

Rivista Italiana di Filosofia Analitica Junior 5:1 (2014) ISSN 2037-4445 © http://www.rifanalitica.it Sponsored by Società Italiana di Filosofia Analitica

ISRAELI SOCIETY FOR HISTORY AND PHILOSOPHY OF SCIENCE 14TH ANNUAL CONFERENCE

[Jerusalem, December, 22nd 2013]

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In spite of the fact that the conceptual, sometimes necessarily technical equipment of philosophy and of the vast majority of academic fields may elicit shy reactions in a larger public, the Israeli Society for History and Philosophy of Science (from now on: ISHPS) proves it possible for a series of lectures to come across to involve a vastly interdisciplinary framework. The extent of the Society's fourteenth annual conference, held on December 22nd, 2013 in West Jerusalem's Bloomfield Science Museum is impressively ample, involving five panels divided into six parallel sections for a total amount of 25 topics ranging from history, philosophy and sociology of medicine to theoretical physics, to the relationships between science, literature and visual arts. Each section was made up of five 30-minute talks, hence amounting to 125 in total.

Now, the present reportage, fairly enough for the purposes of RIFAJ, focuses on the research contributions in the metaphysics of science and philosophy of physics that were provided in two sections devoted to scientific realism, representation and the significance of settheory for physics. In the opening, Matthias Neuber (Tübingen) raises the historical question as to whether some accepted tenets of logical empiricism make the view incompatible with some metaphysical issues raised by scientific realism. Subsequently, Joseph Berkovitz (Toronto) and Boaz Miller (Bar-Ilan), investigate arguments for the existence and the nature of scientific facts. The former relies on mathematical structures as a significant ontological basis to support them and agues that if physical systems are to be represented in higher abstract mathematical patterns, then their identity is fixed by relevant mathematical properties. The latter analyzes and defends an argument of Ian Hacking for entity realism (ER), the idea that only experimentally manipulable entities exist, as an inference to the best explanation in conjunction with Hilary Putnam's contention that empirical success is better explained by truth than by a "miracle". Beginning the second section, **Elay Sheck** (Pittsburgh) tackles and contests on methodological grounds the recently debated paradox of phase transition, based upon the suspicion that some physical systems, while gradually changing their state, display contradictory properties. Following, a talk by Orly Shenker (Hebrew University) focuses on the role of set-theory and of sets of macro-states in classical and quantum statistical mechanics, arguing that they enjoy partial physical significance within certain ap-

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propriate modeling assumptions. Closing the afternoon section, **Danny November** (Hebrew University) discusses the appropriate conditions to define a notion of set-theoretic size that manages to avoid a contradiction, firstly discovered by Cantor, that arises from making of countability and cardinality the determinant of the size of infinite sets. He also draws upon intriguing connections between the theory of size and the physical applications of set-theory.

Finally, allow me to remark that hardly any trip I went to can match the breathtaking sight of the old city of Jerusalem, and so, I think, independently of one's religious affinity. The strenuous conquest march to the Holy Land is, less romantically, replaced with the commodities of excellent pubic transport and the awe-inspiring landscape frames can be (undeservedly?) enjoyed on large panoramic terraces. My visit to the old city's quarters has also been constitutive part of my attendance at Bloomfield Museum, as the whole conference was not very far from a spectacular and dramatic – a constant from Jerusalem's history – coexistence of clashing faiths.

0 Introduction. Scientific realism and representation

The debate on the existence of those entities that appear in scientific theories benefits from a relevantly standard contemporary framework, so that a short introduction should easily elicit a sufficient grasp of the present instances. Philosophers are said, in general, to be realist about a certain domain D if they argue in favor of the view that D is a (fundamental) ontological category, that is: exists in reality and cannot be traced back to- (or metaphysically explained by-) any other of reality's components. Philosophy as a discipline is acquainted with a number of domains for whose existence arguments have been cast. Here we are concerned with scientific items, and more precisely with those entities, called *theoretical*, that escape immediate epistemic grasp and though qualify as indispensable for certain characteristics of their theories: predictive power, ideological economy and logical consistency among others. Often, theoretical entities are equated with *unobservables*, that is: those entities, like fermions and quarks, that do not directly appear in experience or are "observed" in the content of experiment. The association is, strictly speaking, not thorough and orthogonal to the full range of theoretical entities, but we can take it to represent the target of scientific realist with good approximation.

Realists stand their ground relying on Hilary Putnam's (1975) contention that science matching mind-independent features of reality provides the best explanation for its overwhelming success, an explanation that is unmatched by any concurrent, which qualifies as "miraculous". The argument from abduction derived from this claim is nowadays known as "no-miracle" argument (NAM).

On the other side of the dispute stand those philosophers that account for science's failure to grasp any mind-independent features. Anti-realism is perhaps an even more variegated position than realism. Though, one issue turned against realists of any kind and developed by Larry Laudan (1981) and Arthur Fine (1991) unifies their opposers: unpredictable theoretical shifts in history of science account for change of reference of theories' terms, though the empirical power of these theories, which is what Putnam wants to abduct via the truth of realism, remains equal. Moreover, since we have no historical reason to believe that (some of) the currently accepted theories really constitute the final step of theory change, it follows that future theories will plausibly also not allow a realist interpretation. In other words, the sequence of theories in history of science and their continuous denial by means of new theo-

ries work as an inductive basis to claim that realism in the theories' ontology and semantics is never achieved.

Finally, to be somewhat more precise, scientific realism (and other realistic variants in general, see Putnam, 1990, Bartels and Stöckler, 2007) can be spelled out as a philosophically tenable position by associating it with three general *desiderata*.

- (O) Ontological realism. Theoretical entities exist independently of the mind's grasp of reality, and reality has a precise structure whose existence is independent of that of scientific theories.
- (S) *Semantic significance*. Sentences of scientific theories are true if, and only if, they match reality in content.
- (E) *Epistemic accessibility*. Some beliefs over scientific facts are reliable and true. Science is an epistemically successful enterprise that aims at- and partially succeeds in grasping truth.

All of these will recur under different forms within the lectures I proceed to present.

