Einstein's exploration of a steady-state model of the universe

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Abstract

We present a translation and analysis of an unpublished manuscript by Albert Einstein in which he explored a 'steady-state' model of the universe. The manuscript, which appears to have been written in early 1931, demonstrates that Einstein once considered an expanding cosmos in which the mean density of matter is maintained constant by a continuous formation of matter from empty space. This model is very different to previously known Einsteinian models of the cosmos (both static and dynamic) but anticipates the later steady-state cosmology of Hoyle, Bondi and Gold in some ways. We find that Einstein's steady-state model contained a fundamental flaw and suggest that it was discarded for this reason. We also suggest that he declined to try again because he realised that a successful steady-state model would require an amendment to the field equations. The abandoned model is of historical significance because it reveals that Einstein debated between steady-state and evolving models of the cosmos decades before a similar debate took place in the cosmological community.

1. Introduction

In the course of our research into Einstein's cosmology in the early 1930s,[1] we recently came upon an Einsteinian model of the cosmos that he did not publish. This model is set out in a signed, four-page handwritten manuscript entitled "*Zum kosmologischen Problem*" in the Albert Einstein Archives of the Hebrew University of Jerusalem (Einstein 1931a). Until now, the manuscript was understood to be a draft of the paper *"Zum kosmologischen Problem der allgemeinen Relativitätstheorie",* published by Einstein in the journal Sitzungsberichte der Königlich Preussischen Akademie der Wissenschaften (Einstein 1931b).[2] However, on examining the manuscript we found that it features a model of the cosmos strikingly different to the *Sitzungsberichte* paper, despite some similarities in title and opening paragraphs (figure 1).

The manuscript indicates that Einstein once explored a model of an expanding cosmos in which the density of matter remains constant due to a continuous formation of matter in empty space. This model is very different to other Einsteinian models of the cosmos, such as his early model of a static universe (Einstein 1917) or his later models of an expanding, evolving universe (Einstein 1931b, Einstein and de Sitter 1932). However, the model anticipates the well-known steady-state theories of Fred Hoyle, Hermann Bondi and Tommy Gold (Hoyle 1948; Bondi and Gold 1948).

We present a guided tour of Einstein's steady-state model in section 2 of this paper and discuss a likely date for the work in section 3. We consider possible reasons for Einstein's abandonment of the model in section 4, not least a fundamental flaw in the derivation. Other steady-state models of the cosmos are discussed in section 5, and we consider Einstein's steady-state theory in the context of his philosophy of cosmology in section 6. Our translation and transcription of the original manuscript is provided as an Appendix by kind permission of the Albert Einstein Archive of the Hebrew University of Jerusalem.

[1] See (O'Raifeartaigh and McCann 2014).
[2] This point has been confirmed by the founding editor of the Einstein Papers John Stachel (Stachel 2013).

2. A guided tour of the manuscript[3]

The manuscript begins with Einstein recalling the well-known problem of gravitational collapse in a Newtonian universe. This starting point is similar to Einstein's seminal cosmological paper of 1917 (Einstein 1917), although he now includes a reference to the work of Hugo Seeliger:

> *"It is well known that the most important fundamental difficulty that emerges when one asks how the stellar matter fills up space in very large dimensions is that the laws of gravity are not in general consistent with the hypothesis of a finite mean density of matter. At a time when Newton's theory of gravity was still generally accepted, Seeliger had for this reason modified the Newtonian law by the introduction of a distance function that, for large distances r, diminished considerably faster than $1/r^2$."*

Einstein points out that a similar problem arises in relativistic models of the cosmos, and recalls his introduction of the cosmological constant to the field equations of relativity to render them consistent with a static universe of constant radius and matter density:

> *"This difficulty also arises in the general theory of relativity. However, I have shown that this can be overcome by the introduction of the so-called "λ–term" to the field equations. The field equations can then be written in the form*

$$\left(R_{ik} - \frac{1}{2}g_{ik}R\right) - \lambda g_{ik} = \kappa T_{ik} \qquad (1)$$

$$R_{ik} = \Gamma^{\sigma}_{ik,\sigma} - \Gamma^{\sigma}_{i\sigma,k} - \Gamma^{\sigma}_{i\tau}\Gamma^{\tau}_{k\sigma} + \Gamma^{\sigma}_{ik}\Gamma^{\tau}_{\sigma\tau} \qquad (1a)$$

> *At that time, I showed that these equations can be satisfied by a spherical space of constant radius over time, in which matter has a density ρ that is constant over space and time."*

In the next part of the manuscript, Einstein suggests that this static model now seems unlikely:

> *"It has since transpired that this solution is almost certainly ruled out for the theoretical comprehension of space as it really is"*

We note that Einstein dismisses his static model for two separate reasons. First, he comments on the existence of dynamic solutions and that his static solution was found to be unstable:

> *"On the one hand, it follows from investigations based on the same equations by - and by Tolman that there also exist spherical solutions with a*

[3] We suggest this section be read in conjunction with our translation of the manuscript in the Appendix

> *world radius P that is variable over time, and that my solution is not stable with respect to variations of P over time."*

The blank in the sentence above representing theoreticians other than Tolman who suggested dynamic solutions is puzzling as Einstein was unquestionably aware of the cosmological models of both Friedman and Lemaître.[4] Einstein also neglects to make it clear from what research it follows that his static solution is unstable, although this is very likely a reference to Eddington's paper on the subject (Eddington 1930). These points are discussed further in section 3.

