Many-qubit quantum state transfer via spin chains

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Abstract. The transfer of an unknown quantum state, from a sender to a receiver, is one of the main requirements to perform quantum information processing tasks. In this respect, the state transfer of a single qubit by means of spin chains has been widely discussed, and many protocols aiming at performing this task have been proposed. Nevertheless, the state transfer of more than one qubit has not been properly addressed so far. In this paper, we present a modified version of a recently proposed quantum state transfer protocol [Phys. Rev. A **87**, 062309 (2013)] to obtain a quantum channel for the transfer of two qubits. This goal is achieved by exploiting Rabi-like oscillations due to excitations induced by means of strong and localized magnetic fields. We derive exact analytical formulae for the fidelity of the quantum state transfer, and obtain a high-quality transfer for general quantum states as well as for specific classes of states relevant for quantum information processing.

1. Introduction

Quantum Information Processing (QIP) is a fundamental resource for the next generation of technological devices based on quantum principles. The field of QIP is attracting a continuously increasing interest since its birth, a few decades ago, and branches into several subfields: quantum computation, quantum communication, quantum algorithms, and quantum cryptography, just to name a few [1]. Possible broad-reaching applications of such devices are triggering experimental and theoretical efforts, which aim at controlling and manipulating the systems on a quantum scale and characterizing their many-body properties.

In this paper, we focus on Quantum State Transfer (QST), perhaps the simplest communication protocol, designed to send a quantum state through interconnected on-chip architectures, from one node to another. QST can be useful in short distance communications to avoid interfacing problems with flying qubits. In its simplest version, a QST task is performed between parties connected via quantum spin channels, that is, 1D interacting spin-$\frac{1}{2}$ chains working as data buses, see Fig. 1.

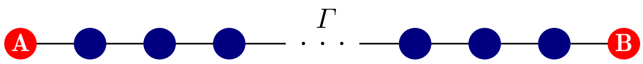

Figure 1. (Color online) The quantum state $|\Psi(0)\rangle_1$, encoded on spin A, and residing on the first site of a spin chain, is meant to be transferred to spin B, which resides on site N, at some time t^*, by exploiting the coherent dynamics of the quantum channel Γ.

Since the seminal paper by Bose [2], numerous protocols acting on spin-$\frac{1}{2}$ chains have been proposed and investigated in order to achieve a high QST fidelity. These can be classified into two broad groups, namely time-dependent and time-independent protocols (see Refs. [3] and references therein for review). In the former protocols, the transfer operation is performed by a time-modulation of the interaction parameters of the spin model; whereas, in the latter scenario, the interaction parameters are kept fixed during the execution of the QST protocol. Notwithstanding the rapid improvement of the ability to control the interactions between quantum spins (especially in cold atom set-ups [4]), the accuracy required to perform time-dependent protocols seems to be still out of reach. Hence, in this paper, we will focus on time-independent QST protocols.

As far as the transfer of the quantum state of a single qubit is concerned, it has been shown [5] that perfect-QST and perfect entanglement distribution are completely equivalent. In other words, if a quantum channel is capable of transferring a quantum state with unit fidelity from a sender to a receiver, then it will also allow for a maximally entangled state to be distributed between these two parties. A high-fidelity QST data bus can be employed as an entangling gate between two spins [6], producing, e.g., any one of the maximally entangled Bell states with two qubits: $|\Psi^{\pm}\rangle = \frac{1}{\sqrt{2}}\left(|01\rangle \pm |10\rangle\right)$ and $|\Phi^{\pm}\rangle = \frac{1}{\sqrt{2}}\left(|00\rangle \pm |11\rangle\right)$, here expressed in the (logical) basis of eigenstates of the Pauli matrix σ^z. The shared entangled state can be then exploited, via the celebrated teleportation protocol (TP) [7], as a resource to teleport an unknown quantum state from one location to another, thus achieving the desired QST. The TP requires the sender to perform a two-qubit measurement, followed by the transmission of two bits of classical communication to the receiver, which report the result of the measurement. Finally, a conditional unitary operation on the target spin is carried out by the receiver himself. As a consequence, one-qubit QST can be achieved deterministically by the use of a two-qubit Bell state teleportation channel.

