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Production of negative parity baryons in the holographic Sakai-Sugimoto model

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Abstract

We extend our investigation of resonance production in the Sakai-Sugimoto model to the case of negative parity baryon resonances. Using holographic techniques we extract the generalized Dirac and Pauli baryon form factors as well as the helicity amplitudes for these baryonic states. Identifying the first negative parity resonance with the experimentally observed $S_{11}(1535)$, we find reasonable agreement with experimental data from the JLab-CLAS collaboration. We also estimate the contribution of negative parity baryons to the proton structure functions.

Contents

1. Introduction

In the past decade, gauge/gravity dualities inspired by the original Maldacena conjecture [1] have been successfully applied to a wide range of problems in Quantum Chromodynamics (QCD) as well as condensed matter theory. For QCD, two classes of models are of particular interest: The phenomenological bottom-up approach based on five-dimensional effective actions, and the more stringent top-down models based on compactifications of ten-dimensional string theories[1]. A prominent and widely studied model of the latter class clearly is the Sakai-Sugimoto model [10,11]. Its popularity can be attributed to its close resemblance to large-N_c QCD, its computational simplicity and the fact that it provides a geometric model that allows to study both the confinement/deconfinement transition (at finite temperature) and chiral symmetry breaking in the same unified framework.

Holographic techniques have also emerged as a very fruitful complementary tool for studying hadronic scattering in QCD where non-perturbative effects become important, namely in the regime of low momentum transfer ($\sqrt{q^2}$ lower than a few GeVs). In this paper we investigate the production of negative parity baryon resonances in proton electromagnetic scattering within the framework of the Sakai-Sugimoto model. This is a natural continuation of our previous work on positive parity baryonic resonances [12]. Production of baryonic resonances is a very important and timely problem in hadronic physics for the following reasons: i) Many baryonic resonances are excited nucleon states (N^*) and their structure is relevant to understand the physics of quark confinement, ii) there is a huge experimental effort at JLab [13] to extract the electromagnetic form factors and helicity amplitudes of baryonic resonances in the regime where non-perturbative effects are dominant and perturbative QCD predictions fail.

The paper is organized as follows: In section 2 we introduce the current matrix decomposition for resonance production in proton electromagnetic scattering and present our theoretical results for electromagnetic form factors and helicity amplitudes in the Breit frame. Moreover, we study the contributions of resonance production to the proton structure functions defined in Deep Inelastic Scattering (DIS). Here, we focus on the production of negative parity baryonic resonances and their contributions, but for completeness, we also provide a review of our previous results for positive parity baryon resonances. Section 3 contains a detailed computation of the electromagnetic currents in the holographic Sakai-Sugimoto model and a subsequent calculation of Dirac and Pauli form factors in the holographic setup. This is done in a unified manner for both positive and negative parity baryon resonances. In section 4 we present our numerical results for the generalized form factors, helicity amplitudes and proton structure functions for the special case of negative parity baryon resonances and compare them to experimental results. Section 5 offers some conclusions and an outlook. Appendix A reviews different frames utilized in this article while appendix B gives technical details on the limits relevant to the model at hand.

Previous holographic calculations on electromagnetic form factors of baryons can be found in [14,15,16,17,18,20]. DIS structure functions from holography were first obtained in [21]. Further developments include the large x regime [22,23,24,25] as well as the small x regime

[1]Recommended reviews are [2,3,4,5] for the bottom-up approach and [6,7,8,9] for the top-down approach.

[26,27,28,29,30,31]; DIS structure functions have also been calculated for strongly coupled plasmas [32,33,34,35].

2. Form factors and helicity amplitudes

2.1. Dirac and Pauli form factors

We want to describe the electromagnetic interaction of a spin 1/2 baryon in the case where, as a result of the interaction, a spin 1/2 baryonic resonance is produced. This baryonic transition is described by an electromagnetic current evaluated between the initial and final states. In our approach, we embed the electromagnetic current in a vectorial $U(2)$ symmetry present in any effective description of large-N_c QCD with chiral symmetry breaking. Then we define the electromagnetic current as a linear combination of flavour currents:

$$\mathcal{J}^\mu = \sum_a c_a J_V^{\mu,a} \qquad c_0 = 1/N_c\,,\; c_3 = 1\,,\; c_1 = c_2 = 0\,. \tag{2.1}$$

Now we evaluate the flavour currents $J_V^{\mu,a}$ between the initial and final baryonic states.

Positive parity resonances

When the final baryonic state has positive parity we can expand the current matrix element as

$$\begin{aligned}\langle p_X, B_X, s_X | J_V^{\mu,a}(0) | p, B, s\rangle &= \frac{i}{2(2\pi)^3}(\tau^a)_{I_3^X I_3}\left(\eta^{\mu\nu} - \frac{q^\mu q^\nu}{q^2}\right)\bar{u}(p_X, s_X)\Big[\gamma_\nu F^{D,a}_{BB_X}(q^2) \\ &+ \kappa_B \sigma_{\nu\lambda} q^\lambda F^{P,a}_{BB_X}(q^2)\Big] u(p,s)\,,\end{aligned} \tag{2.2}$$

where

$$\begin{aligned} q^\mu &= (p_X - p)^\mu \quad , \quad \kappa_B = \frac{1}{m_B + m_{B_X}}\,, \\ (\tau^0)_{I_3^X I_3} &= \delta_{I_3^X I_3} \quad , \quad (\tau^a)_{I_3^X I_3} = (\sigma^a)_{I_3^X I_3} \quad a = (1,2,3)\,,\end{aligned} \tag{2.3}$$

and σ^a are the Pauli matrices. Here we are using the metric $\eta^{\mu\nu} = \mathrm{diag}(-,+,+,+)$ and we adopt the following convention for spinors and gamma matrices:

$$\begin{aligned} u(p,s) &= \frac{1}{\sqrt{2E}}\begin{pmatrix} f\chi_s(\vec{p}) \\ \frac{\vec{p}\cdot\vec{\sigma}}{f}\chi_s(\vec{p})\end{pmatrix} \quad , \quad u(p_X, s_X) = \frac{1}{\sqrt{2E_X}}\begin{pmatrix} f_X \chi_{s_X}(\vec{p}_X) \\ \frac{\vec{p}_X\cdot\vec{\sigma}}{f_X}\chi_{s_X}(\vec{p}_X)\end{pmatrix}\,, \\ \gamma^0 &= -i\begin{pmatrix} 1 & 0 \\ 0 & -1\end{pmatrix} \quad , \quad \gamma^i = -i\begin{pmatrix} 0 & \sigma^i \\ -\sigma^i & 0\end{pmatrix}\,, \\ \sigma^{\mu\nu} &= \frac{i}{2}[\gamma^\mu, \gamma^\nu] \quad , \quad \gamma_5 = i\gamma^0\gamma^1\gamma^2\gamma^3 = \begin{pmatrix} 0 & 1 \\ 1 & 0\end{pmatrix}\,,\end{aligned} \tag{2.4}$$

where

$$f = \sqrt{E + m_B} \quad , \quad f_X = \sqrt{E_X + m_{B_X}}\,. \tag{2.5}$$

In this article we are interested in the case where $I_3^X = I_3 = 1/2$. In this case the baryonic states satisfy the relation [12] :

$$\langle p_X, B_X, S_X | \vec{p}, B, s \rangle \;=\; \delta^3(\vec{p}_X - \vec{p}) \delta_{s_X s} \delta_{B_X B} \,. \tag{2.6}$$

The spinors $\chi_s(\vec{p})$ are defined as the eigenstates of the helicity equation of the initial state:

$$\vec{p} \cdot \vec{\sigma} \chi_s(\vec{p}) = s |\vec{p}| \chi_s(\vec{p}), \quad s = (+, -) \,, \tag{2.7}$$

Similary, the spinors $\chi_{s_X}(\vec{p}_X)$ are defined by the helicity equation for the final state:

$$\vec{p}_X \cdot \vec{\sigma} \chi_{s_X}(\vec{p}_X) = s_X |\vec{p}_X| \chi_{s_X}(\vec{p}_X) \,. \tag{2.8}$$

Using the helicity equations we can obtain the following spinor relations:

$$\begin{aligned}
\bar{u}(p_X, s_X) \gamma^0 u(p, s) &= -\frac{i}{2\sqrt{E E_X}} \left(\frac{f}{f_X} \right) \left[f_X^2 + \frac{s_X s |\vec{p}_X| |\vec{p}|}{f^2} \right] \chi^\dagger_{s_X}(\vec{p}_X) \chi_s(\vec{p}) \,, \\
\bar{u}(p_X, s_X) \gamma^i u(p, s) &= -\frac{i}{2\sqrt{E E_X}} \left(\frac{f}{f_X} \right) \left[\frac{f_X^2}{f^2} s |\vec{p}| + s_X |\vec{p}_X| \right] \chi^\dagger_{s_X}(\vec{p}_X) \sigma^i \chi_s(\vec{p}), \\
\bar{u}(p_X, s_X) \sigma^{0i} q_i u(p, s) &= -\frac{i}{2\sqrt{E E_X}} \left(\frac{f}{f_X} \right) \left[\frac{f_X^2}{f^2} s |\vec{p}| - s_X |\vec{p}_X| \right] q_i \, \chi^\dagger_{s_X}(\vec{p}_X) \sigma^i \chi_s(\vec{p}) \,, \\
\bar{u}(p_X, s_X) \sigma^{i0} q_0 u(p, s) &= \frac{i q_0}{2\sqrt{E E_X}} \left(\frac{f}{f_X} \right) \left[\frac{f_X^2}{f^2} s |\vec{p}| - s_X |\vec{p}_X| \right] \chi^\dagger_{s_X}(\vec{p}_X) \sigma^i \chi_s(\vec{p}) \,, \\
\bar{u}(p_X, s_X) \sigma^{ij} q_j u(p, s) &= -\frac{1}{2\sqrt{E E_X}} \left(\frac{f}{f_X} \right) \left[f_X^2 - \frac{s_X s |\vec{p}_X| |\vec{p}|}{f^2} \right] \epsilon^{ijk} q_j \chi^\dagger_{s_X}(\vec{p}_X) \sigma_k \chi_s(\vec{p}) \,.
\end{aligned}$$

Finally, in order to get the standard relativistic normalizations

$$\bar{u}(p, s) u(p, s) = 2 m_B \quad , \quad \langle p_X, B_X, S_X | p, B, s \rangle = 2\sqrt{E E_X} (2\pi)^3 \delta^3(\vec{p}_X - \vec{p}) \delta_{s_X s} \,, \tag{2.9}$$

we need to transform the spinors and baryon states as

$$u(p, s) \to \frac{1}{\sqrt{2E}} u(p, s) \quad , \quad |p, B, s\rangle \to \frac{1}{\sqrt{2E} (2\pi)^{3/2}} |p, B, s\rangle \,. \tag{2.10}$$

Using (2.1), (2.2) and (2.10) we obtain for $I_3 = I_3^X = 1/2$,

$$\begin{aligned}
\langle p_X, B_X, s_X | \mathcal{J}^\mu(0) | p, B, s \rangle \;=\; & i \left(\eta^{\mu\nu} - \frac{q^\mu q^\nu}{q^2} \right) \bar{u}(p_X, s_X) \Big[\gamma_\nu F^D_{B B_X}(q^2) \\
& + \kappa_B \sigma_{\nu\lambda} q^\lambda F^P_{B B_X}(q^2) \Big] u(p, s) \,,
\end{aligned} \tag{2.11}$$

where

$$F^D_{B B_X}(q^2) = \frac{1}{2} \sum_a c_a F^{D,a}_{B B_X}(q^2) \quad , \quad F^P_{B B_X}(q^2) = \frac{1}{2} \sum_a c_a F^{P,a}_{B B_X}(q^2) \,, \tag{2.12}$$

are the generalized Dirac and Pauli form factors that describe the production of positive parity baryons.

Negative parity resonances

A good expansion for the flavor current matrix element in the case when the final baryonic state has negative parity is given by

$$\begin{aligned} {}_5\langle p_X, B_X, s_X | J_V^{\mu,a}(0) | p, B, s\rangle &= \frac{i}{2(2\pi)^3}(\tau^a)_{I_3^X I_3}\left(\eta^{\mu\nu} - \frac{q^\mu q^\nu}{q^2}\right)\bar{u}(p_X, s_X)\Big[\gamma_\nu \tilde{F}^{D,a}_{BB_X}(q^2) \\ &+ \kappa_B \sigma_{\nu\lambda} q^\lambda \tilde{F}^{P,a}_{BB_X}(q^2)\Big]\gamma_5 u(p,s)\,. \end{aligned} \tag{2.13}$$

Alternatively, if we want to associate the chirality matrix γ_5 with the final state (which is the non-trivial state) we can write the current matrix element as

$$\begin{aligned} {}_5\langle p_X, B_X, s_X | J_V^{\mu,a}(0) | p, B, s\rangle &= \frac{i}{2(2\pi)^3}(\tau^a)_{I_3^X I_3}\left(\eta^{\mu\nu} - \frac{q^\mu q^\nu}{q^2}\right)\bar{u}(p_X, s_X)\gamma_5 \\ &\times\Big[-\gamma_\nu \tilde{F}^{D,a}_{BB_X}(q^2) + \kappa_B \sigma_{\nu\lambda} q^\lambda \tilde{F}^{P,a}_{BB_X}(q^2)\Big]u(p,s)\,. \end{aligned} \tag{2.14}$$

Transforming the spinors and states as (2.10), we get for $I_3 = I_3^X = 1/2$,

$$\begin{aligned} {}_5\langle p_X, B_X, s_X | \mathcal{J}^\mu(0) | p, B, s\rangle &= i\left(\eta^{\mu\nu} - \frac{q^\mu q^\nu}{q^2}\right)\bar{u}(p_X, s_X)\Big[\gamma_\nu \tilde{F}^{D}_{BB_X}(q^2) \\ &+\kappa_B \sigma_{\nu\lambda} q^\lambda \tilde{F}^{P}_{BB_X}(q^2)\Big]\gamma_5 u(p,s)\,, \end{aligned} \tag{2.15}$$

where

$$\tilde{F}^D_{BB_X}(q^2) = \frac{1}{2}\sum_a c_a \tilde{F}^{D,a}_{BB_X}(q^2) \quad , \quad \tilde{F}^P_{BB_X}(q^2) = \frac{1}{2}\sum_a c_a \tilde{F}^{P,a}_{BB_X}(q^2)\,, \tag{2.16}$$

are the generalized Dirac and Pauli form factors that describe the production of negative parity baryons.

