\theta)(\check{v}\sigma_a)^\mu, \\ E_\alpha{}^\mu &= \delta_\alpha{}^\mu + \frac{i}{2}\theta^\lambda(\sigma^n)_{\lambda\alpha}\chi_n{}^\mu \\ &\quad + \frac{1}{8}(\theta\theta)(\sigma^l\sigma^n\chi_l)_\alpha\chi_n{}^\mu - \frac{i}{4}(\theta\theta)(\sigma^n\bar{\sigma})_\alpha{}^\mu\hat{\omega}_n - \frac{i}{8}(\theta\theta)\delta_\alpha{}^\mu A. \end{aligned} \tag{B.20}$$

- connections:

$$\begin{aligned}\Omega_m &= \hat{\omega}_m + \frac{i}{2}(\theta\bar{\sigma}\sigma_m\check{v}) + \frac{i}{2}(\theta\bar{\sigma}\chi_m)A \\ &\quad -\frac{i}{4e}(\theta\theta)g_{mn}\varepsilon^{nl}\partial_l A + \frac{i}{4}(\theta\theta)\hat{\omega}_m A - \frac{1}{8}(\theta\theta)(\check{v}\bar{\sigma}\sigma_n\sigma_m\chi^n), \\ \Omega_\mu &= \frac{i}{2}\theta^\lambda(\bar{\sigma})_{\lambda\mu}A.\end{aligned} \tag{B.21}$$

- auxiliary field:

$$\hat{S} = A + i(\theta\check{v}) + \frac{i}{2}(\theta\theta)B. \tag{B.22}$$

In the above relations, we denote

$$\begin{aligned}\hat{\omega}_m &= \omega_m - \frac{i}{2}(\chi_m\bar{\sigma}\sigma^n\chi_n), \\ \check{v}_\mu &= -\frac{2}{e}\varepsilon^{mn}(\bar{\sigma}\hat{\nabla}_m\chi_n)_\mu - \frac{1}{2}(\sigma^m\chi_m)_\mu A, \\ B &= \frac{2}{e}\varepsilon^{mn}\partial_m\hat{\omega}_n - \frac{i}{2}(\check{v}\sigma^m\chi_m) - \frac{i}{4e}\varepsilon^{mn}(\chi_m\bar{\sigma}\chi_n)A + \frac{1}{2}A^2,\end{aligned}$$

where $\omega_m(x)\Big(= \frac{1}{e}e_{am}\varepsilon^{nl}\partial_n e_l{}^a(x)\Big)$ is the usual torsion free spin connection defined via (A.18). The covariant derivative on spinor fields $\hat{\nabla}_m$ is defined by (A.20), except for using the connection $\hat{\omega}_m(x)$ instead of the torsion free connection $\omega_m(x)$. It is worth to mention that since $E_\mu{}^\alpha(z)$ is essentially Kronecker delta between μ and α indices, spinor indices might be written either μ or α in the Wess-Zumino gauge.

In addition to the above super-general coordinate and super local Lorentz transformations, one can define the super-Weyl scaling transformation [17], which is consistent with the kinematic constraints (B.13),

$$\begin{aligned}\delta E_M{}^a &= -SE_M{}^a, \\ \delta E_M{}^\alpha &= -\frac{1}{2}SE_M{}^\alpha + \frac{i}{2}E_M{}^a(\sigma_a)^{\alpha\beta}\mathcal{D}_\beta S, \\ \delta\Omega_M &= E_M{}^a\varepsilon_a{}^b\mathcal{D}_b S + E_M{}^\alpha(\bar{\sigma})_\alpha{}^\beta\mathcal{D}_\beta S, \\ \delta\hat{S} &= S\hat{S} - i\mathcal{D}^\alpha\mathcal{D}_\alpha S,\end{aligned} \tag{B.23}$$

where $S(z)$ is a scalar superfield parameter for the super-Weyl scaling. If one expands the scalar superfield $S(z)$ by the following way,

$$S = s + i(\theta\check{s}) + \frac{i}{2}(\theta\theta)\bar{s}, \tag{B.24}$$

the auxiliary field $A(x)$ can be gauged away by using the gauge degree of freedom $\bar{s}(x)$. At this stage, we have $8(=9-1)$ degrees of freedom and $8(=5+3)$ gauge degrees of freedom.

Now let us specify residual gauge symmetries. As we have explained, we still have 8 gauge degrees of freedom and then identify these to the following gauge parameters,

$$\begin{aligned}
K^n|_{\theta=0} &\equiv k^n, && \text{(general coordinate transformation)},\\
K^\nu|_{\theta=0} &\equiv \zeta^\nu, && \text{(local supersymmetry transformation)},\\
L|_{\theta=0} &\equiv l, && \text{(local Lorentz transformation)},\\
S|_{\theta=0} &\equiv s, && \text{(Weyl scaling transformation)},\\
-i\partial_\mu S|_{\theta=0} &\equiv \check{s}_\mu, && \text{(super-Weyl scaling transformation)}.
\end{aligned} \tag{B.25}$$

In order to preserve the Wess-Zumino gauge, the other components of the gauge parameters should be determined as

$$\begin{aligned}
K^n &= k^n + i(\theta\sigma^n\zeta) + \frac{1}{4}(\theta\theta)(\chi_l\sigma^n\sigma^l\zeta),\\
K^\nu &= \zeta^\nu - \frac{i}{2}(\theta\sigma^l\zeta)\chi_l{}^\nu - \frac{1}{2}l(\theta\bar{\sigma})^\nu + \frac{1}{2}\theta^\nu s + \frac{i}{4}(\theta\theta)(\bar{\sigma}\sigma^l\zeta)^\nu\hat{\omega}_l\\
&\quad -\frac{1}{8}(\theta\theta)(\chi_k\sigma^l\sigma^k\zeta)\chi_l{}^\nu,\\
L &= l - i(\theta\sigma^l\zeta)\hat{\omega}_l - i(\theta\bar{\sigma}\check{s}) - \frac{i}{4}(\theta\theta)(\check{v}\bar{\sigma}\zeta) - \frac{1}{4}(\theta\theta)(\chi_k\sigma^l\sigma^k\zeta)\hat{\omega}_l,\\
S &= s + i(\theta\check{s}) + \frac{1}{2e}(\theta\theta)\varepsilon^{mn}(\zeta\bar{\sigma}\hat{\nabla}_m\chi_n).
\end{aligned} \tag{B.26}$$