Unfortunately, there is not such a clear-cut equivalence when multipartite QST is addressed, that is, when the quantum state to be transferred has $n \geq 2$ qubits. The reason for this difficulty in connecting the n-QST with multipartite entangled states, which would be useful as a resource for implementing a teleportation channel, is mainly due to the in-equivalence of entangled states in the multipartite regime. For instance, the entanglement of teleportation E_T [8] has been adopted as a quantifier of the capability of an entangled state to act as a teleportation channel. It turned out that 4-qubits GHZ-states, $|GHZ\rangle = \frac{1}{2}\left(|0000\rangle + |1111\rangle\right)$, have $E_T = \frac{1}{2}$ and only special classes of 2-qubit states can be deterministically teleported [9]. On the other hand, a different class of maximally entangled 4-qubit entangled states, the so-called W-states, $|W\rangle = \frac{1}{2}\left(|0001\rangle + |0100\rangle + |0100\rangle + |1000\rangle\right)$, have $E_T = 0$, and no deterministic 2-QST is possible. Actually, it is possible to teleport a 2-qubit state by using a 4-qubit entangled state [10], and an explicit protocol for generic n-QST has been introduced in terms of the so-called $2n$ generalized Bell states [8], which, indeed, have unit teleportation entanglement.

Notwithstanding the possibility to teleport an n-qubit state by means of suitable entangled states, shared by the sender and the receiver, in this paper we address the question in terms of the *transport* of the state. This presents some advantages with respect to the use of a teleportation channel: first of all, in order to perform the n-QST via TP, there is the need to implement generalized Bell measurements, which up to now seem to be highly non-trivial; furthermore, sender and receiver need to protect efficiently their shared pure entangled state from the environment, as the entanglement of teleportation has to be unity in order to achieve a successful TP. If this is not the case, little can be said. Indeed, to be best of our knowledge, an analysis of n-QST via TPs employing mixed states has yet to be performed, and not much is known about the efficiency of such tasks. Therefore, we consider the case of QST in a setting where the state is encoded at one end of the spin chain. Then, by exploiting the natural dynamics of the quantum channel, we aim at retrieving the same state (up to some unitaries) at the other end, as shown schematically in Fig. 2.

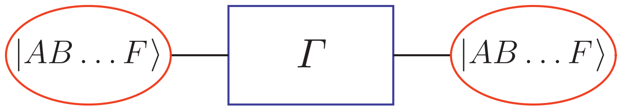

Figure 2. (Color online) The quantum state $|AB...F\rangle$ of the sender qubits is aimed at being transferred to the receiver spins via the quantum channel Γ.

The paper is organized as follows: in Sec. 2 we review some of the main single-qubit QST mechanisms based on spin chains; in Sec. 3 we solve the many-body dynamics for our model, which will be used in Sec. 4 to present our main results on the QST fidelity. Finally, in Sec. 5 we draw some conclusions.

2. Overview of single-qubit quantum state transfer protocols

In this Section, we will focus on 1-QST as performed by means of an open and finite spin-$\frac{1}{2}$ chain, like the one depicted in Fig. 1. The Hamiltonian describing its dynamics is taken to be of the XX-Heisenberg type, with nearest-neighbour interactions only, plus a magnetic field along the z-axis:

$$H = -\sum_{l=1}^{N} J_l(\sigma_l^x\sigma_{l+1}^x + \sigma_l^y\sigma_{l+1}^y) + \sum_{l=1}^{N} h_l\sigma_l^z \ . \tag{1}$$

The initial state, encoded on the sender spin A, reads $|\psi(0)\rangle = a\,|0\rangle + b\,|1\rangle$, with $|a|^2 + |b|^2 = 1$, whereas the rest of the chain, including the quantum channel Γ and the receiver B, is initialized in $|\Gamma\rangle|B\rangle = \otimes_{j=2}^{N}|0\rangle_j$. The evolution of the overall state is $|\Psi(t)\rangle = a \otimes_{j=1}^{N}|0\rangle_j + b\,e^{-iHt}\,|1\rangle_1 \otimes_{j=2}^{N}|0\rangle_j$. By tracing out all of the spins but B, one obtains the (generally mixed) state of the receiver spin on site N: $\rho^B(t) = Tr_{\neq N}\left(|\Psi(t)\rangle\langle\Psi(t)|\right)$. The fidelity between the state transferred to the receiver on site N, and the state encoded initially on spin 1 by the sender, is given by [11]:

$$F\left(|\psi(0)\rangle\langle\psi(0)|_A\,, \rho_B(t)\right) = \sqrt{{}_A\langle\psi(0)|\,\rho_B(t)\,|\psi(0)\rangle_A} \ . \tag{2}$$