Einstein's second reason for ruling out his former static solution concerned the recent astronomical observations of Edwin Hubble:

> *"On the other hand, Hubbel's* [sic] *exceedingly important investigations have shown that the extragalactic nebulae have the following two properties:*
>
> 1) *Within the bounds of observational accuracy they are uniformly distributed in space*
> 2) *They possess a Doppler effect proportional to their distance*"

We note that Hubble's name is misspelt throughout the manuscript, as in the case of Einstein's *Sitzungsberichte* paper of 1931 (1931b). This may indicate that Einstein was not fully familiar with Hubble's work, as has been argued for the latter case (Nussbaumer and Bieri 1996, Nussbaumer 2014). We also note that Einstein uses the term 'Doppler effect' rather than radial velocity, suggesting a qualified acceptance of Hubble's observations as evidence for a cosmic expansion.[5] Remarking that the dynamic models of de Sitter and Tolman are consistent with Hubble's observations, Einstein then articulates a problem associated with their models:

> *"De Sitter and Tolman have already shown that there are solutions to equation (1) that can account for these observations. However the difficulty arose that the theory unvaryingly led to a beginning in time about 10^{10}-10^{11} years ago, which for various reasons seemed unacceptable."*

We note that there is again no reference to the evolving models of Friedman or Lemaître (Friedman 1922, 1924; Lemaître 1927). The *"various reasons"* is almost certainly a reference to the fact that the estimated timespan of dynamic models was not larger than the ages of

[4] Einstein's reaction to each is well-known as discussed in (Nussbaumer 2014).
[5] Two of us have argued elsewhere that this caution is typical of Einstein's approach to cosmology in these years (O'Raifeartaigh and McCann 2014).

stars as estimated from astrophysics or the age of the earth as estimated from radioactivity.[6] However, we note that the sentence is a little ambiguous; it is possible that Einstein's 'difficulty' also refers to the notion of a beginning for the universe. Indeed, it is quite curious that the problem of the singularity is not specifically cited as a motivation for the model of this manuscript, as discussed in section 6.

In the third part of the manuscript, Einstein explores an alternative solution to the field equations that could also be compatible with Hubble's observations – namely, an expanding universe in which the density of matter does not change over time:

> *"In what follows, I would like to draw attention to a solution to equation (1) that can account for Hubbel's* [sic] *facts, and in which the density is constant over time. While this solution is included in Tolman's general scheme, it does not appear to have been taken into consideration thus far."*

The reference to "Tolman's general scheme'' is significant as the analysis to follow bears some similarities to a paper by Tolman in which the cosmic expansion is suggested to arise from a continuous transformation of matter into radiation (Tolman 1930).

Einstein starts his analysis by choosing the metric of flat space expanding exponentially:

> *"I let*
>
> $$ds^2 = -e^{\alpha t}(dx_1^2 + dx_2^2 + dx_3^2) + c^2 dt^2 \quad .... \qquad (2)$$
>
> *This manifold is spatially Euclidean. Measured by this criterion, the distance between two points increase over time as $e^{\frac{\alpha}{2}t}$; one can thus account for Hubbel's Doppler effect by giving the masses (thought of as uniformly distributed) constant co-ordinates over time."*

To modern eyes, equation (2) represents the metric of the de Sitter universe.[7] A similar line element was employed by Tolman in the paper mentioned above (Tolman 1930) and Einstein's choice of metric may owe something to the latter work. However, it should be noted that the hypothesis of a constant rate of matter creation in any case implies a metric that is spatially flat and exponentially expanding.[8] Einstein also notes that the metric is invariant:

[6] Einstein's view of the timescale problem is spelt out in detail in his later review of dynamic models (Einstein 1945).

[7] This was first shown by Robertson in 1928 (Robertson 1928).

[8] A constant rate of matter creation implies spatial flatness because the creation rate is affected by spatial curvature (k/R^2) and the radius is not constant. A constant Hubble parameter $\dot{R}/R$ is also implied, from which it

"Finally, the metric of this manifold is constant over time. For it is transformed by applying the substitution

$$t' = t - \tau \quad (\tau = const)$$

$$\frac{x_1'}{x_1} = \frac{x_2'}{x_2} = \frac{x_3'}{x_3} = e^{-\frac{\alpha}{2}\tau}$$

$$ds^2 = e^{\alpha t'}(dx_1'^2 + dx_2'^2 + dx_3'^2) + c^2 dt'^2$$

We note the apparent sign error in the last equation above, an error that may have led to a miscalculation in the analysis described below.

