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HISTORY AND BECOMING OF SCIENCE IN JEAN CAVAILLÈS

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ABSTRACT. This paper is focused on Cavaillès' theory of science and his peculiar epistemology. In order to understand the position of Cavaillès concerning the becoming of mathematics, it is necessary to start from the way he utilizes the historical method inherited from Brunschvicg. In Cavaillès' works, historical analysis is not reduced to a mere reconstruction of the past, but is regarded as an instrument to find the necessity that characterizes the movement of science. This movement is originated by the tensions between a necessary internal push and historical contingency, and it goes at its own pace, being determined by nothing else but the mathematics itself. Therefore, Cavaillès also states the failure of all foundational projects and affirms the complete autonomy of the becoming of mathematics, which develops as a dialectic unforeseeable concatenation of concepts.

KEYWORDS. History, Becoming, Dialectic, Necessity, Movement, Cavaillès, Bolzano.

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1 Jean Cavaillès: a life between mathematics, philosophy and Resistance

Ambivalent figure of mathematician and philosopher, thanks to his double background and cross-sectional competences in both fields, Jean Cavaillès (1903-1944) has left a great theoretical contribution to the 20th century French epistemology. His works[1]are extremely complex and synthetic; maybe for this reason his epistemological work has often been the object of sketched researches, while bigger attention had been dedicated to his biography, especially to his Resistance activity in France. Just because of his militancy in the Resistance movement, Jean Cavaillès was imprisoned and then shot by Nazis, when he was only 41 years old. In the light of this biographical element, we can understand why his works are scarce.[2] Although he only had few years to work, he however succeeded in leaving an important and original contribution to the philosophy of mathematics of the last century. We can start sharing the Canguilhem point of view: Jean Cavaillès has written too little to be summed up and enough to make us catch the meaning of his philosophical issue.[3] We especially think that it is really interesting to study the epistemological bridge that he was able to build to unify what were, in his understanding of the knowledge, two banks of the same river: mathematics and philosophy. To remain in this metaphor, the nature of water that flows into these two banks emerges when inquiring characteristics of the becoming of science.

2 The historical epistemology

2.1 The historical method

According to Cavaillès, the becoming issue of mathematics must be assessed starting from the massive methodological and epistemological value of history. The topic of history and a certain style of "historical epistemology" are an inheritance of his master Brunschvicg. In his whole production, Cavaillès will keep them as crucial clues, as unavoidable elements and necessarily coexistent with the occurrence of mathematics. Therefore, only in history we can find moments of evolution, links, epistemological gaps which characterize mathematical dialectic, i.e. dialectic as a becoming along a path that is a necessary chain:

> Cavaillès bases his researches on historical analysis. He focuses on mathematics becoming, on overtaking processes, thanks to its objects systems enlarge, turn

[1]The whole Cavaillès' work, consisting in only 686 pages altogether, is gathered up in (Cavaillès 1994). This text collects the reprint of *Méthode axiomatique et formalisme* (1938), the main doctoral thesis, directed by Léon Brunschvicg; *Philosophie mathématique* (1962), wich consists in: the second doctoral thesis *Remarques sur la formation de la théorie abstraite des ensembles* (1938), *Correspondance Cantor-Dedekind* (1937), translated by Jean Cavaillès and Emmy Noether, and the article *Transfini et continu* (1947); the posthumous writing *Sur la logique et la théorie de la science* (1947); the articles *L'école de Vienne au congrès de Prague* (1932), *Logique mathématique et syllogisme* (1935), *Réflexions sur le fondement des mathématiques* (1937), *La pensée mathématique* (1946), *Du collectif au pari* (1940), *La théorie de la science chez Bolzano* (1946), *Mathématiques et formalisme* (1949) and *In memoriam* di G. Canguilhem (*Inauguration de l'amphithêatre Jean Cavaillès*, 1967; *Commémoration à l'ORTF*, 1969; *Commémoration à la Sorbonne*, 1974; *Une vie, une Œuvre*: 1903-1944, Jean Cavaillès, *philosophe et résistant*, 1989). There is only a translation in Italian of *Sur la logique et la théorie de la science*, edited by V. Morfino and L. M. Scarantino (2006), Jean Cavaillès, *Sulla logica e la teoria della scienza*, Milano, Mimesis.

[2]Concerning the destiny of Cavaillès' work, the words of Bruno Huisman are emblematic: "When he died at the beginning of 1944 executed by the Germans, in Arras, Jean Cavaillès left a work that we can't consider as incomplete; it is an unfinished work, or better assassinated". See Présentation in (Cavaillès 1994).

[3](Canguilhem 1984, p. 23).

> more abstract and, starting from there, integrate the infinity. Inside the becoming, the overtaking processes prove an outgrowing faculty which belongs to the thought, a power to create.[4]

From the Cavaillès' historiographic reconstruction work, a peculiarity arises: the historical perspective does not aim at a mere reconstruction of the past, since it would be fruitless in itself, as dealing with the mathematics becoming. The historical research aims at considering movement complexity and wholeness starting from the mathematics achievement till a certain point, in order to observe the present position in that present time, as a starting point in the future becoming. Furthermore, the historical issue is not focused on the accomplished path to obtain a certain outcome, but more likely on the achieved goal, which includes, and at the same time leaves in the background, the whole previous path. The historical analysis lies in its core consisting of thought in act during the present time:

> In a certain way, the homogeneity of materials of the thinking process and the simultaneity of mathematics working on its present time are here asserted: empiricism as describing the actual work, but empiricism of the thought in act, referring only to the unpredictable mathematics becoming.[5]

In Cavaillès' point of view, there is a need to take a closer look at mathematics as the actual work in the actual time, like the actual operation accomplished by "militant" mathematicians every day.[6]The act of thinking completely revolves on *hic et nunc* and, instead of spreading towards what has been gained before, it suggests the unpredictable mathematics becoming channelling into the future. The accomplished act in the present is not considered only as an outcome of the past, but as a foundational moment for a movement forward, as a root for future work. The defining work exemplifies that channelling into the future:

> Defining ways are subjected to variations and demands of its movement: for every new acquirement, new possibilities rise up. The gaining of nameable concepts matches with scientific enrichment itself.[7]

2.2 A history which is not a history

To some extent, Cavaillès' statement leads to define what he himself attests as "a history that is not a history"[8]: the methodological choice of the use of history does not consist in a simple work of reconstruction of the past and does not confine in an unfruitful backwards look. In other words, it is not focused on understanding how notions and definitions used at some point have come out, but it is rather open to future perspectives, possible innovations that

[4](Cassou-Noguès 2001, p. 132).