2.2. The Breit frame

It is usually convenient to work in the Breit frame where

$$p^\mu = (E,0,0,p) \quad , \quad q^\mu = (0,0,0,-2xp) \quad , \quad p_X^\mu = (E,0,0,p(1-2x))\,. \tag{2.17}$$

The details of this frame are given in the Appendix. In the Breit frame, we obtain the following helicity equation:

$$\begin{aligned} \vec{p}_X \cdot \vec{\sigma}\chi_{s_X}(\vec{p}) &= (1-2x)p\sigma^3\chi_{s_X}(\vec{p}) \\ &= s_X(1-2x)|\vec{p}|\chi_{s_X}(\vec{p})\,, \end{aligned} \tag{2.18}$$

Using the relation $|\vec{p}_X| = |1-2x||\vec{p}|$ and (2.8), we identify two situations:

$$\text{If } 1-2x > 0 \;\;\rightarrow\;\; \chi_{s_X}(\vec{p}_X) = \chi_{s_X}(\vec{p})\,,$$

$$\text{If } 1-2x<0 \quad \rightarrow \quad \chi_{s_X}(\vec{p}_X)=\chi_{-s_X}(\vec{p})\,. \tag{2.19}$$

Positive parity resonances

Using the spinor relations of the previous subsection we can calculate the current matrix elements in the Breit frame. For the time component, we get

$$\begin{aligned}
&\langle p_X, B_X, s_X|J_V^{0,a}(0)|p,B,s\rangle \\
&= \frac{1}{2(2\pi)^3}(\tau^a)_{I_3^X I_3}\left(\frac{1}{2E}\right)\left(\frac{f}{f_X}\right)\left\{\left[f_X^2+s_X s|1-2x|\frac{|\vec{p}|^2}{f^2}\right]F^{D,a}_{BB_X}(q^2)\right. \\
&\left.-\left(\frac{1}{2x}\right)\left[\frac{f_X^2}{f^2}-s_X s|1-2x|\right]q^2\kappa_B F^{P,a}_{BB_X}(q^2)\right\}\chi^\dagger_{s_X}(\vec{p}_X)\chi_s(\vec{p})\,,
\end{aligned} \tag{2.20}$$

where we have used the helicity equation $p\sigma^3\chi_s = s|\vec{p}|\chi_s$. If we consider the two cases in (2.19) and the orthogonality property $\chi^\dagger_{-s}(\vec{p})\chi_s(\vec{p}) = 0$, we can easily get rid of the dependence on the helicity indices and the absolute value at the same time. We obtain

$$\begin{aligned}
\langle p_X, B_X, s_X|J_V^{0,a}(0)|p,B,s\rangle &= \frac{1}{2(2\pi)^3}(\tau^a)_{I_3^X I_3}\chi^\dagger_{s_X}(\vec{p}_X)\chi_s(\vec{p}) \\
&\times \left[\alpha F^{D,a}_{BB_X}(q^2)-\beta q^2\kappa_B F^{P,a}_{BB_X}(q^2)\right]\,,
\end{aligned} \tag{2.21}$$

where

$$\alpha = \left(\frac{1}{2E}\right)\left(\frac{f}{f_X}\right)\left[f_X^2+(1-2x)\frac{|\vec{p}|^2}{f^2}\right]\,, \tag{2.22}$$

$$\beta = \left(\frac{1}{2E}\right)\left(\frac{f}{f_X}\right)\left(\frac{1}{2x}\right)\left[\frac{f_X^2}{f^2}+2x-1\right]\,. \tag{2.23}$$

Similarly, for the spatial components of the current, we obtain:

$$\begin{aligned}
\langle p_X, B_X, s_X|J_V^{i,a}(0)|p,B,s\rangle &= -\frac{i}{2(2\pi)^3}(\tau^a)_{I_3^X I_3}\epsilon^{ijk}q_j\chi^\dagger_{s_X}(\vec{p}_X)\sigma_k\chi_s(\vec{p}) \\
&\times \left[\beta F^{D,a}_{BB_X}(q^2)+\alpha\kappa_B F^{P,a}_{BB_X}(q^2)\right]\,,
\end{aligned} \tag{2.24}$$

where we used $\sigma^1 = -i\sigma^2\sigma^3$, $\sigma^2 = i\sigma^1\sigma^3$ and $p\sigma^3\chi_s = s|\vec{p}|\chi_s$. We have also used some properties of $\chi^\dagger_{s_X}(\vec{p})\sigma_2\chi_s(\vec{p})$ and $\chi^\dagger_{s_X}(\vec{p})\sigma_1\chi_s(\vec{p})$.

Negative parity resonances

Note that

$$\gamma_5 u(p,s) = \frac{1}{\sqrt{2E}}\begin{pmatrix}\frac{s|\vec{p}|}{f}\chi_s(\vec{p}) \\ f\chi_s(\vec{p})\end{pmatrix} = \frac{1}{\sqrt{2E}}\begin{pmatrix}\tilde{f}\chi_s(\vec{p}) \\ \frac{s|\vec{p}|}{\tilde{f}}\chi_s(\vec{p})\end{pmatrix}\,, \tag{2.25}$$

where

$$\tilde{f} := \frac{s|\vec{p}|}{f}\,. \tag{2.26}$$

Therefore, we can recycle the spinor identities obtained in the positive parity case by substituting f by $\tilde{f}$ in all the calculations. Then it is not difficult to check that in the Breit frame the current matrix element takes the form

$$\begin{aligned} {}_5\langle p_X, B_X, s_X|J_V^{0,a}(0)|p, B, s\rangle &= \frac{1}{2(2\pi)^3}(\tau^a)_{I_3^X I_3}\chi^\dagger_{s_X}(\vec{p}_X)\chi_s(\vec{p}) \\ &\times \left[\tilde{\alpha}\tilde{F}^{D,a}_{BB_X}(q^2) - \tilde{\beta}\, q^2\kappa_B\tilde{F}^{P,a}_{BB_X}(q^2)\right], \end{aligned} \tag{2.27}$$

$$\begin{aligned} {}_5\langle p_X, B_X, s_X|J_V^{i,a}(0)|p, B, s\rangle &= -\frac{i}{2(2\pi)^3}(\tau^a)_{I_3^X I_3}\epsilon^{ijk}q_j\chi^\dagger_{s_X}(\vec{p}_X)\sigma_k\chi_s(\vec{p}) \\ &\times \left[\tilde{\beta}\tilde{F}^{D,a}_{BB_X}(q^2) + \tilde{\alpha}\kappa_B\tilde{F}^{P,a}_{BB_X}(q^2)\right], \end{aligned} \tag{2.28}$$

where

$$\begin{aligned} \tilde{\alpha} &= \left(\frac{1}{2E}\right)\left(\frac{\tilde{f}}{f_X}\right)\left[f_X^2 + (1-2x)\frac{|\vec{p}|^2}{\tilde{f}^2}\right] \\ &= s|\vec{p}|\left(\frac{f}{f_X}\right)\left(\frac{1}{2E}\right)\left[\frac{f_X^2}{f^2} + 1 - 2x\right] =: s|\vec{p}|\hat{\alpha}\,, \end{aligned} \tag{2.29}$$

$$\begin{aligned} \tilde{\beta} &= \left(\frac{1}{2E}\right)\left(\frac{\tilde{f}}{f_X}\right)\left(\frac{1}{2x}\right)\left[\frac{f_X^2}{\tilde{f}^2} + 2x - 1\right] \\ &= \frac{s}{|\vec{p}|}\left(\frac{f}{f_X}\right)\left(\frac{1}{2E}\right)\left(\frac{1}{2x}\right)\left[f_X^2 - (1-2x)\frac{|\vec{p}|^2}{f^2}\right] =: s|\vec{p}|\hat{\beta}\,. \end{aligned} \tag{2.30}$$

Using the helicity equation

$$s|\vec{p}|\chi_s(\vec{p}) = \vec{p}\cdot\vec{\sigma}\chi_s(\vec{p}) = -\frac{1}{2x}\vec{q}\cdot\vec{\sigma}\chi_s(\vec{p})\,, \tag{2.31}$$

and the identity

$$\sigma^i\sigma^j = \delta^{ij} + i\epsilon^{ijk}\sigma_k\,, \tag{2.32}$$

we get

$$\begin{aligned} {}_5\langle p_X, B_X, s_X|J_V^{0,a}(0)|p, B, s\rangle &= -\left(\frac{1}{2x}\right)\frac{1}{2(2\pi)^3}(\tau^a)_{I_3^X I_3}q^i\chi^\dagger_{s_X}(\vec{p}_X)\sigma_i\chi_s(\vec{p}) \\ &\times \left[\hat{\alpha}\tilde{F}^{D,a}_{BB_X}(q^2) - \hat{\beta}\, q^2\kappa_B\tilde{F}^{P,a}_{BB_X}(q^2)\right], \end{aligned} \tag{2.33}$$

$$\begin{aligned} {}_5\langle p_X, B_X, s_X|J_V^{i,a}(0)|p, B, s\rangle &= \left(\frac{1}{2x}\right)\frac{\vec{q}^2}{2(2\pi)^3}(\tau^a)_{I_3^X I_3}\left(\delta^{ij} - \frac{q^iq^j}{\vec{q}^2}\right)\chi^\dagger_{s_X}(\vec{p}_X)\sigma_j\chi_s(\vec{p}) \\ &\times \left[\hat{\beta}\tilde{F}^{D,a}_{BB_X}(q^2) + \hat{\alpha}\kappa_B\tilde{F}^{P,a}_{BB_X}(q^2)\right]. \end{aligned} \tag{2.34}$$

2.3. Helicity amplitudes

In order to establish a simple connection between the Dirac and Pauli form factors and the more commonly used helicity amplitudes, we first need to review some Gordon identities. We start with a generalized Gordon identity

$$\begin{aligned} p^X_\nu \gamma^\nu\gamma^\mu + p_\nu\gamma^\mu\gamma^\nu &= p^X_\nu\left(\frac{1}{2}\{\gamma^\nu,\gamma^\mu\}+\frac{1}{2}[\gamma^\nu,\gamma^\mu]\right)+p_\nu\left(\frac{1}{2}\{\gamma^\mu,\gamma^\nu\}+\frac{1}{2}[\gamma^\mu,\gamma^\nu]\right) \\ &= p^X_\nu\left(\eta^{\mu\nu}+i\sigma^{\mu\nu}\right)+p_\nu\left(\eta^{\mu\nu}-i\sigma^{\mu\nu}\right) \\ &= (p_X+p)^\mu + i\sigma^{\mu\nu}q_\nu\,. \end{aligned} \tag{2.35}$$

Evaluating (2.35) on the initial and final spinor and recalling the Dirac equation

$$p_\mu\gamma^\mu u(p,s) = im_B u(p,s) \quad , \quad \bar u(p_X,s_X)p^X_\mu\gamma^\mu = im_{B_X}\bar u(p_X,s_X)\,, \tag{2.36}$$

we get the Gordon decomposition for positive parity resonances:

$$\bar u(p_X,s_X)\gamma^\mu u(p,s) = -\frac{i}{m_{B_X}+m_B}\bar u(p_X,s_X)\left[(p_X+p)^\mu + i\sigma^{\mu\nu}q_\nu\right]u(p,s)\,, \tag{2.37}$$

On the other hand, if we multiply (2.35) by γ_5 on the right, and evaluate it on the initial and final spinors, and finally use the Dirac equation (2.36), we get the Gordon decomposition for the negative parity case,

$$\bar u(p_X,s_X)\gamma^\mu\gamma_5 u(p,s) = -\frac{i}{m_{B_X}-m_B}\bar u(p_X,s_X)\left[(p_X+p)^\mu + i\sigma^{\mu\nu}q_\nu\right]\gamma_5 u(p,s)\,. \tag{2.38}$$

Here we used the fact that $\{\gamma^5,\gamma^\mu\}=0$. Another useful spinor identity is

$$\bar u(p_X,s_X)q_\nu\gamma^\nu\gamma_5 u(p,s) = i(m_{B_X}+m_B)\bar u(p_X,s_X)\gamma_5 u(p,s)\,. \tag{2.39}$$

This identity was obtained from the Dirac equation (2.36).

Positive parity resonances

First we define the $G_1(q^2)$ and $G_2(q^2)$ form factors through the vector current decomposition [36]

$$\begin{aligned} \langle p_X,B_X,s_X|\mathcal{J}^\mu(0)|p,B,s\rangle &= i\,\bar u(p_X,s_X)\Big\{\left[\eta^{\mu\nu}-\frac{q^\mu q^\nu}{q^2}\right]\gamma_\nu q^2 G_1(q^2) \\ &\quad +\frac{1}{2}\left[(p_X^2-p^2)\gamma^\mu - q_\nu\gamma^\nu(p_X+p)^\mu\right]G_2(q^2)\Big\}u(p,s)\,. \end{aligned} \tag{2.40}$$

Using the Gordon identity (2.37) and the Dirac equation (2.36), we obtain

$$\begin{aligned} \langle p_X,B_X,s_X|\mathcal{J}^\mu(0)|p,B,s\rangle &= i\left(\eta^{\mu\nu}-\frac{q^\mu q^\nu}{q^2}\right)\bar u(p_X,s_X)\Big[\gamma_\nu q^2 G_1(q^2) \\ &\quad - \frac{1}{2}(m_{B_X}-m_B)\sigma_{\nu\lambda}q^\lambda G_2(q^2)\Big]u(p,s)\,. \end{aligned} \tag{2.41}$$

Recalling the current decomposition (2.11), we get the Dirac and Pauli form factors in terms of the $G_1(q^2)$ and $G_2(q^2)$ form factors:

$$\begin{aligned} F^D_{BB_X}(q^2) &= q^2 G_1(q^2) \\ F^P_{BB_X}(q^2) &= -\frac{1}{2}(m^2_{B_X} - m^2_B) G_2(q^2)\,. \end{aligned} \tag{2.42}$$

According to [36], the transverse helicity amplitude $\mathcal{A}_{1/2}(q^2)$ is defined by

$$\begin{aligned} \mathcal{A}_{1/2}(q^2) &= \sqrt{\frac{E_R - m_B}{2m_B K}} \left[q^2 G_1(q^2) - \frac{1}{2}(m^2_{B_X} - m^2_B) G_2(q^2) \right] \\ &= \sqrt{\frac{E_R - m_B}{2m_B K}} \left[F^D_{BB_X}(q^2) + F^P_{BB_X}(q^2) \right]\,, \end{aligned} \tag{2.43}$$

where

$$K = \frac{m^2_{B_X} - m^2_B}{2m_{B_X}}\,, \tag{2.44}$$

and E_R is the proton energy in the resonance rest frame, i.e.

$$E_R = \frac{1}{2m_{B_X}}(m^2_{B_X} + m^2_B + q^2)\,. \tag{2.45}$$

Details of the resonance rest frame are given in appendix A.2.

The helicity amplitude $\mathcal{A}_{1/2}(q^2)$ can be rewritten as [37]

$$\mathcal{A}_{1/2}(q^2) = \sqrt{\frac{m_B}{m^2_{B_X} - m^2_B}} G^+_{BB_X}(q^2)\,, \tag{2.46}$$

where

$$G^+_{BB_X}(q^2) = \frac{\zeta}{m_B} \left[F^D_{BB_X}(q^2) + F^P_{BB_X}(q^2) \right]\,, \tag{2.47}$$

and

$$\zeta := \sqrt{m_{B_X}(E_R - m_B)} = \frac{1}{\sqrt{2}} \left[(m_{B_X} - m_B)^2 + q^2 \right]^{1/2}\,. \tag{2.48}$$

The longitudinal helicity amplitude $\mathcal{S}_{1/2}(q^2)$ is given by [36],

$$\begin{aligned} \mathcal{S}_{1/2}(q^2) &= \sqrt{\frac{E_R - m_B}{m_B K}} \frac{|\vec{q}_R|}{2} \left[(m_{B_X} + m_B) G_1(q^2) + \frac{1}{2}(m_{B_X} - m_B) G_2(q^2) \right] \\ &= \sqrt{\frac{E_R - m_B}{m_B K}} \frac{|\vec{q}_R|}{2} \left[\frac{m_{B_X} + m_B}{q^2} F^D_{BB_X}(q^2) - \frac{1}{m_{B_X} + m_B} F^P_{BB_X}(q^2) \right]\,, \end{aligned} \tag{2.49}$$

where $\vec{q}_R$ is the spatial momentum of the virtual photon in the resonance rest frame. According to [38], this amplitude can be rewritten as

$$\mathcal{S}_{1/2}(q^2) = \sqrt{\frac{m_B}{m^2_{B_X} - m^2_B}} \frac{|\vec{q}_R|}{\sqrt{q^2}} G^0_{BB_X}(q^2)\,, \tag{2.50}$$

where

$$G^0_{BB_X}(q^2) = \sqrt{\frac{q^2}{2}}\frac{\zeta}{m_B}\left[\frac{m_{B_X}+m_B}{q^2}F^D_{BB_X}(q^2) - \frac{1}{m_{B_X}+m_B}F^P_{BB_X}(q^2)\right]. \tag{2.51}$$

Negative parity resonances

In analogy with the previous case we define the $\tilde{G}_1(q^2)$ and $\tilde{G}_2(q^2)$ negative parity form factors through the vector current decomposition as in [36],

$$\begin{aligned} {}_5\langle p_X, B_X, s_X|\mathcal{J}^\mu(0)|p, B, s\rangle &= -i\,\bar{u}(p_X, s_X)\Big\{\left(\eta^{\mu\nu} - \frac{q^\mu q^\nu}{q^2}\right)\gamma_\nu q^2 \tilde{G}_1(q^2) \\ &+\frac{1}{2}\left[(p_X^2 - p^2)\gamma^\mu - q_\nu\gamma^\nu(p_X+p)^\mu\right]\tilde{G}_2(q^2)\Big\}\gamma_5 u(p,s)\,. \end{aligned} \tag{2.52}$$

Using the Gordon identity (2.38) and the spinor identity (2.39) in (2.52), we find

$$\begin{aligned} {}_5\langle p_X, B_X, s_X|\mathcal{J}^\mu(0)|p, B, s\rangle &= -i\,\bar{u}(p_X, s_X)\left(\eta^{\mu\nu} - \frac{q^\mu q^\nu}{q^2}\right)\Big[\gamma_\nu q^2\tilde{G}_1(q^2) \\ &- \frac{1}{2}(m_{B_X}+m_B)\sigma_{\nu\lambda}q^\lambda\tilde{G}_2(q^2)\Big]\gamma_5 u(p,s)\,. \end{aligned} \tag{2.53}$$

Using (2.15) we obtain the relations

$$\begin{aligned} \tilde{F}^D_{BB_X}(q^2) &= -q^2\tilde{G}_1(q^2) \\ \tilde{F}^P_{BB_X}(q^2) &= \frac{1}{2}(m_{B_X}+m_B)^2\tilde{G}_2(q^2)\,. \end{aligned} \tag{2.54}$$

Now let us write the expressions for the helicity amplitudes. According to [36], the helicity amplitudes $\mathcal{A}_{1/2}(q^2)$ are given by

$$\begin{aligned} \tilde{\mathcal{A}}_{1/2}(q^2) &= \sqrt{\frac{E_R+m_B}{2m_BK}}\left[q^2\tilde{G}_1(q^2) - \frac{1}{2}(m^2_{B_X} - m^2_B)\tilde{G}_2(q^2)\right] \\ &= -\sqrt{\frac{E_R+m_B}{2m_BK}}\left[\tilde{F}^D_{BB_X}(q^2) + \frac{m_{B_X}-m_B}{m_{B_X}+m_B}\tilde{F}^P_{BB_X}(q^2)\right], \end{aligned} \tag{2.55}$$

where K and E_R are given by (2.44) and (2.45), respectively.