The quality of a QST-bus, however, cannot be simply evaluated by considering the fidelity of the transfer of a single, specific input state, but rather by an average QST-fidelity obtained over some classes of states. We will consider here the average fidelity $\bar{F}(t)$, evaluated by integration over the Bloch sphere of all possible pure input states, as a QST figure of merit. In Ref. [2] it has been shown that the average fidelity is given (up to a phase that can be adjusted by a suitable magnetic field) by the expression

$$\bar{F}(t) = \frac{1}{2} + \frac{|f_{N1}(t)|}{3} + \frac{|f_{N1}(t)|^2}{6} \ , \tag{3}$$

where $f_{N1}(t)$ is the transition amplitude of the excitation (or, equivalently, of the spin-flipped state $|1\rangle$) from site 1 to site N. The total magnetization along the z-axis commutes with the hamiltonian given by Eq. 1, then, the subspaces with a fixed number of excitations are invariant under its action. As a consequence, the amplitude is straightforwardly evaluated via

$$f_{N1}(t) = \langle N|\,e^{-iHt}\,|1\rangle = \sum_{k=1}^{N} \langle N|\,\mathbf{a}_k\rangle\langle\mathbf{a}_k\,|1\rangle\,e^{-i\lambda_k t} \ , \tag{4}$$

where $\{\lambda_k, |\mathbf{a}_k\rangle\}$ are the single-particle eigenvalues and eigenvectors of Eq. 1. Now, the average fidelity $\bar{F}(t)$ depends (monotonically) only on the modulus of the excitation transition amplitude $|f_{N1}(t)|$ from site 1 to site N. Besides, $f_{N1}(t)$ (Eq. 4) is a sum of products of the overlaps of any (single particle) eigenstate with the initial and final states $|1\rangle$ and $|N\rangle$. Each these overlaps bring in a phase factor determined by the eigenvalues λ_k and the k-sum in Eq. 4 runs over all the eigenstates of Eq. 1. It turns out that the different components of $f_{N1}(t)$ interfere destructively. Accordingly, a very low quality of 1-QST is obtained for uniformly coupled spins in long chains, unless a specific procedure [3] is applied to single-out some of the terms in the sum given in Eq. 4.

From Eq. 3 it is evident that, as far as 1-QST is concerned, high-fidelity protocols aim at maximizing the transfer of the spin-flipped state $|1\rangle$ from site 1 to site N. In order to maximize $f_{N1}(t)$, within time-independent hamiltonian protocols, several strategies have been adopted, which can be broadly classified in: a.) couplings-engineering methods [12, 13, 14, 15], b.) ballistic transfer [17, 18, 19, 20, 21, 22], and c.) Rabi-like dynamics [16, 23, 24, 25, 26, 27, 28].

In protocols relying on methods of the a.)-type, the nearest-neighbor spin couplings are chosen in such a way that the energy spectrum becomes linear, thus allowing a dispersion-less transfer of the excitation; an instance of such a coupling set is given by $J_n = \sqrt{n\,(N-n)}$, which yields a perfect QST, that is $F = 1$. With regards to ballistic transfer settings of the b.)-type, the main idea is to modify only the couplings of the sender and the receiver to the rest of the spin chain. These couplings are chosen in such a way that, even tough the overall energy spectrum is non linear, most of the modes prominently involved in the dynamics reside in a portion of the spectrum, which is approximately linear. For example, by tuning $J_1 = J_{N-1} = c\,N^{-\frac{1}{6}}$ (with c reading $c = 1.030$ for $N \gg 1$) average fidelities higher than 99% are achieved [18]; furthermore, allowing also for modulations of the second and last-but-one couplings, $\bar{F}$ gets as high as 99.9% in arbitrarily long chains [19]. Finally, methods of c.)-type consist in restricting the dynamics to just two (or three) modes of the spectrum in such a way that effective Rabi oscillations of the excitations take place between the endpoints of the chain.