In particular, the residual symmetries of the supergravity multiplet $e_m{}^a(x)$ and $\chi_m{}^\alpha(x)$ are given by

$$\delta e_m{}^a = k^n\partial_n e_m{}^a + \partial_m k^n e_n{}^a + i(\zeta\sigma^a\chi_m) + l e_m{}^b\varepsilon_b{}^a - s e_m{}^a, \tag{B.27a}$$

$$\delta\chi_m{}^\alpha = k^n\partial_n\chi_m{}^\alpha + \partial_m k^n\chi_n{}^\alpha + 2(\hat{\nabla}_m\zeta)^\alpha - \frac{1}{2}l(\bar{\sigma}\chi_m)^\alpha - \frac{1}{2}s\chi_m{}^\alpha - (\sigma_m\check{s})^\alpha. \tag{B.27b}$$

One may introduce a scalar superfield $\Phi(z)$ and a spinor superfield $\tilde{\Psi}_\alpha(z)$ as

$$\Phi = \phi + i(\theta\kappa) + \frac{i}{2}(\theta\theta)G, \tag{B.28a}$$

$$\tilde{\Psi}_\alpha = i\hat{\psi}_\alpha + i\theta_\alpha X' + i(\bar{\sigma}\theta)_\alpha X + i(\sigma^m\theta)_\alpha\tilde{A}_m + (\theta\theta)\psi_\alpha. \tag{B.28b}$$

The super-general coordinate, super-Lorentz and super-Weyl transformations are given by

$$\delta\Phi = K^N\partial_N\Phi, \tag{B.29a}$$

$$\delta\tilde{\Psi}_\alpha = K^N\partial_N\tilde{\Psi}_\alpha - \frac{1}{2}L(\bar{\sigma})_\alpha{}^\beta\tilde{\Psi}_\beta + \frac{1}{2}S\tilde{\Psi}_\alpha, \tag{B.29b}$$

In the Wess-Zumino gauge, one can express these transformations for the component fields,

$$\begin{aligned}
\delta\phi &= k^n\partial_n\phi + i(\zeta\kappa), \\
\delta\kappa_\alpha &= k^n\partial_n\kappa_\alpha + (\sigma^m\zeta)_\alpha\Big(\partial_m\phi - \frac{i}{2}(\chi_m\kappa)\Big) + \zeta_\alpha G - \frac{1}{2}l(\bar\sigma\kappa)_\alpha + \frac{1}{2}s\kappa_\alpha, \\
\delta G &= k^n\partial_n G - \frac{i}{2}(\zeta\sigma^m\sigma^n\chi_m)\Big(\partial_n\phi - \frac{i}{2}(\chi_n\kappa)\Big) + i(\zeta\sigma^m\hat\nabla_m\kappa) - \frac{i}{2}(\zeta\sigma^m\chi_m)G + sG,
\end{aligned} \tag{B.30a}$$

and

$$\begin{aligned}
\delta\hat\psi_\alpha &= k^n\partial_n\hat\psi_\alpha + \Big((X' + X\bar\sigma + \tilde A_m\sigma^m)\zeta\Big)_\alpha - \frac{1}{2}l(\bar\sigma\hat\psi)_\alpha + \frac{1}{2}s\hat\psi_\alpha, \\
\delta X' &= k^n\partial_n X' + i(\zeta\psi) + \frac{i}{2}(\zeta\sigma^m\hat\nabla_m\hat\psi) - \frac{i}{4}\Big(\zeta\sigma^m(X' + X\bar\sigma + \tilde A_n\sigma^n)\chi_m\Big) + sX', \\
\delta X &= k^n\partial_n X + i(\zeta\bar\sigma\psi) - \frac{i}{2}(\zeta\bar\sigma\sigma^m\hat\nabla_m\hat\psi) + \frac{i}{4}\Big(\zeta\bar\sigma\sigma^m(X' + X\bar\sigma + \tilde A_n\sigma^n)\chi_m\Big) + sX, \\
\delta\tilde A_m &= \partial_m k^n\tilde A_n + k^n\partial_n\tilde A_m + i(\zeta\sigma_m\psi) + \frac{i}{2}(\zeta\sigma^n\sigma_m\hat\nabla_n\hat\psi) + i(\zeta\sigma^n\chi_m)\tilde A_n \\
&\quad -\frac{i}{4}\Big(\zeta\sigma^n\sigma_m(X' + X\bar\sigma + \tilde A_l\sigma^l)\chi_n\Big) - \frac{i}{2}(\check s\sigma_m\hat\psi), \\
\delta\psi_\alpha &= k^n\partial_n\psi_\alpha + \frac{i}{4}(\zeta\sigma^m\sigma^n\chi_m)(\hat\nabla_n\hat\psi)_\alpha - \frac{i}{2}(\zeta\sigma^m\chi_m)\psi_\alpha \\
&\quad +\frac{1}{2}(\sigma^m\zeta)_\alpha\partial_m X' + \frac{1}{2}(\bar\sigma\sigma^m\zeta)_\alpha\partial_m X + \frac{1}{2}(\sigma^m\sigma^n\zeta)_\alpha e_m{}^a\hat\nabla_n\tilde A_a \\
&\quad -\frac{i}{8}(\zeta\sigma^m\sigma^n\chi_m)\Big((X' + X\bar\sigma + \tilde A_l\sigma^l)\chi_n\Big)_\alpha \\
&\quad -\frac{i}{4e}\varepsilon^{mn}(\zeta\hat\nabla_m\chi_n)(\bar\sigma\hat\psi)_\alpha + \frac{i}{4e}\varepsilon^{mn}(\zeta\bar\sigma\hat\nabla_m\chi_n)\hat\psi_\alpha \\
&\quad -\frac{1}{2}l(\bar\sigma\psi)_\alpha + \frac{3}{2}s\psi_\alpha + \frac{1}{2}\Big((X' + X\bar\sigma)\check s\Big)_\alpha,
\end{aligned} \tag{B.30b}$$

where we denote the covariant derivative for the vector field $\tilde A_a(x) \equiv e_a{}^m\tilde A_m(x)$ as

$$\hat\nabla_m\tilde A_a = \partial_m\tilde A_a + \hat\omega_m\varepsilon_a{}^b\tilde A_b.$$