Using the analog of (2.46), we get

$$\tilde{G}^+_{BB_X}(q^2) = -\frac{\tilde{\zeta}}{m_B}\left[\tilde{F}^D_{BB_X}(q^2) + \frac{m_{B_X}-m_B}{m_{B_X}+m_B}\tilde{F}^P_{BB_X}(q^2)\right], \tag{2.56}$$

where

$$\tilde{\zeta} := \sqrt{m_{B_X}(E_R+m_B)} \;=\; \frac{1}{\sqrt{2}}\left[(m_{B_X}+m_B)^2 + q^2\right]^{1/2}. \tag{2.57}$$

The helicity amplitude $\tilde{\mathcal{S}}_{1/2}(q^2)$ is given by [36]

$$\begin{aligned}
\tilde{\mathcal{S}}_{1/2}(q^2) &= -\sqrt{\frac{E_R+m_B}{m_B K}}\frac{|\vec{q}_R|}{2}\left[(m_{B_X}-m_B)\tilde{G}_1(q^2)+\frac{1}{2}(m_{B_X}+m_B)\tilde{G}_2(q^2)\right] \\
&= \sqrt{\frac{E_R+m_B}{m_B K}}\frac{|\vec{q}_R|}{2}\left[\frac{m_{B_X}-m_B}{q^2}\tilde{F}^D_{BB_X}(q^2)-\frac{1}{m_{B_X}+m_B}\tilde{F}^P_{BB_X}(q^2)\right], \quad (2.58)
\end{aligned}$$

where $\vec{q}_R$ is the spatial momentum of the virtual photon in the resonance rest frame. Using the negative parity analog of (2.50) we get

$$\tilde{G}^0_{BB_X}(q^2) = \sqrt{\frac{q^2}{2}}\frac{\tilde{\zeta}}{m_B}\left[\frac{m_{B_X}-m_B}{q^2}\tilde{F}^D_{BB_X}(q^2)-\frac{1}{m_{B_X}+m_B}\tilde{F}^P_{BB_X}(q^2)\right]. \quad (2.59)$$

2.4. The proton structure functions

A typical deep inelastic scattering (DIS) process is illustrated in figure 1. The corresponding differential cross section is determined by the hadronic tensor,

$$W^{\mu\nu} = \frac{1}{8\pi}\sum_s \int d^4x\, e^{iq\cdot x}\langle p,s|\left[\mathcal{J}^\mu(x),\mathcal{J}^\nu(0)\right]|p,s\rangle, \quad (2.60)$$

where $\mathcal{J}^\mu(x)$ is the electromagnetic current, q^μ and p^μ are the momenta of the virtual photon and the initial hadron, respectively. Inserting the sum of the final states X with momentum p^μ_X we can rewrite the hadronic tensor as

$$W^{\mu\nu} = \frac{1}{8\pi}\sum_s\sum_X (2\pi)^4\delta^4(p+q-p_X)\langle p,s|\mathcal{J}^\mu(0)|X\rangle\langle X|\mathcal{J}^\nu(0)|p,s\rangle\,. \quad (2.61)$$

One usually parametrizes DIS using as dynamical variables the Bjorken parameter $x=-\frac{q^2}{2p\cdot q}$ and the photon virtuality q^2. The hadronic tensor can be decomposed in terms of the Lorentz invariant scalar structure functions $\mathcal{F}_1(x,q^2)$ and $\mathcal{F}_2(x,q^2)$:

$$W^{\mu\nu} = \mathcal{F}_1(x,q^2)\Big(\eta^{\mu\nu}-\frac{q^\mu q^\nu}{q^2}\Big)+\frac{2x}{q^2}\mathcal{F}_2(x,q^2)\Big(p^\mu+\frac{q^\mu}{2x}\Big)\Big(p^\nu+\frac{q^\nu}{2x}\Big). \quad (2.62)$$

The standard limit of DIS corresponds to the Bjorken limit of large q^2 and fixed x. In this paper we are interested in the regime of small q^2 where non-perturbative contributions are relevant (for a review of DIS, see e.g., [39]).

The baryonic tensor for a spin 1/2 baryon, in the case where one particle is produced in the final state, can be written as

$$\begin{aligned}
W^{\mu\nu} &= \frac{1}{8\pi}\sum_{s,s_X}\sum_{m_{B_X}}\int\frac{d^4p_X}{(2\pi)^3}\theta(p^0_X)\delta(p^2_X+m^2_{B_X})(2\pi)^4\delta^4(p+q-p_X) \\
&\quad\times\Big[\langle p,B,s|\mathcal{J}^\mu(0)|p_X,B_X,s_X\rangle\langle p_X,B_X,s_X|\mathcal{J}^\nu(0)|p,B\rangle \\
&\qquad+\langle p,B,s|\mathcal{J}^\mu(0)|p_X,B_X,s_X\rangle_{5\,5}\langle p_X,B_X,s_X|\mathcal{J}^\nu(0)|p,B\rangle\Big]
\end{aligned}$$

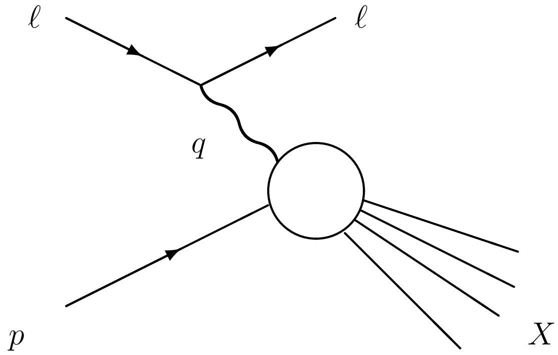

Figure 1: Diagram for a deep inelastic scattering process. A lepton ℓ exchanges a virtual photon with a hadron of momentum p.

$$
\begin{aligned}
&= \frac{1}{4}\sum_{s,s_X}\sum_{m_{B_X}} \delta\left[(p+q)^2 + m_{B_X}^2\right]\Big[\langle p,B,s|\mathcal{J}^\mu(0)|p_X,B_X,s_X\rangle\langle p_X,B_X,s_X|\mathcal{J}^\nu(0)|p,B,s\rangle \\
&\quad +\langle p,B,s|\mathcal{J}^\mu(0)|p_X,B_X,s_X\rangle_{5\,5}\langle p_X,B_X,s_X|\mathcal{J}^\nu(0)|p,B\rangle\Big]. \qquad (2.63)
\end{aligned}
$$

Note that we are including the contribution from positive parity resonances as well as negative parity resonances. Substituting (2.11) and (2.15) into (2.63), we obtain

$$
\begin{aligned}
W^{\mu\nu} &= -\frac{1}{4}\sum_{m_{B_X}} \delta\left[(p+q)^2 + m_{B_X}^2\right]\left(\eta^{\mu\rho} - \frac{q^\mu q^\rho}{q^2}\right)\left(\eta^{\nu\sigma} - \frac{q^\nu q^\sigma}{q^2}\right) \\
&\quad \times\Big\{ F^D_{BB_X}(q^2)F^D_{BB_X}(q^2)\mathcal{A}_{\rho\sigma} + F^P_{BB_X}(q^2)F^P_{BB_X}(q^2)\mathcal{B}_{\rho\sigma} \\
&\quad + F^P_{BB_X}(q^2)F^D_{BB_X}(q^2)\mathcal{C}_{\rho\sigma} + F^D_{BB_X}(q^2)F^P_{BB_X}(q^2)\mathcal{D}_{\rho\sigma} \\
&\quad +\tilde{F}^D_{BB_X}(q^2)\tilde{F}^D_{BB_X}(q^2)\tilde{\mathcal{A}}_{\rho\sigma} + \tilde{F}^P_{BB_X}(q^2)\tilde{F}^P_{BB_X}(q^2)\tilde{\mathcal{B}}_{\rho\sigma} \\
&\quad +\tilde{F}^P_{BB_X}(q^2)\tilde{F}^D_{BB_X}(q^2)\tilde{\mathcal{C}}_{\rho\sigma} + \tilde{F}^D_{BB_X}(q^2)\tilde{F}^P_{BB_X}(q^2)\tilde{\mathcal{D}}_{\rho\sigma}\Big\}, \qquad (2.64)
\end{aligned}
$$

where

$$
\begin{aligned}
\mathcal{A}_{\rho\sigma} &= 4\Big\{\left[m_B m_{B_X} + p^2 - \frac{q^2}{2x}\right]\eta_{\rho\sigma} - 2p_\rho p_\sigma - p_\rho q_\sigma - p_\sigma q_\rho\Big\}, \qquad (2.65) \\
\mathcal{B}_{\rho\sigma} &= 4\kappa_B^2 q^2\Big\{\left[-m_B m_{B_X} + p^2 + \frac{q^2}{2x}\left(1 - \frac{1}{x}\right)\right]\eta_{\rho\sigma} - 2p_\rho p_\sigma
\end{aligned}
$$

$$-\frac{1}{x}(q_\rho p_\sigma + q_\sigma p_\rho) + \left[m_B m_{B_X} - p^2 - \frac{q^2}{2x}\right]\frac{q_\rho q_\sigma}{q^2}\Big\}\,, \tag{2.66}$$

$$\mathcal{C}_{\rho\sigma} = 4\kappa_B q^2\Big\{-\left[m_B + \frac{1}{2x}(m_{B_X} - m_B)\right]\eta_{\rho\sigma} + \left[m_B q_\rho + (m_B - m_{B_X})p_\rho\right]\frac{q_\sigma}{q^2}\Big\}\,, \tag{2.67}$$

$$\mathcal{D}_{\rho\sigma} = \mathcal{C}_{\sigma\rho}\,, \tag{2.68}$$

$$\tilde{\mathcal{A}}_{\rho\sigma} = 4\Big\{\left[-m_B m_{B_X} + p^2 - \frac{q^2}{2x}\right]\eta_{\rho\sigma} - 2p_\rho p_\sigma - p_\rho q_\sigma - p_\sigma q_\rho\Big\}\,, \tag{2.69}$$

$$\begin{aligned}\tilde{\mathcal{B}}_{\rho\sigma} &= 4\kappa_B^2 q^2\Big\{\left[m_B m_{B_X} + p^2 + \frac{q^2}{2x}\left(1 - \frac{1}{x}\right)\right]\eta_{\rho\sigma} - 2p_\rho p_\sigma \\ &\quad -\frac{1}{x}(q_\rho p_\sigma + q_\sigma p_\rho) - \left[m_B m_{B_X} + p^2 + \frac{q^2}{2x}\right]\frac{q_\rho q_\sigma}{q^2}\Big\}\,,\end{aligned} \tag{2.70}$$

$$\tilde{\mathcal{C}}_{\rho\sigma} = 4\kappa_B q^2\Big\{\left[m_B - \frac{1}{2x}(m_B + m_{B_X})\right]\eta_{\rho\sigma} - \left[m_B q_\rho + (m_B + m_{B_X})p_\rho\right]\frac{q_\sigma}{q^2}\Big\}\,, \tag{2.71}$$

$$\tilde{\mathcal{D}}_{\rho\sigma} = \tilde{\mathcal{C}}_{\sigma\rho}\,, \tag{2.72}$$

and we used the sum over spin formula

$$\sum_s u(p,s)\bar{u}(p,s) = -i\gamma^\mu p_\mu + m_B\,, \tag{2.73}$$

and the gamma trace identities

$$\begin{aligned}
\mathrm{tr}(\gamma_\mu\gamma_\nu) &= +4\eta_{\mu\nu}\,,\\
\mathrm{tr}(\gamma_\mu\gamma_\nu\gamma_\rho\gamma_\sigma) &= +4(\eta_{\mu\nu}\eta_{\rho\sigma} - \eta_{\mu\rho}\eta_{\nu\sigma} + \eta_{\mu\sigma}\eta_{\nu\rho}) =: 4\tilde{\eta}_{\mu\nu\rho\sigma}\,,\\
\mathrm{tr}(\gamma_\mu\gamma_\nu\gamma_\rho\gamma_\sigma\gamma_\lambda\gamma_\tau) &= +4\Big[\eta_{\mu\nu}\tilde{\eta}_{\rho\sigma\lambda\tau} - \eta_{\mu\rho}\tilde{\eta}_{\nu\sigma\lambda\tau} + \eta_{\mu\sigma}\tilde{\eta}_{\nu\rho\lambda\tau} - \eta_{\mu\lambda}\tilde{\eta}_{\nu\rho\sigma\tau} + \eta_{\mu\tau}\tilde{\eta}_{\nu\rho\sigma\lambda}\Big]\,,\\
\mathrm{tr}(\sigma_{\mu\nu}\gamma_\rho\gamma_\sigma) &= -\,\mathrm{tr}(\sigma_{\mu\nu}\gamma_\sigma\gamma_\rho) = 4i(-\eta_{\mu\rho}\eta_{\nu\sigma} + \eta_{\mu\sigma}\eta_{\nu\rho})\,,\\
\mathrm{tr}(\sigma_{\mu\nu}\sigma_{\rho\sigma}) &= -4(-\eta_{\mu\rho}\eta_{\nu\sigma} + \eta_{\mu\sigma}\eta_{\nu\rho})\,,\\
\mathrm{tr}(\sigma_{\mu\nu}\gamma_\rho\sigma_{\sigma\lambda}\sigma_\tau) &= -4\Big[-\eta_{\mu\rho}(\eta_{\nu\sigma}\eta_{\lambda\tau} - \eta_{\nu\lambda}\eta_{\sigma\tau}) + \eta_{\mu\sigma}\tilde{\eta}_{\nu\rho\lambda\tau} - \eta_{\mu\lambda}\tilde{\eta}_{\nu\rho\sigma\tau}\\
&\qquad +\eta_{\mu\tau}(-\eta_{\nu\sigma}\eta_{\rho\lambda} + \eta_{\nu\lambda}\eta_{\rho\sigma})\Big]\,.
\end{aligned} \tag{2.74}$$