[5](Cavaillès 1938c).

[6]The expression militant mathematician has a very strong meaning in Jean Cavaillès' works. He uses it at the end of *Méthode axiomatique et formalisme*, asserting that the last word on mathematical issues doesn't belong to the philosopher, but the militant mathematician. The militant mathematician is, in Cavaillès' point of view, the person who has a mathematical education and works with numbers, formulas, theories as well as, with a pencil on his hand, on symbols, figures, he makes demonstrations advisedly. Furthermore, the adjective militant reminds the military environment, word that Cavaillès knew well, at first as official in the French army, then as leader of a network of the Resistance. Being militant for Cavaillès has first of all a moral meaning: it implies the refusal of passivity, taking the social, political and everyday life in hand. At the same time, it also means to obey a necessity at a theoretical level. This necessity also correspond to the immanent necessity of the reasoning, to whom the thinking individual can't escape.

[7](Cavaillès 1938c, pp. 17-18).

[8](Cavaillès 1949, p. 664).

are going to enrich different scientific theories. Historical analysis looks backwards, but it is completely projected in the forwarding movement of science. This projection in the future corresponds to a need that comes from the nature of scientific work itself:

> If an element of irremovable uncertainty is left [...], its action does not lead back, the accomplished gesture remains actually valid (final validity of statements), but it leads forward to transform what is set (modification of the notions).[9]

Obviously the perspective interest centred on what is coming afterwards does not expect to be more than a sort of privileged point of you, as the "old" and the "new" ones are connected in an indissoluble way: the previous moment sustains the subsequent one, and *vice versa*, the subsequent moment justifies and explains the former one. As Gilles-Gaston Granger remarks, the old one subsists as it is only within the new one and, at the same time, necessary new contents follow the old ones.[10] Progress is a continuous generation where the inner movement produces the rise of new theories starting from the previous ones. On the one hand, mathematics becoming is characterized by adopting new theories and, on the other hand, by transforming old theories[11]: "Mathematics is an odd building whose progressive construction shifts and reshuffles its foundations".[12] In the history of mathematics, there is no substitution of a theory with another, but a deepening reflection. Mathematical notions and theories are not totally rejected, but gradually modified: the mathematical becoming implies transformation and permanence at the same time. [13]

Between the different moments (the old theory, the overcoming of that theory, the new theory) there is a continuity and it is up to the historian to analyze and reconstruct *a posteriori* the connection between the various stages of mathematical becoming and therefore to follow the necessary development of mathematical rationality. On the opposite, the role of the mathematician is different: he works on theories and concepts more innovative than the ones which are contemporary to him, he does not need to know the past at all, to some extent, he even "denies that by vocation".[14] Precisely because the history of mathematics is projected forward, the mathematician's work consists in leaning towards the future, which implies a rooting in the past at the same time, but also a denial of antecedent moments:

> The history of mathematics appears, among all histories, the less connected to its vehicle; if there is a connection, it is *a parte post* and it is only for curiosity, not for the intelligence of result: what is precedent explains what comes afterwards. The mathematician does not need to know the past because his vocation is to reject it: to the extent that he does not submit to what seems to go by itself because it is, to the extent that he rejects the traditional authority, and disregards the intellectual climate so as to be a real mathematician, he unfolds necessities. By doing so, what are the means he uses? The work that denies history takes place in history.[15]

[9](Cavaillès 1938c, p. 187).

[10](Granger 1998, pp. 65-77 and p. 70).

[11]«Double link: with the set and studied problems at that precise moment -choise to rebel-, with yet present methods, materials to make new instruments. In both cases, either the individual will or the style of an environment are sufficient explications: even if mathematics is conceived as a system in itself, the winding course of the revealing process should be related to the structure of the revealed elements». Cfr. (Cavaillès 1938b, p. 226).

[12](Cassou-Noguès 2001, p. 135).

[13]About this, also see Monti Mondella (1962, p. 532): «The recent past of this science [mathematics], he [Cavaillès] remarks, is not the history of results and contributions, that have been added for mere juxtaposition to the previous ones, but it is also a critical revision of its own foundations and of the structure of its own theories in a radical sense».

[14](Cavaillès 1938b, p. 226).

[15](Cavaillès 1938b, pp. 225-226).

2.3 The mathematician and the epistemologist: the mathematical development between necessity and contingence

The mathematician has no theoretical interest in reconstructing the historical circumstances that led to the current situation of mathematics. The history of concepts and theories, in fact, owes nothing to the contingency, "in spite of the winding process of revelation" [16]: the mathematician is the revealing person of needs, his curiosity and his research are all forward-becoming, directing his attention to find out what mathematical needs still have to reveal.

