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A new vortex method for modelling two-dimensional, unsteady, incompressible, viscous flows.

by

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SUMMARY

This report presents details of a new mathematical model of viscous, incompressible, unsteady flow around multiple closed bodies. A system of vortex particles is employed in the model, on all solid boundaries and in the wake, to represent quantities of vorticity. The method is a Lagrangian technique and does not require the generation of a flow mesh.

A review of recent advances in vortex modelling is provided in the Introduction. Many of these ideas are incorporated into the model or are planned for future inclusion.

Section 2 is the main core of the report where the theoretical development of the model is presented. The extensive numerical details have been omitted and will be presented in a future report.

Conclusions are made regarding the development of the model and the verification procedure required to validate the algorithm.

CONTENTS

1. INTRODUCTION.

1.1 Overview.

The main objective of this report is to provide details of a new two dimensional vortex algorithm which has been proposed as a means of enhancing the modelling capabilities of the low speed aerodynamics group within the Department of Aerospace Engineering. Starting initially with investigations into the dynamic stall of helicopter rotors, research activities have been expanded into the areas of blade-vortex interaction, low Reynolds number aerofoils and the aerodynamic analyses of vertical and horizontal axis wind turbines. The development of algorithms capable of modelling both the free and contricted flow fields should result in clarification of this situation.

The decision to employ vortex methodology as opposed to other techniques was based on three main factors:

- (i) lower computational cost as compared with full Navier-Stokes (N-S) solvers (e.g. F.V., F.E.) - especially when the problem involves many unsteady cycles, multiple bodies in relative motion and numerous test cases;
- (ii) greater amount of flowfield information as compared with methods based heavily on empirical "fits" of experimental data;
- (iii) previous experience with vortex methods - many of the advantages and limitations of the methodology are already known.

1.2 Recent Developments in Vortex Methodology.

The starting point for vortex methods is the representation of the incompressible flow equations in vorticity/streamfunction form as opposed to pressure/velocity form. The velocity field is obtained from the vorticity field via the Biot-Savart law, and the vorticity field is updated by solving the associated transport equation.

The most popular methods approximate the continuous vorticity field by employing a finite number of small particles, each of which is tracked with time. It is this Lagrangian treatment of the vorticity field which is one of the main advantages of this approach, i.e. there is no mesh, body fitted or otherwise, to be generated and hence no fluxes across mesh boundaries. The vortices are normally defined by radially symmetric functions with either infinitesimal support (dirac distribution), finite support (e.g. constant vorticity core) or infinite support (e.g. Gaussian core). The latter two are more commonly employed today because of their greater stability and the availability of error estimates for both Euler and N-S solutions (e.g. Beale and Majda,1982; Fishelov, 1990).

One of the main disadvantages is the requirement, according to the error estimates, for a large number of vortices with overlapping cores. Since the vortex interactions dominate the computational cost, this is a serious drawback if neither a limitation is put on the number of vortices (e.g. coallescing device) nor an efficient, accurate approximation scheme is employed. The two main types of approximation scheme available are:

(i) _Particle-Mesh (PM) methods_ - vorticity is distributed over a previously generated mesh, and a "fast" Poisson solver is employed for the streamfunction. Numerical differentiation and interpolation are then used to obtain particle velocities (Hockney and Eastwood, 1981);

(ii) _Taylor expansion (TE) methods_ - analytic expressions for the potential/velocity field are expanded in a Taylor series and truncated after a few terms. Advanced versions involve multipole expansions, combining the use of both Taylor and Binomial series to achieve high accuracy (Greengard and Rokhlin, 1987).

The main disadvantages of the PM methods are twofold. Firstly, there is the deterioration in accuracy due to interpolation, especially with large variations in particle density. Secondly, the requirement for a body-fitted mesh destroys one of the benefits of the Lagrangian method. In contrast the more advanced TE methods have controllable accuracy and adaptivity (Carrier and Greengard, 1988) and, despite the use of a grid (_not_ body-fitted), retain the benefits of the Lagrangian formulation.

The inclusion of the viscous terms in the N-S equations has been tackled in two main ways:

(i) using Gaussian cores that expand with time, based on the exact solution of the pure diffusion equation (e.g. Leonard, 1980);

(ii) randomly perturbing the position of the vortices to reproduce the diffusion process statistically (e.g. Chorin, 1973).

The expanding core technique has been proven to approximate the wrong equation when convection is present (Greengard, 1985). The random vortex method, however, has been shown to converge, albeit with low order, to the solution of the N-S equations (Roberts, 1985). Fishelov has recently presented a deterministic method for modelling the viscous terms which involves directly differentiating the vortex core functions. Vortex strengths and positions are updated as a result of interactions with other vortices, including some with zero strength initially. This suggests, however, that the process would be very costly computationally, although higher order convergence appears to be obtainable.