1 Is logical empiricism compatible with scientific realism? Matthias Neuber (University of Tübingen)

The conference's section on realism and scientific representation appears to open with an historical, if not directly exegetic, question. Or at least this could have been the impression of those who caught a glimpse of the program. But I personally know Matthias Neuber from my studies in Tübingen well enough to suppose with good precision that his attitude would not have been simply that of looking for evidences of compatibility between logical empiricism (LE) and scientific realism (SR), or simply provide a fresh interpretation, in realist terms, of the former. Rather, he strongly believes that historical enquiry is oriented towards solving present issues, to the extent that the discovery that logical empiricism is indeed compatible with a realist framework leads not only to an enhancement of the interest towards the former, but to the elicitation of further strategies in favor of the latter. The two main instances that Neuber raises in his talk are that of determining whether there are realistic claims in LE and whether it can adopt the contemporary vocabulary of the metaphysics of science, despite its being standardly conceived as a severely anti-metaphysical package.

In brief, his strategy consists in observing that some authors, *in primis* the Finnish philosopher and psychologist Eino Kaila (1890-1958) managed to combine some of LE's main tenets – chiefly the verificationist semantics – with theses that, if not directly imply, at least leave the door open to an ontology of scientific facts. To begin with, he observes that LE may be read as containing, especially in the work of Moritz Schlick, some claims that contemporary philosophers of science would deem as realist. Crucially, it appears to account for all three of the standard standpoints listed in $\S 0$: mind-independence, semantic significance and epistemic accessibility. More than providing a testimony of modifications of the philosophical vocabulary in the last century, the inclusion of (O) - (E) in LE signifies that the domain of anti-realism does not necessarily coincide with that of empiricism, as was traditionally suggested, and that empiricism is not sufficient nor necessary for- anti-realism.

Widespread interpretations (Niiniluoto, 1996) view LE as rejecting these principles, to the extent that metaphysical independence is a meaningless thesis, that theoretical terms ought to be reduced to terms referring to observables and that knowability is extended only to the phenomenal realm. In particular, if the latter requirement contradicts (E) in virtue of the sole definition of "empiricism", it directly follows that an empiricist position cannot be realist in the sense specified above. In other words, the virtual success of scientific realism should mark the breakdown, and not the re-evocation, of LE – an idea that Neuber calls the *strong incompatibility thesis*.

The most immediate way out of the contradiction consists in weakening (O) – (E) to make SR affordable for an interpretation of LE to qualify as realist, or at least as "quasi-realist" (Gibbard, 2003). Relatedly, Neuber recalls that Carnap and Schlick themselves are seen by some as defending a moderated form of "empirical realism", accounting for the existence of some theoretical entities that can reliably be derived from the content of experience¹.

A first attempt in this direction has been undertaken, according to Neuber, with Reichenbach's allegation to replace LE's verificationist semantics — which holds that a sentence p is true *iff* there is or there could be a method to verify p's content — with a probability theory of meaning, based on abductive inferences that allow a sentence containing theoretical terms q to be true *iff* it is highly probable that an inference from the content of an experience to the content of q is valid.

Neuber believes, though, that a similarly psychological justification of realist semantics falls down of determining with certainty the truth of (O), which for its part is meant as a necessary statement, should not be assigned any stochastic interpretation and should not account for any form of projective construction, based on the probability assigned to initial states of the theoretical world.

A second step was made in 1950 by Herbert Feigl, who criticized Reichenbach for not seeing that abductive justification in favor of probabilistic frameworks is implausible and argued that semantic realism (in our terminology: (S)) is compatible to Carnap's distinction between internal (theory-relative) and external question², according to which the realists' struggles on existence questions (e.g. on numbers, theoretical entities, values, etc.) easily turn up to make sense only within relevant theoretical frameworks, but are meaningless if addressed to the world itself.

A contemporary view that closely resembles Carnap's frame of interest is advocated by the realist Stathis Psillos, who also thinks that fundamental ontic questions are questions of framework, and there is no framework-free standpoint to decide what there is. Given this,

¹The label "empirical realism" dates back to Kant and refers in the *Critique of Pure Reason* (A369) to a view that "regards space and time as something given in themselves (independent of our sensibility) [...] represents outer appearances (if their reality is conceded) as things in themselves, which would exist independently of us and our sensibility and thus would also be outside us according to pure concepts of the understanding». Moreover (A370), the empirical realist "can concede the existence of matter without going beyond mere self-consciousness and assuming something more than the certainty of representations in me, hence the cogito ergo sum. For because he allows this matter and even its inner possibility to be valid only for appearance — which, separated from our sensibility, is nothing — matter for him is only a species of representations (intuition), which are call external, not as if they related to objects that are external in themselves but because they relate perceptions to space, where all things are external to one another, but that space itself is in us".

²See Hofweber (2011): «According to Carnap one crucial project in philosophy is to develop frameworks that can be used by scientists to formulate theories of the world. Such frameworks are formal languages that have a clearly defined relationship to experience or empirical evidence as part of their semantics. For Carnap it was a matter of usefulness and practicality which one of these frameworks will be selected by the scientists to formulate their theories in, and there is no one correct framework that truly mirrors the world as it is in itself».

an argument from the indispensability of theoretical entities follows to the extent that introducing unobservables is indispensable to have a consistent causal picture of the world. Just add to this the "weak" variant of (O) that to satisfy this requirement is to be real, and we obtain the philosophically significant upshot that the no-miracle contention NAM follows only at the condition that theoretical entities exist. Neuber believes that NAM has not been shaped to *prove* this conclusion, but to employ it as a premise. And the premise should be defended with naturalistic metaphysics along the lines of Ladyman and Ross (2007) asseveration that the aim of metaphysics is that of finding out valid and unifying generalizations for variegated and apparently incompatible scientific hypotheses. In this sense, following a recent suggestion by structural realists like Ladyman and Ross, Neuber observes that invariances are the most basic and privileged medium of bringing about unification of categorically different *phenomena*.

At this stage, it suffices to point to a logical empiricist who held the appropriate views about invariances to substantiate the talk's main point. This philosopher is Eino Kaila, whose anti-conventionalist theory of measurement (1942, 1960) accounts for invariance as being grounded in objective laws of nature and helping determine our assignment of non-arbitrary numerical differences to measured elements of physical systems. This should in turn inform the construction of real and fundamental structures³, thus configuring in a way similar to the constructional system of rules described by Carnap (1928) so as to obtain the fixation of psycho-scientific objects as disposed in a hierarchy of levels that moves from phenomenological (pure sensory impression determined via the relational system they appear into) to the theoretical.