The superdeterminant $E(z)$ is defined by

$$\begin{aligned}
E &\equiv \mathrm{sdet}E_M{}^A = \det E_m{}^a \det E_\alpha{}^\mu \\
&= e + \frac{i}{2}e(\theta\sigma^m\chi_m) + \frac{i}{4}e(\theta\theta)A + \frac{1}{8}(\theta\theta)\varepsilon^{mn}(\chi_m\bar\sigma\chi_n).
\end{aligned} \tag{B.31}$$

It might be useful to give the following relation for the checking of the gauge invariance of the action in the superfield formulation,

$$\int \mathrm{d}^2\theta E\mathcal{D}^\alpha\tilde\Psi_\alpha = \partial_m\Big(eg^{mn}\tilde A_n - \frac{i}{2}\varepsilon^{mn}(\hat\psi\bar\sigma\chi_n)\Big), \tag{B.32}$$

for an arbitrary spinor superfield $\tilde{\Psi}_\alpha(z)$ defined by (B.28b). We also give the following relation,

$$(\bar{\sigma})^{\alpha\beta}\mathcal{D}_\alpha\mathcal{D}_\beta\Phi = 0, \tag{B.33}$$

for an arbitrary scalar superfield $\Phi(z)$.

Appendix C. Generalized Poisson bracket

A generalized Poisson bracket [26] is defined by

$$\{F, G\} \equiv \left(\frac{\delta_{\rm L}F}{\delta\varphi^i}\frac{\delta_{\rm L}G}{\delta P_{\varphi^i}} - \frac{\delta_{\rm L}F}{\delta P_{\varphi^i}}\frac{\delta_{\rm L}G}{\delta\varphi^i}\right) + (-)^{|F|}\left(\frac{\delta_{\rm L}F}{\delta\theta^\alpha}\frac{\delta_{\rm L}G}{\delta P_{\theta^\alpha}} + \frac{\delta_{\rm L}F}{\delta P_{\theta^\alpha}}\frac{\delta_{\rm L}G}{\delta\theta^\alpha}\right), \tag{C.1}$$

where canonical variables φ^i and P_{φ^i} are bosonic, and θ^α and P_{θ^α} are fermionic. In the above definition the contraction of the indices contains the integration of space or spacetime and $|F|$ is the Grassmann parity of F. This generalized Poisson bracket will be replaced by the graded commutation relation multiplied by $-i$ upon quantization, as usual,

$$\{\quad,\quad\} \to \frac{1}{i}[\quad,\quad\}. \tag{C.2}$$

The explicit forms of the basic Poisson brackets are given by

$$\{\varphi^i, P_{\varphi^j}\} = -\{P_{\varphi^j}, \varphi^i\} = \delta^i_j,$$
$$\{\theta^\alpha, P_{\theta^\beta}\} = \{P_{\theta^\beta}, \theta^\alpha\} = -\delta^\alpha_\beta.$$

The algebraic properties of the Poisson bracket are as follows:

$$\{F, G\} = -(-)^{|F||G|}\{G, F\},$$
$$\{F, G_1G_2\} = \{F, G_1\}G_2 + (-)^{|F||G_1|}G_1\{F, G_2\},$$
$$\{F_1F_2, G\} = F_1\{F_2, G\} + (-)^{|G||F_2|}\{F_1, G\}F_2.$$

Appendix D. Lorentz generators in the light-cone gauge formulation

We give bellow some of the explicit operator forms of the Lorentz generators in the light-cone gauge formulation:

• (NS, NS) sector:

$$
\begin{aligned}
M^{i-} = x^i\Big(&\frac{(p^{\hat{+}})^2}{\phi^{\hat{+}}(\phi^{\hat{+}}p^+ - \phi^+ p^{\hat{+}})}\Big(-\phi^+\phi^- + \frac{1}{2}\phi^j\phi_j\Big) - \frac{p^{\hat{+}}}{\phi^{\hat{+}}p^+ - \phi^+ p^{\hat{+}}}\Big(-\phi^- p^+ + \phi^j p_j\Big) \\
&+\frac{2\pi\phi^{\hat{+}}}{\phi^{\hat{+}}p^+ - \phi^+ p^{\hat{+}}}\Big(:L_0^{\rm tr}: + :\tilde{L}_0^{\rm tr}: - a_0 - \tilde{a}_0\Big)\Big) \\
&-x^- p^i \\
&+i\frac{p^{\hat{+}}\phi_k}{\phi^{\hat{+}}p^+ - \phi^+ p^{\hat{+}}}\Big(\sum_{m=1}^{\infty}\frac{1}{m}\Big(\alpha^i_{-m}\alpha^k_m - \alpha^k_{-m}\alpha^i_m\Big) + \sum_{r=\frac{1}{2}}^{\infty}\Big(b^i_{-r}b^k_r - b^k_{-r}b^i_r\Big)\Big) \\
&-i\frac{2\sqrt{\pi}\phi^{\hat{+}}}{\phi^{\hat{+}}p^+ - \phi^+ p^{\hat{+}}}\Big(\sum_{m=1}^{\infty}\frac{1}{m}\Big(\alpha^i_{-m}L^{\rm tr}_m - L^{\rm tr}_{-m}\alpha^i_m\Big) + \sum_{r=\frac{1}{2}}^{\infty}\Big(b^i_{-r}G^{\rm tr}_r - G^{\rm tr}_{-r}b^i_r\Big)\Big) \\
&+i\frac{p^{\hat{+}}\phi_k}{\phi^{\hat{+}}p^+ - \phi^+ p^{\hat{+}}}\Big(\sum_{m=1}^{\infty}\frac{1}{m}\Big(\tilde{\alpha}^i_{-m}\tilde{\alpha}^k_m - \tilde{\alpha}^k_{-m}\tilde{\alpha}^i_m\Big) + \sum_{r=\frac{1}{2}}^{\infty}\Big(\tilde{b}^i_{-r}\tilde{b}^k_r - \tilde{b}^k_{-r}\tilde{b}^i_r\Big)\Big) \\
&-i\frac{2\sqrt{\pi}\phi^{\hat{+}}}{\phi^{\hat{+}}p^+ - \phi^+ p^{\hat{+}}}\Big(\sum_{m=1}^{\infty}\frac{1}{m}\Big(\tilde{\alpha}^i_{-m}\tilde{L}^{\rm tr}_m - \tilde{L}^{\rm tr}_{-m}\tilde{\alpha}^i_m\Big) + \sum_{r=\frac{1}{2}}^{\infty}\Big(\tilde{b}^i_{-r}\tilde{G}^{\rm tr}_r - \tilde{G}^{\rm tr}_{-r}\tilde{b}^i_r\Big)\Big) \\
&+\phi^i p_\phi{}^- - \phi^- p_\phi{}^i, \qquad\qquad \text{(D.1a)}
\end{aligned}
$$