Note that the terms with q_ρ or q_σ will vanish when contracting with the transverse tensors. Using (2.65),(2.66),(2.67),(2.68) in (2.64) and comparing to (2.62), we obtain the proton structure functions

$$\mathcal{F}_1(q^2,x) = F_1(q^2,x) + \tilde{F}_1(q^2,x)\,, \tag{2.75}$$

$$\mathcal{F}_2(q^2,x) = F_2(q^2,x) + \tilde{F}_2(q^2,x)\,, \tag{2.76}$$

where

$$\begin{aligned}F_1(q^2,x) &= \sum_{m_{B_X}}\delta\left[(p+q)^2 + m_{B_X}^2\right]\zeta^2\Big[F^D_{BB_X}(q^2,x) + F^P_{BB_X}(q^2,x)\Big]^2\\ &= \sum_{m_{B_X}}\delta\left[(p+q)^2 + m_{B_X}^2\right]m_B^2(G^+_{BB_X}(q^2))^2\,,\end{aligned} \tag{2.77}$$

$$F_2(q^2,x) = \left(\frac{q^2}{x}\right)\sum_{m_{B_X}}\delta\left[(p+q)^2 + m_{B_X}^2\right]$$

$$
\begin{aligned}
&\times\left[(F^D_{BB_X}(q^2))^2+\kappa_B^2 q^2 (F^P_{BB_X}(q^2))^2\right] \\
= &\sum_{m_{B_X}} \delta\left[(p+q)^2+m_{B_X}^2\right]\left(\frac{q^2}{2x}\right)\left(1+\frac{q^2}{4m_B^2x^2}\right)^{-1} \\
&\times\left[(G^+_{BB_X}(q^2))^2+2(G^0_{BB_X}(q^2))^2\right] ,
\end{aligned} \tag{2.78}
$$

are the positive parity contributions to the proton structure functions and

$$
\begin{aligned}
\tilde{F}_1(q^2,x) &= \sum_{m_{B_X}} \delta\left[(p+q)^2+m_{B_X}^2\right]\tilde{\zeta}^2\left[\tilde{F}^D_{BB_X}(q^2,x)+\frac{m_{B_X}-m_B}{m_{B_X}+m_B}\tilde{F}^P_{BB_X}(q^2,x)\right]^2 \\
&= \sum_{m_{B_X}} \delta\left[(p+q)^2+m_{B_X}^2\right] m_B^2 (\tilde{G}^+_{BB_X}(q^2))^2 ,
\end{aligned} \tag{2.79}
$$

$$
\begin{aligned}
\tilde{F}_2(q^2,x) &= \left(\frac{q^2}{x}\right)\sum_{m_{B_X}} \delta\left[(p+q)^2+m_{B_X}^2\right] \\
&\quad\times\left[(\tilde{F}^D_{BB_X}(q^2))^2+\kappa_B^2 q^2 (\tilde{F}^P_{BB_X}(q^2))^2\right] \\
&= \sum_{m_{B_X}} \delta\left[(p+q)^2+m_{B_X}^2\right]\left(\frac{q^2}{2x}\right)\left(1+\frac{q^2}{4m_B^2x^2}\right)^{-1} \\
&\quad\times\left[(\tilde{G}^+_{BB_X}(q^2))^2+2(\tilde{G}^0_{BB_X}(q^2))^2\right] ,
\end{aligned} \tag{2.80}
$$

are the negative parity contributions to the proton structure functions. Here ζ, $G^+_{BB_X}(q^2)$ and $G^0_{BB_X}(q^2)$ were defined in (2.48), (2.47) and (2.51) respectively while $\tilde{\zeta}$, $\tilde{G}^+_{BB_X}(q^2)$ and $\tilde{G}^0_{BB_X}(q^2)$ were given in (2.57), (2.56) and (2.59) respectively.

3. Dirac and Pauli form factors from holography

3.1. Review of the Sakai-Sugimoto model

3.1.1. Generalities and meson effective action

The Sakai-Sugimoto model [10,11] is the most widely studied string-theoretic model of large-N_c QCD and has been successfully applied to investigate many of its phenomenological aspects. Its holographic limit describes a stable configuration of $D8-\overline{D8}$ branes embedded into Witten's $D4$ model [40]. In the following section we will briefly review those features of this model which are important for the investigations carried out in this article. The geometry of Witten's model is generated by N_c coincident $D4$ branes with a compact spatial direction τ in type IIA supergravity with the following metric, dilaton and four-form,

$$
ds^2=\frac{u^{3/2}}{R^{3/2}}\left(\eta_{\mu\nu}dx^\mu dx^\nu+f(u)d\tau^2\right)+\frac{R^{3/2}}{u^{3/2}}\frac{du^2}{f(u)}+R^{3/2}u^{1/2}d\Omega_4^2, \tag{3.1}
$$

$$
f(u)=1-\frac{u_{KK}^3}{u^3},\quad e^\phi=g_s\frac{u^{3/4}}{R^{3/4}},\quad F_4=\frac{(2\pi l_s)^3N_c}{V_{S^4}}\varepsilon_4,
$$

where u_{KK} is the radial position of the tip of the cigar geometry generated by the $D4$ branes and $R = (\pi g_s N_c)^{1/3}\sqrt{\alpha'}$. To incorporate fundamental (quark and anti-quark) degrees of freedom, one needs to introduce two stacks of N_f coincident $D8$ and $\overline{D8}$ flavor branes into the background generated by the N_c $D4$ branes. The probe condition $N_f \ll N_c$ ensures that the back reaction of the flavor branes on the geometry can be safely neglected. It turns out that the solution to the DBI equations merges the two stacks of $D8$ and $\overline{D8}$ branes in the infrared region (small u), resulting in a geometrical realization of chiral symmetry breaking $U(N_f) \times U(N_f) \to U(N_f)$.

The dynamics of the gauge field fluctuations on the $D8/\overline{D8}$ brane embedding is also described by the Dirac-Born-Infeld action, which yields a vector meson effective field theory given by a five dimensional $U(N_f)$ Yang-Mills-Chern-Simons theory in a curved background. Its effective action reads [11]

$$\begin{aligned} S_{\text{eff}} &= S_{\text{YM}} + S_{\text{CS}}, \qquad (3.2)\\ S_{\text{YM}} &= \kappa \int d^4x \int dz \ \text{tr}\left[\frac{1}{2}h(z)\eta^{\mu\lambda}\eta^{\nu\rho}\mathcal{F}_{\lambda\rho}\mathcal{F}_{\mu\nu} + M_{KK}^2 k(z)\eta^{\mu\nu}\mathcal{F}_{\mu z}\mathcal{F}_{\nu z}\right], \\ S_{\text{CS}} &= \frac{N_c}{24\pi^2}\int_{M^4\times\mathbb{R}} \omega_5, \qquad (3.3) \end{aligned}$$

where z is a dimensionless variable with domain $(-\infty, +\infty)$ that combines the original left and right chiral sectors ($D8$ and $\overline{D8}$ branes), $\omega_5 = \text{tr}\left(\mathcal{A}\mathcal{F}^2 - \frac{i}{2}\mathcal{A}^3\mathcal{F} - \frac{1}{10}\mathcal{A}^5\right)$, and the two 'warp factors' are

$$h(z) := (k(z))^{-1/3} = (1+z^2)^{-1/3}, \quad k(z) := 1 + z^2, \quad \kappa = \frac{\lambda N_c}{216\pi^3}, \qquad (3.4)$$

where $\lambda = g_{YM}^2 N_c$ is the t'Hooft coupling and M_{KK} is a mass parameter related to the $D4$-brane background. The quantity $\mathcal{A} = \mathcal{A}_\alpha dx^\alpha = \mathcal{A}_\mu dx^\mu + \mathcal{A}_z dz \ \ (\alpha = 0,1,2,3,z)$ represents the five-dimensional $U(N_f)$ gauge field and $\mathcal{F} = \frac{1}{2}\mathcal{F}_{\alpha\beta}dx^\alpha \wedge dx^\beta = d\mathcal{A} + i\mathcal{A}\wedge\mathcal{A}$ is its field strength. Expanding the gauge fields in the $A_z = 0$ gauge as in [11]:

$$\mathcal{A}_\mu(x,z) = \hat{\mathcal{V}}_\mu(x) + \hat{\mathcal{A}}_\mu(x)\psi_0(z) + \sum_{n=1}^{\infty} v_\mu^n(x)\psi_{2n-1}(z) + \sum_{n=1}^{\infty} a_\mu^n(x)\psi_{2n}(z) \quad , \qquad (3.5)$$

where $\psi_0(z) := (2/\pi)\arctan z$ and

$$\begin{aligned} \hat{\mathcal{V}}_\mu(x) &= \frac{1}{2}e^{-\frac{i\Pi(x)}{f_\pi}}\left[A_{L\mu}(x) + \partial_\mu\right]e^{\frac{i\Pi(x)}{f_\pi}} + \frac{1}{2}e^{\frac{i\Pi(x)}{f_\pi}}\left[A_{R\mu}(x) + \partial_\mu\right]e^{\frac{-i\Pi(x)}{f_\pi}} \\ \hat{\mathcal{A}}_\mu(x) &= \frac{1}{2}e^{-\frac{i\Pi(x)}{f_\pi}}\left[A_{L\mu}(x) + \partial_\mu\right]e^{\frac{i\Pi(x)}{f_\pi}} - \frac{1}{2}e^{\frac{i\Pi(x)}{f_\pi}}\left[A_{R\mu}(x) + \partial_\mu\right]e^{\frac{-i\Pi(x)}{f_\pi}} . \qquad (3.6) \end{aligned}$$

The field $\Pi(x)$ is interpreted as the pion field. The modes ψ_n satisfy the following conditions

$$\begin{aligned} \kappa \int dz h(z)\psi_m(z)\psi_n(z) &= \delta_{mn} , \qquad (3.7)\\ -h(z)\partial_z\left[k(z)\partial_z\psi_n(z)\right] &= \lambda_n\,\psi_n(z) , \qquad (3.8) \end{aligned}$$

where n, m are positive integers.

In order to represent vector and axial-vector mesons, one performs the following field redefinitions

$$\tilde{v}^n_\mu = v^n_\mu + \frac{g_{v^n}}{M^2_{v^n}}\mathcal{V}_\mu \quad , \quad \tilde{a}^n_\mu = a^n_\mu + \frac{g_{a^n}}{M^2_{a^n}}\mathcal{A}_\mu \,, \tag{3.9}$$

$$\mathcal{V}_\mu = \frac{1}{2}(A_{L\mu} + A_{R\mu}) \quad , \quad \mathcal{A}_\mu = \frac{1}{2}(A_{L\mu} - A_{R\mu}) \,, \tag{3.10}$$

introducing the constants

$$M^2_{v^n} = \lambda_{2n-1} M^2_{KK} \quad , \quad M^2_{a^n} = \lambda_{2n} M^2_{KK} \,, \tag{3.11}$$

$$g_{v^n} = \kappa\, M^2_{v^n} \int dz\, k(z)^{-1/3} \psi_{2n-1}(z) \,, \tag{3.12}$$

$$g_{a^n} = \kappa\, M^2_{a^n} \int dz\, k(z)^{-1/3} \psi_{2n}(z)\psi_0(z) \,. \tag{3.13}$$

This way, one obtains the 4d effective Lagrangian

$$\begin{aligned}\mathcal{L}^{4d}_{eff} &= \frac{1}{2}\,\mathrm{tr}\left(\partial_\mu \tilde{v}^n_\nu - \partial_\nu \tilde{v}^n_\mu\right)^2 + \frac{1}{2}\,\mathrm{tr}\left(\partial_\mu \tilde{a}^n_\nu - \partial_\nu \tilde{a}^n_\mu\right)^2 + \mathrm{tr}\left(i\partial_\mu \Pi + f_\pi \mathcal{A}_\mu\right)^2 \\ &+ M^2_{v^n}\,\mathrm{tr}\left(\tilde{v}^n_\mu - \frac{g_{v^n}}{M^2_{v^n}}\mathcal{V}_\mu\right)^2 + M^2_{a^n}\,\mathrm{tr}\left(\tilde{a}^n_\mu - \frac{g_{a^n}}{M^2_{a^n}}\mathcal{A}_\mu\right)^2 + \sum_{j\geq 3}\mathcal{L}_j\end{aligned} \tag{3.14}$$

where $\mathcal{L}_j$ represents the interaction terms of order j in the fields and divergent terms were disregarded. The massive fields $\tilde{v}^n_\mu$, $\tilde{a}^n_\mu$ represent vector and axial-vector mesons, respectively. The decay constant g_{v^n} describes the coupling of the vector mesons $\tilde{v}^n_\mu$ to an external massless vectorial field $\mathcal{V}_\mu$ (the photon), while the decay constant g_{a^n} couples the axial-vector meson $\tilde{a}^n_\mu$ to an external massless axial vector field $\mathcal{A}_\mu$. Note that g_{v^n} is the only interaction between photons and mesons, which implies that vector meson dominance is realized in the Sakai-Sugimoto model.

It is important to remark that in (3.14), the terms that depend only on the pion field and external fields join to form the Skyrme and Wess-Zumino-Witten terms, as expected in any effective description of non-perturbative QCD in the large-N_c limit.

3.1.2. Baryons in the Sakai-Sugimoto model

Let us describe the ideas behind the construction of holographic baryons. Recall that, in the confined phase, the Sakai-Sugimoto model reduces to a five-dimensional $U(N_f)$ Yang Mills-Chern Simons (YM-CS) theory with an action given by (3.2). In this article, we restrict ourselves to the $N_f = 2$ case. Then, the $U(2)$ gauge field $\mathcal{A}$ can be decomposed as

$$\mathcal{A} = A + \widehat{A}\,\frac{\mathbf{1}_2}{2} = A^i \frac{\tau^i}{2} + \widehat{A}\,\frac{\mathbf{1}_2}{2} = \sum_{a=0}^{3} \mathcal{A}^a\,\frac{\tau^a}{2} \;, \tag{3.15}$$

where τ^i $(i = 1, 2, 3)$ are Pauli matrices and $\tau^0 = \mathbf{1}_2$ is a unit matrix of dimension 2. Thus, the equations of motion are given by

$$
\begin{aligned}
&-\kappa \left(h(z)\partial_\nu \widehat{F}^{\mu\nu} + \partial_z(k(z)\widehat{F}^{\mu z}) \right) + \frac{N_c}{128\pi^2}\epsilon^{\mu\alpha_2\ldots\alpha_5}\left(F^a_{\alpha_2\alpha_3}F^a_{\alpha_4\alpha_5} + \widehat{F}_{\alpha_2\alpha_3}\widehat{F}_{\alpha_4\alpha_5} \right) = 0, \\
&-\kappa \left(h(z)\nabla_\nu F^{\mu\nu} + \nabla_z(k(z)F^{\mu z}) \right)^a + \frac{N_c}{64\pi^2}\epsilon^{\mu\alpha_2\ldots\alpha_5}F^a_{\alpha_2\alpha_3}\widehat{F}_{\alpha_4\alpha_5} = 0, \\
&-\kappa k(z)\partial_\nu \widehat{F}^{z\nu} + \frac{N_c}{128\pi^2}\epsilon^{z\mu_2\ldots\mu_5}\left(F^a_{\mu_2\mu_3}F^a_{\mu_4\mu_5} + \widehat{F}_{\mu_2\mu_3}\widehat{F}_{\mu_4\mu_5} \right) = 0, \\
&-\kappa k(z)\left(\nabla_\nu F^{z\nu} \right)^a + \frac{N_c}{64\pi^2}\epsilon^{z\mu_2\ldots\mu_5}F^a_{\mu_2\mu_3}\widehat{F}_{\mu_4\mu_5} = 0,
\end{aligned}
\tag{3.16}
$$

where $\nabla_\alpha = \partial_\alpha + iA_\alpha$ is the covariant derivative. The baryon in the Sakai-Sugimoto holographic model is represented by a soliton with nontrivial instanton number in the four-dimensional space parameterized by x^M $(M = 1, 2, 3, z)$. Consequently, the instanton number is interpreted as the baryon number N_B, and reads

$$
N_B = \frac{1}{64\pi^2}\int d^3x dz\, \epsilon_{M_1M_2M_3M_4}F^a_{M_1M_2}F^a_{M_3M_4}\ . \tag{3.17}
$$