The aim of exact science is to discover the higher invariances of the domain of experience in question. We shall show that "physico-scientific reality" (as to its content) consists in nothing other than the system of higher invariances of the everyday physical world and thus (in the last analysis) "immediate experience". (Kaila, 1942, p. 152), cited in (Neuber, 2013, p. 363)

The "real" is what is in some respect (relatively) invariant. [...] [P]hysico-scientific reality, which is represented by the system of real-descriptions, is in logical respects the highest reality we can attain. The disclosure and representation of this reality – and not, say, the "analysis of sensations" – is the aim of physical research which determines the formation of its concepts and theories. (Kaila, 1942, p. 185), cited in (Neuber, 2013, p. 369)

I believe that two of the many questions emerged in the commentary section are particularly relevant to frame Neuber's stance within the qualification of LE as a realist position. Firstly, in Neubers' reading, Psillos' defense of SR goes through only if there is a fact of the matter as to whether humans have good reasons to suppose that a causally consistent conception of the world is required, for this is what is actually achieved by postulating the existence of theoretical entities. Despite this, there are some philosophers of physics who believe that consistent causal pictures are essentially local and do not account for the whole of the natural world (for instance, see Torretti, 1997, ch. 3), with selected surveys into the history of science to epitomize this. Consequently, the thesis that NAM qualifies as a framework response at the condition of- and not to the conclusion that entities are there and this is justified, is

³See Neuber (2013, p. 362): «Kaila asserts that knowable reality must be equated with the realm of invariances, or, as he himself contends, "The 'essence' of a thing consists of the invariances of this thing" (Kaila, 1942, p. 228).»

blocked due to insufficient information in the premise that theoretical unity suffices for the acceptance of unobservables' existence.

Secondly, the appeal to the naturalistic procedure of hypothesis unification is correlated in the literature and even by Ladyman and Ross (2007) with physical reductionism. One of the main criticism against the naturalistic reference to the priority of physical explanations is that of neglecting the ontological import of further sciences, *in primis* social disciplines and non-directly physical sciences. Though it may be that some invariances regulate the measurement practice in the physical context, do these suffice to account for and establish larger domains of invariance?

2 Mathematical constitution of physical facts Joseph Berkovitz (Toronto)

Mathematics qualifies as indispensable for the theory-generating affair of theoretical physics. A pressing question arises, though, as to whether the explanation of concrete physical facts' identity involves mathematical properties at all. Berkovitz starts off by depicting *descriptivism*, a view according to which physical facts are not in the first place mathematical because mathematics as a language is eligible only for descriptive and not constitutive aims. The mathematical and the physical are addressed by this view as two incompatible categories, whereby between the two realms lies an ontological fracture that allows at most to correlate them, but not to ground the one in the other.

Apparently, we make use of similarly descriptive means in using geometrical properties to characterize ordinary objects. For we tend to say, for instance, that a lamp is parabolic (like a parable) and not a parable, that a spring roll is cylindrical (like a cylinder) and not a cylinder. Minimal scientific literacy, though, instructs us that abstract mathematical structures represent concrete physical systems. And of course the notion of representation is open to different intuitive characterizations, matching both descriptivism and its concurrent. Berkovitz believes that once we get into serious philosophical contemplation of the representation issue, every view of how this phenomenon takes place, even instrumentalism, commit us to the constitutivity of mathematics, this meaning that mathematical properties are embedded in some of the most fundamental features of reality: no representation without constitution. To begin with, notice that the simple fact that physical systems display abstract mathematical patterns does not suffice at the semantic level to claim that physical facts have constitutive mathematical properties⁴, since the explanation of this phenomenon is equally compatible with descriptivism. But we can agree with Berkovitz in holding that this pre-condition is at least necessary to support ontic explanations of mathematical representation.

2.1 The applicability of mathematics to the physical

The most commonly widespread account of how mathematical structures match physical facts is called the "mapping account" (Pincock, 2004; Bueno and Colyvan, 2011) and develops along the lines of a theory of correspondence⁵: portions of the mathematical models show structural

⁴See (Tegmark, 2008, 2014) for contemporary examples.

⁵See also Pincock (2010): «In the first stage, the mathematical domain is identified with a particular abstract structure. Then, in the second stage, applications such as counting are explained in terms of structurally specified mappings between the objects in some non-mathematical domain and the elements of the mathematical structure. For example, counting objects can be thought of as establishing a one-one correspondence between the objects to

similarities with physical systems. That is, they represent physical facts because there is an appropriate mapping between a structure and a system. The most intuitive way to think of this representation is to associate each concrete object $o_1, \ldots o_n$ of the target system O to a multiply-realizable variable x_1, \ldots, x_n and then build an abstract structure X of these variables that mirrors the relational pattern of O. The relationship of the concrete system to the abstract structure is then comparable to that of a token to its type. Physical systems that differ in the identity of their elements but respect the same arrangement pattern define what is called an "isomorphism class".

This schema does still not suffice, though, to establish mathematical realism. For although correspondence theories of truth are generally seen as a straightforward mark of the existence of the representation target, and here this role is certainly played by the physical systems, mathematical representation is still compatible with descriptivism and does not necessarily identify any property of its same kind in that which corresponds to, by virtue of solely corresponding to it. In other words, physical components of the system can be intuitively appointed as relata of the corresponding relation without thereby being committed to treating them as mathematical. One motivation here, on which Berkovitz does not touch upon directly, is that represented physical objects are not sufficiently explained by (or do not supervene upon) mathematical properties and that even if the latter can be pinned down to the properties constituting an isomorphism class, it is implausible to build-up a relation of constitution from a similarly poor and coarse-grained patch of properties (e.q. the cardinality of the abstract structure and the mathematical properties of structural relations) and the physical system. If the picture of type and token is a satisfying one to describe this relationship, then too many instantiation possibilities of a physical system over an abstract structure make the mathematical properties too gerrymandered and thus irrelevant for the purposes of setting out an identity-fixing relation between the mathematical properties and the system. This form of underdetermination of concrete systems over commonly shared abstract structures is known in the literature⁶ as the Newman problem, and is due to M.H.A. Newman's (1928) response to Bertrand Russell's (1928) reconstruction of relativity in analogously structuralist terms⁷.