$$
\begin{aligned}
M^{i\hat{-}} = x^i\Big(&\frac{p^+ p^{\hat{+}}}{\phi^{\hat{+}}(\phi^{\hat{+}}p^+ - \phi^+ p^{\hat{+}})}\Big(\phi^+\phi^- - \frac{1}{2}\phi^j\phi_j\Big) - \frac{p^+}{\phi^{\hat{+}}p^+ - \phi^+ p^{\hat{+}}}\Big(\phi^- p^+ - \phi^j p_j\Big) \\
&-\frac{2\pi\phi^+}{\phi^{\hat{+}}p^+ - \phi^+ p^{\hat{+}}}\Big(:L_0^{\rm tr}: + :\tilde{L}_0^{\rm tr}: - a_0 - \tilde{a}_0\Big)\Big) \\
&-x^{\hat{-}} p^i \\
&-i\frac{p^+\phi_k}{\phi^{\hat{+}}p^+ - \phi^+ p^{\hat{+}}}\Big(\sum_{m=1}^{\infty}\frac{1}{m}\Big(\alpha^i_{-m}\alpha^k_m - \alpha^k_{-m}\alpha^i_m\Big) + \sum_{r=\frac{1}{2}}^{\infty}\Big(b^i_{-r}b^k_r - b^k_{-r}b^i_r\Big)\Big) \\
&+i\frac{2\sqrt{\pi}\phi^+}{\phi^{\hat{+}}p^+ - \phi^+ p^{\hat{+}}}\Big(\sum_{m=1}^{\infty}\frac{1}{m}\Big(\alpha^i_{-m}L^{\rm tr}_m - L^{\rm tr}_{-m}\alpha^i_m\Big) + \sum_{r=\frac{1}{2}}^{\infty}\Big(b^i_{-r}G^{\rm tr}_r - G^{\rm tr}_{-r}b^i_r\Big)\Big) \\
&-i\frac{p^+\phi_k}{\phi^{\hat{+}}p^+ - \phi^+ p^{\hat{+}}}\Big(\sum_{m=1}^{\infty}\frac{1}{m}\Big(\tilde{\alpha}^i_{-m}\tilde{\alpha}^k_m - \tilde{\alpha}^k_{-m}\tilde{\alpha}^i_m\Big) + \sum_{r=\frac{1}{2}}^{\infty}\Big(\tilde{b}^i_{-r}\tilde{b}^k_r - \tilde{b}^k_{-r}\tilde{b}^i_r\Big)\Big) \\
&+i\frac{2\sqrt{\pi}\phi^+}{\phi^{\hat{+}}p^+ - \phi^+ p^{\hat{+}}}\Big(\sum_{m=1}^{\infty}\frac{1}{m}\Big(\tilde{\alpha}^i_{-m}\tilde{L}^{\rm tr}_m - \tilde{L}^{\rm tr}_{-m}\tilde{\alpha}^i_m\Big) + \sum_{r=\frac{1}{2}}^{\infty}\Big(\tilde{b}^i_{-r}\tilde{G}^{\rm tr}_r - \tilde{G}^{\rm tr}_{-r}\tilde{b}^i_r\Big)\Big) \\
&+\phi^i p_\phi{}^{\hat{-}} + \frac{1}{\phi^{\hat{+}}}\Big(\phi^+\phi^- - \frac{1}{2}\phi^j\phi_j\Big)p_\phi{}^i, \qquad\qquad \text{(D.1b)}
\end{aligned}
$$

$$
\begin{aligned}
M^{-\hat{-}} = x^-\Big(&\frac{p^+ p^{\hat{+}}}{\phi^{\hat{+}}(\phi^{\hat{+}}p^+ - \phi^+ p^{\hat{+}})}\Big(\phi^+\phi^- - \frac{1}{2}\phi^j\phi_j\Big) - \frac{p^+}{\phi^{\hat{+}}p^+ - \phi^+ p^{\hat{+}}}\Big(\phi^- p^+ - \phi^j p_j\Big) \\
&-\frac{2\pi\phi^+}{\phi^{\hat{+}}p^+ - \phi^+ p^{\hat{+}}}\Big(:L_0^{\rm tr}: + :\tilde{L}_0^{\rm tr}: - a_0 - \tilde{a}_0\Big)\Big) \\
&+x^{\hat{-}}\Big(\frac{(p^{\hat{+}})^2}{\phi^{\hat{+}}(\phi^{\hat{+}}p^+ - \phi^+ p^{\hat{+}})}\Big(\phi^+\phi^- - \frac{1}{2}\phi^j\phi_j\Big) - \frac{p^{\hat{+}}}{\phi^{\hat{+}}p^+ - \phi^+ p^{\hat{+}}}\Big(\phi^- p^+ - \phi^j p_j\Big)
\end{aligned}
$$

$$
\begin{aligned}
&\quad -\frac{2\pi\phi^{\hat{+}}}{\phi^{\hat{+}}p^{+}-\phi^{+}p^{\hat{+}}}\Big(:L_0^{\rm tr}:+:\tilde{L}_0^{\rm tr}:-a_0-\tilde{a}_0\Big)\Big)\\
&+i\frac{2\sqrt{\pi}\phi_j}{\phi^{\hat{+}}p^{+}-\phi^{+}p^{\hat{+}}}\Big(\sum_{m=1}^{\infty}\frac{1}{m}\Big(\alpha_{-m}^{j}L_m^{\rm tr}-L_{-m}^{\rm tr}\alpha_m^{j}\Big)+\sum_{r=\frac{1}{2}}^{\infty}\Big(b_{-r}^{j}G_r^{\rm tr}-G_{-r}^{\rm tr}b_r^{j}\Big)\Big)\\
&+i\frac{2\sqrt{\pi}\phi_j}{\phi^{\hat{+}}p^{+}-\phi^{+}p^{\hat{+}}}\Big(\sum_{m=1}^{\infty}\frac{1}{m}\Big(\tilde{\alpha}_{-m}^{j}\tilde{L}_m^{\rm tr}-\tilde{L}_{-m}^{\rm tr}\tilde{\alpha}_m^{j}\Big)+\sum_{r=\frac{1}{2}}^{\infty}\Big(\tilde{b}_{-r}^{j}\tilde{G}_r^{\rm tr}-\tilde{G}_{-r}^{\rm tr}\tilde{b}_r^{j}\Big)\Big)\\
&+\phi^{-}{p_\phi}^{\hat{-}}+\frac{1}{\phi^{\hat{+}}}\Big(\phi^{+}\phi^{-}-\frac{1}{2}\phi^{j}\phi_j\Big){p_\phi}^{-}.
\end{aligned}
\tag{D.1c}
$$