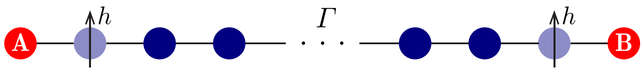

Figure 3. (Color online) The state of qubit A is transferred to qubit B by means of Rabi-like oscillations between eigenstates localized on A and B, due to a strong magnetic field on the neighboring "barrier" spins.

Up to now, these schemes have been applied mainly to the QST of a single qubit (even if this does not mean, in general, that the dynamics is restricted to the single excitation subspace, as the chain could be initialized in a state different from $|0\rangle^{\otimes N}$, [30]). However, it would be of uttermost importance to have a *single* channel able to perform different QIP tasks, such as, for instance, a QST of arbitrary n-qubits or a QST involving more than just a single sender and receiver.

In the next section we will focus on a specific Rabi-like 1-QST protocol (Fig. 3), presented in Ref.[23], with the idea to investigate its capability to be used as an n-QST bus. The relevant feature of the protocol is given by the presence of (strong) magnetic fields applied only on the spins sitting at sites 2 and $N-1$, whereas the spin-spin coupling is assumed to be uniform. These strong magnetic fields on the next-neighboring spins to the sender and the receiver, named hereafter *barrier* spins, play a crucial role in effectively decoupling the end-spins from the rest of the chain. Then, an effective Rabi-like hamiltonian $H_R \simeq \omega_1 |\Psi_1\rangle\langle\Psi_1| + \omega_2 |\Psi_2\rangle\langle\Psi_2|$ can be written down, with the two modes $|\Psi_i\rangle$ bi-localized on sender and receiver: $|\Psi_{1,2}\rangle \simeq \frac{1}{\sqrt{2}}(|01\rangle \pm |10\rangle)_{1N}$. In the following, we will investigate if such a scheme succeeds also in the n-QST scenario.

It is worth mentioning that such a setup has been already extended in Ref. [29] to perform a one-to-many routing protocol.

3. Dynamics in the many-body regime

In the usual single qubit QST scenario (sketched in Fig. 1), a pure state $|\Psi\rangle$ is encoded at time $t=0$ in a spin located at site 1 (the sender). The aim is to retrieve, at some time $t=t^*$, the very same state in a spin located at site N (the receiver), via the natural (coherent) dynamics of the channel ruled by H. As the number of excitations (flipped spins) remains constant during the evolution, the dynamics takes place in the single excitation subspace, as witnessed by Eqs. 3 and 4. Though, one of the applications of the QST protocol is, for instance, to transfer the output of a QIP task, performed by a quantum processor, to another quantum device, and often it is the case that such an output consists of more than just one qubit. As a consequence, the output has to be encoded in a larger Hilbert space, say, that of n spins; so the resulting dynamics of the n-QST may take place simultaneously in all the subspaces with $m \leq n$ excitations. These components of the initial state have to evolve in such a way that there exists a certain time t^* at which the initial state is rebuilt at the n receiver spins. It is evident that not only the analytical, or numerical, complexity of the problem increases exponentially with the number of spin flipped in the initial state, but also the achievement of high-quality transfer may become considerably more difficult w.r.t. the 1-QST case.

One way to circumvent the problem would be to employ n *identical*, non-interacting quantum channels, each transferring one component of the state [5]. However, there are some drawbacks in this configuration: all of the n channels have to posses exactly the same technical specifics, as each of the component of the whole n-qubit quantum state has to be delivered at the same time $t=t^*$ to the corresponding receiver. Unfortunately, experimental imperfections are likely to modify the coupling strength between spins in the different chains, thus yielding different optimal transfer times t^*. These times could also depend on the state to be transferred, thus making the n-qubit QST a quite involved task in the presence of disorder; see, e.g., Refs. [31]. Furthermore, in order to fulfil the assumption of coherent dynamics, the n quantum channels have to be efficiently protected from detrimental environmental effects, which may result in a demanding technical (and economical) request. Also endowing the QIP devices by moving parts give rise to easily presumable difficulties. It would be therefore interesting to explore the possibility to use a single quantum channel to perform arbitrary n-qubit QST, as shown schematically in Fig. 2.