2.2 Common wisdom about the role of mathematics in science

Berkovitz's response against the matching account consists in a re-qualification of the notion of mapping. In particular, he insists that both concrete and abstract structures ought to be further qualified by some natural properties that allow matching to work between structures that "share" not only an isomorphism class, but also a common "nature". In brief, his allegation is that in order for the representation to qualify as precise, there must be a commonality of the properties shared by both of its two sides (the mathematical and the physical structures), and that these are mathematical.

The notion of the physical is often disregarded and assumed as obviously clear, but this is a

be counted and an initial segment of the structure of natural numbers. Other applications for other domains may involve different kinds of mappings».

⁶See Demopoulos and Friedman (1988).

⁷For an analogous formulation, see Reichenbach (1916, p. 127) (quoted in Padovani, 2011, p. 5): «While mathematical judgments determine variables in such a way that they are the same for all their individual objects in all places at all times, the variables in a physical judgment are not equal for all individual objects in their class, but rather subject to a law of distribution in space and time. Instead of the general validity of mathematical claims, we have in the case of physical judgments the subsumption under the law of distribution».

mistake. In the practice of physics, its domain is *determined* by an experimental framework, whereby some of its general fundamental features are in fact mathematical. For instance, sometimes the object under enquiry in experiment are quantities by their very essence, and are identified as for the kind they belong to thanks to their displaying a certain position in the measuring scale of their order of magnitude⁸. So the issue of applicability of mathematics to physics draws relevant advantage from the deeper idea (with regard to the matching account) that only similarly mathematical elements of reality can match abstract structure to give a sufficiently precise representation.

2.3 What is fact constitution?

A relation of constitution is hence required in the process of metaphysical explanation targeting concrete systems, to the extent that the latter would not qualify as such if their identity were not (at least necessarily, if not sufficiently) explained by a bunch of mathematical properties. Two different frameworks for mathematical representation are in charge of clarifying constitution: the Pythagorean and the neo-Kantian, for which Berkovitz does not express an immediate preference. Aristotle (*Met.* A5, 986 b3-29)⁹ appears to ironize on the Pythagorean belief that physical facts are *essentially* mathematical. This solution is embraced, though, by a great lot of contemporary physicists and leads straightly to mathematical realism.

According to the Neo-Kantians, on the contrary, science describes phenomena in a mathematical fashion because this is necessary as a form of our understanding, although it remains hopeless to raise the question as to whether the properties that we ascribe as encoded into the physical systems account for some fundamental aspect of reality. This behavior reflects a typically Kantian approach towards the exclusion of metaphysical properties from ontology and a conservation of their part in our mind-dependent epistemic grasp. In other words, although these are mathematical in virtue of our categorical necessity to think of them as mathematical, this does not alone imply that *there is* something as for the of reality unabridged by the categories which is by itself mathematical. Rather, we are (and ought to be) left in the darkness as for providing a final answer to the question. And more roughly, the physical reality we discuss about is a construction made in mathematical terms 1^{10} .

To sum up, Berkovitz thinks that for the only purposes of mathematical representation, the scientist that developed the relevant set of physical facts according to certain mathemati-

⁸See again the references to invariances in the process of measuring described in §1.

⁹«The so-called Pythagoreans, who were the first to take up mathematics, not only advanced this study, but also having been brought up in It they thought its principles were the principles of all things. Since of these principles numbers are by nature the first, and in numbers they seemed to see many resemblances to the things that exist and come into being [...] all other things seemed in their whole nature to be modeled on numbers, and numbers seemed to be the first thing in the whole of nature, they supposed the elements of numbers to be the elements of all things, and the whole heaven to be a musical scale and a number [...] and whatever characteristics in numbers and harmonics they could show were in agreement with the properties of the heavens and its parts and with its whole arrangement, these they collected and adapted; and if there chanced to be any gap anywhere, they eagerly sought that the whole system might be connected with these (stray phenomena). To give an example of my meaning: inasmuch as ten seemed to be the perfect number and to embrace the whole nature of numbers, they asserted that the number of bodies moving through the heavens were ten, and when only nine were visible, for the reason just stated they postulated the counter-earth as the tenth. [...] They certainly seem to consider number as the first principle and as it were the matter in things and in their conditions and states; and the odd and the even are elements of number, and of these the one is infinite and the other finite, and unity is the product of both of them, for it is both odd and even, and number arises from unity, and the whole heaven, as has been said, is numbers».

¹⁰See Padovani (2011, p. 5): «[A] two-step "con-structional" interpretation, in which the two levels, the set of formal assumptions and the set of empirical (approximated) data, cooperate in order to form a solid ground for scientific knowledge».

cal patterns is thereby committed to the existence of mathematical properties that constitute these facts in a relevant sense. He leaves the question open as to whether the explanation of this constitution ought to be Phythagorean. Naturally, there are substantive differences between the two positions, whereby the acceptance of Phythagoreanism is seemingly brings about a number of stronger consequences.

A brief comment is thus in order. Assume that we succeeded in modeling a Phythagorean relation of constitution that is wholly identity-fixing for the physical facts, so that the existence of a fully "Phythagorean universe" (Tegmark, 2008) is accounted for. Combining this mathematically realist perspective with Berkovitz' approach, we are forced to accept a certain amount of mathematical entities, presumably those displayed by the best physical theories we have available. Quine (1960, 1981) and Putnam (1979) notoriously argued that mathematical commitment is indispensable in one's ontology if one accepts being committed to all and only the entities that figure in the quantificational domain of our best theories (§3)11. A problem could be that, when theories are considered with empirical power and explanatory range on a par, as is intuitively possible, but display divergent (if not even contradictory) mathematics, it seems that also a bunch of physical facts, whose existence we are entitled to support, display these inadmissible properties. Of course a more detailed justification of why (or even whether!) we should opt for the Pythagorean case and of which among many mathematical properties are those we ought to allow in our ontology is in order. And of course, having the conference's talk a fixed duration and scope, this cannot be appointed as a deficiency in the original account but only as one of its further lines of expansion.

3 What is Hacking's Argument for Entity Realism Anyway? Boaz Miller (Tel Aviv)

Boaz Miller enquires into a notorious argument in favor of a weaker realism given by Ian Hacking in his *Representing and Intervening: Introductory Topics in the Philosophy of Natural Science* (1983). He identifies it and defends it as an inference to the best explanation which receives positive enhancement from Putnam's (NAM). Pointing out the debate framework, he interprets the structure of NAM as a *dilemma* for the anti-realist: either miraculous affairs or truth can explain the overwhelming success of science. Of course, this remodulation does not make the argument immune from Laudan's criticism that (translated accordingly) the first horn has been proven plausible by simple enquiries in the history of science (for all theories, including actual ones, achieve empirical power but no truth-matching) and, above all, that the systematical success of the first horn should lead one to believe that no theory really achieves truth, on pain of accepting that every empirically adequate theory, even those that have been proven false, are abductively true.