Assuming a low velocity of masses relative to the co-ordinate system and that the gravitational effects of the radiation pressure are negligible, Einstein constructs the matter-energy tensor in a manner analogous to his seminal paper of 1917 (Einstein 1917):

"We ignore the velocities of the masses relative to the co-ordinate system as well as the gravitational effect of the radiation pressure. The matter tensor is then to be expressed in the form

$$T^{ik} = \rho u^i u^k \quad (u^i = \frac{dx^i}{ds})$$

or

$$T_{ik} = \rho u^\sigma u^\tau g_{\sigma i} g_{\tau k} \qquad (3)$$

where $u^1 = u^2 = u^3 = 0;\ u^4 = \frac{1}{c}$ *"*

From equations (1) - (3), Einstein derives two simultaneous equations and, eliminating the cosmological constant, deduces a relation between the expansion coefficient α and the matter density ρ:

" Equations (1) yield:

$$\frac{\mathbf{-3}(\not{9})}{4}\alpha^2 + \lambda c^2 = 0$$

$$\frac{3}{4}\alpha^2 - \lambda c^2 = \kappa\rho c^2$$

or $$\alpha^2 = \frac{\kappa c^2}{3}\rho \qquad (4)$$ *"*

We note an inconsistency regarding the coefficient of α^2 in the first of the simultaneous equations above. While a value of +9/4 is implied by the equations that follow, it appears to have been amended to **-3**/4 (see figure 2). The correction leads to the null result $\rho = 0$ instead of equation (4), exposing a fundamental flaw in the model as discussed in section 4.

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follows that the expansion must be exponential. It is very possible that Einstein realised this independently of Tolman.

In the original draft of the model, Einstein concludes from equation (4) that the density of matter remains constant and is related to the cosmic expansion:

> *"The density is therefore constant and determines the expansion apart from its sign*."

Hence Einstein assumes that the expansion is determined by the creation of matter, while Tolman suggested that it was driven by a transformation of matter into radiation (Tolman 1930). However, Einstein does not attempt to use Hubble's measurements to quantify the rate of expansion, unlike Tolman.

In the final part of the manuscript, Einstein proposes a mechanism to allow the density of matter remain constant in a universe of expanding radius - namely, the continuous formation of matter from empty space:

> *"If one considers a physically bounded volume, particles of matter will be continually leaving it. For the density to remain constant, new particles of matter must be continually formed in the volume from space*."

This proposal anticipates the 'creation field' or 'C-field' of Fred Hoyle in some ways (see section 5 below). However, Einstein has not introduced a term representing this process into the field equations (1). Instead he associates the continuous formation of matter with the cosmological constant, commenting that the latter ensures that space is not empty of energy:

> "*The conservation law is preserved since by setting the λ-term, space itself is not empty of energy; its validity is well-known to be guaranteed by equations (1)."*

Thus, in this model of the cosmos, Einstein proposes that the cosmological constant assigns an energy to empty space that is associated with the creation of matter. However, the proposal is fundamentally flawed because the lack of a specific term representing matter creation in fact leads to the null solution $\rho = 0$. We suggest that Einstein recognized this problem on revision and set the model aside rather than pursue more contrived steady-state solutions, as discussed in section 6.

3. Historical remarks: dating the manuscript

The manuscript under discussion has been assigned the year 1931 at the Albert Einstein Archive. However, this dating is no longer certain as the manuscript was mistaken for a draft of a different paper until now (see introduction).

It is instructive to attempt to date the manuscript from its contents. The statement*"…Hubbel's exceedingly important investigations have shown that the extra-galactic nebulae… possess a Doppler effect proportional to their distance"* (see section 2) gives confidence that it was written after Hubble's seminal publication of 1929 (Hubble 1929).[9] Indeed, it is generally thought that Einstein's interest in cosmology was rekindled by Hubble's seminal observations of the recession of the galaxies, and by his three-month stay in the United States from December 1930 to March 1931 (Nussbaumer and Bieri 2009, chapter 14; Bartusiak 2009, chapter 16; Eisinger 2012, chapter 5) . Much of this trip was spent at Caltech, and included a meeting with Edwin Hubble and other astronomers at the Mount Wilson Observatory.[10] Press reports of seminars given by Einstein at Caltech certainly suggest that he viewed Hubble's observations as likely evidence for an expanding universe. For example, *The New York Times* reported Einstein as commenting that *"New observations by Hubble and Humason concerning the redshift of light in distant nebulae make the presumptions near that the general structure of the universe is not static*" (AP 1931a) and "*The redshift of the distant nebulae have smashed my old construction like a hammer blow"*(AP 1931b).

Einstein had many interactions with the theoretician Richard Tolman at Caltech and greatly admired Tolman's work on relativity.[11] Thus, Einstein may have been influenced by Tolman's cosmology and we have already noted in section 2 that the manuscript under discussion bears some similarities to Tolman's 'annihilation' model of 1930 (Tolman 1930). Finally, Einstein's manuscript is written on American paper, making it highly improbable that it was written before his arrival in the United States in January 1931.[12]

As regards an upper bound for the date of the manuscript, it is different in both style and content to the cosmic models published by Einstein in April 1931 (Einstein 1931b)[13] and 1932 (Einstein and deSitter 1932). In the latter papers (known as the Friedman-Einstein and the Einstein-deSitter models respectively), Einstein assumes that the mean density of matter decreases with increasing cosmic radius and removes the cosmological constant term from the field equations, pointing out that it was only introduced to keep the universe static.

[9]It is very unlikely that Einstein knew of Hubble's observations before this date, see (Nussbaumer 2014).
[10] See (Bartusiak 2009, p251-256; Eisinger 2012, p 109-115),
[11]See (Nussbaumer and Bieri 2014; Eisinger 2012, p114).
[12] We thank Barbara Wolff of the Albert Einstein Archives for confirming this point
[13]It is known that this model was written in early April 1931 (see Eisinger 2012) chap 7.