To achieve this goal, we now turn our attention to a detailed analysis of the dynamics in the many-body regime. Let's consider the spin model given in Eq. 1, with uniform couplings ($J_i = J$, $\forall i$). This model can be mapped to a spinless fermion model via a Jordan-Wigner transformation [32]

$$\hat{\mathcal{H}} = -2\sum_{i=1}^{N} \hat{c}_i^\dagger \hat{c}_{i+1} + \text{h.c.} - \sum_{i=b_1,b_2} 2h\hat{c}_i^\dagger \hat{c}_i \ . \tag{5}$$

where we have taken $J=1$ as our energy unit. Notice that the magnetic field is zero everywhere but at the barrier spins, on sites $b_1 = n+1$ and $b_2 = N-n-1$, where it takes the same value $h>0$. The most general initial n-qubit pure state reads [33]

$$\begin{aligned}|\Psi(0)\rangle_{12..n} =& a_0 |0\rangle + \sum_{n_1=1}^{n} a_{n_1} |n_1\rangle + \sum_{n_1<n_2=1}^{n} a_{n_1 n_2} |n_1 n_2\rangle + ... \\ &... + \sum_{n_1<n_2<...<n_j=1}^{n} a_{n_1 n_2 ... n_j} |n_1 n_2 ... n_j\rangle + \\ &\ldots + a_{n_1 n_2 ... n_n} |n_1 n_2 ... n_n\rangle \ , \end{aligned} \tag{6}$$

The last spin n of the sender string is coupled to the first barrier spin on site $n+1$, which is the first spin of the quantum channel Γ, made out of $N-2n$ spins; the last spin of Γ (the second barrier) is coupled to the first spin of the n receiver string, located at the other end of the chain. In Fig. 4 an instance of such a setting is shown for $n = 2$. Finally, both the spin bus Γ and the receiver spins are initialized in $|0\rangle$. We now derive the time-evolved state of the whole system, sender spins, quantum channel, and receiver spins. Because $\hat{\mathcal{H}}$ commutes with the total number of excitations, each n-fermions sector is an invariant subspace, and the analysis can be performed separately in each subspace. A lengthy but straightforward calculation shows that

$$|\Psi(t)\rangle_{12..N} = a_0\,|0\rangle + \sum_{n_1,m_1,k_1=1}^{N} a_{n_1}(t) D_{n_1}^{k_1} U_{k_1}^{m_1}\,|m_1\rangle \quad (7)$$

$$+ \sum_{n_1<n_2;k_1<k_2;n,m=1}^{N} a_{n_1 n_2}(t) D_{n_1 n_2}^{k_1 k_2} U_{k_1}^{m_1} U_{k_2}^{m_2}\,|m_1 m_2\rangle + \ldots$$

$$\ldots + \sum_{n^\uparrow=1}^{N} \sum_{k^\uparrow=1}^{N} \sum_{m_i=1}^{N} a_{n^\uparrow} D_{k_i^\uparrow}^{n_i^\uparrow} U_{m_i}^{k_i^\uparrow} \left|\{m_i^\uparrow\}\right\rangle \; .$$

Here the arrow in $x^\uparrow$ indicates that the set of x's are ordered with the location label increasing from left to right, i.e., $x^\uparrow \equiv x_1 < x_2 < \ldots < x_N$, whereas $U_{m_i}^{k_i^\uparrow} = U_{m_1}^{k_1} \ldots U_{m_N}^{k_N}$ and $a_{n_1 n_2}(t) = \prod_i e^{-itE_{k_i}}$. $D_{k_1 \ldots k_r}^{n_1 \ldots n_r}$ is the determinant of the minor made up by taking the $\{n_1, \ldots, n_r\}$ rows and the $\{k_1, \ldots, k_r\}$ columns of the matrix U, which diagonalizes the $N \times N$ hamiltonian matrix given by Eq. 5, in the single particle sector, while E_{k_i} denotes the k_i-th eigenvalue. Finally, by tracing out all of the spins but the receivers, the reduced density matrix $\rho^{(N-n+1,\ldots,N)}(t)$, describing the spins located on sites $m = N - n + 1, \ldots, N$ and embodying the QST target string, is obtained. Its fidelity with the state given by Eq. 6 can be evaluated via Eq. 2.