Miller follows Charkravarrty (2007) in labelling his response strategy against PMI *selective skepticism* and depicts it as the attempt to build a distinction into empirically powerful theories, with the purpose of identifying those which tend to be preserved and those that are rejected by a scientific community.

¹¹See also Cole (2010): «A collection of entities is indispensable to a scientific theory if and only if, when that theory is optimally formulated in canonical notation, the entities in question fall within the range of the first-order bound variables of that theory».

As in life generally, so too in science: do not believe anything that you are told. Not all aspects of scientific theories are to be believed. Theories can be interpreted as to making many claims about the nature of reality, but at best one has good grounds, or epistemic warrant, to believe some of these claims. Only some aspects of theories are likely to be retained as the sciences march on. (Charkravarrty, 2007, p. 29)

Notice that this is only one of the many instances of the realism-preserving strategy developing along with selective skepticism¹². A large variety of recent approaches in philosophy of science embody a selection criterion for true theories, once empirical strength is set on a par. Among these we find (1) structural realism (the idea that mathematical relational systems and isomorphism classes are preserved in theory change, whereas theoretical ontologies are lost); (2) semi-realism (the idea that some causal properties link the existence of theoretical entities to our true belief in them, see Charkravarrty, 2007); (3) whig realism (the idea that the truth of an empirically successful theory is detected with temporally successive combination of the same theory with other theories); (4) critical scientific realism (mainly rejecting (S) and re-qualifying scientific realism's relationship to truth as avoiding to assume correspondentistic lines). Among these selectively skeptical responses lies also Miller's target of enquiry, entity realism (ER), accounting for the primacy of experimental entities in the realist's categorical target. The notion of an experimental entity has been introduced by Hacking to denote all of the components of a theory with which scientists can achieve a causal connection (e.g. they can manipulate them) through experimental practice. The slogan that accompanied his approach's foundation, "If you can spray them [electrons], they are real", became largely influential. Nevertheless, Miller recognizes it as worth-noticing that a similarly structured defense of ER radically narrows the scope of realism's domain, and as such may in the last analysis leave a fringe of strong realists unsatisfied.

Miller's asseveration, at this point, resides in Hacking's discourse being elusive on the specification of the argumentative form employed. Accordingly, he draws out three possible interpretations differing in how strongly the epistemic warrant for the conclusion is carried out. By arguing that one of these, namely that ER is justified via an inference to the best explanation, succeeds in providing solid warrant, ER will also be advocated with a plausible argument.

To begin with, Hacking conflates two separate issues: the type of scientific activity (*i.e.* laboratory experimental practice) that supports the existence of electrons, and the argument that must be given in order to account for the existence of electrons, namely IBE as opposed to something else. This suggests that merely exercising some leverage over the fact that scientists manipulate experimental entities does not suffice to qualify as an argument to the extent that they are real. Indeed, Miller argues against the interpretation of Hacking as providing "no-argument", that the following deductive inference fails due to insufficient premise information:

No argument interpretation (NAI)

(A) Contemporary experimental physicists' success in manipulating experimental entities in order to study other phenomena is much more impressive than,

¹²See Rickles (2009, p. 262): «The realist will be sceptical about just those aspects that are left behind in such changes, retaining commitment only to that which is retained or recoverable (in some sense) from the successor theory».

and different in kind from previous empirical successes in the history of science.

- (B) If experimental entities can be manipulated, than they are real.
- (C) Thus, ER is true.

The arguments' failure is given, I suppose following Miller's employment of inferences to the best explanation, by the fact that no real qualification is given in support of (B), and that this qualifies as false. Thus, Miller insists that Hacking's intuitions nonetheless, the question is still open as to why ER should be true.

ER as an inference to the best explanation (IBE)

- (1) Contemporary experimental physicists' success in manipulating experimental entities in order to study other phenomena is much more impressive than, and different in kind from previous empirical successes in the history of science.
- (2) ER is the only philosophy that doesn't make this success a miracle.
- (3) Therefore, ER is true.

IBE is Miller's favorite candidate to both support the interpretation of Hacking and, consequently, to mount up a proper defense of entity realism. He confesses that a proper commitment to this argument is hardly traceable in Hacking's texts, but he also remarks that the few passages in which Hacking mentions a "direct' proof" to the conclusion that electrons are real are particularly significative¹³. The main point in favor of IBE as the most favorable reading of ER's justification, though, is its epistemic superiority with respect to the other two proposed interpretations.

As first, Miller proposes to read ER as generated by the rational behavioral indispensability to think of experimental entities *as existent* once their manipulation is possible.

ER as an indispensability argument

- (4) For the purpose of experimentally studying other phenomena (as opposed to other scientific activities), experimental physicists must treat electrons as real, on pain of the consistency of their behavior.
- (5) Therefore, scientists are rationally compelled to believe that they are real.

Similar argumentative forms are common (§2) among mathematical Platonists and moral realists, and share certain features with inferences to the best explanation, to the extent that IBE qualifies as a special case of an indispensability argument. Crucially, however,

¹³ See Hacking (2012, p. 757): "The argument [...] is not that we infer the reality of electrons from our success. We do not make the instruments and then infer the reality of the electrons, as when we test a hypothesis, and then believe it because it passed the test. That gets the time-order wrong. By now we design apparatus relying on a modest number of home truths about electrons to produce some other phenomenon that we wish to investigate"; (Hacking, 2012, p. 763): "Anti-realism about atoms was very sensible when Bain wrote a century ago. Anti-realism about any sub-microscopic entities was a sound doctrine those days. Things are different now. The 'direct' proof of electrons and the like is our ability to manipulate them using well understood low-level causal properties [...]. The best evidence for this kind of understanding is that we can set out, from scratch, to build machines that will work fairly reliably"; (Hacking, 1983, p. 271): "Once upon a time it made good sense to doubt that there are electrons. [...] Once upon a time the best reason to think that there are electrons might have been success in explanation [...] I said that ability to explain carries little warrant of truth. [...] Luckily we no longer have to pretend from explanatory success (i.e., from what makes our minds feel good)".