- (R, R) sector:

$$
\begin{aligned}
M^{i-}=x^{i}\Big(&\frac{(p^{\hat{+}})^2}{\phi^{\hat{+}}(\phi^{\hat{+}}p^{+}-\phi^{+}p^{\hat{+}})}\Big(-\phi^{+}\phi^{-}+\frac{1}{2}\phi^{j}\phi_j\Big)-\frac{p^{\hat{+}}}{\phi^{\hat{+}}p^{+}-\phi^{+}p^{\hat{+}}}\Big(-\phi^{-}p^{+}+\phi^{j}p_j\Big)\\
&+\frac{2\pi\phi^{\hat{+}}}{\phi^{\hat{+}}p^{+}-\phi^{+}p^{\hat{+}}}\Big(:L_0^{\rm tr}:+:\tilde{L}_0^{\rm tr}:-a_0-\tilde{a}_0\Big)\Big)\\
&-x^{-}p^{i}\\
&+i\frac{p^{\hat{+}}\phi_k}{\phi^{\hat{+}}p^{+}-\phi^{+}p^{\hat{+}}}\Big(\sum_{m=1}^{\infty}\frac{1}{m}\Big(\alpha_{-m}^{i}\alpha_m^{k}-\alpha_{-m}^{k}\alpha_m^{i}\Big)\\
&\qquad\qquad+\sum_{r=1}^{\infty}\Big(b_{-r}^{i}b_r^{k}-b_{-r}^{k}b_r^{i}\Big)+\frac{1}{2}\Big(b_0^{i}b_0^{k}-b_0^{k}b_0^{i}\Big)\Big)\\
&-i\frac{2\sqrt{\pi}\phi^{\hat{+}}}{\phi^{\hat{+}}p^{+}-\phi^{+}p^{\hat{+}}}\Big(\sum_{m=1}^{\infty}\frac{1}{m}\Big(\alpha_{-m}^{i}L_m^{\rm tr}-L_{-m}^{\rm tr}\alpha_m^{i}\Big)\\
&\qquad\qquad+\sum_{r=1}^{\infty}\Big(b_{-r}^{i}G_r^{\rm tr}-G_{-r}^{\rm tr}b_r^{i}\Big)+\frac{1}{2}\Big(b_0^{i}G_0^{\rm tr}-G_0^{\rm tr}b_0^{i}\Big)\Big)\\
&+i\frac{p^{\hat{+}}\phi_k}{\phi^{\hat{+}}p^{+}-\phi^{+}p^{\hat{+}}}\Big(\sum_{m=1}^{\infty}\frac{1}{m}\Big(\tilde{\alpha}_{-m}^{i}\tilde{\alpha}_m^{k}-\tilde{\alpha}_{-m}^{k}\tilde{\alpha}_m^{i}\Big)\\
&\qquad\qquad+\sum_{r=1}^{\infty}\Big(\tilde{b}_{-r}^{i}\tilde{b}_r^{k}-\tilde{b}_{-r}^{k}\tilde{b}_r^{i}\Big)+\frac{1}{2}\Big(\tilde{b}_0^{i}\tilde{b}_0^{k}-\tilde{b}_0^{k}\tilde{b}_0^{i}\Big)\Big)\\
&-i\frac{2\sqrt{\pi}\phi^{\hat{+}}}{\phi^{\hat{+}}p^{+}-\phi^{+}p^{\hat{+}}}\Big(\sum_{m=1}^{\infty}\frac{1}{m}\Big(\tilde{\alpha}_{-m}^{i}\tilde{L}_m^{\rm tr}-\tilde{L}_{-m}^{\rm tr}\tilde{\alpha}_m^{i}\Big)\\
&\qquad\qquad+\sum_{r=1}^{\infty}\Big(\tilde{b}_{-r}^{i}\tilde{G}_r^{\rm tr}-\tilde{G}_{-r}^{\rm tr}\tilde{b}_r^{i}\Big)+\frac{1}{2}\Big(\tilde{b}_0^{i}\tilde{G}_0^{\rm tr}-\tilde{G}_0^{\rm tr}\tilde{b}_0^{i}\Big)\Big)\\
&+\phi^{i}{p_\phi}^{-}-\phi^{-}{p_\phi}^{i},
\end{aligned}
\tag{D.2a}
$$

$$
\begin{aligned}
M^{i\hat{-}}=x^{i}\Big(&\frac{p^{+}p^{\hat{+}}}{\phi^{\hat{+}}(\phi^{\hat{+}}p^{+}-\phi^{+}p^{\hat{+}})}\Big(\phi^{+}\phi^{-}-\frac{1}{2}\phi^{j}\phi_j\Big)-\frac{p^{+}}{\phi^{\hat{+}}p^{+}-\phi^{+}p^{\hat{+}}}\Big(\phi^{-}p^{+}-\phi^{j}p_j\Big)\\
&-\frac{2\pi\phi^{+}}{\phi^{\hat{+}}p^{+}-\phi^{+}p^{\hat{+}}}\Big(:L_0^{\rm tr}:+:\tilde{L}_0^{\rm tr}:-a_0-\tilde{a}_0\Big)\Big)\\
&-x^{\hat{-}}p^{i}\\
&-i\frac{p^{+}\phi_k}{\phi^{\hat{+}}p^{+}-\phi^{+}p^{\hat{+}}}\Big(\sum_{m=1}^{\infty}\frac{1}{m}\Big(\alpha_{-m}^{i}\alpha_m^{k}-\alpha_{-m}^{k}\alpha_m^{i}\Big)
\end{aligned}
$$