The equations of motion (3.16) are complicated nonlinear differential equations in a curved space-time. In general, it will be too difficult to find an analytic solution corresponding to baryons. However, working in the large λ regime, one can utilize a $1/\lambda$-expansion. Note that in this limit S_{CS} will be subleading compared to S_{YM}, and therefore the leading contribution for the instanton mass comes from the YM action. As discussed in [41], one can work with a small instanton ansatz where the instanton is localized at $z = 0$ (because the instanton size scales as $\lambda^{-1/2}$) and the warp factors $h(z), k(z)$ are approximately one. Thus, the corresponding field equations will be solved by a BPST instanton with infinitesimal size $\rho \to 0$. Including the contributions to the field equations of the CS term induces a $U(1)$ electric field $\widehat{A}_0$ and will stabilize the size of the instanton at a finite value. The complete classical solution,

$$
A^{\text{cl}}_M = -\,if(\xi)g\partial_M g^{-1}\ ,\quad \widehat{A}^{\text{cl}}_0 = \frac{N_c}{8\pi^2\kappa}\frac{1}{\xi^2}\left[1 - \frac{\rho^4}{(\rho^2+\xi^2)^2} \right]\ ,\quad A_0 = \widehat{A}_M = 0. \tag{3.18}
$$

thus corresponds to a static baryon configuration with

$$
f(\xi) = \frac{\xi^2}{\xi^2+\rho^2}\ ,\quad g(x) = \frac{(z-Z) - i(\vec{x}-\vec{X})\cdot\vec{\tau}}{\xi}\ ,\quad \xi = \sqrt{(z-Z)^2 + |\vec{x}-\vec{X}|^2}, \tag{3.19}
$$

where $X^M = (X^1, X^2, X^3, Z) = (\vec{X}, Z)$ gives the position in the spatial $\mathbb{R}^4$ direction. The effective potential for ρ and Z reads

$$
V_{\text{eff}}(\rho, Z) = M_0\left(1 + \frac{\rho^2}{6} + \frac{N_c^2}{5M_0^2}\frac{1}{\rho^2} + \frac{Z^2}{3} \right), \tag{3.20}
$$

where $M_0 = 8\pi^2\kappa M_{\text{KK}}$ is the minimal, i.e. ground state, mass of the baryons. The effective potential is minimized at

$$
\rho^2_{\text{cl}} = \frac{N_c}{M_0}\sqrt{\frac{6}{5}}\ ,\quad Z_{\text{cl}} = 0. \tag{3.21}
$$

The quantization of the solitons is facilitated by utilizing the moduli space approximation method to study a quantum mechanical problem on the instanton moduli space. For a more detailled discussion of the quantization and on how to extend the solution to the large z region, the interested reader is referred to refs. [12,15,41]. The resulting baryon eigenstates are characterized by quantum numbers $B = (l, I_3, n_\rho, n_z)$ in addition to their spin s. For example, the baryon wave functions with quantum numbers $B_n = (1, +1/2, 0, n)$ are given by

$$|B_n \uparrow\rangle \propto R(\rho)\psi_{B_n}(Z)(a_1 + ia_2), \tag{3.22}$$

where

$$\begin{aligned} R(\rho) &= \rho^{-1+2\sqrt{1+N_c^2/5}}\, e^{-\frac{M_0}{\sqrt{6}}\rho^2}, \\ \psi_{B_n}(Z) &= \left(\frac{(2M_0)^{1/4}}{6^{1/8}\pi^{1/4}2^{n/2}\sqrt{n!}}\right) H_n\left(\sqrt{2M_0}6^{-1/4}Z\right)\, e^{-\frac{M_0}{\sqrt{6}}Z^2}. \end{aligned} \tag{3.23}$$

The mass formula for the baryonic eigenstates (obtained from the quantized Hamiltonian of the system) reads

$$M = M_0 + \sqrt{\frac{(\ell+1)^2}{6} + \frac{2}{15}N_c^2} + \frac{2(n_\rho + n_z) + 2}{\sqrt{6}} =: \tilde{M}_0 + \frac{2n_z}{\sqrt{6}}\,. \tag{3.24}$$

The classical solution (3.18) is valid only near $z = 0$. This solution can be extended to large z as long as we require $\rho \ll \xi$ which is the condition of small size for the skyrmion. Under this condition the equations of motion linearize and the solutions can be found by defining Green's functions corresponding to the curved space generated by $k(z)$:

$$\begin{aligned} G(\vec{x}, z, \vec{X}, Z) &= \kappa \sum_{n=1}^{\infty} \psi_n(z)\psi_n(Z)Y_n(|\vec{x} - \vec{X}|) \\ H(\vec{x}, z, \vec{X}, Z) &= \kappa \sum_{n=0}^{\infty} \phi_n(z)\phi_n(Z)Y_n(|\vec{x} - \vec{X}|)\,, \end{aligned} \tag{3.25}$$

where $\psi_n(z)$ is the complete set of vector meson eigenfunctions, and $\phi_n(z)$ is another set defined by

$$\phi_0(z) = \frac{1}{\sqrt{\kappa\pi}k(z)} \quad , \quad \phi_n(z) = \frac{1}{\sqrt{\lambda_n}}\partial_z\psi_n(z) \quad (n = 1, 2, \dots)\,, \tag{3.26}$$

and $Y_n(r)$ is the Yukawa potential

$$Y_n(r) = -\frac{1}{4\pi}\frac{e^{-\sqrt{\lambda_n}r}}{r}\,. \tag{3.27}$$

The gauge field solutions found in [15] for the case $\rho \ll \xi$ can be written as

$$\hat{A}_0 = -\frac{N_c}{2\kappa}G(\vec{x}, z, \vec{X}, Z)\,,$$

$$
\begin{aligned}
\hat{A}_i &= \frac{N_c}{2\kappa}\left\{\dot{X}^i + \frac{\rho^2}{2}\left[\frac{\chi^a}{2}\left(\epsilon^{iaj}\frac{\partial}{\partial X^j} - \delta^{ia}\frac{\partial}{\partial Z}\right) + \frac{\dot{\rho}}{\rho}\frac{\partial}{\partial X^i}\right]\right\} G(\vec{x}, z, \vec{X}, Z)\,, \\
\hat{A}_z &= \frac{N_c}{2\kappa}\left[\dot{Z} + \frac{\rho^2}{2}\left(\frac{\chi^a}{2}\frac{\partial}{\partial X^a} + \frac{\dot{\rho}}{\rho}\frac{\partial}{\partial Z}\right)\right] H(\vec{x}, z, \vec{X}, Z)\,, \\
A_0^\Lambda &= 2\pi^2\rho^2\left\{2i\,\mathbf{a}\,\dot{\mathbf{a}}^{-1} + 2\pi^2\rho^2\mathbf{a}\tau^a\mathbf{a}^{-1}\left[\dot{X}^i\left(\epsilon^{iaj}\frac{\partial}{\partial X^j} - \delta^{ia}\frac{\partial}{\partial Z}\right) + \dot{Z}\frac{\partial}{\partial X^a}\right]\right\} G(\vec{x}, z, \vec{X}, Z)\,, \\
A_i^\Lambda &= -2\pi^2\rho^2\mathbf{a}\tau^a\mathbf{a}^{-1}\left(\epsilon^{iaj}\frac{\partial}{\partial X^j} - \delta^{ia}\frac{\partial}{\partial Z}\right) G(\vec{x}, z, \vec{X}, Z)\,, \\
A_z^\Lambda &= -2\pi^2\rho^2\mathbf{a}\tau^a\mathbf{a}^{-1}\frac{\partial}{\partial X^a} H(\vec{x}, z, \vec{X}, Z)\,,
\end{aligned} \tag{3.28}
$$

where

$$
A_\alpha^\Lambda = \Lambda A_\alpha \Lambda^{-1} - i\Lambda\partial_\alpha\Lambda \quad , \quad \Lambda = \mathbf{a} g^{-1} V^{-1}\,. \tag{3.29}
$$

3.2. Electromagnetic currents in the Sakai-Sugimoto model

The holographic currents in the Sakai-Sugimoto model, denoted here by $J^{\mu,a}_{V(SS)}$, can be obtained using the holographic relations [15] :

$$
J^{\mu,a}_{V(SS)} = -\kappa\left\{\lim_{z\to\infty}\left[K_z \mathcal{F}^{\rm cl}_{\mu z}\right] + \lim_{z\to-\infty}\left[K_z \mathcal{F}^{\rm cl}_{\mu z}\right]\right\}, \tag{3.30}
$$

where $\mathcal{F}^{\rm cl}_{\mu z}$ is the field strength associated with the classical field (3.28). From (3.28) and (3.30), one gets [15] :

$$
\begin{aligned}
J^{0,0}_{V(SS)}(x) &= \frac{N_c}{2} G_V\,, \\
J^{i,0}_{V(SS)}(x) &= -\frac{N_c}{2}\left\{\dot{Z}\partial^i H_V - \dot{X}^i G_V - \frac{S_a}{16\pi^2\kappa}\left[(\partial^i\partial^a - \delta^{ia}\partial^2)H_V + \epsilon^{ija}\partial_j G_V\right]\right\}, \\
J^{0,c}_{V(SS)}(x) &= 2\pi^2\kappa\left\{\rho^2\,\mathrm{tr}[\tau^c\partial_0(\mathbf{a}\tau^a\mathbf{a}^{-1})]\partial_a H_V + \frac{I^c}{2\pi^2\kappa} G_V\right. \\
&\quad \left. - \rho^2\,\mathrm{tr}[\tau^c\mathbf{a}\tau^a\mathbf{a}^{-1}]\dot{X}^i\left[(\partial_a\partial_i - \delta_{ia}\partial^2)H_V + \epsilon^{ija}\partial_j G_V\right]\right\}, \\
J^{i,c}_{V(SS)}(x) &= -2\pi^2\kappa\rho^2\,\mathrm{tr}[\tau^c\mathbf{a}\tau_a\mathbf{a}^{-1}]\left[(\partial^i\partial^a - \delta^{ia}\partial^2)H_V + \epsilon^{ija}\partial_j G_V\right]\,,
\end{aligned} \tag{3.31}
$$

where

$$
\begin{aligned}
G_V &= -\sum_n g_{v^n}\psi_{2n-1}(Z) Y_{2n-1}(|\vec{x} - \vec{X}|)\,, \\
H_V &= -\sum_n \frac{g_{v^n}}{\lambda_{2n-1}}\partial_Z\psi_{2n-1}(Z) Y_{2n-1}(|\vec{x} - \vec{X}|)\,, \\
\dot{Z} &= -\frac{i}{M_0}\partial_Z = \frac{P_Z}{M_0} \quad , \quad \dot{X}^i = -\frac{i}{M_0}\frac{\partial}{\partial X^i} = \frac{P^i}{M_0}\,,
\end{aligned} \tag{3.32}
$$

and

$$
S^a = 4\pi^2\kappa\rho^2\chi_a = -i4\pi^2\kappa\rho^2\,\mathrm{tr}(\tau^a\mathbf{a}^{-1}\dot{\mathbf{a}}) \quad , \quad I^a = -i4\pi^2\kappa\rho^2\,\mathrm{tr}(\tau^a\mathbf{a}\dot{\mathbf{a}}^{-1})\,, \tag{3.33}
$$

are the spin and isospin operators. Note that

$$\dot{Z}(\partial^i H_V) - \dot{X}^i G_V = \frac{1}{M_0}\Big[(\partial^i H_V)P_Z - G_V P^i\Big] , \tag{3.34}$$

where we used the relation $\partial_Z H_V = -G_V$.

Defining the Fourier transform as

$$\tilde{J}^{\mu,a}_{V(SS)}(\vec{k}) = \int d^3\vec{x} e^{-i\vec{k}\cdot x} J^{\mu,a}_{V(SS)}(x) , \tag{3.35}$$

and using the identity

$$\int d^3\vec{x} e^{-i\vec{k}\cdot x} Y_{2n-1}(|\vec{x} - \vec{X}|) = -\frac{e^{-i\vec{k}\cdot\vec{X}}}{\vec{k}^2 + \lambda_{2n-1}} , \tag{3.36}$$

we find

$$\tilde{J}^{0,0}_{V(SS)}(\vec{k}) = \frac{N_c}{2} e^{-i\vec{k}\cdot\vec{X}} \sum_n \frac{g_{v^n}\psi_{2n-1}(Z)}{\vec{k}^2 + \lambda_{2n-1}} , \tag{3.37}$$

$$\begin{aligned} \tilde{J}^{i,0}_{V(SS)}(\vec{k}) &= \frac{N_c}{2} e^{-i\vec{k}\cdot\vec{X}} \Big\{ \sum_n \frac{g_{v^n}\psi_{2n-1}(Z)}{\vec{k}^2 + \lambda_{2n-1}} \left[\frac{P^i}{M_0} + \frac{i}{16\pi^2\kappa}\epsilon^{ija} k_j S_a \right] \\ &\quad - \sum_n \frac{g_{v^n}\partial_Z\psi_{2n-1}(Z)}{\lambda_{2n-1}(\vec{k}^2 + \lambda_{2n-1})} \left[\frac{k^i}{M_0}\partial_Z + \frac{1}{16\pi^2\kappa}(k^i k^a - \vec{k}^2\delta^{ia}) S_a \right] \Big\} , \end{aligned} \tag{3.38}$$

$$\begin{aligned} \tilde{J}^{0,c}_{V(SS)}(\vec{k}) &= 2\pi^2\kappa\, e^{-i\vec{k}\cdot\vec{X}} \Big\{ \sum_n \frac{g_{v^n}\psi_{2n-1}(Z)}{\vec{k}^2 + \lambda_{2n-1}} \left[\frac{I^c}{2\pi^2\kappa} - \frac{i}{M_0}\epsilon^{ija} P_i k_j \rho^2 \operatorname{tr}(\tau^c \mathbf{a}\tau_a \mathbf{a}^{-1}) \right] \\ &\quad + \sum_n \frac{g_{v^n}\partial_Z\psi_{2n-1}(Z)}{\lambda_{2n-1}(\vec{k}^2 + \lambda_{2n-1})} \Big[i k_i \rho^2 \operatorname{tr}[\tau^c \partial_0(\mathbf{a}\tau^i \mathbf{a}^{-1})] \\ &\quad + \frac{1}{M_0}(\vec{P}\cdot\vec{k} k_i - \vec{k}^2 P_i)\rho^2 \operatorname{tr}[\tau^c \mathbf{a}\tau^i \mathbf{a}^{-1}] \Big] \Big\} , \end{aligned} \tag{3.39}$$

$$\begin{aligned} \tilde{J}^{i,c}_{V(SS)}(\vec{k}) &= 2\pi^2\kappa\, e^{-i\vec{k}\cdot\vec{X}} \Big[-i \sum_n \frac{g_{v^n}\psi_{2n-1}(Z)}{\vec{k}^2 + \lambda_{2n-1}} \epsilon^{ija} k_j \\ &\quad + \sum_n \frac{g_{v^n}\partial_Z\psi_{2n-1}(Z)}{\lambda_{2n-1}(\vec{k}^2 + \lambda_{2n-1})} (k^i k^a - \vec{k}^2\delta^{ia}) \Big] \rho^2 \operatorname{tr}(\tau^c \mathbf{a}\tau_a \mathbf{a}^{-1}) . \end{aligned} \tag{3.40}$$

Note that one term arising from $\dot{Z}$ cancels with another from $\dot{X}^i$ and we have used the relation $\partial_Z^2\psi_n(Z) \approx -\lambda_n\psi_n(Z)$. Now we calculate the expectation values of the Sakai-Sugimoto currents:

$$\langle p_X, B_X, s_X | J^{\mu,a}_{V(SS)}(0) | p, B, s\rangle = \int \frac{d^3\vec{k}}{(2\pi)^3} \langle p_X, B_X, s_X | \tilde{J}^{\mu,a}_{V(SS)}(\vec{k}) | p, B, s\rangle . \tag{3.41}$$

We define the baryon states as

$$
\begin{aligned}
|\vec{p}, B, s, I_3\rangle &= \frac{1}{(2\pi)^{3/2}} e^{i\vec{p}\cdot\vec{X}} |n_B\rangle |n_\rho\rangle |s, I_3\rangle_R \,, \\
|\vec{p}_X, B_X, s_X, I_3^X\rangle &= \frac{1}{(2\pi)^{3/2}} e^{i\vec{p}_X\cdot\vec{X}} |n_{B_X}\rangle |n_\rho\rangle |s_X, I_3^X\rangle_R \,.
\end{aligned}
\tag{3.42}
$$