The theme of the peculiar role of a mathematician is of great interest to Cavaillès, who has been talking about it several times during his career. On February 4^{th}, 1939, the Société Française de Philosophie invited Jean Cavaillès and Albert Lautman (close friend and colleague[17]) to present and discuss together the results of their respective doctoral dissertations. From this meeting came the article *La pensée mathématique*.[18] From the very start, it is obvious that Cavaillès gives particular importance to the division of tasks between the epistemologist (also historian of mathematics) and the mathematician, because of two characteristic features of mathematical becoming: on the one hand, it is a process that unfolds in time, throughout history, through contingencies and accidents; on the other hand, it is an independent progress, in ceaseless becoming and unpredictability. The epistemologist and historian of mathematics has to question the path of history, the role of contingencies and interaction between disciplines in science becoming, the mathematician has to realize the becoming itself. Although the two elements of historical contingency and mathematical necessity are inseparable aspects, they are irreducible to each other:

> Mathematics is a peculiar becoming. It is impossible to reduce it to nothing but itself; but also, each definition at a given time, is connected to that very time, i.e. to the history it is the conclusion of: there is no definition that can be valid forever. Discussions about mathematics cannot be anything else, but re-doing it. This becoming seems independent; the epistemologist can realize the necessary sequencing [*enchaînement*] under the historical circumstances, concepts are introduced because of the inner needs of a problem solution and, thanks to their being earlier concepts, they raise new issues.[19]

Even if it seems paradoxical, there is no contradiction in what we might call the "non-historical history" of mathematics. After all, Cavaillès stated it clearly: "There is nothing so little historical – meaning the opaque becoming, perceivable only through artistic intuition – such as the history of mathematics".[20] The history of mathematics is very little historical precisely because mathematics is a true becoming: history marks stages and periods, but mathematics is a living movement, an organic whole,[21] which can not be reduced to a perio-

[16](Cavaillès 1938b, p. 226).

[17]The two young philosophers were tied not only by the common interest in the relationship between philosophy and mathematics, but also by friendship and the shared fight in French Resistance. On the relationship between Cavaillès and Lautman see (Aglan, Azéma 2002), (AAVV 1985), (AAVV 2003), «La Lettre de la Fondation de la Résistence», n. 34, (Bloch 2002), (Ferrières 1950), (Granger 2002), (Sinaceur 1987b).

[18](Cavaillès 1946a).

[19](Cavaillès 1946).

[20](Cavaillès 1938c, p. 184).

[21]About the organic unity of mathematic, see (Cavaillès 1938b, p. 225): These processes are linked in a organic body: a state of mind constitutes the secret base where the processes are founded with an outward unforeseeability. We must come back to this unforseeability if we really want to understand, without exposing, but continuing. There is a deep kinship that links the researchers (without realizing, as a sort of harmony) on the same subject in a precise moment, *vis a tergo*, objective impulse of the research itself.

dization or temporal systematization. Mathematics is a becoming that produces history and it is produced in history, nevertheless, mathematics and history have different contingent features that clarify the distance between them. History is mainly the place of contingency, intended as accidental: the history of mathematics is undoubtedly contingent, but its contingency has nothing to do with randomness. Mathematical becoming is a contingent and necessary becoming at the same time. Contingent, as it runs through the process of the history of human thought, but at the same time necessary, due to the nature of mathematics itself, which finds the reasons of its development within an inner autonomy, indifferent to any kind of requirement referred to problems external to mathematics becoming. Mathematics progresses and develops in a way that is intrinsically necessary, according to an independent movement in the light of which every solution, each discovery and each moment of forward progress emerges from the single stimulus of an inner need. Due to this inner impetus, the following moments generated from the problem raised in the required manner and within this movement, there is nothing accidental meant at random. Mathematics is an authentic rigorous discipline as mathematical methods are not arbitrarily isolated according to the needs of a particular problem in a given time. On the contrary, it requires a certainty to be obliged to strip every method used of any "*vêtement accidentel*"[22] of some particular cases and clarify the necessary and sufficient conditions for its application. Due this extent, the unpredictability of mathematics is just apparent.[23]

Although Cavaillès states in a certain way the unpredictability is only apparent, at the same time, he also thinks the mathematical becoming is authentically, formerly unpredictable. How to explain this unpredictability's dual nature, apparent and authentic at the same time, of the results of the mathematical progress? Precisely because this becoming is a real becoming, it develops unpredictably: "There is becoming for real: the mathematician is involved in an endeavour which he cannot stop but arbitrarily, in facts, every moment can bring a radical innovation".[24] As Houria Sinaceur highlights, at every moment, mathematics develops bringing out elements that are actually innovative and the necessary element appears *a posteriori*, for this reason it is a historical [25] achievement. The necessity of becoming must not be considered at all as a sort of determinism or teleology: "The development of mathematics is necessary, not because it follows established and predictable paths, or because it obeys schemes, but as it unfolds through the construction of relations between the results that their rational connections withdraw, so to say, from contingency".[26]

Just because mathematics is unpredictable, but not contingent, it "is subjected to an authentic becoming, which consists in setting new theories and especially in the transformation of previous theories. Now, the becoming, once recognized, gains primacy over succeeding periods it connects".[27] Cavaillès goes further, saying that mathematics is not only subjected to a ceaseless becoming, but it is a movement itself, it is in a peculiar becoming itself.

[22](Cavaillès 1938b, p. 225).
[23](Cavaillès 1938b, p. 225).
[24](Cavaillès 1946a, p. 594).
[25](Sinaceur 1994, pp. 31-32).
[26](Sinaceur 1994, pp. 31-32).
[27](Cassou-Noguès 2001, p. 136).