$$
\begin{aligned}
&\qquad\qquad + \sum_{r=1}^{\infty} \Big(b^i_{-r} b^k_r - b^k_{-r} b^i_r\Big) + \frac{1}{2}\Big(b^i_0 b^k_0 - b^k_0 b^i_0\Big)\Big) \\
&+ i \frac{2\sqrt{\pi}\phi^+}{\phi^{\hat{+}} p^+ - \phi^+ p^{\hat{+}}} \Big(\sum_{m=1}^{\infty} \frac{1}{m}\Big(\alpha^i_{-m} L^{\rm tr}_m - L^{\rm tr}_{-m}\alpha^i_m\Big) \\
&\qquad\qquad + \sum_{r=1}^{\infty} \Big(b^i_{-r} G^{\rm tr}_r - G^{\rm tr}_{-r} b^i_r\Big) + \frac{1}{2}\Big(b^i_0 G^{\rm tr}_0 - G^{\rm tr}_0 b^i_0\Big)\Big) \\
&- i \frac{p^+ \phi_k}{\phi^{\hat{+}} p^+ - \phi^+ p^{\hat{+}}} \Big(\sum_{m=1}^{\infty} \frac{1}{m}\Big(\tilde{\alpha}^i_{-m} \tilde{\alpha}^k_m - \tilde{\alpha}^k_{-m}\tilde{\alpha}^i_m\Big) \\
&\qquad\qquad + \sum_{r=1}^{\infty} \Big(\tilde{b}^i_{-r} \tilde{b}^k_r - \tilde{b}^k_{-r} \tilde{b}^i_r\Big) + \frac{1}{2}\Big(\tilde{b}^i_0 \tilde{b}^k_0 - \tilde{b}^k_0 \tilde{b}^i_0\Big)\Big) \\
&+ i \frac{2\sqrt{\pi}\phi^+}{\phi^{\hat{+}} p^+ - \phi^+ p^{\hat{+}}} \Big(\sum_{m=1}^{\infty} \frac{1}{m}\Big(\tilde{\alpha}^i_{-m} \tilde{L}^{\rm tr}_m - \tilde{L}^{\rm tr}_{-m}\tilde{\alpha}^i_m\Big) \\
&\qquad\qquad + \sum_{r=1}^{\infty} \Big(\tilde{b}^i_{-r} \tilde{G}^{\rm tr}_r - \tilde{G}^{\rm tr}_{-r} \tilde{b}^i_r\Big) + \frac{1}{2}\Big(\tilde{b}^i_0 \tilde{G}^{\rm tr}_0 - \tilde{G}^{\rm tr}_0 \tilde{b}^i_0\Big)\Big) \\
&+ \phi^i p_\phi{}^{\hat{-}} + \frac{1}{\phi^{\hat{+}}}\Big(\phi^+\phi^- - \frac{1}{2}\phi^j\phi_j\Big) p_\phi{}^i, \qquad \text{(D.2b)}
\end{aligned}
$$

$$
\begin{aligned}
M^{-\hat{-}} = x^- \Big(&\frac{p^+ p^{\hat{+}}}{\phi^{\hat{+}}(\phi^{\hat{+}} p^+ - \phi^+ p^{\hat{+}})}\Big(\phi^+\phi^- - \frac{1}{2}\phi^j\phi_j\Big) - \frac{p^+}{\phi^{\hat{+}} p^+ - \phi^+ p^{\hat{+}}}\Big(\phi^- p^+ - \phi^j p_j\Big) \\
&- \frac{2\pi\phi^+}{\phi^{\hat{+}} p^+ - \phi^+ p^{\hat{+}}}\Big(:L^{\rm tr}_0: + :\tilde{L}^{\rm tr}_0: - a_0 - \tilde{a}_0\Big)\Big) \\
+ x^{\hat{-}} \Big(&\frac{(p^{\hat{+}})^2}{\phi^{\hat{+}}(\phi^{\hat{+}} p^+ - \phi^+ p^{\hat{+}})}\Big(\phi^+\phi^- - \frac{1}{2}\phi^j\phi_j\Big) - \frac{p^{\hat{+}}}{\phi^{\hat{+}} p^+ - \phi^+ p^{\hat{+}}}\Big(\phi^- p^+ - \phi^j p_j\Big) \\
&- \frac{2\pi\phi^{\hat{+}}}{\phi^{\hat{+}} p^+ - \phi^+ p^{\hat{+}}}\Big(:L^{\rm tr}_0: + :\tilde{L}^{\rm tr}_0: - a_0 - \tilde{a}_0\Big)\Big) \\
+ i &\frac{2\sqrt{\pi}\phi_j}{\phi^{\hat{+}} p^+ - \phi^+ p^{\hat{+}}} \Big(\sum_{m=1}^{\infty} \frac{1}{m}\Big(\alpha^j_{-m} L^{\rm tr}_m - L^{\rm tr}_{-m}\alpha^j_m\Big) \\
&\qquad\qquad + \sum_{r=1}^{\infty} \Big(b^j_{-r} G^{\rm tr}_r - G^{\rm tr}_{-r} b^j_r\Big) + \frac{1}{2}\Big(b^j_0 G^{\rm tr}_0 - G^{\rm tr}_0 b^j_0\Big)\Big) \\
+ i &\frac{2\sqrt{\pi}\phi_j}{\phi^{\hat{+}} p^+ - \phi^+ p^{\hat{+}}} \Big(\sum_{m=1}^{\infty} \frac{1}{m}\Big(\tilde{\alpha}^j_{-m} \tilde{L}^{\rm tr}_m - \tilde{L}^{\rm tr}_{-m}\tilde{\alpha}^j_m\Big) \\
&\qquad\qquad + \sum_{r=1}^{\infty} \Big(\tilde{b}^j_{-r} \tilde{G}^{\rm tr}_r - \tilde{G}^{\rm tr}_{-r} \tilde{b}^j_r\Big) + \frac{1}{2}\Big(\tilde{b}^j_0 \tilde{G}^{\rm tr}_0 - \tilde{G}^{\rm tr}_0 \tilde{b}^j_0\Big)\Big) \\
+ \phi^- &p_\phi{}^{\hat{-}} + \frac{1}{\phi^{\hat{+}}}\Big(\phi^+\phi^- - \frac{1}{2}\phi^j\phi_j\Big) p_\phi{}^-. \qquad \text{(D.2c)}
\end{aligned}
$$