Here we make use of the results and definitions of a recent publication [42], in which a relativistic generalization of baryon states and wave functions was discussed in detail. In particular, the spin and isospin part was defined as

$$
\begin{aligned}
|s, I_3\rangle_R &= \frac{1}{\sqrt{2E}} \begin{pmatrix} f\,|s, I_3\rangle \\ \frac{s|\vec{p}|}{f}\,|s, I_3\rangle \end{pmatrix} , \\
\langle s_X, I_3^X|_R &= \frac{1}{\sqrt{2E_X}} \left(f_X\,\langle s_X, I_3^X| \quad - \frac{s_X|\vec{p}_X|}{f_X}\,\langle s_X, I_3^X| \right) ,
\end{aligned}
\tag{3.43}
$$

where $|s, I_3\rangle$ and $\langle s_X, I_3^X|$ are the non-relativistic initial and final states associated with the spin and isospin operators. Evaluating the currents in these states, we find

$$
\langle J^{0,0}_{V(SS)}(0)\rangle = \frac{1}{(2\pi)^3}\frac{N_c}{2}\langle s_X, I_3^X|s, I_3\rangle_R F^1_{BB_X}(\vec{q}^2)\,, \tag{3.44}
$$

$$
\begin{aligned}
\langle J^{i,0}_{V(SS)}(0)\rangle &= \frac{1}{(2\pi)^3}\frac{N_c}{2}\langle s_X, I_3^X|_R\Big\{ F^1_{BB_X}(\vec{q}^2)\left[\frac{p^i}{M_0} - \frac{i}{16\pi^2\kappa}\epsilon^{ija}q_j S_a\right] \\
&+ \frac{q^i}{M_0}F^3_{BB_X}(\vec{q}^2) - \frac{1}{16\pi^2\kappa}F^2_{BB_X}(\vec{q}^2)(q^i q^a - \vec{q}^2\delta^{ia})S_a\Big\}|s, I_3\rangle_R\,,
\end{aligned}
\tag{3.45}
$$

$$
\begin{aligned}
\langle J^{0,c}_{V(SS)}(0)\rangle &= 2\pi^2\kappa\,\frac{1}{(2\pi)^3}\langle n_\rho|\langle s_X, I_3^X|_R\Big\{ F^1_{BB_X}(\vec{q}^2)\left[\frac{I^c}{2\pi^2\kappa} + \frac{i}{M_0}\epsilon^{ija}p_i q_j \rho^2\,\mathrm{tr}(\tau^c\mathbf{a}\tau_a\mathbf{a}^{-1})\right] \\
&+ F^2_{BB_X}(\vec{q}^2)\Big[- i q_i \rho^2\,\mathrm{tr}[\tau^c\partial_0(\mathbf{a}\tau^i\mathbf{a}^{-1})] \\
&+ \frac{1}{M_0}(\vec{P}\cdot\vec{q}q_i - \vec{q}^2 P_i)\rho^2\,\mathrm{tr}[\tau^c\mathbf{a}\tau^i\mathbf{a}^{-1}]\Big]\Big\}|n_\rho\rangle|s, I_3\rangle_R\,,
\end{aligned}
\tag{3.46}
$$

$$
\begin{aligned}
\langle J^{i,c}_{V(SS)}(0)\rangle &= 2\pi^2\kappa\,\frac{1}{(2\pi)^3}\Big[iF^1_{BB_X}(\vec{q}^2)\epsilon^{ija}q_j + F^2_{BB_X}(\vec{q}^2)(q^i q^a - \vec{q}^2\delta^{ia})\Big] \\
&\times \langle n_\rho|\rho^2|n_\rho\rangle\langle s_X, I_3^X|_R\,\mathrm{tr}(\tau^c\mathbf{a}\tau_a\mathbf{a}^{-1})|s, I_3\rangle_R\,.
\end{aligned}
\tag{3.47}
$$

where

$$
\begin{aligned}
F^1_{BB_X}(\vec{q}^2) &= \sum_n \frac{g_{v^n}\langle n_{B_X}|\psi_{2n-1}(Z)|n_B\rangle}{\vec{q}^2 + \lambda_{2n-1}} \\
F^2_{BB_X}(\vec{q}^2) &= \sum_n \frac{g_{v^n}\langle n_{B_X}|\partial_Z\psi_{2n-1}(Z)|n_B\rangle}{\lambda_{2n-1}(\vec{q}^2 + \lambda_{2n-1})}
\end{aligned}
$$

$$F^3_{BB_X}(\vec{q}^2) \;=\; \sum_n \frac{g_{v^n}\langle n_{B_X}|\partial_Z\psi_{2n-1}(Z)\partial_Z|n_B\rangle}{\lambda_{2n-1}(\vec{q}^2+\lambda_{2n-1})}\,, \tag{3.48}$$

the momentum $\vec{q}$ is the photon momentum defined by $\vec{q}=\vec{p}_X-\vec{p}$ and we have used

$$\langle\vec{p}_X|e^{-i\vec{k}\cdot\vec{X}}|\vec{p}\rangle = \delta^3(\vec{k}-\vec{p}+\vec{p}_X)\,. \tag{3.49}$$

In order to calculate the expectation values of the holographic currents we need the following identities:

$$\begin{aligned}
\langle s_X, I_3^X|s,I_3\rangle_R &= \frac{1}{2\sqrt{E_XE}}(ff_X-\tfrac{ss_X|\vec{p}||\vec{p}_X|}{ff_X})\,\delta_{I_3^XI_3}\chi^\dagger_{s_X}(\vec{p_X})\,\chi_s(\vec{p})\,,\\
\langle s_X, I_3^X|_R\,\mathrm{tr}(\tau^c\mathbf{a}\tau^a\mathbf{a}^{-1})|s,I_3\rangle_R &= -\frac{1}{3\sqrt{E_XE}}(ff_X-\tfrac{ss_X|\vec{p}||\vec{p}_X|}{ff_X})\,\tau^c_{I_3^XI_3}\,\chi^\dagger_{s_X}(\vec{p_X})\,\sigma^a\chi_s(\vec{p})\,,\\
\langle s_X, I_3^X|_R\,I^c\,|s,I_3\rangle_R &= \frac{1}{4\sqrt{E_XE}}(ff_X-\tfrac{ss_X|\vec{p}||\vec{p}_X|}{ff_X})\,(\tau^c)_{I_3^XI_3}\chi^\dagger_{s_X}(\vec{p_X})\,\chi_s(\vec{p})\,,\\
\langle s_X, I_3^X|_R\,S_a\,|s,I_3\rangle_R &= \frac{1}{4\sqrt{E_XE}}(ff_X-\tfrac{ss_X|\vec{p}||\vec{p}_X|}{ff_X})\,\delta_{I_3^XI_3}\chi^\dagger_{s_X}(\vec{p_X})\,\sigma_a\chi_s(\vec{p})\,,\\
\langle s_X, I_3^X|_R\,\mathrm{tr}(\tau^3\partial_0(\mathbf{a}\tau^i\mathbf{a}^{-1}))|s,I_3\rangle_R &= \frac{i}{M_0\rho^2\sqrt{E_XE}}(ff_X-\tfrac{ss_X|\vec{p}||\vec{p}_X|}{ff_X})\\
&\times\; (\tau^3)_{I_3^XI_3}\,\chi^\dagger_{s_X}(\vec{p_X})\,\sigma^i\chi_s(\vec{p})\,.
\end{aligned} \tag{3.50}$$

The last identity can be obtained by first noticing that

$$\begin{aligned}
\mathrm{tr}(\tau^c\partial_0(\mathbf{a}\tau^i\mathbf{a}^{-1})) &= -\frac{2i}{M_0\rho^2}\Big\{\left(a_4\frac{\partial}{\partial a_4}-a_a\frac{\partial}{\partial a_a}\right)\delta^{ic}+a_i\frac{\partial}{\partial a_c}+a_c\frac{\partial}{\partial a_i}\\
&-\;\epsilon^{ica}\left(a_a\frac{\partial}{\partial a_4}-a_4\frac{\partial}{\partial a_a}\right)\Big\}\,.
\end{aligned} \tag{3.51}$$

Using (3.50), we get in the Breit frame

$$\begin{aligned}
\langle J^{0,0}_{V(SS)}(0)\rangle &= \frac{N_c}{2(2\pi)^3}\xi\delta_{I_3^XI}\chi^\dagger_{s_X}(\vec{p_X})\chi_s(\vec{p})F^1_{BB_X}(\vec{q}^2)\,,\\
\langle J^{i,0}_{V(SS)}(0)\rangle &= \frac{N_c}{2(2\pi)^3M_0}\delta_{I_3^XI}\chi^\dagger_{s_X}(\vec{p_X})\Big\{q^i\left[F^3_{BB_X}(\vec{q}^2)-\frac{1}{2x}F^1_{BB_X}(\vec{q}^2)\right]\xi\\
&-\;\frac{i}{4}\alpha\epsilon^{ija}q_j\sigma_aF^1_{BB_X}(\vec{q}^2)\Big\}\chi_s(\vec{p})\,,\\
\langle J^{0,c}_{V(SS)}(0)\rangle &= \frac{\xi}{2(2\pi)^3}(\tau^c)_{I_3^XI_3}\chi^\dagger_{s_X}(\vec{p_X})\chi_s(\vec{p})F^1_{BB_X}(\vec{q}^2)\,,\\
\langle J^{i,c}_{V(SS)}(0)\rangle &= -i\frac{\alpha}{2(2\pi)^3}\left(\frac{M_0}{3}\right)\langle n_\rho|\rho^2|n_\rho\rangle\,(\tau^c)_{I_3^XI_3}\\
&\times\;\epsilon^{ija}q_j\chi^\dagger_{s_X}(\vec{p_X})\sigma_a\chi_s(\vec{p})F^1_{BB_X}(\vec{q}^2)\,,
\end{aligned} \tag{3.52}$$

when the final state is a positive parity resonance, and

$$\begin{aligned}
{}_5\langle J^{0,0}_{V(SS)}(0)\rangle &= 0\,,\\
{}_5\langle J^{i,0}_{V(SS)}(0)\rangle &= \frac{N_c}{8(2\pi)^3M_0}\vec{q}^2\left(\delta^{ia}-\frac{q^iq^a}{\vec{q}^2}\right)\delta_{I_3^XI}\chi^\dagger_{s_X}(\vec{p_X})\,\sigma_a\chi_s(\vec{p})\,\alpha F^2_{BB_X}(\vec{q}^2)\,,
\end{aligned}$$

$$
\begin{aligned}
{}_5\langle J^{0,3}_{V(SS)}(0)\rangle &= \frac{1}{2(2\pi)^3}(\tau^3)_{I_3^X I_3}\, q_i \chi^\dagger_{s_X}(\vec{p_X})\,\sigma_i\chi_s(\vec{p})\,\xi F^2_{BB_X}(\vec{q}^2),\\
{}_5\langle J^{i,3}_{V(SS)}(0)\rangle &= \frac{1}{2(2\pi)^3}(\tau^3)_{I_3^X I_3}\left(\frac{M_0}{3}\right)\langle n_\rho|\rho^2|n_\rho\rangle\\
&\times\ \vec{q}^2\left(\delta^{ia}-\frac{q^i q^a}{\vec{q}^2}\right)\chi^\dagger_{s_X}(\vec{p_X})\sigma_a\chi_s(\vec{p})\alpha F^2_{BB_X}(\vec{q}^2)\,,
\end{aligned}
\tag{3.53}
$$

when the final state has negative parity. In (3.52) and (3.53) we used the definitions

$$
\begin{aligned}
\xi &= \left(\frac{1}{2E}\right)\left(\frac{f}{f_X}\right)\left[f_X^2+\frac{\vec{p}^2}{f^2}(2x-1)\right],\\
\left(\frac{M_0}{3}\right)\langle n_\rho|\rho^2|n_\rho\rangle &= \frac{1}{\sqrt{6}M_{KK}}\left[1+2\sqrt{1+\frac{N_c^2}{5}}\right] =: \frac{g_{I=1}}{4m_B}\,,
\end{aligned}
\tag{3.54}
$$

and α was defined in (2.22).

3.3. Dirac and Pauli form factors in the Sakai-Sugimoto model

We are going to use the holographic prescription

$$
\eta_\mu\langle p_X, B_X, s_X|J^{\mu,a}_V(0)|p,B,s\rangle = \eta_\mu\langle p_X, B_X, s_X|J^{\mu,a}_{V(SS)}(0)|p,B,s\rangle\,,
\tag{3.55}
$$

where $\eta_\mu=(\eta_0,\vec{\eta})$ is the polarization of the photon and we choose to work with transverse photons satisfying the relation $\eta_\mu q^\mu=0$ in order to avoid the discussion of current anomalies.

Using (3.55) we can compare, the kinematic currents (2.21), (2.24), (2.33) and (2.34) with the Sakai-Sugimoto currents (3.52) and (3.53). For positive parity resonances we get

$$
\begin{aligned}
F^{D,0}_{BB_X}(q^2) &= \left[\frac{\xi\alpha+\beta\alpha\frac{q^2}{4M_0}}{\alpha^2+\beta^2q^2}\right]N_cF^1_{BB_X}(q^2)\,,\\
F^{P,0}_{BB_X}(q^2) &= -\frac{1}{\kappa_B}\left[\frac{\beta\xi-\frac{\alpha^2}{4M_0}}{\alpha^2+\beta^2q^2}\right]N_cF^1_{BB_X}(q^2)\,,\\
F^{D,3}_{BB_X}(q^2) &= \left[\frac{\xi\alpha+\beta\alpha q^2\left(\frac{M_0}{3}\right)\langle\rho^2\rangle}{\alpha^2+\beta^2q^2}\right]F^1_{BB_X}(q^2)\,,\\
F^{P,3}_{BB_X}(q^2) &= -\frac{1}{\kappa_B}\left[\frac{\beta\xi-\alpha^2\left(\frac{M_0}{3}\right)\langle\rho^2\rangle}{\alpha^2+\beta^2q^2}\right]F^1_{BB_X}(q^2)\,,
\end{aligned}
\tag{3.56}
$$

where α and β are given in (2.22), (2.23) and ξ is given in (3.54). For negative parity resonances, we can write

$$
\begin{aligned}
\tilde{F}^{D,0}_{BB_X}(q^2) &= x\left(\frac{q^2}{2M_0}\right)\left[\frac{\hat{\beta}\alpha}{\hat{\alpha}^2+\hat{\beta}^2q^2}\right]N_cF^2_{BB_X}(q^2)\,,\\
\tilde{F}^{P,0}_{BB_X}(q^2) &= x\left(\frac{1}{2M_0\kappa_B}\right)\left[\frac{\hat{\alpha}\alpha}{\hat{\alpha}^2+\hat{\beta}^2q^2}\right]N_cF^2_{BB_X}(q^2)\,,
\end{aligned}
$$

$$
\begin{aligned}
\tilde{F}^{D,3}_{BB_X}(q^2) &= 2x\left[\frac{\frac{M_0}{3}\langle\rho^2\rangle\hat{\beta}\alpha q^2-\hat{\alpha}\xi}{\hat{\alpha}^2+\hat{\beta}^2q^2}\right]F^2_{BB_X}(q^2)\,,\\
\tilde{F}^{P,3}_{BB_X}(q^2) &= 2x\left(\frac{1}{\kappa_B}\right)\left[\frac{\frac{M_0}{3}\langle\rho^2\rangle\hat{\alpha}\alpha+\hat{\beta}\xi}{\hat{\alpha}^2+\hat{\beta}^2q^2}\right]F^2_{BB_X}(q^2)\,,
\end{aligned}
\tag{3.57}
$$

where $\hat{\alpha}$ and $\hat{\beta}$ are given in (2.29) and (2.30), respectively. We relegate the details of the large λ expansions relevant to get the dominant contribution to the form factors in the non-elastic case to appendix B. The large λ limit in the elastic case, corresponding to $n_X = 0$, $m_{B_X} = m_B$ was already considered in [15]. In the non-elastic case, the positive parity resonances correspond to $n_{B_X} = 2, 4, 6, \ldots$. In this case, we get in the large λ limit [12] :

$$
\begin{aligned}
F^{D,0}_{BB_X}(q^2) &= \left[\frac{m_B}{E}+\mathcal{O}\left(\frac{1}{\lambda N_c}\right)\right]N_cF^1_{BB_X}(q^2)\,,\\
F^{P,0}_{BB_X}(q^2) &= \left[\frac{g_{I=0}}{2}-\frac{m_B}{E}+\mathcal{O}\left(\frac{1}{\lambda N_c}\right)\right]N_cF^1_{BB_X}(q^2)\,,\\
F^{D,3}_{BB_X}(q^2) &= \left[\frac{m_B}{E}+\mathcal{O}\left(\frac{1}{\lambda}\right)\right]F^1_{BB_X}(q^2)\,,\\
F^{P,3}_{BB_X}(q^2) &= \frac{g_{I=1}}{2}\left[1+\mathcal{O}\left(\frac{1}{\lambda N_c}\right)\right]F^1_{BB_X}(q^2)\,.
\end{aligned}
\tag{3.58}
$$

For negative parity resonances, we have $n_{B_X} = 1, 3, 5, \ldots$. Using the expansions in appendix B is not difficult to show that in the large λ limit the form factors reduce to

$$
\tilde{F}^{D,0}_{BB_X}(q^2) = \frac{q^2}{4E}g_{I=0}\left[1+\mathcal{O}\left(\frac{1}{\lambda N_c}\right)\right]N_cF^2_{BB_X}(q^2)\,, \tag{3.59}
$$

$$
\tilde{F}^{P,0}_{BB_X}(q^2) = \left(\frac{1}{x}\right)\frac{q^2}{4E}g_{I=0}\left[1+\mathcal{O}\left(\frac{1}{\lambda N_c}\right)\right]N_cF^2_{BB_X}(q^2)\,, \tag{3.60}
$$

$$
\tilde{F}^{D,3}_{BB_X}(q^2) = \frac{q^2}{4E}g_{I=1}\left[1+\mathcal{O}\left(\frac{1}{N_c}\right)\right]F^2_{BB_X}(q^2)\,, \tag{3.61}
$$

$$
\tilde{F}^{P,3}_{BB_X}(q^2) = \left(\frac{1}{x}\right)\frac{q^2}{4E}g_{I=1}\left[1+\mathcal{O}\left(\frac{1}{N_c}\right)\right]F^2_{BB_X}(q^2)\,. \tag{3.62}
$$

4. Numerical results for negative parity baryons

We present in this section our numerical results for the negative parity baryons. These include the wave functions, Dirac and Pauli form factors, helicity amplitudes and their contribution to the proton structure function. We are using the Sakai-Sugimoto parameters $M_{KK} = 949$MeV and $\kappa = 7.45\times10^{-3}$ [11]. We also choose $\tilde{M}_0 = 940$MeV, for phenomenological reasons.

