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## GESTURES, PEIRCE, AND THE FRENCH PHILOSOPHY OF MATHEMATICS

The idea of ‘gesture’ is present in the philosophical world in various forms. All of them might find an important theoretical grounding in pragmatist philosophy.<sup>1</sup> Some years ago there were only a few pragmatist scholars who kept the concept alive, and then only as a phase in Mead’s philosophy, and even fewer recalled the notion of gesture as an important but obscure concept in the French philosophy of mathematics (Zalamea 2012; 2019).

In recent years, the notion of gesture has gained some prominence in the philosophical world, sometimes starting from a pragmatist background, sometimes not. However, any author referring to the concept eventually must deal with the views of the pragmatists. These authors include Barbara Formis (2010), who founded a ‘*laboratoire du geste*’ at the University of Paris 1 in 2009 (begun in 2006 as a website), tackling the notion of gesture from an aesthetical perspective; Maurizio Ferraris (2016) in Italy has used the notion of gesture in his thinking about technology; and Irene Mittelberg and Alexander Gerner have edited a book on gesture as an epistemic tool (2019). Gerner has explored the idea of gesture for its semiotic implications in treating schizophrenia. In Italy, a couple of long seminars and one new book focuses on gesture from a political and economic perspective – Matteo Santarelli and Guido Baggio are the leading Italian scholars in this area. I contributed my own book looking at gesture from an epistemic standpoint, *The Philosophy of Gesture*. Also studies on Mead, such as Roberta Dreoni’s (2019) most recent papers, have put a new emphasis on gesture. Outside the realm of philosophy, the pragmatist view of gesture has been treated by Esa Kirkkopelto (2010) in his studies on theater, and also by artists of various stripes, mathematicians, and people involved in the development of communication and technology.

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<sup>1</sup> For an introduction to Pragmatism see Calcaterra, Maddalena, Marchetti 2015.

What is offered to philosophy and other disciplines by the notion of gesture? Why should time and effort be spent in clarifying the term? The answer to this question is suggested first by the French philosophy of mathematics, in particular in the work of Jean Cavailles (section 1). A second important clue comes from Peirce's philosophy and, in particular, from his semiotic research on Existential Graphs (section 2). These two parts will form the core of this article, and in the conclusion I will attempt to answer the question and to suggest new roads of inquiry (section 3).

### **1. Cavailles and resistance**

Jean Cavailles (1903-1944) was an inspiring character. A scholar of philosophy and mathematics and a *normalien*, he took on various roles in the resistance to Nazism. Caught and imprisoned in 1942, he wrote a short book on logic while in prison. He escaped, directed several actions of sabotage, and was finally arrested and executed in 1944, a few months before liberation.

Cavaillès's writings are extremely important for our discussion; here I will sum up the crucial philosophical issues he addressed and where his thought on those issues led him. In his book *On the Logic and Theory of Science*, Cavailles illustrates the weakness of the epistemic Kantian defense of mathematical and logical thought. Cavailles argues that the shortcomings of Kant's conception of mathematical objects stems from the analytic and formal grounds he insists on. Kant requires mathematical knowledge to start from apriorism and to be general; according to him, only these characteristics can justify the universality and necessity of mathematics. However, in order to complete his conception of mathematics, he is forced to move into transcendentalism so that apriorism and generality can work without breaking up at any particular empirical, concrete step. According to Cavailles, Kant maintained that «logic has to be transcendental or not be» (Cavaillès 2008, 26). However, once this transcendental construction has been erected, new difficulties arise. On the one hand, the empirical content is always there, and it becomes either useless or a threat to the profound analyticity of the paradigm. On the other, the unicity of space, which is highly questionable on mathematical grounds, is the only guarantee for the entire system (Cavaillès 2008, 26-30). Many years later, Robert Hanna pointed out that Kant's notion of syntheticity was constructed on the same analytic ground (Hanna 2001, 224), and following him, I argued that the

weakness of this alleged unicity is its underlying ‘whole-part’ me-reological scheme (Maddalena 2015, 38).