3 Characteristics of mathematics

3.1 Mathematics as movement and «enchaînement»

Mathematical becoming is characterized as singular becoming, totally autonomous, unforeseeable and historical. Let's start questioning what it means to define mathematics as a movement in a never ending and unremitting becoming. When Cavaillès deals with this becoming, he talks about a fact inside mathematics, an essential element inscribed into the mathematical reality itself: "The mathematical becoming has its own objectivity mathematically established".[28] Mathematics is a movement, a never-ending process of redefinition of problems and elaboration of new theories beginning from the revision of the previous ones, following a ceaseless chain appearing as an intrinsically and necessary linear progress. On this continuous path, gap moments could be individuated as epistemological breaks. Therefore, there are fractures only in appearance, because each moment is essentially connected to the previous one: "One of the problems of the doctrine of science is precisely that the advancement was not increased in volume by juxtaposition, so that the previous one subsists along with the new one, but the advancement is an uninterrupted revision of contents through close examinations and deletions".[29]

Therefore the becoming occurs as a *continuum* in a never-ending becoming, neither as alternating theories in contrast which develop by denial and opposition between the more recent and the outdated ones, nor as a simple quantitative increase of knowledge.

In the light of this uninterrupted activity of revision, the history of science outcome lies in the image of a chain, where any link is indissolubly connected to its previous one forming a chain extending to infinity through the addition of new links joining the former ones. The definition of Mathematic as «*enchaînement*» is the crucial basis of Cavaillès' thinking. As Hourya Sinaceur states:

> court history, but it is more likely the system that it reveals, i.e. the architectonic set of connections between notions, problems and mathematical methods, the layered and shifting network coherent with different moments, each of them is unpredictable, but still indissoluble. History is a necessary practise if we reckon the whole mathematics and thinking in general is a becoming. Nothing is set nor is acquired once and for all.[30]

Rationality develops according to the chain modality, both rationality in general and specific mathematics rationality. Reasoning evolves as a chain, like a gradual linking of operations; in other words, reasoning is facing problematic clues set in a necessary order of further development.

The modality along which reasoning develops mirrors the modality of production and progress of knowledge, transferring its peculiarities to it. In fact, theories themselves develop according to this modality, as "the establishment of a certain structure of concepts".[31]

3.2 Mathematics as an object *sui generis*. Bolzano's heritage

Cavaillès inherits from Bolzano the theme of mathematics as an object *sui generis*, irreducible to anything but mathematics itself. In fact, Cavaillès dedicates to Bolzano the last pages of

[28](Cavaillès 1938b, p. 226).
[29](Cavaillès 1947b, p. 560).
[30]Sinaceur, *Philosophie et histoire* in (Aglan & Azéma 2002, pp. 208-209).
[31](Cavaillès 1938c, p. 85).

the first part of his major posthumous work. This part of *Sur la logique et la théorie de la science* is a key step, and it was so accurate that it was published under the title *La théorie de la science selon Bolzano* in the first issue of «Deucalion» in 1946, edited by Canguilhem and Ehresmann. Bolzano is an author mentioned several times in Cavaillès' works, although during the years Cavaillès dealt with Bolzano, he was still unknown, despite his recognition by Husserl (to this extent a good example is in the *Appendix* of Chapter X of *the Prolegomena of Pure Logic*). It was Cavaillès who, between the two world wars, recognized the importance of Bolzano, particularly because he understood the meaning and role of a theory of science as a theory of the structure of science and of operations included within itself.

What is specifically interesting for Cavaillès in Bolzano's works is that he makes a critic of science, i.e. he fathoms what constitutes science as science and what the driving force behind science is. The main point is that the big merit of Bolzano consists in considering for the first time science as science, separating himself at first from Kant (who subordinates science to conscience and defines science starting from conscience), and then from Brunschvicg (who defines science as the starting point between the experience and the world).[32] The science, free from this double subordination, can finally emerge through its autonomy and independence and it can be defined as itself:

> For the first time, perhaps, science is not considered as a mere intermediary between the human mind [*esprit*] and the being in itself, depending on both of them and without a reality of its own, but it is considered as an object sui generis, getting an original essence and autonomy through its own movement. [33]

Science is a *sui generis object*, that can not be placed into universe of cultural objects because they inherently depend on their way to be produced and they are linked to the accidental exteriority of a sensible system. Furthermore science, unlike cultural objects, does not take form as a multiplicity of singular realisations: science is one and it demands this unity.

This extract from Cavaillès refers to a dissertation present in Bolzano's *Wissenschaftslehre* (1837), namely the distinction, on the one hand, between the sentences in themselves, the sentences of science, entities that have their own being independently from thought and statement and, on the other hand, the sentences expressed or thought. Consequently to this distinction, it follows the independence of science both from consciousness (and the thought propositions)[34] and from the world (and propositions stated)[35]. This independence and autonomy of science gives unity, which is shaped, first of all, as architecture: there are no different sciences anymore nor separated moments of a science, but one unique science shaped as unified, cohesive and organic. This unity of science does not mean that there is unity of principles or methods, but it means that the various disciplines interpenetrate, are connected, influence each other and interact, giving rise to science as a system, an architectural structure, a living organism. This unity implies interdependence between the various parts which partake in a unified whole. For this reason, "a theory of science can be nothing else than the theory of the unity of science".[36]

The unity of science is characterized as a necessary and indefinite progress[37], enclosed in itself, and its movement and dynamism:

[32]Cfr. (Cassou-Noguès 2001, p. 265).
[33](Cavaillès 1947b, p. 503).
[34](Cavaillès 1947b, p. 504).
[35]Cfr. (Cassou-Noguès 2001, p. 266).
[36](Cavaillès 1947b, p. 504).
[37]Cfr. (Cavaillès 1947b, p. 506).