Einstein also employs the analysis of Alexander Friedman to quantify the models of these papers, i.e., uses the Friedman differential equations in conjunction with Hubble's observations to determine values for the radius and matter density of the universe.[14] By contrast, the cosmological constant is not removed from the field equations in the manuscript under discussion, but assigned a new role; there is no reference to Friedman's analysis, no attempt to quantify the model and no reference to Einstein's published models of 1931 and 1932. Thus, it seems very likely that this manuscript precedes the Friedman-Einstein and Einstein-deSitter models, i.e., was written sometime in early 1931, and represents Einstein's first attempt at a cosmic model in the wake of astronomical evidence for an expansion on the largest scales.

4. Why was Einstein's steady-state model not published?

We note first that there is no mention of steady-state solutions in Einstein's later discussions of cosmic models (Einstein 1931b, Einstein and de Sitter 1932, Einstein 1945), nor have we been able to find a reference to the manuscript under discussion in Einstein's letters, diaries or other personal papers. This silence indicates that he decided against the model, rather than simply mislaid the manuscript during his travels or neglected to publish it.

We suggest that the model was discarded because it contains a fundamental flaw. In particular, we have been unable to reproduce Einstein's derivation of equation (4) from equations (1) - (3). Although a value of 9/4 is implied for the coefficient of α^2 in the first of the simultaneous equations by the two equations after it, we find a value of -3/4 for this coefficient on deriving the equations from first principles. The latter value renders the left-hand sides of the simultaneous equations identical, giving the null solution $\rho = 0$ instead of equation (4). Close scrutiny of figure 2 suggests that the co-efficient of α^2 in the first equation has indeed been amended to **-3**/4; it seems that Einstein discovered the error at a later point, realised the model led to a trivial solution and set the work aside without correcting equation (4).[15]

With modern eyes, it is easy to see why Einstein's steady-state model leads to a null solution; the fundamental problem is that he has not included a specific term in the analysis

[14]Two of us have analyzed the paper of 1931in some detail (O'Raifeartaigh and McCann 2014).

[15] Deriving the equations from first principles, we find that that Einstein's error probably arose from a sign error in his calculation of the Christoffel coefficients, an error that may be a further indication that the manuscript represents Einstein's first attempt at a relativistic model of the cosmos in many years.

representing the hypothesised creation of matter. This leads one to ask why Einstein did not attempt a more sophisticated steady-state solution by introducing a 'matter-creation' term into the field equations. We suggest that Einstein may have decided that such an approach was rather contrived in comparison with evolving models, as discussed in section 6 below.

5. On steady-state models of the cosmos

We note first that steady-state models of the universe were considered many times in 20th century cosmology. In 1918, the American physicist William MacMillan proposed a continuous creation of matter from radiation in order to avoid a gradual 'running down' of the universe due to the conversion of matter into energy in stellar processes (MacMillan 1918, 1925). MacMillan's proposal was enthusiastically received by Robert Millikan, who suggested that the process might be the origin of cosmic rays (Millikan 1928). The idea of a continuous creation of matter from radiation was also considered by Richard Tolman as a means of introducing matter into the empty de Sitter universe, although he saw the idea as rather improbable (Tolman 1929).

Other physicists considered the possibility of a continuous creation of matter from empty space. In 1928, James Jeans speculated that matter was continuously created in the centre of the spiral nebulae: *"The type of conjecture which presents itself, somewhat insistently,is that the centres of the nebulae are of the nature of "singular points", at which matter is poured into our universe from some other spatial dimension….so that they appear as points at which matter is poured into our universe from some other, and entirely extraneous spatial dimension, so that, to a denizen of our universe, they appear as points at which matter is continually created"* (Jeans 1928, p 360). Similar ideas of continuous creation were explored by the Swedish scientist Svante Arrhenius and the German chemist Walther Nernst (Arrhenius 1909; Nernst 1928).[16] However, these theories did not concern the creation of matter in an expanding universe.

The concept of an expanding universe that remains in a steady-state due to a continuous creation of matter is most strongly associated with the Cambridge physicists Fred Hoyle, Hermann Bondi and Thomas Gold. In the late 1940s, these physicists became concerned with well-known problems associated with evolving models of the expanding cosmos. In particular, they noted that the evolving models predicted a cosmic age that was problematic, and disliked Lemaître's idea of a universe with an explosive beginning

[16] See (Kragh 1996) p143-162 for a review of steady-state cosmologies in the early 20th century.

(Lemaître 1931).[17] In order to circumvent these, and other problems,[18] the trio explored the idea of an expanding universe that does not evolve over time, i.e., a cosmos in which the mean density of matter is maintained constant by a continuous creation of matter from the vacuum (Hoyle 1948; Bondi and Gold 1948). [19]

In the case of Bondi and Gold, the proposal of a steady-state model followed from their belief in the 'perfect cosmological principle', a principle that stated that the universe should appear the same to all observers at all times. This principle led to the postulate of a continuous creation of matter in order to sustain an unchanging universe. While the idea bears some similarity to Einstein's steady-state model, it is difficult to compare the models directly because the Bondi-Gold theory was not formulated in the context of general relativity.[20]

On the other hand, Fred Hoyle constructed a steady-state model of the cosmos by means of a daring modification of the Einstein field equations (Hoyle 1948, Mitton 2011, chapter 5). Replacing Einstein's cosmological constant with a new 'creation-field' term C_{ik}, representing the continuous formation of matter from the vacuum, Hoyle obtained the equation

$$\left(R_{ik} - \frac{1}{2} g_{ik} R\right) - C_{ik} = \kappa T_{ik} \qquad (5)$$

The creation-field term allowed for an unchanging universe but was of importance only on the largest scales, in the same manner as the cosmological constant. (Note that the perfect cosmological principle followed as a consequence of Hoyle's model, rather than a starting assumption; we also note that Hoyle also demonstrated that his model necessitated a line element of de Sitter form).