4. Results

In this Section we apply the above formalism to the first non-trivial case, i.e., the transfer of a 2-qubit state residing at sites 1 and 2, $|\Psi(0)\rangle_{12} = \alpha\,|00\rangle + \beta\,|01\rangle + \gamma\,|10\rangle + \delta\,|11\rangle$, to the spins located at the other end of the chain, $m = N-1, N$. To this purpose, we modify the scheme depicted in Fig. 3 by shifting the strong magnetic field on qubits 3 and $N-2$, with the idea to perform a 2-QST, as shown in Fig. 4.

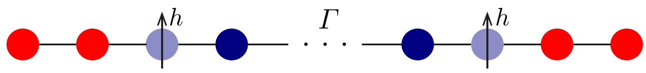

Figure 4. (Color online) 2-QST via Rabi-like mechanism between the two ends of the spin chain.

The reduced density matrix of the last two spins reads

$$\begin{pmatrix} \mathcal{G}_N^{N-1}\mathcal{G}^{*N-1}_N & \mathcal{F}^*_{N-1}\mathcal{G}_N^{N-1} & \mathcal{F}^*_N\mathcal{G}_N^{N-1} & \alpha^*\mathcal{G}_N^{N-1} \\ \mathcal{F}_{N-1}\mathcal{G}^{*N-1}_N & |\mathcal{G}_{N-1}^m|^2 + |\mathcal{F}_{N-1}|^2 & \mathcal{G}_{N-1}^m\mathcal{G}^{*m}_N + \mathcal{F}_{N-1}\mathcal{F}^*_N & \mathcal{F}^*_m\mathcal{G}_{N-1}^m + \alpha^*\mathcal{F}_{N-1} \\ \mathcal{F}_N\mathcal{G}^{*N-1}_N & \mathcal{G}_N^m\mathcal{G}^{*m}_{N-1} + \mathcal{F}_N\mathcal{F}^*_{N-1} & |\mathcal{G}_N^m|^2 + |\mathcal{F}_N|^2 & \mathcal{F}^*_N\mathcal{G}_N^m + \alpha^*\mathcal{F}_N \\ \alpha\mathcal{G}^{*N-1}_N & \mathcal{F}_m\mathcal{G}^{*m}_{N-1} + \alpha\mathcal{F}^*_{N-1} & \mathcal{F}_m\mathcal{G}^{*m}_N + \alpha\mathcal{F}^*_N & 1 - |\mathcal{G}_{N-1}^m|^2 - |\mathcal{G}_N^m|^2 - |\mathcal{F}_{N-1}|^2 - |\mathcal{F}_N|^2 \end{pmatrix} \quad (8)$$

Here the matrix elements $\mathcal{F}_r = \beta\,\langle r|\,U\,|1\rangle + \gamma\,\langle r|\,U\,|2\rangle$ and $\mathcal{G}_s^r = \delta\,\langle s,r|\,U\,|1,2\rangle$ depend on the time-evolution operator $U = e^{-iHt}$ and the m-sum is intended to range between 1 and $N-2$.

In order to obtain the average fidelity over all possible pure initial states, we conveniently adopt the following parametrization for 2-qubit pure states

$$|\Psi(0)\rangle_{12} = \mathcal{R}_1\mathcal{R}_2\left(\sqrt{\frac{s+1}{2}}\,|00\rangle + \sqrt{\frac{s-1}{2}}\,|11\rangle\right) \quad (9)$$

where $\mathcal{R}_{1,2}$ are local rotations acting on the first and second spins, and s characterize the initial amount of entanglement (as the concurrence is given by $C = \sqrt{1-s^2}$).

4.1. Average Fidelity for general states

With the reduced density matrix written above, we obtained an analytic expression for the average fidelity $\bar{F}(t)$ (not reported here for the sake of brevity). The results are shown in Fig. 5, where we report the maximum average fidelity $\bar{F}(t)$ achieved at an optimal time t^*, scanned over the time interval $[0, t_{max}]$, for different values of the magnetic field h on the barrier spins. The length of the quantum channels chosen in the figures is a minimal one, that is $N = 7$ and $N = 8$, in order to demonstrate a proof-of-principle of the setup. Longer chains will be addressed in the following subsection. We see that the fidelity increases if larger and larger values values of h are taken, and that spin-buses with an odd number of sites perform better that even-numbered ones, both because they require smaller barrier magnetic fields and because the optimal times where the maximum fidelity is achieved are shorter.