> This unity is movement: since it does not deal with a scientific ideal, but with concrete science, the incompleteness and the need for the progress are a fundamental part of the definition. Autonomous pure progress, dynamism in itself enclosed, without absolute beginning nor end, science moves out of the time – if time means reference to the experience of a consciousness.[38]

About this characterisation of science, Jean Sebestik defines Cavaillès as the dynamical side of the static universe of Bolzano,[39] which is able to answer to the difficulty consisting in how to determinate the relationship between science as a whole of out of time truths and actual science, historically realized. Sebestik precisely underlines that Cavaillès develops his conception of mathematical becoming, starting and basing it on Bolzano's theory. In fact, in this passage of *Sur la Logique et la théorie de la science*, Cavaillès not only proposes a mere reading of the science theory of Bolzano, but he also gives an original interpretation, where sometimes Bolzano's and Cavaillès's characterisation of science are mixed up. Furthermore, it is very difficult to establish, in this extract, which definitions of science are from Cavaillès or from Bolzano; it is also difficult to understand if some sentences are Cavaillès's interpretations of Bolzano's theories or if they are Cavaillès' original sentences founded on Bolzano's epistemology. To give an example: "The true meaning of a theory lies in its unending conceptual becoming, not in a particular and essentially provisional aspect".[40] In this sentence it plainly emerges that expressions as «dynamism», «autonomy», «indefinite progress» of science, etc..., are equally appropriate to the science theory of Bolzano as to Cavaillès' philosophical conception of the becoming of mathematics.

Anyhow, it is not necessary to distinguish between the thought of these two authors: what is really important in this excerpt is the praise that Cavaillès shows for Bolzano's epistemology, because he sets science free from the primacy of consciousness and from the world. Scientific experience consists in including the world in the scientific universe. The value of science is right in its necessity and so in its capacity to free itself from the empirical world (that is singularity, exteriority, heterogeneity) to unify science itself. Science organisation is independent from experience, that is autonomous and that Cavaillès metaphorically defines as a "Riemann volume, that can at the same time be closed and self-contained".[41] Even if Cavaillès appreciates Bolzano's epistemology so much, some difficulties arise. The first problem that emerges is the relationship between theory of science and science itself:

> Science doctrine is also a demand of intelligibility and of validity; it is a science of science, so it is a part of itself. So its statements cannot be essential to a particular development, but it has to directly appear in the self-illumination of the scientific movement. Statements differ from the scientific movement because of their continuous surfacing.[42]

Cavaillès can not accept a conception which assumes a sort of self illumination, a science transparency in itself: in this way, Bolzano falls into the same trap as Kant, when he puts the transparency in itself of the thought into the logic. Bolzano's theory of science is not a part of science among other parts, but, in a way, it is the essence of science, the immanent

[38](Cavaillès 1947b, p. 504).

[39]Cfr. (Sebestik 1997).

[40](Cavaillès 1947b, p. 505). It is not easy to correctly translate this passage, therefore I quote here the original text: «Le sens véritable d'une théorie est non pas dans un aspect compris par le savant lui-même comme essentiellement provisoire, mais dans un devenir conceptuel qui ne peut s'arrêter».

[41](Cavaillès 1947b, p. 506).

[42](Cavaillès 1947b, p. 506).

structure of the science chain, it is the structure that knows the chain in an immediate way, by self illumination. This theory of science on the one hand avoids the subservience, the dependence on the world, on the consciousness and on a historical being. But, on the other hand, if the becoming of science consists in a self-revealing process in the construction of the demonstration, the theory of science has to put down by itself the whole of what it produces.

Finally, in spite of the difficulties in Bolzano's epistemology, it is necessary to underline that Cavaillès, in this part of his work, deals with the main issue of his whole work: how to determinate de relationship between science as a set of out of time truths and historical science, actually achieved.[43]

3.3 The refusal of any foundational attempt

3.3.1 Formalism, logicism and intuitionism

In fact, all Cavaillès' research is specifically focused on the characteristics of mathematical progress, as an emblematical model of knowledge in general. According to Jean Cavaillès, mathematics is a singular becoming, irreducible to anything else but the mathematics, that originates from itself. At the bottom of this characterisation of mathematics there is the refusal of any foundational attempt, referring to external causes rather than to the dialectical movement inside mathematics itself.

In his main doctoral thesis, *Méthode axiomatique et formalisme. Essai sur le problème du fondement des mathématiques*,[44] Jean Cavaillès specifically deals with mathematical foundation's problem and analyses solutions proposed by formalism, logicism and intuitionism, coming to the conclusion that none of them can really solve this problem.

First of all, Cavaillès states the failure of the formalism's foundational project (at least of what he defines "Von Neumann's radical formalism").[45] According to Von Neumann, mathematics obtains its objective validity through its representation as a system or a collection of a system of signs, endowed with a meaning by its rules or structure and of deduction. In other words, mathematics is considered as a system of signs and rules to be employed, that governs the mechanical chain of formulas. The formalism's program consists in founding mathematics on a metamathematics that demonstrates the non-contradiction of formal systems. The Gödel theorem puts offside the formalist conception of mathematics, because it makes impossible to define mathematics as an hypothetical-deductive system. If it is impossible to bring a proof of non-contradiction evidence inside the system, the edifice collapses as "the notion of formal demonstration that gave to the system its only meaning is not yet able to be specified (because it is not possible to prove that everything is demonstrable in a formal system)".[46]

Therefore, Cavaillès takes into consideration logicism foundational attempt. First of all, he considers "logicism" as it was conceived by the Vienna Circle until 1929 and by Carnap. Cavaillès thinks that logicism is a sort of formalism's development and that the formalism's failure implies logicism failure too, even if the latter does not base mathematics on systems' internal non-contradiction, but on logical evidence. Logicism boundary, in Cavaillès' opinion, consists in being led by the same hidden realism that in Frege and Russell's view puts an "in itself" of the universe.[47] According to Cavaillès, logicism, can not solve the mathematical foundation problem.

[43]Cfr. (Sebestik 1997, p. 104).
[44](Cavaillès 1938c).
[45](Cavaillès 1938c, p. 172).
[46](Cavaillès 1938c, p. 173).
[47]Cfr. (Cavaillès 1938c, p. 177).