Hoyle's steady-state model violated the conservation of energy. However, the theory was more quantitative than the Bondi-Gold model and he extracted estimates of 1.8 x 10^{27} cm and 5x10^{-28} g/cm^3 for the radius and mean matter density of the universe respectively; these values were very similar to the predictions of evolving models (Eddington 1930, Einstein and

[17] That Einstein did not consider the problem of cosmic origins in this manuscript is another indication that it was written before 1932 (see Eddington 1931 for example).

[18] Hoyle was also unconvinced by Gamow's postulate of nucelosynthesis in the infant universe and concerned about the problem of the formation of galaxies in an expanding universe (Hoyle 1948).

[19] That the problem of cosmic origins is not a motivation for in Einstein's steady –state model is another indication that the manuscript was written before 1932, i.e. before the debate of Eddington and Lemaître's on the subject (Eddington 1931; Lemaître 1931)

[20] Bondi and Gold took the view that it was not known whether it was appropriate to apply general relativity to the cosmos on the largest scales (see Bondi 1952, p146).

de Sitter 1932). In the years that followed, Hoyle's steady-state model became the dominant steady-state theory because it could be quantified and refined within the context of the general theory of relativity.[21]

It could be argued that Einstein's steady-state theory anticipates that of Hoyle in some aspects; however, they are crucially different because of the lack of a specific term in the field equations in Einstein's model to enable the continuous formation of matter. Another possibility for Einstein would have been to include a term representing the continuous formation of matter within the matter-energy tensor of the field equations, as suggested by William McCrea in a later steady-state theory (McCrea 1951). Instead, Einstein set his steady-state model to one side, never to return.

As is well known, a significant debate was waged between steady-state and evolving models of the cosmos during the 1950s and 1960s (Kragh 1996, chapter 5; Mitton 2011, chapter 7). Eventually, steady-state models were ruled out by astronomical observations[22] that showed unequivocally that we inhabit a universe that is evolving over time.[23] We note that there is no evidence to suggest that any of the steady-state theorists were aware of the manuscript under discussion; indeed, it is likely that they would have been greatly intrigued to learn that Einstein had once attempted a steady-state model.

6. On Einstein's philosophy of cosmology

It should come as no great surprise that when confronted with empirical evidence for an expanding universe, Einstein once considered a stationary or steady-state model of the expanding cosmos. There is a great deal of evidence that Einstein's philosophical preference was for an unchanging universe, from his introduction of the cosmological constant to the field equations in 1917 to keep the universe static[24] to his well-known hostility to the dynamic models of Friedman and Lemaître when they were first suggested.[25] Indeed, a model

[21] One problem was that the model became very complicated when density perturbations were introduced.
[22] The principle observations were the discovery that the distribution of galaxies was significantly different in the distant past, and the discovery of the cosmic microwave background. See (Kragh 1996) chaper 7 for a review.
[23] Alternative versions of steady-state models were suggested, but failed to convince the community (Kragh 1996) chapter 7.
[24]It could be argued that this choice was as much philosophical as empirical because there was no guarantee that an expansion on the largest scales would be detectable by astronomy.
[25] An account of Einstein's reaction to the models of Friedman and Lemaitre can be found in chapters 7 and 9 respectively of (Nussbaumer and Bieri 2009).

of an expanding cosmos in which the density of matter remains unchanged seems a natural successor to Einstein's static model of 1917, at least from a philosophical point of view.

However, such a steady-state theory requires the assumption of a continuous creation of matter and, as Einstein discovered in this manuscript, a successful model of the latter process was not possible without some alteration to the field equations. On the other hand, an expanding universe of varying matter density could be described without any such assumption – and indeed without the cosmological constant,[26] as Einstein suggested in the Friedman-Einstein and Einstein-deSitter models (Einstein 1931b; Einstein and de Sitter 1932). Thus it seems very probable that Einstein decided against steady-state solutions because they were more contrived than evolving models of the cosmos. This suggestion fits very well with our view of Einstein's pragmatic approach to cosmology in these years.[27]

It is also possible that Einstein decided against steady-state models on empirical grounds, i.e., on the grounds that there was no observational evidence to support the postulate of a continuous formation of matter from empty space. It is interesting that, when asked to comment on Hoyle's steady-state model many years later, Einstein is reported to have dismissed the theory as "*romantic speculation*" (Michelmore 1962, p253). This criticism is confirmed in a letter written by Einstein to the physicist Jean Jacques Fehr in 1952. Einstein seems highly sceptical of Hoyle's model, and in particular of the postulate of a continuous creation of matter from the vacuum: "*Die kosmologischen Spekulationen von Herrn Hoyle, welche eine Entstehung von Atomen aus dem Raum voraussetzen, sind nach meiner Ansicht viel zu wenig begründet, um ernst genommen zu werden"* or *"The cosmological speculations of Mr Hoyle, which presume the creation of atoms from empty space, are in my view much too poorly grounded to be taken seriously"* (Einstein 1952).[28]