Cavaillès understood that all subsequent steps in logic, the philosophy of mathematics, and epistemology are an attempt to resolve the problems caused by Kant’s fundamental insight of a ‘philosophy of consciousness,’ as he called it (Cavaillès 2008, 17-9). Cavaillès examines intuitionism, logicism, and phenomenology as keys to a possible solution. As much as he likes intuitionism, referring to Brunschvicg for his defense of the autonomy of mathematics, he finds it naïve in its conception of realism and weak in grasping a technical difference between mathematics and other intellectual, cultural enterprises (Cavaillès 2008, 30-4). Cavaillès is even more critical of logicism and phenomenology, which were both somehow grounded in Bolzano’s insight of transforming mathematics into the science of demonstration (Cavaillès 2008, 34-90). Both Husserl’s construction of a theory of theories and the attempt at formalization that culminated in Hilbert’s project to find ground for mathematics failed, as Gödel demonstrated in his theorems. The basic idea of these philosophies of mathematics was that either a judgment is a consequence of a set of axioms or of a set of primitive phenomenological structures; otherwise it will be proved inconsistent. ‘*Tertium non datur*,’ but Gödel showed that *tertium is datur*. «On peut y énoncer une proposition qui n’est ni conséquence des axiomes, ni en contradiction avec eux» (Cavaillès 2008, 84).

Facing the debacle of his attempts to shore up Kant’s philosophy of consciousness, Cavaillès became one of the few authors to realize that we simply have to move on to another paradigm, to a post-Kantian philosophy and especially philosophy of mathematics. But which paradigm?

Two options were available, which were discussed in the interesting debate held at the Sorbonne University in 1939 between Cavaillès and another important young mathematician, Albert Lautman (Cavaillès 1994, 593-630). Lautman defended a Heideggerian approach to mathematics (Cavaillès 1994, 605-9), understanding that the logicist solution had failed and that it was impossible to go back to Platonic ontologies. He found Heidegger’s idea of understanding «the genesis of ontic reality starting from the ontological analysis of the Idea» (Cavaillès 1994, 608) to be a good way to demonstrate that mathematics moved between essence and existence. In this way, mathematics would disclose its

«genesis in its eternal way of creating itself» (Cavaillès 1994, 608). In a nutshell, this is what Heidegger would later call an 'event.' The event is capable of creating a relationship among the various structural layers of mathematics, especially between universality and particularity, and analyticity and syntheticity. Mathematics as event would account for the possible existence of these otherwise impossible relationships.

In opposition to Lautman, Cavaillès contended for a different view, which he had elaborated some years before (Cavaillès 1994, 593-605). The characteristics that Cavaillès attributes to mathematics are 1) construction is an act and rejects any apriorism; 2) the foundation of mathematics is in the tradition of mathematics, that is, in the becoming of mathematics; 3) mathematics originates in experience (in transition) and is a transition; and 4) the construction that is part of this tradition is justified by its success. In a word, mathematics is a gesture, a word in the French philosophy of mathematics that, as Fernando Zalamea pointed out, is not always sufficiently highlighted or understood (Zalamea 2019). The suggestive expression employed by Cavaillès is «attraper le geste et pouvoir continuer» (Cavaillès 1994, 186).

Cavaillès did not read William James but he understood mathematical objects as correlating to our acts. Mathematical objects cannot have an ontology severed from epistemology, describable prior to epistemology. The mathematical object is 'inseparable' from the operation that gives birth to it. That is why to understand is to grasp the gesture and to be able to continue, «attraper le geste, et pouvoir continuer». A gesture is not an event because the mathematical gesture follows a rule, must have a physical aspect, and can fail (Cavaillès 1994, 594). As for rules, they must come from the tradition of mathematics, so that gestures become gestures by virtue of their belonging to, and changing, a tradition of thought. Finally, as in pragmatism, the success (*la réussite*) of our gestures is the criterion of the reality of the corresponding mathematical object:

La nécessité de l'engendrement d'un objet n'est jamais saisissable qu'à travers la constatation d'une réussite; l'existence dans le champ thématique n'a de sens qu'en tant que corrélat d'un acte effectif. (Cavaillès 1994, 185)

Therefore, Cavaillès's definitive description of experience is a system of gestures governed by a rule and subject to conditions that

are independent of the gestures (Cavaillès 1994, 601). This idea of gesture is the nucleus of a new constructivism or a new form of deep realism, as we shall see.