4.1. Baryon wave functions

First we present in fig. 2 the results for the wave functions of the first excited baryons with negative parity. These wave functions have quantum numbers $B_n = (1, +1/2, 0, n)$

with $n = 2k - 1$ and are odd functions in the radial coordinate z. Table 1 shows the mass spectrum of the first negative parity baryonic resonances. The spectrum of positive parity baryonic resonances can be found in [12].

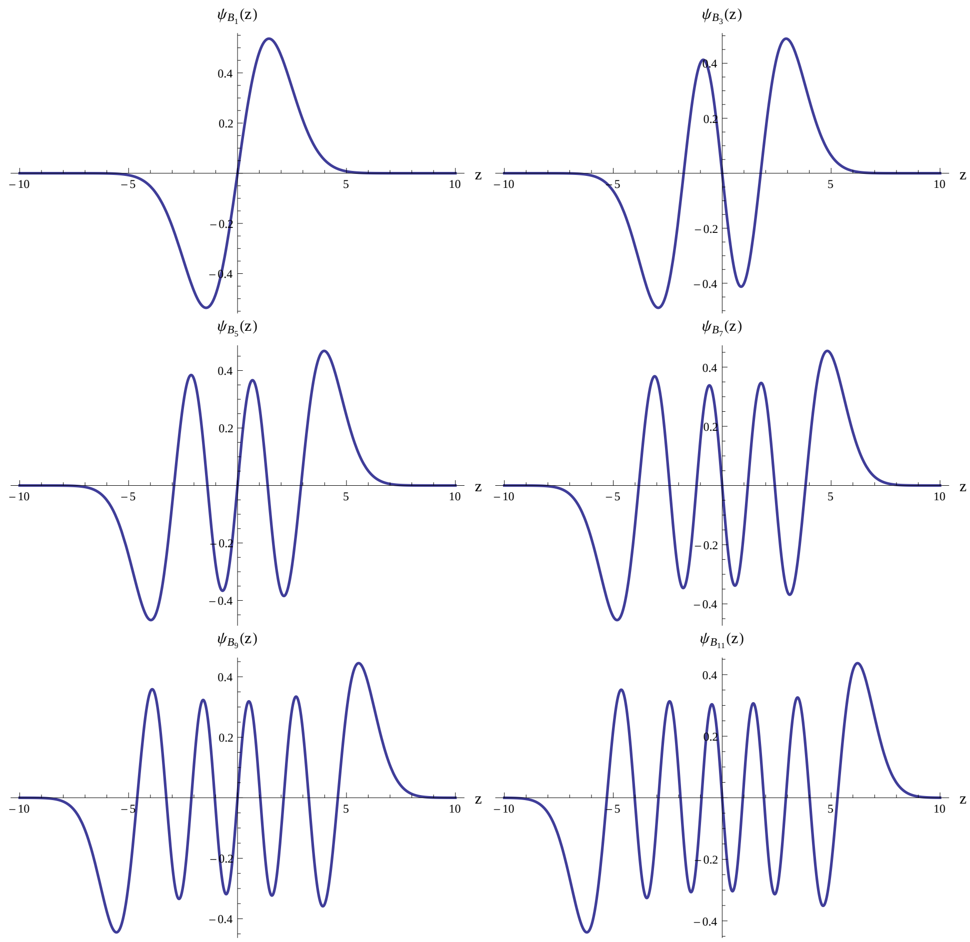

Figure 2: (Normalized) wave functions $\Psi_{B_{2k-1}}(z)$ for the first six parity odd baryon states.

4.2. Dirac and Pauli Form factors

In the previous section we extracted from holography the Dirac and Pauli form factors that describe the production of negative parity baryons. Interestingly, our results (3.62) show that the Dirac and Pauli form factors depend on only one form factor $F^2_{BB_X}(\vec{q}^2)$ defined by (3.48). This is a feature that has also appeared in previous holographic approaches to electromagnetic scattering[2]. The form factor $F^2_{BB_X}(\vec{q}^2)$ in (3.48) can be written as

$$F^2_{BB_X}(\vec{q}^2) = \sum_n \frac{g_{v^n} g_{v^n B_0 B_X}}{\vec{q}^2 + \lambda_{2n-1}} \tag{4.1}$$

[2]See [43,44] for a similar result for vector meson form factors in the holographic approach.

n	1	3	5	7	9	11	13	15
m_{B_n}/GeV	1.715	3.265	4.814	6.364	7.914	9.463	11.013	12.563

Table 1: Some numerical values for the masses of negative parity baryon states

where

$$g_{v^n B_0 B_X} := \frac{1}{\lambda_{2n-1}} \langle n_{B_X} | \partial_Z \psi_{2n-1}(Z) | n_B \rangle \,, \tag{4.2}$$

are the effective couplings between a vector meson, a negative parity baryon and the proton. We show in table 2 our numerical results for these effective couplings. Identifying the first negative parity resonance with the experimentally observed $S_{11}(1535)$, our numerical result for the coupling constant $g_{v1B_0B_1} = -1.889$ should be useful to describe the decay of $S_{11}(1535)$ into a ρ meson and a proton. This result is compatible with recent analysis from experimental data [45] where $0.79 < |g_{v1B_0B_1}| < 2.63$. The vector meson squared masses λ_{2n-1} and decay contants g_{v^n} are also shown in table 2 .

n	1	2	3	4	5	6	7	8
λ_{2n-1}	0.6693	2.874	6.591	11.80	18.49	26.67	36.34	47.49
$\frac{g_{v^n}}{\sqrt{\kappa}M_{KK}^2}$	2.109	9.108	20.80	37.15	58.17	83.83	114.2	149.1
$g_{v^n B_0 B_1}$	-1.889	1.182	-0.562	0.1381	0.04057	-0.05213	0.01239	0.009893
$g_{v^n B_0 B_3}$	1.038	-0.841	0.6132	-0.3325	0.08703	0.04209	-0.05382	0.01409
$g_{v^n B_0 B_5}$	-0.6432	0.5892	-0.5217	0.3802	-0.1907	0.02706	0.05097	-0.04458
$g_{v^n B_0 B_7}$	0.429	-0.4239	0.4223	-0.3644	0.2416	-0.09421	-0.01629	0.05276
$g_{v^n B_0 B_9}$	-0.3005	0.3132	-0.3386	0.3273	-0.2571	0.1417	-0.0266	-0.0404

Table 2: Coupling constants between vector mesons and baryons when the initial state is the proton and the final state has negative parity.

The Dirac and Pauli form factors depend on the magnetic g_I factors whose numerical values in the Sakai-Sugimoto model are given by

$$g_{I=0} \approx 1.684 \,, \qquad g_{I=1} \approx 7.031 \,. \tag{4.3}$$

Using (4.3) and our results for the couplings, masses and decay constants (shown in table 2) we can calculate the Dirac and Pauli form factors describing the production of negative parity baryon states. We show our results for the first three excited states in figure 3. As a general feature, the form factors go to zero as $q^2 \to 0$, reach a maximum and then decay for large q^2. Note that some of the form factors are non-positive.

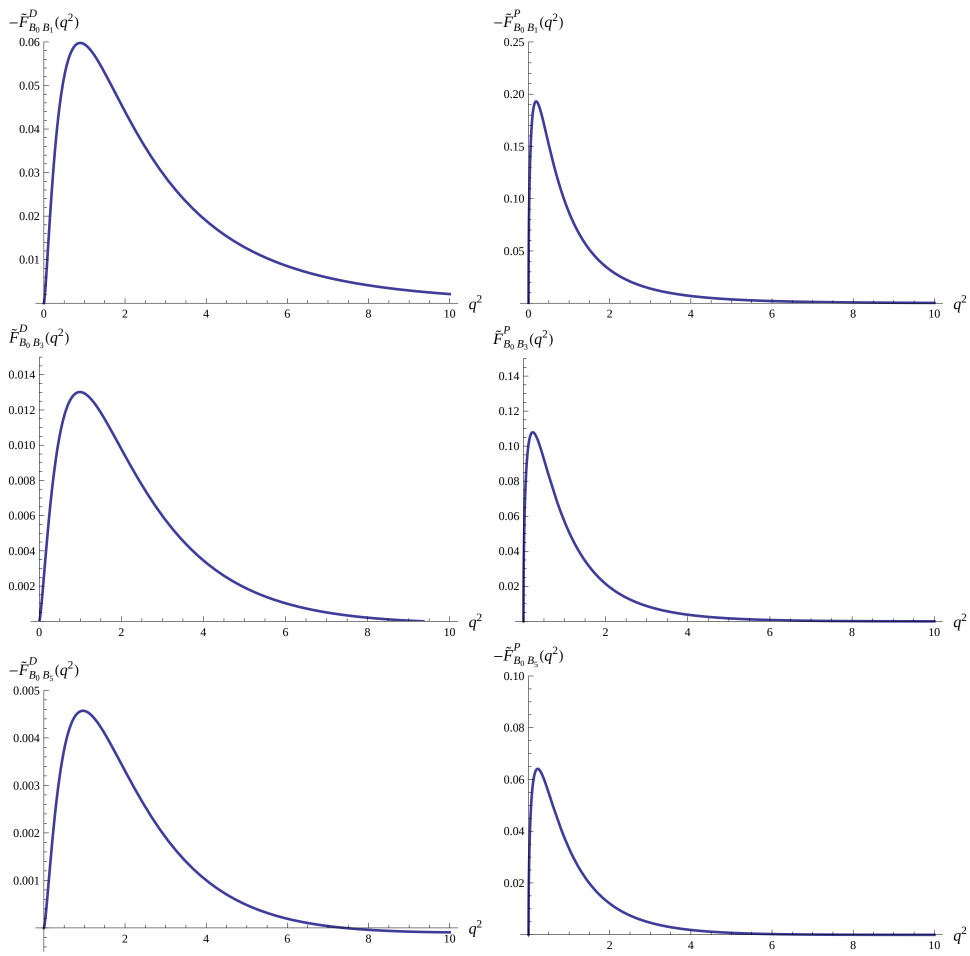

Figure 3: Dirac and Pauli form factors $\tilde{F}^{D,P}_{B_0B_{2j-1}}(q^2)$ for the first three negative parity baryon states. The momentum transfer q^2 is given in (GeV)^2.

4.3. Helicity amplitudes: comparison with JLab-CLAS data

In the large λ limit, the transverse helicity amplitudes take the form

$$\begin{aligned} \tilde{G}^{+}_{BB_X}(q^2) &\approx -\sqrt{2}\left[\tilde{F}^{D}_{BB_X}(q^2) + \frac{m_{B_X}-m_B}{2m_B}\tilde{F}^{P}_{BB_X}(q^2)\right], \\ \tilde{\mathcal{A}}^{1/2}_{BB_X}(q^2) &\approx \frac{e}{\sqrt{2(m_{B_X}-m_B)}}\tilde{G}^{+}_{BB_X}(q^2), \end{aligned} \tag{4.4}$$

In figure 4 we show our results for the transverse helicity amplitudes $\tilde{G}^{+}_{BB_X}(q^2)$ and $\tilde{\mathcal{A}}^{1/2}_{BB_X}(q^2)$ for the first negative parity resonance. As stated above, this resonance can be identified with the experimentally observed $S_{11}(1535)$. In the right panel of figure 4 we compare our results for $\tilde{\mathcal{A}}^{1/2}_{BB_X}(q^2)$ with recent experimental data from the JLAB-CLAS collaboration [46]. In spite of the model limitations (the large λ limit), we find reasonable agreement with experimental data. This is a very encouraging result for our long-term project of investigating resonance production in holographic models.

Unfortunately, in the large λ limit we cannot say too much about the longitudinal helicity amplitudes because we obtain

$$\begin{aligned} \tilde{G}^0_{BB_X}(q^2) &\approx \sqrt{q^2}\left[\frac{m_{B_X}-m_B}{q^2}\tilde{F}^D_{BB_X}(q^2)-\frac{1}{2m_B}\tilde{F}^P_{BB_X}(q^2)\right]\approx 0\,, \\ \tilde{\mathcal{S}}^{1/2}_{BB_X}(q^2) &\approx e\sqrt{\frac{m_B}{q^2}}\tilde{G}^0_{BB_X}(q^2)\approx 0\,. \end{aligned} \tag{4.5}$$

This result seems to be consistent with the fact that the experimental data available for these helicity amplitudes indicate a strong contribution from meson clouds [46]. These kinds of effects would require the investigation of loop corrections in electromagnetic scattering. The $1/\lambda$ corrections would not only modify our results but also the standard results on the elastic electromagnetic form factors[3]. We leave this interesting issue for future work.

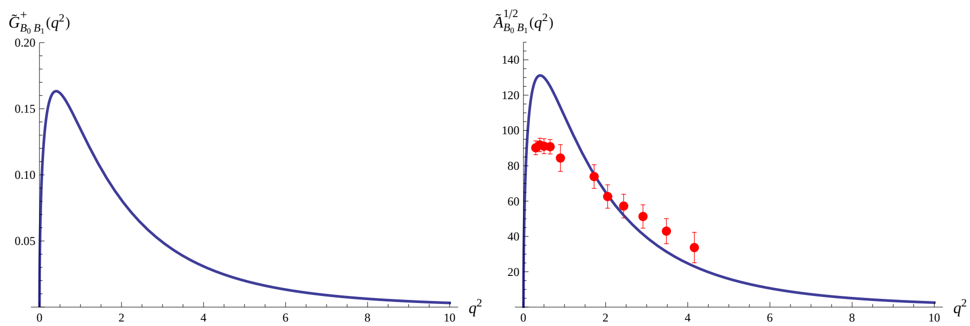

Figure 4: Helicity amplitudes $\tilde{G}^+_{B_0B_1}(q^2)$ and $\tilde{\mathcal{A}}^{1/2}_{BB_X}(q^2)$ (in units $10^{-3}(\mathrm{GeV})^{-1/2}$) plotted versus q^2 in $(\mathrm{GeV})^2$. The experimental data was taken from ref. [46].