Finally, he focuses on the intuitionism, by pointing out the problem that emerges concerning the infinite subject. First of all, it is impossible to explain, by an intuitionist point of view, how infinite can be introduced in mathematics. Secondly, also in stating infinite in mathematics, the argument that brings to infinite must be explained and founded.

3.3.2 Psychological or social causality

Besides the fact that the mathematical movement can not be founded on these three approaches, it takes place in a total independence from any kind of social causality and from the pressure that the needs of other scientific domains exert. Cavaillès admits that in the contingent events' following it could happen that what he named the psychological or social causality phenomenon can delay or divert the mathematical course. However, despite this slowing down risk, it is impossible to stop this course, as it has a sort of internal drive in the direction of progress: The historical, contingent mathematician can stop himself, can be tired, but a problem's requirement imposes the act that will solve it'.[48] While presenting his doctoral thesis, his master of thesis, Brunschvicg complains about the fact that he ignored mathematical psychology, Cavaillès crisply answers that this problem was not what he had to consider and that he was not interested in this kind of issues.[49] What really mattered for him, were objective cores, relationships of intelligibility: the historical method can bring out this kind of issues. Cavaillès thinks that it is possible to perceive the internal unity in the central intuition from which the necessary mathematical chains proceed and we can discover it, methodically disclose it through the only description phenomena, as it happened in history. Therefore, there is a sort of virtuous circle that makes history come out of a theory unity and it is this unity that gives history its objectivity and justifies its function.

3.4 Mathematics is necessary and unforeseeable. How Cavaillès explains the paradox

So two fundamental issues of Cavaillès's thought emerge. On the one hand, he states that we can not separate the mathematical object from the concrete experience of its elaboration in the course of time, in a historical moment. In other words, mathematical progress depends on the operational act of the mathematician, which, at a specific time, elaborates a notion or defines a new mathematical object. On the other hand, mathematics can only be defined each time in the present, in the moment when a definition is given: at any time in history, giving a definition of mathematics that doesn't strictly concern this time is impossible, when the present moment is the achievement of the scientific progress. So mathematics is temporally and historically characterized and it is definable only in the present time. It can also be constantly re-defined, revisited and it is impossible to know *a priori* its final result. An everlasting definition of mathematics does not exist, and it is not possible to determinate what is true and what is false outside actual mathematics, in its historical context. Mathematicians, as human being, is irremediably involved in this adventure, he can not foresee its progress, mathematics is carried out by the scientific movement, in the time when every moment acquires its radical innovation.[50]

That is the way mathematics can, at the same time, be necessary but not predetermined: a becoming, in order to be an authentic becoming, has to be unforeseeable. Cavaillès underli-

[48](Cavaillès 1946a, p. 627).
[49]Letter of 28th April 1938, in (Ferrière 1950).
[50]Cfr. (Cavaillès 1946a, p. 594).

nes that this unforeseeability is not placed in the intuition of the "militant" mathematician. In fact, when he works, he obviously knows (or better, he can imagine) in what direction it is necessary to carry out his search: his operational act is not casual. Unforeseeability is instead placed in an original level, it is based on the foundation of mathematics: "It is what can be called the basic dialectics of mathematics: if new notions seem to be needed by given problems, this novelty itself is really an absolute novelty".[51] It seems that both necessity and unforeseeability are a paradox in their development, but we can explain this by characterising mathematical progress as an achievement through an experimental method, and not as the development of a logical demonstration.[52] The unforeseeability of mathematics can be really considered an absolute novelty if the peculiar way of the becoming of mathematics is considered as an operation that mathematicians do in an experimental context, working towards a discovery. Unforeseeability is not set into theories, as if it was a sort of unforeseeability of the mathematical operations: unforeseeability of the becoming of mathematics shows itself when a theory is overcome by dialectical movements. Unforeseeability is original, fundamental, placed at the heart of the becoming[53] and because of it, science gains its meaning: "The real meaning of a theory is not in what the scientist considers as a temporary step, but it is in an unstoppable conceptual becoming".[54]

4 Dialectics of becoming of science

The last issue that must be taken into account belongs to the determination of the dialectics of the becoming of science. "A doctrine of science could be provided not by a philosophy of science, but by a philosophy of the concept. The generating necessity is not that of an activity, but of dialectics"[55]: these words conclude Jean Cavaillès' *Sur la logique et la théorie de la science*, that could be considered as his philosophical will. Cavaillès did not have the time to deepen the philosophical meaning of the dialectics of concept, that he endorsed as a model for the elaboration of a correct theory of science, in opposition to the philosophies of knowledge. However, it is possible to reconstruct the gist of the dialectics of concept without misinterpreting Cavaillès' thought since the author left, scattered in his texts, various elements that can be detected and made coherent and systematic. Cavaillès' theory of science, being a philosophy of concept, is interpreted by most part of critics as a program rather than a thorough theory, as a need rather than as a doctrine.[56] Yet, the dialectics of concept is to be considered the gist of Cavaillès' philosophy of science, by means of which it is possible to tie together all its different aspects and, in particular, the history and the nature of the becoming of knowledge. Consequently, it is possible to elucidate the peculiar use of the notion of dialectics held by Cavaillès. In spite of its rare use, this notion has a deeper and more original meaning than it might appear at fist glance.[57] This is the interpretation given by Henri Mougin, who claims that:

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[51](Cavaillès 1946a, p. 601).

[52]"This means that is impossible to find new notions into the demonstration, just analising already utilised notions: for example, [analizing] generalisations which have generated new processes" (Cavaillès 1946a, p. 601).

[53](Sinaceur 1994, p. 33).

[54](Cavaillès 1947b, p. 505).

[55](Cavaillès 1947b, p. 560).