Certainly, Cavaillès places a clear boundary between mathematical experience and regular (physical) experience. According to Cavaillès, even if a baby is a mathematician in any of his gestures, the baby's experience cannot be structured according to rules (Cavaillès 1994, 180). Therefore, the baby's experience is an inchoate mathematics to which we cannot extend the methodology of mathematics. However, as Kant showed, the paradigm that we apply to mathematics will account at different levels for other objects of our experience, including any socially constructed object (including the arts) and, indeed, any communication, that is, any aspect of reality in which human beings are involved.

Is it possible to find a rule for describing and defining gestures also in our everyday experience? Finding this would trespass on Cavaillès's boundary, but would also respect his basing of mathematics on successful activity and on its traditions. Unfortunately, Cavaillès was shot by the Nazis and could not follow his insight, astonishingly ahead of the philosophy of his time, in which he forecasted the impasse that analytic philosophy had reached in every field of endeavor.

The French philosophy of mathematics during the last fifty years forecasted, or put more poetically, longed for, a trespass on Cavaillès's prohibition. Gilles Châtelet goes back to the notion of gesture, describing mathematics as a «major gesture within the gesture of humanity» (Châtelet 2010, 178). The dimension of gesture prevents mathematics from being considered as either the queen of sciences or a mere tool, locating it among human practices. Moreover, in commenting on revolutionary developments in mathematics, for example Grothendieck's idea of passing from a world of points to a world of arrows, Châtelet highlights the most serious flaw of contemporary philosophy.

La rationalité philosophique n'a pas trouvé la forme moderne de rendre compte de cet événement où des terminologies mathématiques changent, où des mouvements de pensées se brisent, où précisément la formalisation apparaît comme un'espèce d'iceberg absolument (je vais inventer une horreur) 'incompréhensible' où tout est devenu cristal. (Châtelet 2010, 178-9)

Châtelet sees that mathematics is moving forward and that philosophy has not elaborated a corresponding form of rationality. He discards any possibility that cognitivism can provide a fair accounting for the powerful rationality shown in mathematics. At a certain point in his argument, he asks for a new form of rationality in which everything is both evident to the heart and clear to the mind. For Châtelet, ‘what has happened’ while we were reasoning is exactly the philosophical question we must tackle. «What happened?» «Que s’est-il passé?» (Châtelet 2010, 181) is the threshold between experience and reasoning, a question that can sometimes appear as suddenly as lightning but is not irrational, a question that happens when ‘we draw a line’, like in geometry, which was not there before. However, Châtelet bases this view on a restoration of the term ‘event’ and references to that term in Heidegger and phenomenology, even though he is aware that we are talking about some sort of «preparation of matter or of figure to the event» («une preparation de la matière ou une preparation de la figure à l’événement,» Châtelet 2010, 181). Interestingly enough, Châtelet underlines the relevance of the border between regular experience and mathematical experience. Unfortunately, he lacks a broad pragmatist view of experience so that, in the end, he is forced to say that this creative human capacity for experience imagines what is not in experience, while machines must stay within its limits (Châtelet 2010, 182). From a pragmatist point of view, the realm of possibility as well as the realm of necessity are part of experience so that mathematical gestures are not required to dwell in has not to look for any surrealism, but only in a rich realism.