4.2. Average Fidelity for subsets of states

In the previous subsection we dealt with the transfer of the most general two-qubit state, and the dynamics of the chain involved all of these (invariant) subspaces, in presence of $n = 0, 1, 2$ excitations. We now switch to cases where the quantum state to be transferred is not completely unknown, but it belongs to a known subset. We will see that the above results can be further improved and, in addition, the analytical formula for the average fidelity simplifies. Indeed, for states of the

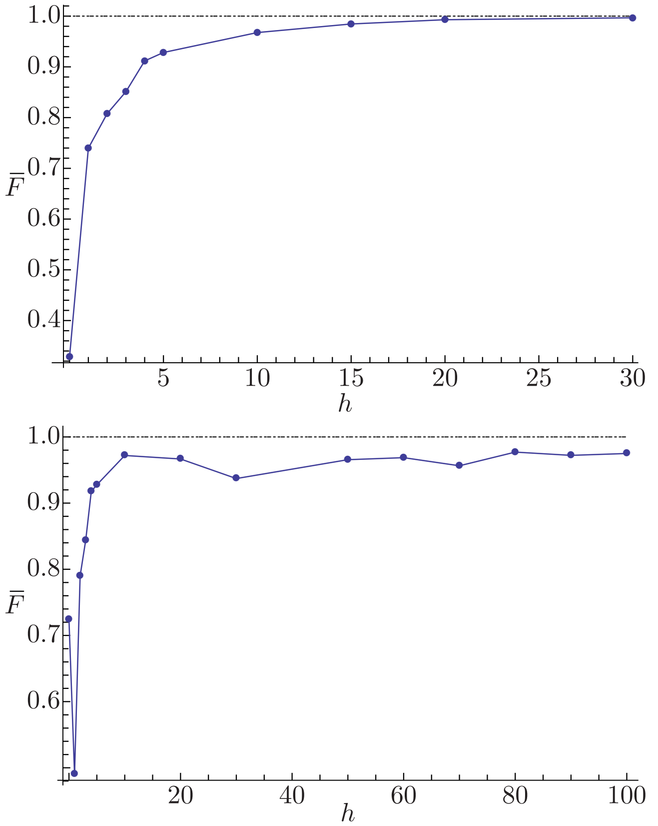

Figure 5. (Color online) Maximum average fidelity $\bar{F}$ for the 2-QST through chains made of N=7 and N=8 spins, for time scans $t\in[0, 2\times 10^4]$ and $t\in[0, 6\times 10^4]$, respectively (upper and lower panels). Notice how the presence of strong magnetic fields greatly enhances the transfer efficiency both for even and odd chains. Nevertheless, the protocol requires weaker magnetic fields and less time in order to be performed on odd-N chains.

form $|\Omega_1(0)\rangle_{12} = b\,|01\rangle + c\,|10\rangle$ and $|\Omega_2(0)\rangle_{12} = a\,|00\rangle + d\,|11\rangle$, the QST exploits the invariance of the respective subspaces, and interference effects in the average fidelity are strongly suppressed.

Restricting our considerations to states of the form $|\Omega_1(0)\rangle$, the average fidelity reads

$$\bar{F}(t) = \frac{1}{3}\left(|f_{N-1,1}|^2 + |f_{N,2}|^2 + \frac{|f_{N-1,2}|^2}{2} + \frac{|f_{N,1}|^2}{2}\right) + \frac{1}{3} Re\left[f_{N,2} f^*_{N-1,1}\right] \quad (10)$$

where $f_{j,i}$ denotes the transition's amplitude of the excitation from site i to site j. In Figs. 6 we report the threshold value of the magnetic field h^* as a function of the chain's length N. This yields an average fidelity, over the class of input states described by $|\Omega_1\rangle$, larger than 0.95. It turns out that, also for quite long chains, a high-quality 2-QST can be achieved with our method provided that suitable values of h are chosen.