[56]For instance, this view is endorsed by Schwartz (1998, p. 82); in (Sinaceur 1985, pp. 977-978): «Reading again the last lines of Cavaillès' «philosophical will», for the reader these lines are similar to an oasis, because they propose a program at least»; finally, in (Cassou-Noguès 2011, p. 312): «In my opinion, the last part of the text of the posthumous work it's quite a sketch, the outline of his plan and it's not really a task that is over».

[57]Cfr. (Dubarle 1948b, p. 359).

> The results of Cavaillès' reflection are all oriented towards the elaboration of this notion of dialectics [. . .]; the dialectics hides itself behind failures and requires, at every step, a positive and authentic effort towards a new experience. If I should sum up in a word the whole philosophical reflection of Cavaillès – after the remembrance of the friendly controversies [between Cavaillès and myself] –, I would say it essentially consists in a dialogue with the notion of dialectics, a dialogue that was already developed between 1933 and 1937, years during which the reflections on dialectics became fundamental; a dialogue that became even more intense after 1937.[58]

Cavaillès had already made use of the term 'dialectics' in his doctoral thesis both as a substantive and as an adjective referring to progress, movement and to the chain of concepts: "internal dialectics of the intuitive activity", "the dialectics of the history of mathematics", "the dialectic development of mathematics"[59], "dialectic generation of concepts" (referring to Cantor's works), "dialectic movement that generates the ∞"[60], "dialectic passage from a theory to a superior one", "dialectic development of the mathematical experience", "the creative dialectics", "the dialectic necessity"[61] "dialectic concatenation of concepts", "dialectic of concept", "dialectic moment of the position of object".[62] In all these different contexts, the term dialectics is always linked to the notions of necessity, of radical novelty or unexpectedness, of movement, progress, or concept.[63]

So, we can start considering the definition provided by Cavaillès in the article *Réflexions sur le fondement des mathématiques*: "Dialectics of the history of mathematics: liberation of the contingent by mean of the actual, but the actual in its becoming actual is contingent".[64] The importance of the notion of dialectics in Cavaillès consists especially in its dynamical essence: dialectics is movement. Indeed, being a process, the dialectics unfolds in time, and so in history and contingency. Notwithstanding this, this movement is not in turn contingent or arbitrary, because it allows the passage from a moment of becoming to another towards a sequence that happens necessarily. In fact, the dialectic movement starts from a contingent moment and, in its development, characterizes it as necessary, since the passage from a moment to another is pre-determined by the internal needs of the moments itself. To put it differently, the contingent moment is freed by the actual one because it loses its accidental nature when it gets tied to the following moment. However, this necessary correlation takes place in the historical present, and so, in the contingency. For this reason, the dialectic becoming of the history of mathematics is at the same time necessary and unforeseeable.

4.1 An example of dialectical becoming: thematization

It is possible to find an example of this in the actual mathematical experience and, for instance, in what Cavaillès calls "thematization". Thematization is, like paradigmatization, the process which allows to abstract and analyse the forms of demonstrations. In particular, thematization is a process of abstraction thanks to which an operation becomes in turn the basis of a superior one. Let us provide an example of how thematization works for Cavaillès.

[58] (Mougin 1945, p. 75).
[59] (Cavaillès 1937c, p. 578 and p.580).
[60] (Cavaillès 1938b, p. 280 and p. 281).
[61] Cavaillès 1938c, pp. 180, 185, 190, 191).,
[62] (Cavaillès 1946a, p. 601).
[63] Cfr. (Sinaceur 1994, p. 116).
[64] (Cavaillès 1937c, p. 578).

Geometrical transformations, for instance rotation and translation, are gestures, operations in a broad sense, that apply to some points in the geometrical space. Thanks to thematization, the geometrical transformations become a field of objects and undergo a further operation, that is, composition. Composition provides to the geometrical transformations that Cavaillès calls "the group structure", that is, in other words, the composition of geometrical figures. Composition, thus, is an operation of superior degree than rotation and translation. In fact, composition is an operation that applies to geometrical transformations, assuming them as its own object.[65] According to Cavaillès, this passage has a dialectical nature:

> The development of conscience and the dialectical development of experience coincide, generating the indefinite set of objects in what we call a thematic field. [. . .] The necessity of generating an object can only be grasped by the observation of an outcome; the existence in the thematic field has sense in so far as it is correlated to a real act.[66]

The new objects are generated, but the necessary nature of their generation in the thematic field emerges a posteriori, that is to say, when the movement is done and has determined the existence of the objects: "At this point, an internal necessity compels the theory to overcome itself, by means of an unforeseeable development that reveals itself at the end of the process".[67] The development of the thematic field, that is, the generation of new objects, takes place by means of a real act, that is, the act of mathematical experience. This act is performed by a mathematician in flesh and blood, and the thematization, as a dialectical moment, occurs in the passage from a theory to another. Indeed, as Cavaillès specifies, it is impossible to know a priori the results of the necessary process of becoming of mathematics precisely because finding the solution to a problem is an authentic experience, and the solution itself assumes all the features of an experience. Being an experience, the search in the mathematical field is "a construction that eventually takes place in the sensible world and thus it is exposed to the risk of a possible failure, but it is performed according to a rule (that is to say, reproducible and so a non-event)".[68] To put it differently, all the discoveries in the mathematical field are possible because the mathematical experience, being an authentic experience, occurs in the sensible world. There is no abstract thought (or, better, no abstraction in thought) without a concrete act, there is no necessary becoming without starting from a contingent experience. As Cavaillès stresses:

> There is no really-thought representation (as distinctive of the pure experience) which, being truly thought – that is to say, organization of the sensible world according to rules (in virtue of the continuity of mathematical gestures, starting from the most elementary ones) – is not at the same time a mathematical system. The existence of objects is correlative to the actualization of a method and, thus, not categorical, but always depending on the fundamental experience of an actual thought.[69]

[65]Cfr. (Cassou-Noguès 2001, p. 146). Here we can find another example of thematisation, taken from *Remarques sur la théorie abstraite des ensambles*. When Cavaillès remarks on Dedekind's demonstration of an infinite system existance and when he analises the intuitive numeration, he asserts that thematisation is an operation into a theory, consisting in making a tought as an object of a thought, i.e. in making a multiplicity as an object similar to an unit.