In any case, Châtelet explored the frontier pointed out by Cavallès. Another attempt in this direction is Giuseppe Longo’s view. Longo is Italian but he is also heir to the same French tradition due to his many years spent at the École Normale Supérieure. Longo, a mathematician and expert in informatics and biology, underlines the intertwining of the material conditions of possibility and the conceptual construction of the mathematical gesture. Longo traces the history of this phenomenological and semiotic interplay back to the Lascaux caves, where primitive human beings invented the idea of border, a line that holds and limits content, in both a material and a conceptual way (Longo 2016). He then explores the idea of the universal into the particular through Euclid’s drawing of a line without thickness, Lorenzetti’s paint-

ings, Galileo's principle of inertia, Riemann surfaces, and Grothendieck's geometry. Mathematical constructions are rooted in a «practice of gesture» going far beyond any possible formalization, connected with human political and social life. According to Longo, there is a continuity between biology, pre-cognitive roots, and potential universality.

Tuttavia, il loro radicamento biologico e cognitivo, pre-umano, rende accessibili queste costruzioni a tutti gli uomini, produce la loro potenziale universalità per gli esseri umani, tutti. (Longo 2016, 39)

Interestingly, Longo attempts to employ Kant to explain conceptuality as being grounded in space and time. Again, I think he does not see the degree to which Kantian space and time analytically organize and limit knowledge; however, he does see the need for a change in those Kantian conditions when discussing biological rather than mathematical objects. Biology seems to require dynamic principles describing a story that generates spaces. I think that the asymmetry Longo correctly sees as a problem for his Kantian reading hides the deeper need to consider continuous change and historical a posteriority as grounds for our gestures. In the line of development I am drawing, Longo trespasses on Cavallès's prohibition, broadening Cavallès's view of mathematics to apply to any science, and underlining the mutual interpenetration of phenomenology and semiotics in any cognitive activity. Moreover, he proposes this interpenetration as the only way to reason about universals in particulars, showing the pernicious effects of both a bottom-up and a top-down approach to sciences. When we engage in science, no differently from any other form of creative work, we are neither simply defining and drawing consequences from definitions nor simply generalizing from examples. We are always gesturing; namely, working on a history of universals within particulars.

However, in the end, Châtelet and Longo rely, respectively, on Heidegger's form of phenomenology and Kant's transcendentalism. They broaden Cavallès's view of gesture but they do not see an alternative to escape from the dramatic need to abandon Kant's paradigm of the philosophy of consciousness.

## 2. Peirce and gesture

For this alternative, one needs different tools permitting the overthrow of the main foundation of the philosophy of conscious-

ness: what is needed is to overthrow the idea of representation, or better, the dualism between representation and represented object or, as Cavaillès put it, the *sens posant* and the *sens posée* (Cavaillès 2008, 45). Peirce did not talk of gestures but he found the way out of this dualism by figuring out a grounding for gestures as the elemental component of both mathematical and common experience.

Peirce, who significantly thought that mathematics grounded logic and not the other way around, found the escape from this dualism in the semiotic and phenomenological study of Existential Graphs (EG).<sup>2</sup> In EG, a chiefly iconic logic developed by means of diagrams or graphs, he found the unity of represented and representing. Reasoning happens while we are creating the graphs: there is no gap between reasoning and the representation of reasoning. The phenomenological and semiotic structures of the graphs are the conditions for a development that occurs according to the rule of different relationships among different semiotic and phenomenological elements. From Peirce's analysis of EG we understand that any reasoning is a transition that happens through changes; that is, as a form of continuity on a continuous ground (Maddalena 2015, 43-55). We understand also that when one draws the graphs, he/she must use all three phenomenological categories (firstness, secondness, and thirdness) and all three main kinds of signs (icons, indexes, and symbols) at the same time. Peirce thinks that the line of identity, which works as a quantifier in betagraphs, is a «perfect sign»; namely, a sign that blends together icons, indexes, and symbols in an equal or dense way (CP 4.448). I use the word 'gesture,' almost foreign to Peirce, to signify any action phenomenologically and semiotically structured with a beginning and an end that extends a meaning, exactly as EG do (Maddalena 2015, 68-77). Understood in this sense, there are 'gestures' in everyfield. There are gestures in the form of public and private rites, scientific experiments, and artistic creations or performances. Gestures are more apt to be epistemic tools when the blend of their semiotic and phenomenological structures is dense. In that case, they synthesize more, that is, they acquire new knowledge as they are performed.