Similar results are obtained by restricting the QST to the states $|\Omega_2\rangle$ (involving the 2-particle subspace). The average fidelity, in this case, is

$$\bar{F}(t) = \frac{1}{2} - \frac{1}{6}\sum_{n=1}^{N-2}\left(|g_{1,2}^{n,N-1}|^2 + |g_{1,2}^{n,N}|^2\right) + \frac{1}{3}\left(|g_{1,2}^{N-1,N}|^2 + Re\left[g_{1,2}^{N-1,N}\right]\right), \quad (11)$$

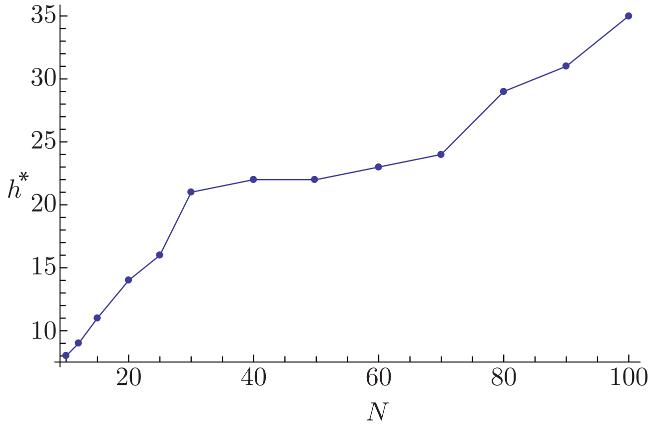

Figure 6. (Color online) Values of h as a function of N for which the average fidelity $\bar{F}$ of the 2-QST of states, belonging to the subset $|\Omega_1\rangle$, is larger than 0.95. The time scan has been performed over the interval $t\in[0, 1.3\times 10^4]$.

where $g_{1,2}^{r,s}$ is the transition's amplitude in the two excitations subspace.

Is is worthwhile to mention that the present work, as far as we are aware of, relates for the first time 2-QST and transition amplitudes in a functional form via Eqs. 10 and 11. This is achieved in the very same spirit as the framework leading to Eq. 3, which relates functionally 1-QST and single particle excitation transfer amplitudes. The results presented here, which are valid for general quantum channels ruled by Eq. 1, may pave the way to investigate other than Rabi-like protocols for the achievement of high-quality 2-QST.

Finally, we notice that the two subclasses $|\Omega_1\rangle$ and $|\Omega_2\rangle$ span respectively the two-qubit pure states with $\{0,1\}$- and $\{0,2\}$-spin flipped, respectively. This means that the proposed scheme is able to transfer, to the other end of the spin chain, outputs obtained by a magnetization-conserving unitary gate on two-qubits. Examples of such a gate are the Controlled-Z gate, (the target state of) a Fredkin gate, and arbitrary single- and two-qubit phase gates.

5. Conclusions

In this paper we addressed the problem of the transfer of many-body quantum states by means of open, finite 1D spin-$\frac{1}{2}$ chains, interacting via XX-Heisenberg exchange. Based on the protocol introduced in Refs. [23] and [29], we proposed a setting capable to transfer arbitrary 2-qubit quantum states between the end-points of a spin chain. The key element of the protocol is the presence of a strong magnetic field applied on two barrier qubits, on each side of the sender and receiver strings connected to the spin bus. This magnetic field effectively decouples the sender and receiver spins from the quantum channel, and results in an effective hamiltonian supporting Rabi-like oscillations of the excitations between the two ends of the spin chain. We expressed the average fidelity in terms of the corresponding excitation transfer amplitude and found that spin chains made of an odd number of spins generally perform better than even-numbered chains. By this we mean that the maximum average fidelity obtained is higher at fixed intensities of the magnetic field and the time duration of the protocol is shorter.

We also solved the dynamics for general n-qubit sender states, with $n > 2$, and this result may open the way to investigate n-QST as well as distribution of multipartite entangled states, relating in a functional form, amenable to theoretical investigations, the amplitude for multiple excitation transfer and n-QST fidelity. In this direction, we reported the explicit formulas for 2-QST which may trigger the search and optimization of other QST protocols.

6. Acknowledgement

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[33] A multi-spin computational basis notation is here adopted, that is, the ket $|n_1 n_2 ... n_i\rangle$ denotes the positions along the chain of the flipped spins, e.g., $|n_1{=}2\ n_2{=}4\rangle = |01010...\rangle$. Note that, because of the increasing ordering, no phase factor due to the Jordan-Wigner transformations arises. Thus, provided the same increasing ordering is in use when referring to fermion excitations, the same notation indicates the presence of a Jordan-Wigner fermion at postion n_i.