there exists a difference within the epistemic warrant that the two assign to their conclusion. Under the indispensability interpretation, the argument ascribes to the scientists' behavior a certain rational compellence to think that some entities exits because they are manipulable. This makes the conclusion substantially weaker than its concurrent. First of all, that a scientific community comes to *believe* that electrons exist is undesired in the face of ER's thesis that electrons exist *simpliciter*, and are not simply part of the content of someone's beliefs that pictures them *as existent*. Furthermore, given that not everyone is trained to perform experiments nor to take actively part in the scientific community, one does not see why everyone should be required, following this argument, to believe in the existence of electrons – the upshot being that indispensability arguments do not manage to make their conclusion inter-subjective enough to respect a fully realist account. Miller insists that the argument's employment of the term 'we' is ambiguous as it does not specify whose behavior exactly should be compelled to elicit a belief in electrons. Subsequently, he reads Hacking as proposing a transcendental argument targeting the conditions of possibility of experimental manipulation:

ER as a transcendental argument

- (6) Contemporary experimental physicists' success in manipulating experimental entities in order to study other phenomena is different in kind from previous empirical successes in the history of science and is part of their content of experience.
- (7) *Transcendental premise*: we can establish a content correspondence between one's experiences and the features of reality that make these experiences possible, thus making the counterfactual: "necessarily, if there weren't at all experimental entities, experimental physicists' success would not be possible" true.
- (8) But the experimental physicists' success is possible (and indeed, actual).
- (9) So, experimental entities exist.

A transcendental argument moves deductively from the content of certain experiences to the state of affairs that obtains if this experience is to be possible. In this picture Hacking is seen as describing the context of experimental practice with the aim of establishing that the existence of experimental objects is one of its necessary conditions. Moreover, the argument's kernel presents a Kuhnian vein, for it should follow from it that necessary conditions for the possibility of experimental practices are also necessary for the preservation of reference from one paradigm to the next.

The transcendental interpretation, however, is only a sketch and requires to immoderately force Hacking's text. Moreover, there exists a general problem as to whether similar arguments manage to transcend the gap from experiential content to actual features of reality. According to Barry Stroud (1969), for instance, transcendental arguments do not establish that the necessary condition is true in actuality but only that it is (necessarily) believed by us as true, and as such is susceptible of further denigration by the skeptics. In other words, the conclusion of the argument is «'[M]odest', because its transcendental claim extends only to how [experience] must appear to us, and so is not 'world-directed'» (Stern, 2011). Henceforth, given the failure of the latter alternative readings, Miller is allowed to conclude that IBE succeeds on epistemic evaluative standards and qualifies as the most adequate sketch of an argument in favor of entity realism.

4 Phase Transations and Scientific Representation Elay Sheck (Pittsburgh)

Elay Sheck presents a complementary talk with regard to Berkovitz' (§2), bringing into focus a paradox that arises from the representation relation between the mathematical and the physical. Many authors came to believe that contradictory properties are displayed by the phenomenon of phase transition (PT) – the progressive replacement by concrete bodies of different properties linked to their physical state, such as those displayed by a gas condensing into liquid – and its mathematical representation. Whereas the mathematical properties of the transitioning system change abruptely and discontinuously, being modeled by a continuum function f containing a discontinuity f that is differentiable only at the thermodynamic limit, the macroscopic properties change slightly as the system switches its physical state. For instance the representation of a particular gas' pressure in terms of its volume abruptly drops at f(f), although there is no similar discontinuity in the physical properties displayed by the system. As we will see in a while, the properties that the system is required to possess if f is infinitely differentiable are incompatible with how the system is initially presented.

Now, two questions arise as to this divergence of the representational system from the observable phenomenon. First (1), is there a paradox of phase transitions, and are there irreducibly emergent phenomena, which necessitate novel explanatory scheme? And If so (2), is there something philosophically interesting? Sheck wishes to explain that no paradox is present in the first instance and that appearances to the contrary are guided only by further, highly disputable philosophical assumptions – the upshot being that the interest of the problem concerns issues of how concrete phenomena are represented, not their instantiating contradictory properties.

The paradox of phase transition runs as follows:

(PP) Paradox of phase transitions

- 1. The mathematical representation of a phase transiting concrete system requires a system with infinite degrees of freedom that is modeled by a discontinuous non-linear function (*i.e.* a function that is not infinitely differentiable if not at the thermo-dynamic limit, see Yang and Lee, 1952).
- 2. A system that is infinitely differentiable at the thermo-dynamic limit has infinite particles.
- 3. The process is observable and describable as happening in finite systems (e.g. boiling pots).
- 4. But concrete physical systems have a finite number of particles.
- 5. Thus, phase transitions are only reducible to infinite systems, although they emerges from finite systems, and cannot be derived unless we assume that the system is infinite (Callender, 2001).

The difficulty that follows is not worth the name of a paradox if we set concrete systems – representing particles and their properties – and their abstract representations as belonging to different categories (notice the detachment from Berkovitz' account of mathematical constitution), and allow the latter to constitute a *faithful representation* of the former¹⁴. In this

¹⁴See Contessa (2007).

case we are always allowed to make inferences about the nature of the representational target and, consequently, possess a reliable guide to the ontology underlying a physical theory. A faithful representation is comparable to a map, in that it permits its reader to understand in an inferentially significant way that the world is so as it depicts it. From this standpoint, then, the contradiction arises in PP if, and only if, the abstract representational structure is not faithful with respect to its target, for instance due to its being too much "an artifact of an idealization (or an approximation)" Subsequently, the argument reduces to the following form:

(PP)' Phase transition paradox from representation (Sheck, 2013, p. 1176)

- 1'. A concrete system C includes a concrete attribute A and displays a concrete phenomenon P.
- 2'. P arises to an idealizing limit I.
- 3'. $I \rightarrow A' \neq A$.
- 4'. P' faithfully represents P.
- 5'. C has A and A', that is: A and $\neg A$.

Moreover, an ambiguity in the purported paradox arises if we consider that the argument does not make clear whether concrete systems really possess abstract properties in virtue only of their being represented by abstract structures. A further premise, for which severe philosophical justification is in order is 6': "If P' faithfully represents P, then C has and A and A'". Indeed, there is no paradox if infinitary abstract properties are instantiated by the abstract system exclusively. In other words, with the help of an indispensability argument to the extent that commitment to abstract properties is necessary if needed in scientific representation — and more generally, inescapable in a scientific account of the concrete world — is it possible to re-evoke a significant contradiction within PT. But this does not refer anymore to an apparent clash in the practice of physics, as rather to a point that is worth much more unstable philosophical discussion.