As for mathematics, understood as a crucial field for comprehending the normal functioning of our reasoning, a Peirce in-

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<sup>2</sup>For a complete analysis of Existential Graphs from this point of view, see Maddalena 2015, 58-67. See also Zalamea 2010.



spired philosophy arrives at the same conclusions as Cavallès: mathematics does not need any foundation a priori because it grounds itself as it proceeds. Mathematics is a creative and completely autonomous discipline that we discover by doing it, as Wittgenstein would maintain some decades later. Moreover, one cannot find in mathematics an absolute beginning either in a naively conceived external reality or in any transcendental world. There is no absolute beginning because the beginning is located in the tradition of human mathematical practices. Mathematics is an important gesture among the gestures of humanity. And, contrary to Cavallès's prohibition, when one detects the structure of this gesture, he/she can apply it also beyond the realm of logic and mathematics to any major form of understanding or communicating the world, in particular in creative fields, but in any field in which we apply notions and practices belonging to the history of the field itself or to the important artefacts of humanity.

What is the nature of the escape from Cavallès's overthrow of Kant's account of mathematics and science? The escape is the possibility of understanding Kant's syntheticity not on analytic, rationalist grounds, while at the same time avoiding a slip into an irrationalist idea of event. We synthesize by performing an action; that is, by reasoning with our body and spirit at the same time, or rather, by performing actions in the presence of the phenomena and signs that perfuse reality around us and within us. We perform gestures as our contribution to the development of reality. Of course, actions can be more or less synthetic: the actions performed in a conversation like that described by Mead as a primitive language is less synthetic than a work of theater. Sometimes, as I learned from some very interesting performers I met at the Sorbonne in an event organized by Barbara Formis some years ago, we do not want to or we cannot synthesize too much. We want our gestures to be incomplete, to use phenomena and signs at different densities. However, in whatever way we perform gestures, they are our way to understand and communicate the world and to overthrow the dualism between represented and representation.

### **3. Conclusion**

We have seen that the problem of mathematical certainty that Kant initiated is still at the heart of any epistemic enterprise. Cavallès's considerations of the history of the philosophy of mathe-

matics illustrate how the attempts to reformulate or amend Kant's project failed. Among the proposed paths forward, the most realistic seems to be the pragmatist one based on the notion of gesture, an action that unites representation and represented. In order to understand gestures, we used Peirce's phenomenology and semiotics and applied the structure of mathematics to every human activity. We found that the structure, epistemology, and metaphysics that hold in mathematics and logic work in a synthetic way to also explain our ordinary ways of reasoning.

Recalling Cavallès's description of the powerful legacy of Kant's «philosophy of consciousness,» we are led to ask ourselves: are we abandoning that philosophy? Let us say that any philosophy of gesture opens up at least a different conception of consciousness. Maybe, there is space here for another very pragmatist claim. What is overthrown in a philosophy of gesture is the idea of consciousness inherited from Descartes, identified by Rorty as «the mirror of nature» (Rorty 1979).<sup>3</sup> In a philosophy of gesture, consciousness becomes somehow more similar to a medieval, Bonaventurian view (Bonaventura 1994, 141), less egotistical and more fallibilist. In this view, we are free but we are not the masters and even less the owners of the world. We belong to it like every other creature. We can change the world with our assents and dissents – as witnessed by Cavallès resistance to Nazism – but our consciousness is always in movement just like our judgments. Our judgments are a recognition of identities in ever-changing reality, and they move among possibilities and necessities as well as actualities. Our conscience is always in transition and has never exhausted its possibilities.

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<sup>3</sup> On Rorty see also Calcaterra 2016.