[66](Cavaillès 1938c, p. 185).

[67](Cavaillès 1947b, p. 556).

[68](Cavaillès 1946a, p. 594).

[69](Cavaillès 1946a, pp. 594-595).

5 Conclusion. "The return to the origin is the return to the original"

Even the most abstract level, apparently far from the sensible experience, is in fact dialectically tied to its empirical basis, that is to say, to the moment when the mathematical research historically began. In this respect, Cavaillès claims this necessity to be a dialectical one, and not a formal one. In fact, Cavaillès maintains the perception of this necessary order to take place in the concrete, real, and temporal act, and, at the same time, to be all enlightened by its own conquest.[70] Even at the highest level of abstraction, the sensible basis is not to be neglected. In this way, it is possible to understand the fundamental role that Cavaillès appoints to the historical inquiry, which has to reconstruct the connection between the different moments of the process. As Cavaillès puts it in *Sur la logique et la théorie de la science*, "the return to the origin is the return to the original".[71] Looking at the past means giving a reconstruction of the different moments of the process on the basis of the dialectical moment rather than of the temporal succession. The return to the origin is not a mere inquiry on the historical starting point, but consists in grasping the moment when "the broadening of conscience and the dialectical development coincide",[72] which Cavaillès maintains to be the authentic origin of the necessary process of becoming. In this sense, "history reveals the authentic meanings, because allows to find the lost connections, firstly identifying automatisms and sedimentations, and then vivifying them through an immersion in a conscious actuality".[73] History is not a mere chronological reconstruction, but rather makes the original moments re-emerge and gives them a sense because of their re-actualization in the present. In other words, the original moments are such not merely in a temporal sense, but rather and foremost in an essential sense. The preceding moments are the necessary basis of the following ones both from a chronological and a dialectical point of view. The historical reconstruction becomes a reconstruction of the necessary connections because this necessity itself is not abstract from the world but rather it is anchored to the sensible universe and to the concrete experience which takes place in the historical temporality. As Cavaillès writes, "the time has a twofold necessity: that the immediate act becomes habitude and that the new act loads the system of the mute trails of the past, making use of them".[74] Time acquires the necessary feature of the dialectical movement, where present and past are complementary and justify themselves reciprocally in the unity of movement.

> There is no juxtaposition nor a set starting point: it is the entire body of mathematics that develops itself with a single movement, through different stages and forms – the latter, always as an entire body, achieves or not a certain cognitive function, through different stages and forms (technical devices included).[75]

Given this unity of mathematics, a correct historical inquiry should avoid fragmenting or even describing the becoming as divided into stages, but rather should make the complete dialectical unity of it emerge, and so bring into light the necessary connections and ties between the past, the preceding acquisitions and their actualization in the present. Mathematics is an authentic historical becoming and so it is irreducible and can be defined only

[70] Cfr. (Dubarle 1948b, p. 360).
[71] (Cavaillès 1947b, p. 558).
[72] (Cavaillès 1938c, p. 185).
[73] (Cavaillès 1947b, p. 558).
[74] (Cavaillès 1947b, p. 558).
[75] (Cavaillès 1947b, p. 556).

starting from itself: "the mathematical activity is an object of analysis and possess an essence, but as an odour or a sound, the mathematical activity is itself".[76] Mathematics does have an autonomous rational legitimacy which is historically connoted, that is to say, that concretely realizes its essence in subsequent stages. Thus, the comprehension of its becoming must involve a temporal analysis which should be able to connect the past with its actualization in the present:

> Impossibility to characterize the mathematical activity by reducing it to something else; [mathematics] is not only an intuition, but an original historical becoming. The starting objects are not whole numbers nor peculiar element of intuition, but are the result of concrete proceedings meant to organize the action of conscience in the world. The mathematical data is – in every moment of history – the system of objects the actual mathematics deals with.[77]

To conclude, the two essential features of the dialectical movement (necessity and unforeseeability) make the mathematical becoming an authentic becoming and characterize the fundamental dialectics of mathematics. Here, the new notions appear as necessitated by the given problem and this novelty itself is a real radical one.[78]

The history of mathematics is not properly a history but a process of re-emersion and re-actualization of its different moments. It is now possible to understand the apparently enigmatic statement that Cavaillès wrote in 1938, in its main doctoral thesis, *Méthode axiomatique et formalisme*: "There is nothing less historical [. . .] than the history of mathematics. But nothing less reducible, in its radical singularity".[79]

Ed.: translations by the author.

[76](Cavaillès 1949, p. 664).

[77](Cavaillès 1937c, p. 578).

[78]Cfr. (Cavaillès 1946a, p. 601). Cfr. also (Cavaillès 1947a, p.471-472): « The description of these historical reversals is well known. The result of these revearsals makes emerge the method and the whole system it rised: from the processes emerging from an internal solution of a problem need to create (in the same actualisation that gives sense to this need) a big change in the point of view, so that the notions that set up their structure have to be given up. But the relational links go beyond the empirical history: their dialectical movement assures at the same time the links' movement, and, through the links, it assures that the links themselves remain valid. History is characterized by the submission of the trascendental to its steps: the need of a passage emerges in a failure, the necessity of the progress emerges in the indetermination of the discovery. Necessity appears when it is all over».

[79](Cavaillès 1938c, p. 186).

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