Presentation and discussion of these and other aspects of vortex methods can be found in the comprehensive reviews of Leonard (1980), Spalart (1988) and Sarpkaya (1990).

1.3 Proposal for a New Model.

In the following sections the details of a new mathematical model of unsteady, incompressible, viscous flow around closed bodies is presented. Vortex "blobs" are employed exclusively, both in the vortical wake and in all solid boundary zones.

The boundary condition of zero flow across solid boundaries is applied in integral form rather than the more traditional collocation form. The governing constraints are more accurately satisfied in this way, especially when significant amounts of vorticity exist close to the boundary surfaces.

A new scheme for the removal of vortices is proposed. The aim is to cause as little disturbance to the solution as possible when vortices traverse a boundary or when the separation point is prescribed. Details of this and other features are presented herein.

2. MATHEMATICAL MODEL.

The theoretical development of the model pertaining to the exterior flow past multiple closed bodies is given below. The additional details associated with external boundaries, in particular wind tunnel walls, are provided in Appendix 1.

2.1 Governing Equations.

The two dimensional domain, consisting of the fluid region, the solid bodies and boundary surfaces, is illustrated in Fig. 1.

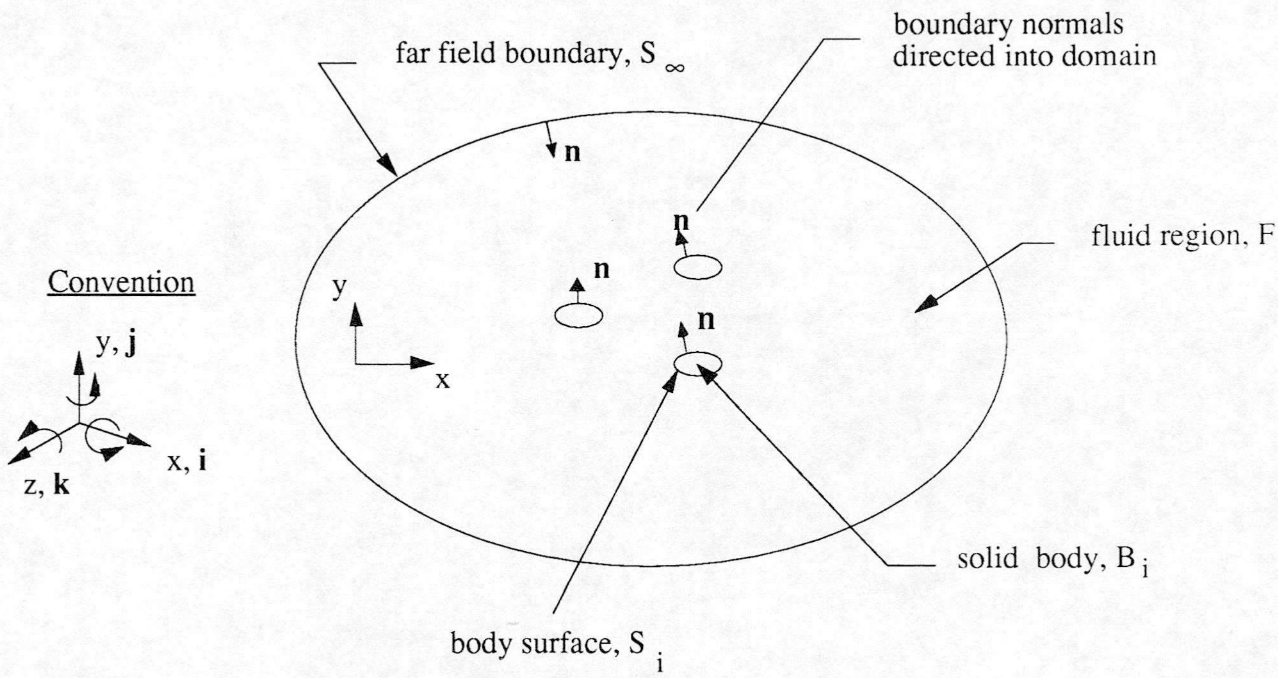

Fig. 1 Domain and boundary definitions.