As pointed out in section 5, steady-state models of the cosmos were eventually ruled out by astronomical observation. Nevertheless, the model of this manuscript is of some interest in the study of 20th century cosmology. In the first instance, it is significant that Einstein retained the cosmological constant in at least one cosmic model he proposed *after*

[26] As Einstein later pointed out, the cosmological constant was introduced solely to render the equations of relativity consistent with a static universe, and was a source of some dissatisfaction to him from a theoretical standpoint (Einstein 1945, p115).

[27] Two of us have argued that Einstein's removal of the cosmological constant in 1931, followed by his removal of spatial curvature in 1932, suggests an Occam's razor approach to cosmology (O'Raifeartaigh and McCann, 2014).

[28] It could be argued that this particular criticism was is a little harsh since the rate of matter creation required for a steady-state universe was far below detectable levels (Hoyle 1948).

Hubble's observations; it seems that the widely held view that Einstein was happy to banish the cosmological constant at the first sign of evidence for a non-static universe is not entirely accurate.[29] It now appears that Einstein's attraction to an unchanging universe at first outweighed his dislike of the cosmological constant, just as it did in 1917 - he simply found a new role for the term. Second, while it could be argued that Einstein's steady-state model was trivial because it didn't work, it is interesting that this was not evident to Einstein on first approach; it seems the de Sitter model remained a source of some confusion at this point. It is even more interesting that, when the model failed, Einstein declined to attempt a more sophisticated version by amending the field equations. Thirdly, Einstein's model reminds us that today's view of an evolving cosmos did not occur as a sudden 'paradigm shift' in the wake of Hubble's observations. Instead, physicists explored a plethora of diverse cosmic models for many years, from the possibility of an expansion caused by a continuous annihilation of matter (Tolman 1930) to one caused by condensation processes (Eddington 1930), from the conjecture that the redshifts of the nebulae represented a loss of energy by photons (Zwicky 1929) to the hypothesis of a steady-state universe. Finally, Einstein's postulate that the continuous formation of matter does not violate the conservation of energy because " *by setting the λ-term, space itself is not empty of energy"* is highly relevant to today's discussions of dark energy. In addition, the metric used of an exponentially expanding flat space anticipates many modern models of cosmic inflation. These points remind us of the relevance of past models of the universe for cosmology today.

7. Conclusions

The manuscript "*Zum kosmologischen Problem*" discussed in this paper was never formally published in the literature but is of historical interest because it proposes a model of the cosmos distinct from Einstein's static model of 1917 (Einstein 1917) or his dynamic models of 1931 and 1932 (Einstein 1931, Einstein and de Sitter 1932). In the manuscript, Einstein explores a steady-state universe of expanding radius, non-zero cosmological constant and constant matter density, where the latter is maintained by the continuous formation of matter from empty space; this model anticipates the well-known 'steady-state' cosmologies of the late 1940s and 1950s.

[29]See for example (Kragh1999) p34, (Nussbaumer and Bieri 2009) p147, (Nussbaumer 2014).

The manuscript was almost certainly written after 1929 and very likely before Einstein's published cosmic models of 1931 and 1932. The analysis was probably discarded because of a fundamental flaw in the specific model proposed, and we suggest that Einstein declined to try again because he came to the view that steady-state models of the cosmos were rather contrived and not realistic. The manuscript is nevertheless of great interest in the study of 20th century cosmology because it suggests that Einstein debated between steady-state and evolving models of the universe decades before a similar debate took place in the wider cosmological community.

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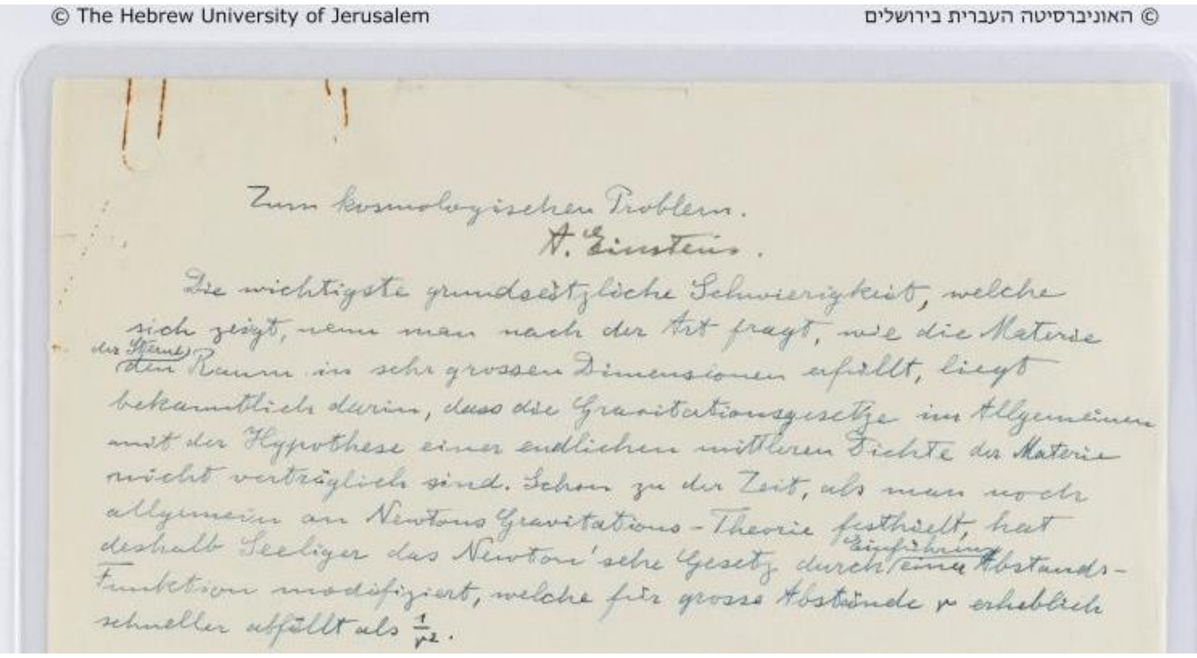
Figure 1