4.4. The proton structure function

4.4.1. A first approximation

Assuming approximate continuity of the mass distribution, we can now approximate the delta distributions in the following way:

$$\begin{aligned} \sum_{B_X}\delta[m^2_{B_X}-s] &= \sum_n \delta[m^2_n-m^2_{\bar{n}}] = \int dn\left[\left|\frac{\partial m^2_n}{\partial n}\right|\right]^{-1}\delta(n-\bar{n}) \\ &= \left[\left|\frac{\partial m^2_n}{\partial n}\right|\right]^{-1}_{n=\bar{n}} =: f(\bar{n}), \end{aligned} \tag{4.6}$$

with the definition

$$s := -(p+q)^2 = m^2_{B_0} + q^2(\frac{1}{x}-1). \tag{4.7}$$

[3]See [47] for a discussion regarding pion loop corrections in baryon electromagnetic form factors.

Therefore we have to evaluate the Regge trajectory of the baryon spectrum in order to calculate $\frac{\partial m_n^2}{\partial n}$. We find from (3.24)

$$\frac{\partial m_n^2}{\partial n} = \left(\frac{4}{\sqrt{6}} \widetilde{M}_0 M_{KK} + \frac{4}{3} n M_{KK}^2 \right), \tag{4.8}$$

where $\widetilde{M}_0$ can be chosen to match, e.g. the proton mass m_{B_0} and $n := n_z$.
Using the approximation (4.6) we get in the large λ limit the structure functions

$$\begin{aligned} \tilde{F}_1(q^2, x) &\approx f(\bar{n}) m_B^2 (\tilde{G}^+_{BB_{\bar{n}}}(q^2))^2 \,, \\ \tilde{F}_2(q^2, x) &\approx f(\bar{n}) \left(\frac{q^2}{2x} \right) \left(1 + \frac{q^2}{4 m_B^2 x^2} \right)^{-1} (\tilde{G}^+_{BB_{\bar{n}}}(q^2))^2 \,. \end{aligned} \tag{4.9}$$

We plot in figures 5 and 6 the structure functions obtained from (4.9) as a function of q^2 and x. We also demonstrate the violation of the Callan-Gross relation at intermediate values of x in 7.

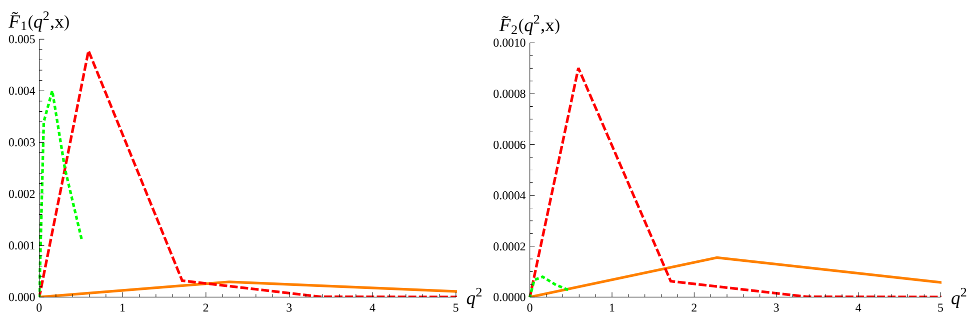

Figure 5: Structure functions $F_{1,2}(q^2)$ for $x = 0.3$ (orange, solid), $x = 0.1$ (red, dashed) and $x = 0.01$ (green, dotted).

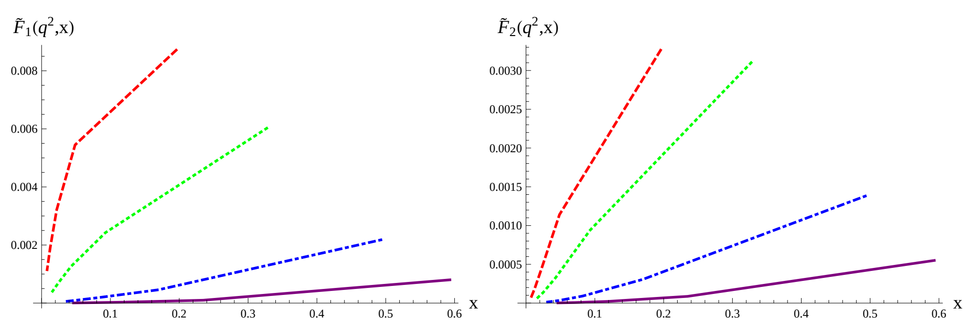

Figure 6: Structure functions $\tilde{F}_{1,2}(x)$ for $q^2 = 3(\text{GeV})^2$ (purple, solid), $q^2 = 2(\text{GeV})^2$ (blue, dotdashed), $q^2 = 1(\text{GeV})^2$ (green, dotted) and $q^2 = 0.5(\text{GeV})^2$ (red, dashed).

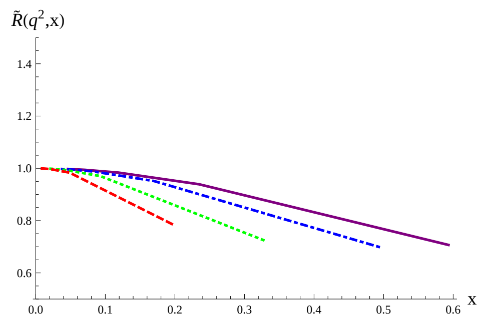

Figure 7: Callan-Gross ratio $R_{\rm CG}(x)$ for $q^2 = 3({\rm GeV})^2$ (purple, solid), $q^2 = 2({\rm GeV})^2$ (blue, dotdashed), $q^2 = 1({\rm GeV})^2$ (green, dotted) and $q^2 = 0.5({\rm GeV})^2$ (red, dashed).

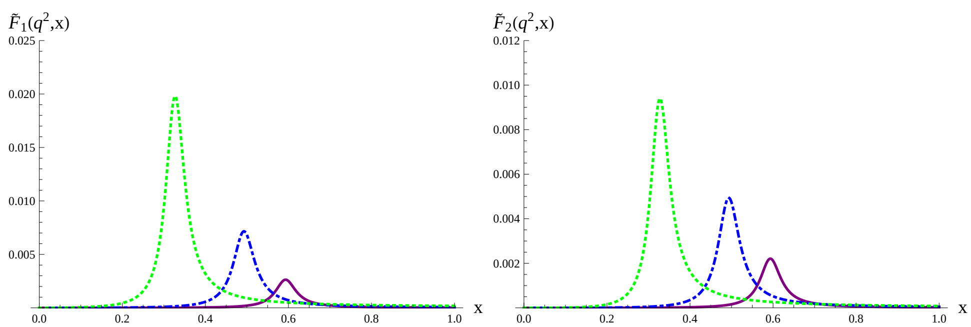

Figure 8: Structure functions $F_{1,2}(x)$ for $q^2 = 3({\rm GeV})^2$ (purple, solid), $q^2 = 2({\rm GeV})^2$ (blue, dotdashed), $q^2 = 1({\rm GeV})^2$ (green, dotted) near the first negative parity resonance B_1

4.4.2. A realistic approach near a resonance peak

The results shown in figures 5 and 6 were obtained using a the naive approximation (4.6). Alternatively, if we are only interested in the region of q^2 where a resonance is produced we can approximate the Delta distribution by a Lorentzian function [37] :

$$\delta[m_{B_X}^2 - s] \approx \frac{\Gamma_{B_X}}{4\pi m_{B_X}} \left[(\sqrt{s} - m_{B_X})^2 + \frac{\Gamma_{B_X}^2}{4} \right]^{-1} , \tag{4.10}$$

where Γ_{B_X} is the decay width of the resonance B_X. Identifying the first negative parity baryonic resonance B_1 with the experimentally observed $S_{11}(1535)$ and using the decay width $\Gamma_{B_1} = 150\,{\rm Mev}$, estimated from experimental data in [48], we obtain the results for the structure functions shown in Figure 8. Note that the structure functions have improved by an order of magnitude. Unfortunately, we cannot follow this procedure for the higher resonances because there are no experimental results available for the decay widths.

The results for the proton structure functions obtained in this paper represent only a small fraction of possible final states, namely single final states with spin 1/2 and negative parity. If we include the contribution from final states with positive parity [12] as well as final states with higher spin[4] and pion production relevant in this kinematical regime, we should get a better/more complete picture of the proton structure functions and significantly improve the comparison with experimental data.

5. Conclusions and Outlook

In this article we have presented a treatment of non-elastic proton electromagnetic scattering for the special case when baryonic resonances of negative parity are produced as the single-particle final state of the scattering process. We have in turn applied the Sakai-Sugimoto model of holographic baryons in the large λ limit to compute the relevant form factors and proton structure functions. Our numerical results show good agreement with available experimental data. One should, however, keep in mind the limitations of the (holographic) description of baryons in large-N_c QCD [49], which fully apply to the non-relativistic (large λ) model discussed herein as well. It would be very interesting to calculate $1/\lambda$- and other corrections to the current model and to study other scattering processes within the Sakai-Sugimoto model. Finally, it would be fruitful to investigate baryons and their resonance production in more recent holographic models, e.g., [50,51,52]. We leave this for future work.

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A. Some frames in inelastic scattering

A.1. The Breit frame

Consider the scattering between a virtual photon and a hadron in the hadron rest frame. After two rotations we can set the spatial momentum of the photon to the x^3 direction so

$$\begin{aligned} p^\mu &= (m_B, 0, 0, 0) \\ q^\mu &= (q_0, 0, 0, q_3)\,, \end{aligned} \tag{A.1}$$

and we choose $q_3 > 0$. The virtuality and Bjorken variable are in this frame given by

$$Q^2 = q_3^2 - q_0^2 \quad , \quad x = -\frac{Q^2}{2m_B q_0}\,. \tag{A.2}$$

Now we perform a boost in the x^3 direction so that

$$p'^\mu = (\gamma m_B, 0, 0, -\beta\gamma m_B)$$

[4] Usually one expects a high contribution coming from the production of Δ resonances. See [18] for Δ resonances in the Sakai-Sugimoto model and [19] for higher spin resonances.

$$q'^{\mu} = (\gamma q_0 - \beta\gamma q_3, 0, 0, -\beta\gamma q_0 + \gamma q_3)\,. \tag{A.3}$$

The Breit Frame is defined by the condition $q'_0 = 0$ so that

$$\beta = \frac{q_0}{q_3} = \frac{q_0}{\sqrt{q_0^2 + Q^2}} \quad , \quad \gamma = \frac{\sqrt{q_0^2 + Q^2}}{Q} \quad , \quad q'_3 = Q\,, \tag{A.4}$$

and we arrive to

$$\begin{aligned} p'^{\mu} &= (\sqrt{m_B^2 + p^2}, 0, 0, p) \\ q'^{\mu} &= (0, 0, 0, Q)\,, \end{aligned} \tag{A.5}$$

with

$$p = -\frac{Q}{2x}\,. \tag{A.6}$$

A.2. The resonant rest frame

In the resonant frame we have

$$\begin{aligned} p^{\mu} &= (E_R\,, -\vec{q})\,, \\ q^{\mu} &= (m_{B_X} - E_R\,, \vec{q}_R)\,, \\ (p+q)^{\mu} &= (m_{B_X}\,, 0)\,. \end{aligned} \tag{A.7}$$

We can write the energy and the momentum squared in terms of the squared masses and virtuality

$$\begin{aligned} E_R &= \frac{1}{2m_{B_X}}\left[q^2 + m_{B_X}^2 + m_B^2\right] \\ |\vec{q}_R|^2 &= (E_R - m_B)(E_R + m_B)\,. \end{aligned} \tag{A.8}$$

B. Expansions at large λ

The relevant large λ expansions for the non-elastic case are given by

$$\begin{aligned} q^2 &\sim \mathcal{O}(1) \quad , \quad m_B \sim \mathcal{O}(\lambda N_c) \quad , \\ m_{B_X} &= m_B + \frac{2}{\sqrt{6}} n_X M_{KK} = m_B\left[1 + \mathcal{O}\left(\frac{1}{\lambda N_c}\right)\right]\,, \\ x &= \left(\frac{\sqrt{6}}{4}\right)\frac{q^2}{m_B M_{KK} n_X}\left[1 + \mathcal{O}\left(\frac{1}{\lambda N_c}\right)\right] = \mathcal{O}\left(\frac{1}{\lambda N_c}\right)\,, \\ E &= m_B\sqrt{1 + \frac{2}{3}\frac{n_X^2 M_{KK}^2}{q^2}}\left[1 + \mathcal{O}\left(\frac{1}{\lambda N_c}\right)\right]\,, \\ \left(\frac{M_0}{3}\right)\langle\rho^2\rangle &= \frac{g_{I=1}}{4m_B} = \mathcal{O}(N_c) \quad , \quad \alpha = 1 + \mathcal{O}\left(\frac{1}{\lambda N_c}\right)\,, \end{aligned}$$

$$
\begin{aligned}
\beta &= \frac{1}{2m_B}\left[1+\mathcal{O}\left(\frac{1}{\lambda N_c}\right)\right]=\mathcal{O}\left(\frac{1}{\lambda N_c}\right) , \\
\hat{\alpha} &= \left(\frac{f}{f_x}\right)\left(\frac{1}{2E}\right)\left[\frac{f_x^2}{f^2}+1-2x\right]=\frac{1}{E}\left[1+\mathcal{O}\left(\frac{1}{\lambda N_c}\right)\right] , \\
\hat{\beta} &= \frac{2x}{q^2}\xi=\left(\frac{2x}{q^2}\right)\frac{m_B}{E}\left[1+\mathcal{O}\left(\frac{1}{\lambda N_c}\right)\right]=\mathcal{O}\left(\frac{1}{\lambda N_c}\right) , \\
\xi &= \frac{m_B}{E}+\mathcal{O}\left(\frac{1}{\lambda N_c}\right) \quad , \quad \alpha^2+\beta^2q^2=1+\mathcal{O}\left(\frac{1}{\lambda N_c}\right) , \\
\xi\alpha+\beta\alpha\frac{q^2}{4M_0} &= \frac{m_B}{E}+\mathcal{O}\left(\frac{1}{\lambda N_c}\right) , \\
-\frac{1}{\kappa_B}\left(\beta\xi-\frac{\alpha^2}{4M_0}\right) &= \frac{g_{I=0}}{2}-\frac{m_B}{E}+\mathcal{O}\left(\frac{1}{\lambda N_c}\right) \\
\xi\alpha+\beta\alpha q^2\left(\frac{M_0}{3}\right)\langle\rho^2\rangle &= \frac{m_B}{E}+\mathcal{O}\left(\frac{1}{\lambda}\right) , \\
-\frac{1}{\kappa_B}\left[\beta\xi-\alpha^2\left(\frac{M_0}{3}\right)\langle\rho^2\rangle\right] &= \frac{g_{I=1}}{2}\left[1+\mathcal{O}\left(\frac{1}{\lambda N_c}\right)\right] , \\
\hat{\alpha}^2+\hat{\beta}^2q^2 &= \frac{4x^2}{q^2}\left[1+\mathcal{O}\left(\frac{1}{\lambda N_c}\right)\right] , \\
x\left(\frac{q^2}{2M_0}\right)\hat{\beta}\alpha &= \frac{x^2}{E}g_{I=0}\left[1+\mathcal{O}\left(\frac{1}{\lambda N_c}\right)\right] , \\
x\left(\frac{1}{2M_0\kappa_B}\right)\hat{\alpha}\alpha &= \frac{x}{E}g_{I=0}\left[1+\mathcal{O}\left(\frac{1}{\lambda N_c}\right)\right] , \\
2x\left[\frac{M_0}{3}\langle\rho^2\rangle\hat{\beta}\alpha q^2-\hat{\alpha}\xi\right] &= \frac{x^2}{E}g_{I=1}\left[1+\mathcal{O}\left(\frac{1}{N_c}\right)\right] , \\
2x\left(\frac{1}{\kappa_B}\right)\left[\frac{M_0}{3}\langle\rho^2\rangle\hat{\alpha}\alpha+\hat{\beta}\xi\right] &= \frac{x}{E}g_{I=1}\left[1+\mathcal{O}\left(\frac{1}{N_c}\right)\right] .
\end{aligned}
\tag{B.1}
$$

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