Appealing to indispensability on the sole basis of a realist intuition, however, is utterly mysterious if the relation of correspondence between representation and reality that is accounted for by 6' is not identified. This amounts to finding out the kind of idealization abstract structures rely upon in their representation work. Within this framework, still assuming that the representation's features are reliable as a guide to the ontology, the paradox arises if every "de-idealization" of the abstract structure that does not contain infinitary abstract properties is no-more an appropriate representation of PT. The purpose of Sheck's talk is not to argue in favor of this requirement, but rather to show that instance as to the emergence of the paradox are properly the domain of philosophical discussion. Though, he suggests that the work of different authors working in this direction can be employed to further support his conclusion. Among these, he mentions, but not straightforwardly accept, Butterfield's (2011) and Norton's (2012) idea that PT's mathematical representation, though being ontologically reliable, it is only irreducibly approximately so and qualifies as an appropriate idealization only in the limit, thus letting physical attributes qualify as more fundamental.

¹⁵Sheck (2013, p. 1175): «The source of the problem of PT seems to be that the mathematical structure that scientifically represents concrete PT – a discontinuity in the partition function – is an artifact of an idealization (or an approximation), the TLD, which is essential in the sense that when one "de-idealizes" said idealization, the mathematical structure representing PT no longer exists».

5 The Status of Sets in Physics Orly Shenker (Hebrew University)

The following talk appertains to the interpretation of statistical mechanics and the employment of set-theory that was employed in this discipline to characterize the stochastic development of physical states over time. Classical mechanics treats particle states $S_1, \ldots, S_j, \ldots, S_n$ as the value of a function $f(t_j)$ over time instants that is meant to model the systems' evolution in their phase space. The interesting cases happen to be those in which the target system coincides with the whole universe, constituting what Shenker calls a "macro-state". Here as before a function takes into account time instants along the universe's evolution and provides values for a complete and informative description of its configuration. Its high informativity is given by the fact that macro-states are defined as the particles' configuration at a time and that no further information is produced by $f(t_j)$ aside from what is firstly available in the basic configuration, so that nothing is altogether lost or ignored. Also, the value of a further function μ that links the informativity of two states located at different times and expresses their overlap is fixed dependently on experimental and measurement results. This expresses the idea that knowledge of the state partition to a future time t_{j+m} is a posteriori.

It is though a more recent discovery that fruitful results in terms of phase space modeling, still lying within the boundaries of classical mechanics, are achieved in the absence of a similarly large amount of information. In fact, when the number of particles is sufficiently large, relatively simple observable regularities appear, whose representation calls for relatively limited information. Sets enter the picture here and are required as collecting states in terms of some relevant properties that they share. We know that basic set-theory involves an axiom of comprehension that allows to select a (limitedly) complex property P and make it pick out the set $\{x \mid Px\}$ of those xs instantiating P. As for its encoded information, then, a set of states ought to be shown more supportable than a group of states. But how can this be? An inference to the best explanation can be easily cast to the extent that there are facts in the world that correspond to sets constructed with the comprehension axiom, something along the lines of Lockean "real essences" matching certain configuration of relevant statistical properties. So it appears after all that even sets of states appear to be overly informative.

As a response, Shenker asks us at this stage to imagine Ludwig [aka. Boltzmann], a gifted scientist who knows and remembers all of the information contained in the macro-states with unlimited computational capabilities; virtually an incarnation of physics. Suppose we asked him whether we, mortals, are going to experience a macro-state configuration of the future universe with certain thermo-dynamical features P associated with a partition $\phi(p)$ of the relevant set dictated by P by extensionality. If we also reveal to him the sets $\langle T_1, \ldots, T_n \rangle$ that correspond to the states which we presently observe and their initial conditions, then he will answer that if the universe happened to belong to $T_j \in \langle T_1, \ldots, T_n \rangle$ and $\phi(p)$ is derivable from T_j , then we are going to experience P. Indeed, knowledge of the sets' characteristic property is sufficient to predict the following states given a certain instant of time. However, there is a problem as to how he answers the question of partitioning the relevant state distributions into sets by selecting P. For even though Ludwig prevision faculties are acceptable, the partition itself is an additional fact and goes beyond Ludwig's embodiment of physics and its fully provided information.

The issue remains that physics contains a non-physical fact about shared properties and set membership. But there is no need to reject it, for Shenker believes that a non-mysterious move is available. Ludwig can appreciate the partitions in two ways, as we want him to describe how humans experience the rest of beings. Since it is exclusively our ignorance that expresses the present difficulty to partition sets, it will suffice to represent humans as physical entities in a relevant state and thereby to introduce observers as part of the picture. Human beings interact with their environment as a matter of actual fact, and this combination creates the ignorance by accounting for a one-to-many interaction which is not epistemically *a priori*, but precisely derived from the act of measurement.

6 Mathematical concepts of size Danny November (Hebrew University)

The talk that closes the afternoon section considers the importance of sets for the determination of equinumerosity relations, counting, and size. November initially shows, relying on Cantor's well-known observations on infinite sets, that the acceptance of a couple of traditional set-theoretic principles generates a contradiction:

(WP) Whole-part principle

For all sets A and B, if B is a proper subset of A, then A is larger (*i.e.* has a greater size) than B.

(SSP) Principle of size (Correspondence principle)

All sets A and B have the same size if, and only if, there is a bijective function f such that for every element $a \in A$ there is a $b \in B$, f(b) = a and there is no $c \in B$ such that $f(c) \neq a$.

Cantor was lead to reject WP in virtue of the fact that bijective functions could be assigned between an infinite set, such as the set of natural numbers, and some of its proper sub-sets, such as the even and the odd numbers, so that the latter should be thought as having the same size of the former. Note that Cantor's argument does not target the case in which infinite sets are generated as a power-set P(A) of A. For a well-known theorem (proven by Cantor himself) shows that the cardinality of P(A) equals 2^c , with c being the cardinality of A, and thus that, although A qualifies as a proper subset of P(A), no bijection can be established between them.