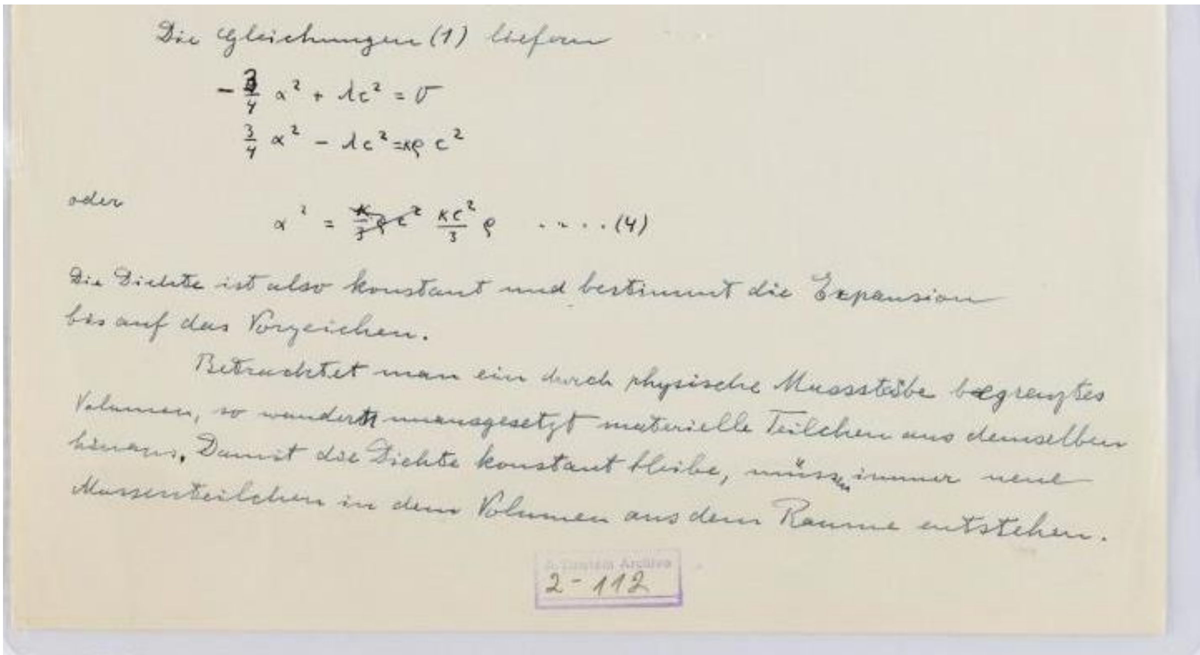


Zum kosmologischen Problem.
A. Einstein.

Die wichtigste grundsätzliche Schwierigkeit, welche sich zeigt, wenn man nach der Art fragt, wie die Materie der Sterne den Raum in sehr grossen Dimensionen erfüllt, liegt bekanntlich darin, dass die Gravitationsgesetze im Allgemeinen mit der Hypothese einer endlichen mittleren Dichte der Materie nicht verträglich sind. Schon zu der Zeit, als man noch allgemein an Newtons Gravitations-Theorie festhielt, hat deshalb Seeliger das Newton'sche Gesetz durch Einführung einer Abstands-Funktion modifiziert, welche für grosse Abstände r erheblich schneller abfällt als $\frac{1}{r^2}$.

Title and opening paragraph of manuscript [2-112], reproduced from the Albert Einstein Archive by kind permission of the Hebrew University of Jerusalem

Figure 2



Die Gleichungen (1) liefern

$$-\frac{3}{4}\alpha^2 + \lambda c^2 = 0$$
$$\frac{3}{4}\alpha^2 - \lambda c^2 = \kappa\rho c^2$$

oder

$$\alpha^2 = \frac{\kappa c^2}{3}\rho \quad \ldots\ldots (4)$$

Die Dichte ist also konstant und bestimmt die Expansion bis auf das Vorzeichen.

Betrachtet man ein durch physische Massstäbe begrenztes Volumen, so wandern unausgesetzt materielle Teilchen aus demselben heraus. Damit die Dichte konstant bleibe, müssen immer neue Massenteilchen in dem Volumen aus dem Raume entstehen.