- (NS, R) sector:

$$
\begin{aligned}
M^{i-} = x^i \Big(&\frac{(p^{\hat{+}})^2}{\phi^{\hat{+}}(\phi^{\hat{+}} p^+ - \phi^+ p^{\hat{+}})}\Big(-\phi^+\phi^- + \frac{1}{2}\phi^j\phi_j\Big) - \frac{p^{\hat{+}}}{\phi^{\hat{+}} p^+ - \phi^+ p^{\hat{+}}}\Big(-\phi^- p^+ + \phi^j p_j\Big) \\
&+ \frac{2\pi\phi^{\hat{+}}}{\phi^{\hat{+}} p^+ - \phi^+ p^{\hat{+}}}\Big(:L^{\rm tr}_0: + :\tilde{L}^{\rm tr}_0: - a_0 - \tilde{a}_0\Big)\Big)
\end{aligned}
$$

$$
\begin{aligned}
&-x^{-}p^{i} \\
&+i\frac{p^{\hat{+}}\phi_k}{\phi^{\hat{+}}p^{+}-\phi^{+}p^{\hat{+}}}\Bigg(\sum_{m=1}^{\infty}\frac{1}{m}\Big(\alpha^{i}_{-m}\alpha^{k}_{m}-\alpha^{k}_{-m}\alpha^{i}_{m}\Big)+\sum_{r=\frac{1}{2}}^{\infty}\Big(b^{i}_{-r}b^{k}_{r}-b^{k}_{-r}b^{i}_{r}\Big)\Bigg) \\
&-i\frac{2\sqrt{\pi}\phi^{\hat{+}}}{\phi^{\hat{+}}p^{+}-\phi^{+}p^{\hat{+}}}\Bigg(\sum_{m=1}^{\infty}\frac{1}{m}\Big(\alpha^{i}_{-m}L^{\mathrm{tr}}_{m}-L^{\mathrm{tr}}_{-m}\alpha^{i}_{m}\Big)+\sum_{r=\frac{1}{2}}^{\infty}\Big(b^{i}_{-r}G^{\mathrm{tr}}_{r}-G^{\mathrm{tr}}_{-r}b^{i}_{r}\Big)\Bigg) \\
&+i\frac{p^{\hat{+}}\phi_k}{\phi^{\hat{+}}p^{+}-\phi^{+}p^{\hat{+}}}\Bigg(\sum_{m=1}^{\infty}\frac{1}{m}\Big(\tilde{\alpha}^{i}_{-m}\tilde{\alpha}^{k}_{m}-\tilde{\alpha}^{k}_{-m}\tilde{\alpha}^{i}_{m}\Big) \\
&\qquad\qquad+\sum_{r=1}^{\infty}\Big(\tilde{b}^{i}_{-r}\tilde{b}^{k}_{r}-\tilde{b}^{k}_{-r}\tilde{b}^{i}_{r}\Big)+\frac{1}{2}\Big(\tilde{b}^{i}_{0}\tilde{b}^{k}_{0}-\tilde{b}^{k}_{0}\tilde{b}^{i}_{0}\Big)\Bigg) \\
&-i\frac{2\sqrt{\pi}\phi^{\hat{+}}}{\phi^{\hat{+}}p^{+}-\phi^{+}p^{\hat{+}}}\Bigg(\sum_{m=1}^{\infty}\frac{1}{m}\Big(\tilde{\alpha}^{i}_{-m}\tilde{L}^{\mathrm{tr}}_{m}-\tilde{L}^{\mathrm{tr}}_{-m}\tilde{\alpha}^{i}_{m}\Big) \\
&\qquad\qquad+\sum_{r=1}^{\infty}\Big(\tilde{b}^{i}_{-r}\tilde{G}^{\mathrm{tr}}_{r}-\tilde{G}^{\mathrm{tr}}_{-r}\tilde{b}^{i}_{r}\Big)+\frac{1}{2}\Big(\tilde{b}^{i}_{0}\tilde{G}^{\mathrm{tr}}_{0}-\tilde{G}^{\mathrm{tr}}_{0}\tilde{b}^{i}_{0}\Big)\Bigg) \\
&+\phi^{i}{p_\phi}^{-}-\phi^{-}{p_\phi}^{i}, \qquad\qquad (\mathrm{D.3a})
\end{aligned}
$$