In the last analysis, Cantor accepts that A and $B \subset A$ can be appointed with equal size, but this proves to be only one out of many ways out of the contradiction. In particular, a different concept of size heading away from cardinality and counting was recently modeled (see Benci and Di Nasso, 2003) allowing the two sets to differ. November shows that the size notion to which we appeal cannot be cardinality by means of epitomizing algebraic examples. For example, generating a sigma algebra Σ with a function G(B) defined on a collection of sub-sets of a set A may result in Σ being the smallest generable algebra, although its smallness does not depend on the cardinality of the sub-sets appearing in G(B)'s image, but, more generally, on the values that the function assigns on them¹⁶.

The contradiction generated by WP and SSP can be tackled more clearly by implementing two further principles, which we can call respectively the conditions of "uniqueness" and

 $^{^{16}}$ The value assignment of an algebra-generating function to subsets of A is called a "measure" of A. The conceptual extent of measure, although it does not match up with cardinality, amounts to displaying some intuitively advisable features of size, such as its operational properties (e.g. size addition is transitive, distributive and associative).

"existence" of a set: (1) every set has a size; and (2) no set has more than one size. Accordingly, four options are available to solve the contradiction that A and B are claimed to be the one larger than the other, although it is not so.

Firstly, one could follow Cantor in rejecting WP and biting the bullet that it is not sound anymore to claim that A and B are different in size, because a bijection can be framed that associates each element of A to exactly one of B, and this suffices to assign them equal size. November rejects this solution to lay claim to the intuitive idea that naturals and evens differ in size.

A second way out of the contradiction involves rejecting SSP. This is the solution embraced by those following the contemporary evolution of the concept of numerosity and accounts for *A* and *B* really having different sizes, *even if* equal cardinality.

Further, the attempt to reject the uniqueness principle (2) countenances the attribution of more size values s_1, \ldots, s_n to sets simultaneously. Accordingly, it turns out that a size s_1 of the naturals is larger than its s_2 , which is the size of the evens. The strategy seems sufficient to solve the contradiction, but it also sounds highly counterintuitive. Supposedly, it displays a certain relevance for some physical applications, as there are for instance in general relativity measurable sets collecting points of space-time whose size is different due to the lack of a uniquely defined notion of simultaneousness, and depending on the assigned reference frame. Other examples can be derived from probability spaces with more than one variable, as described in §5. November observes that the difference between cardinality notions is overlooked in physics, so as to render the acceptance of a similar conceptual revision methodologically viable. On the other hand, however, assumptions regarding cardinality and ordinality are crucial in foundational physics, making it quite common to reduce measuring to counting in systems.

Lastly, the failure of (1) leads us to admit that some sets do *not* have a size at all, and for which any proposition regarding their size is false or meaningless. Clearly the lack of a clash in truth-values makes it the case for the contradiction's avoidance. But some deem this solution as highly counterintuitive or even absurd. After all size is crucial to qualify the sets' identity through the axiom of extensionality and qualifies as necessary to distinguish them not only in virtue of their elements, but also of their largeness.

To sum up, the concept of a size has been shown to rely upon a collection of variegated notions: a principle regulating a parthood relation between sets and sub-sets, the possibility of building element-linking bijections, the uniqueness of the size-property (perhaps connected to the non-ambiguity of the term 'size'), and the assumption that sets necessarily have sizes (which possibly tells them apart from classes). Apparently, only a selection of a proper sub-set of these principles can maintain the notion's consistency. The original Cantorian suggestion that some sets are equinumerous in virtue of their displaying equal cardinality fails, so that some research towards conceptual re-qualification is needed. Two candidates qualify as primarily plausible: that sets have a unique size which is not determined directly by cardinality; and that they have more than one size. The latter especially displays a number of interesting connections with the employment of set-theory in theoretical physics.

I would like to provide a couple of positive remarks over the two suggested solutions, partially relying on the commentary section that followed the talk. As first, the option of rejecting SSP is pleasurably viable with respect to the paradox' elimination. When two infinite sets A and B with $B \subset A$ are compared as for their sizes, but they display a difference in the kind of elements that they contain, there still remains the possibility to provide a metaphysical explanation to the extent that the difference in the sets' identity grounds, in some sense, the

difference in size. It can be urged that the identity of the set of the naturals, in some sense, "contains" *all* of the identity of the set of the even numbers but "goes beyond" it, for it would just be the set of the even numbers if there were no identity differences in its elements and, also, every further information that we can predicate of it once we recognize that the evens are a sub-set of it *surely* does not mention the evens. And since it this discrepancy in the sets' identities that one must explain, it follows that it cannot be that the naturals are like the even in size, if set-theoretic identity is determined extensionally¹⁷.

Secondly and concerning the denial of uniqueness, the range of compatibility that November solution displays with physics might be even larger than expected. With special regard to the application of set theory to quantum mechanics, there is reason to believe that the Cantorian notion of size will not suffice to completely characterize those systems – typically superposed states of fermions and bosons – in which the elements do not qualify as individuals by violating the principle of identity of the indiscernibles $(\forall x \forall y (\forall F \ Fx \leftrightarrow Fy) \rightarrow x = y)$ and the law of identity $(\forall x \exists y \ x = y)$. The developers of a so-called "quasi-set theory" (Da Costa, 1980; Krause, 1992; French and Krause, 2006) explicitly recognized the elements being *discernible* individuals as a necessary condition to define cardinality in the usual Cantorian way, and thus accounted for a primitive of "quasi-cardinal" to express size, thus epitomizing again the talk's argumentative kernel that set-theoretic size can qualify as a conceptually composite notion.

¹⁷The premise that the naturals "contain" the evens and "go beyond" them ought to be justified by implementing some semantical remarks. Recently, many authors contributed to the development of a "logic of totality" or "total logic", whose scope is to regulate claims about the totality of elements comprised in given domains and the fact that no further element is allowed to join the domain and generate further truths about it. The interested reader should take a look at the reportage of Tübingen's workshop on "Existence, Truth and Fundamentality" (July 2013), contained in the present issue. There, Stephan Leuenberger presented a classification of the usages of the totality operator and applied them to the debate on naturalism in the philosophy of mind. In the present case the general idea would be to apply the "that's all"-clause to a sufficiently large set of sentences (mainly instances of axiom schemes, and especially the axiom of extensionality) predicating properties of the set of the evens, but not to those that account for its being a sub-set of the naturals. This should make the general metaphysical remark I presented about the identity of the two set more viable from a logical point of view.

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