Excerpt from the third page of manuscript [2-112], reproduced from the Albert Einstein Archive by kind permission of the Hebrew University of Jerusalem. In the first of the simultaneous equations, the first term has been corrected to **-3**/4 α^2 in different ink. The sentence following equation (4) states *"Die Dichte ist also konstant und bestimmt die Expansion bis auf das Vorzeichen*" or "*The density is therefore constant and determines the expansion apart from its sign"*

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Appendix Translation and transcription of Einstein's manuscript[30]

On the cosmological problem

A. Einstein

It is well known that the most important fundamental difficulty that emerges when one asks how the stellar matter fills up space in very large dimensions is that the laws of gravity are not in general consistent with the hypothesis of a finite mean density of matter. Thus, at a time when Newton's theory of gravity was still generally accepted, Seelinger had already modified the Newtonian law by the introduction of a distance function that, for large distances r, diminished considerably faster than $1/r^2$.

This difficulty also arises in the general theory of relativity. However, I have shown in the past that this can be overcome by the introduction of the so-called "λ–term" to the field equations. The field equations can then be written in the form

$$\left(R_{ik} - \frac{1}{2} g_{ik} R\right) - \lambda g_{ik} = \kappa T_{ik} \qquad \ldots.(1)$$

$$R_{ik} = \Gamma^{\sigma}_{ik,\sigma} - \Gamma^{\sigma}_{i\sigma,k} - \Gamma^{\sigma}_{i\tau}\Gamma^{\tau}_{\tau\sigma} + \Gamma^{\sigma}_{ik}\Gamma^{\sigma}_{\sigma\tau} \qquad \ldots.1(a)$$

At that time, I showed that these equations can be satisfied by a spherical space of constant radius over time, in which matter has a density ρ that is constant over space and time.

It has since transpired that this solution is almost certainly ruled out for the theoretical comprehension of space as it really is.

On the one hand, it follows from investigations based on the same equations by – and by Tolman that there also exist spherical solutions with a world radius P that is variable over time, and that my solution is not stable with respect to variations of P over time. On the other hand, Hubbel's [sic] *exceedingly important investigations have shown that the extragalactic nebulae have the following two properties:*

1) *Within the bounds of observational accuracy they are uniformly distributed*
2) *They possess a Doppler effect proportional to their distance.*

De Sitter and Tolman have already shown that there are solutions to equation (1) that can account for these observations. However the difficulty arose that the theory unvaryingly led to a beginning in time about 10^{10} *-*10^{11} *years ago, which for various reasons seemed unacceptable.*

[30] Translated and transcribed by C.O'Raifeartaigh and B.McCann by kind permission of the Albert Einstein Archives of the Hebrew University of Jerusalem.

In what follows, I wish to draw attention to a solution to equation (1) that can account for Hubbel's [sic] facts, and in which the density is constant over time. While this solution is included in Tolman's general scheme, it does not appear to have been taken into consideration thus far.

I let

$$ds^2 = -e^{\alpha t}(dx_1^2 + dx_2^2 + dx_3^2) + c^2dt^2 \ldots. \qquad (2)$$

This manifold is spatially Euclidean. Measured by this criterion, the distance between two points increases over time as $e^{\frac{\alpha}{2}t}$ *; one can thus account for Hubbel's Doppler effect by giving the masses (thought of as uniformly distributed) constant co-ordinates over time. Finally, the metric of this manifold is constant over time. For it is transformed by applying the substitution*

$$t' = t - \tau \quad (\tau = const)$$

$$\frac{x_1'}{x_1} = \frac{x_2'}{x_2} = \frac{x_3'}{x_3} = e^{-\frac{\alpha}{2}\tau}$$

$$ds^2 = e^{\alpha t'}(dx_1'^2 + dx_2'^2 + dx_3'^2) + c^2dt'^2$$

We ignore the velocities of the masses relative to the co-ordinate system as well as the gravitational effect of the radiation pressure. The matter tensor is then to be expressed in the form

$$T^{ik} = \rho u^i u^k \quad (u^i = \frac{dx^i}{ds})$$

or

$$T_{ik} = \rho u^\sigma u^\tau g_{\sigma i}\, g_{\tau k} \;\ldots.(3)$$

where $u^1 = u^2 = u^3 = 0;\ u^4 = \frac{1}{c}$

Equations (1) yield

$$\frac{-\mathbf{3}[?]}{4}\alpha^2 + \lambda c^2 = 0$$ [31]

$$\frac{3}{4}\alpha^2 - \lambda c^2 = \kappa\rho c^2$$

$$\alpha^2 = \frac{\kappa c^2}{3}\rho \qquad \ldots\ldots \qquad (4)$$

The density is therefore constant and determines the expansion apart from its sign.

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[31] A value of + 9/4 is implied for the coefficient of the α^2 term in the first equation by the two equations that follow. It appears to have been overwritten as **-3**/4, a correction that leads to a null solution instead of equation (4), as discussed in section 4.

If one considers a physically bounded volume, particles of matter will be continually leaving it. For the density to remain constant, new particles of matter must be continually formed in that volume from space. The conservation law is preserved since by setting the λ-term, space itself is not empty of energy; its validity is well-known to be guaranteed by equations (1).