$$
\begin{aligned}
M^{i\hat{-}} = {}& x^{i}\Bigg(\frac{p^{+}p^{\hat{+}}}{\phi^{\hat{+}}(\phi^{\hat{+}}p^{+}-\phi^{+}p^{\hat{+}})}\Big(\phi^{+}\phi^{-}-\frac{1}{2}\phi^{j}\phi_{j}\Big)-\frac{p^{+}}{\phi^{\hat{+}}p^{+}-\phi^{+}p^{\hat{+}}}\Big(\phi^{-}p^{+}-\phi^{j}p_{j}\Big) \\
&\quad-\frac{2\pi\phi^{+}}{\phi^{\hat{+}}p^{+}-\phi^{+}p^{\hat{+}}}\Big(:L^{\mathrm{tr}}_{0}:+:\tilde{L}^{\mathrm{tr}}_{0}:-a_0-\tilde{a}_0\Big)\Bigg) \\
&-x^{\hat{-}}p^{i} \\
&-i\frac{p^{+}\phi_k}{\phi^{\hat{+}}p^{+}-\phi^{+}p^{\hat{+}}}\Bigg(\sum_{m=1}^{\infty}\frac{1}{m}\Big(\alpha^{i}_{-m}\alpha^{k}_{m}-\alpha^{k}_{-m}\alpha^{i}_{m}\Big)+\sum_{r=\frac{1}{2}}^{\infty}\Big(b^{i}_{-r}b^{k}_{r}-b^{k}_{-r}b^{i}_{r}\Big)\Bigg) \\
&+i\frac{2\sqrt{\pi}\phi^{+}}{\phi^{\hat{+}}p^{+}-\phi^{+}p^{\hat{+}}}\Bigg(\sum_{m=1}^{\infty}\frac{1}{m}\Big(\alpha^{i}_{-m}L^{\mathrm{tr}}_{m}-L^{\mathrm{tr}}_{-m}\alpha^{i}_{m}\Big)+\sum_{r=\frac{1}{2}}^{\infty}\Big(b^{i}_{-r}G^{\mathrm{tr}}_{r}-G^{\mathrm{tr}}_{-r}b^{i}_{r}\Big)\Bigg) \\
&-i\frac{p^{+}\phi_k}{\phi^{\hat{+}}p^{+}-\phi^{+}p^{\hat{+}}}\Bigg(\sum_{m=1}^{\infty}\frac{1}{m}\Big(\tilde{\alpha}^{i}_{-m}\tilde{\alpha}^{k}_{m}-\tilde{\alpha}^{k}_{-m}\tilde{\alpha}^{i}_{m}\Big) \\
&\qquad\qquad+\sum_{r=1}^{\infty}\Big(\tilde{b}^{i}_{-r}\tilde{b}^{k}_{r}-\tilde{b}^{k}_{-r}\tilde{b}^{i}_{r}\Big)+\frac{1}{2}\Big(\tilde{b}^{i}_{0}\tilde{b}^{k}_{0}-\tilde{b}^{k}_{0}\tilde{b}^{i}_{0}\Big)\Bigg) \\
&+i\frac{2\sqrt{\pi}\phi^{+}}{\phi^{\hat{+}}p^{+}-\phi^{+}p^{\hat{+}}}\Bigg(\sum_{m=1}^{\infty}\frac{1}{m}\Big(\tilde{\alpha}^{i}_{-m}\tilde{L}^{\mathrm{tr}}_{m}-\tilde{L}^{\mathrm{tr}}_{-m}\tilde{\alpha}^{i}_{m}\Big) \\
&\qquad\qquad+\sum_{r=1}^{\infty}\Big(\tilde{b}^{i}_{-r}\tilde{G}^{\mathrm{tr}}_{r}-\tilde{G}^{\mathrm{tr}}_{-r}\tilde{b}^{i}_{r}\Big)+\frac{1}{2}\Big(\tilde{b}^{i}_{0}\tilde{G}^{\mathrm{tr}}_{0}-\tilde{G}^{\mathrm{tr}}_{0}\tilde{b}^{i}_{0}\Big)\Bigg) \\
&+\phi^{i}{p_\phi}^{\hat{-}}+\frac{1}{\phi^{\hat{+}}}\Big(\phi^{+}\phi^{-}-\frac{1}{2}\phi^{j}\phi_{j}\Big){p_\phi}^{i}, \qquad\qquad (\mathrm{D.3b})
\end{aligned}
$$

$$
\begin{aligned}
M^{-\hat{-}} = {}& x^{-}\Bigg(\frac{p^{+}p^{\hat{+}}}{\phi^{\hat{+}}(\phi^{\hat{+}}p^{+}-\phi^{+}p^{\hat{+}})}\Big(\phi^{+}\phi^{-}-\frac{1}{2}\phi^{j}\phi_{j}\Big)-\frac{p^{+}}{\phi^{\hat{+}}p^{+}-\phi^{+}p^{\hat{+}}}\Big(\phi^{-}p^{+}-\phi^{j}p_{j}\Big) \\
&\quad-\frac{2\pi\phi^{+}}{\phi^{\hat{+}}p^{+}-\phi^{+}p^{\hat{+}}}\Big(:L^{\mathrm{tr}}_{0}:+:\tilde{L}^{\mathrm{tr}}_{0}:-a_0-\tilde{a}_0\Big)\Bigg)
\end{aligned}
$$

$$
\begin{aligned}
&+x^{\hat{-}}\Big(\frac{(p^{\hat{+}})^2}{\phi^{\hat{+}}(\phi^{\hat{+}}p^{+}-\phi^{+}p^{\hat{+}})}\Big(\phi^{+}\phi^{-}-\frac{1}{2}\phi^{j}\phi_{j}\Big)-\frac{p^{\hat{+}}}{\phi^{\hat{+}}p^{+}-\phi^{+}p^{\hat{+}}}\Big(\phi^{-}p^{+}-\phi^{j}p_{j}\Big)\\
&\qquad-\frac{2\pi\phi^{\hat{+}}}{\phi^{\hat{+}}p^{+}-\phi^{+}p^{\hat{+}}}\Big(:L_0^{\mathrm{tr}}:+:\tilde{L}_0^{\mathrm{tr}}:-a_0-\tilde{a}_0\Big)\Big)\\
&+i\frac{2\sqrt{\pi}\phi_j}{\phi^{\hat{+}}p^{+}-\phi^{+}p^{\hat{+}}}\Big(\sum_{m=1}^{\infty}\frac{1}{m}\Big(\alpha_{-m}^{j}L_m^{\mathrm{tr}}-L_{-m}^{\mathrm{tr}}\alpha_m^{j}\Big)+\sum_{r=\frac{1}{2}}^{\infty}\Big(b_{-r}^{j}G_r^{\mathrm{tr}}-G_{-r}^{\mathrm{tr}}b_r^{j}\Big)\Big)\\
&+i\frac{2\sqrt{\pi}\phi_j}{\phi^{\hat{+}}p^{+}-\phi^{+}p^{\hat{+}}}\Big(\sum_{m=1}^{\infty}\frac{1}{m}\Big(\tilde{\alpha}_{-m}^{j}\tilde{L}_m^{\mathrm{tr}}-\tilde{L}_{-m}^{\mathrm{tr}}\tilde{\alpha}_m^{j}\Big)\\
&\qquad\qquad+\sum_{r=1}^{\infty}\Big(\tilde{b}_{-r}^{j}\tilde{G}_r^{\mathrm{tr}}-\tilde{G}_{-r}^{\mathrm{tr}}\tilde{b}_r^{j}\Big)+\frac{1}{2}\Big(\tilde{b}_0^{j}\tilde{G}_0^{\mathrm{tr}}-\tilde{G}_0^{\mathrm{tr}}\tilde{b}_0^{j}\Big)\Big)\\
&+\phi^{-}p_{\phi}{}^{\hat{-}}+\frac{1}{\phi^{\hat{+}}}\Big(\phi^{+}\phi^{-}-\frac{1}{2}\phi^{j}\phi_{j}\Big)p_{\phi}{}^{-}.
\end{aligned}
\tag{D.3c}
$$

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