The equations in *velocity/pressure* form are:

$$\text{INCOMPRESSIBILITY}: \quad \nabla \cdot \mathbf{U} = 0 \quad \text{in} \quad F \qquad (2.1.1)$$

$$\text{MOMENTUM}: \quad \frac{D\mathbf{U}}{Dt} = -\frac{1}{\rho}\nabla p + \nu \nabla^2 \mathbf{U} \quad \text{in} \quad F \qquad (2.1.2)$$

$$\text{SOLID REGION}: \quad \mathbf{U}_i = \mathbf{U}_{i0} + \Omega_i \times (\mathbf{r} - \mathbf{r}_{i0}) \quad \text{in} \quad B_i \qquad (2.1.3)$$

with kinematic boundary conditions:

$$\left.\begin{array}{lll} \mathbf{U}=\mathbf{U}_i & \text{on} \quad S_i & [\text{no penetration and no slip}] \\ \mathbf{U}=\mathbf{U}_\infty & \text{on} \quad S_\infty & \end{array}\right\} \tag{2.1.4}$$

Since both the fluid and body velocity fields are solenoidal, *vector potentials* exist such that:

$$\left.\begin{array}{lll} \text{Velocities} & \mathbf{U}=\nabla\times\boldsymbol{\Psi} \quad \text{and} & \mathbf{U}_i=\nabla\times\boldsymbol{\Psi}_i \\ \text{Vorticities} & \omega=\nabla\times(\nabla\times\boldsymbol{\Psi}) \quad \text{and} & 2\Omega_i=\nabla\times(\nabla\times\boldsymbol{\Psi}_i) \end{array}\right\} \tag{2.1.5}$$

In 2-D the vorticity and vector potential are defined in the **k** direction, i.e. $\mathbf{k}\omega$ and $\mathbf{k}\Psi$, where Ψ is the *stream function.* It can be of benefit to decompose the stream function into *onset* and *perturbation* components:

$$\Psi=\Psi_\infty+\psi$$
$$\Psi_i=\Psi_\infty+\psi_i$$

From relations (2.1.5) we get the governing equations in *vorticity/stream function* form:

$$\text{INCOMPRESSIBILITY}: \quad \nabla^2\psi=-\omega \quad \text{in } F \tag{2.1.6}$$

$$\text{VORTICITY TRANSPORT}: \quad \frac{D\omega}{Dt}=\nu\nabla^2\omega \quad \text{in } F \tag{2.1.7}$$

$$\text{SOLID REGION}: \quad \nabla^2\psi_i=-2\Omega_i \quad \text{in } B_i \tag{2.1.8}$$

with boundary conditions:

$$\left.\begin{array}{l} \mathbf{n}\times\nabla\psi=-\mathbf{n}^*\times\nabla\psi_i \\ \mathbf{n}\cdot\nabla\psi=-\mathbf{n}^*\cdot\nabla\psi_i \end{array}\right\} \quad \text{on } S_i \quad [\mathbf{n}^*=-\mathbf{n}] \tag{2.1.9}$$

$$\psi\rightarrow\text{constant} \quad \text{as } \|\mathbf{r}\|\rightarrow\infty \tag{2.1.10}$$

Conditions (2.1.9) cannot be applied explicitly as only one component can be specified on the boundary. However both conditions are satisfied, for the continuous problem, due to the representation of the internal kinematics of each solid body by the appropriate amount of constant vorticity.

2.2 Solution for the Stream Function/Velocity.

A solution to equation (2.1.6) can be obtained by employing Green's second identity:

$$\int_F [\phi\, \nabla^2\psi - \psi\nabla^2\phi]\,dF \;=\; \int_S [\,\psi\mathbf{n}\cdot\nabla\phi - \phi\mathbf{n}\cdot\nabla\psi]\,dS$$

Function ϕ is the *fundamental solution* to the problem:

$$\nabla^2\phi = -\,\delta(\mathbf{r} - \mathbf{r}_p) \qquad \text{in} \quad F$$

given by: $$\phi = \frac{1}{2\pi}\ln\left\{\frac{1}{\|\,\mathbf{r} - \mathbf{r}_p\|}\right\} \qquad (2.2.1)$$

Hence the value of the stream function at $\mathbf{r} = \mathbf{r}_p$ is given by:

$$\psi_p = \int_S [\,\psi\mathbf{n}\cdot\nabla\phi - \phi\mathbf{n}\cdot\nabla\psi]\,dS \;+\; \int_F \phi\omega\; dF$$

Applying to interior regions B_i also, the combined solution becomes [note $\nabla^2\phi = 0$ in B_i]

$$\psi_p = \int_{S_i} [(\psi - \psi_i)\mathbf{n}\cdot\nabla\phi - \phi\mathbf{n}\cdot\nabla(\psi - \psi_i)]\,dS_i \;+\; \int_{S_\infty} [\psi\mathbf{n}\cdot\nabla\phi - \phi\mathbf{n}\cdot\nabla\psi]\,dS_\infty$$
$$+ \int_F \phi\omega\, dF \;+\; \int_{B_i} 2\phi\Omega_i\, dB_i$$

Implementation of boundary conditions (2.1.9) and (2.1.10), and relation (2.2.1) results in the final equation for the stream function (excluding the arbitrary constant):

$$2\pi\psi_p \;=\; \int_F \omega\,\ln\frac{1}{\|\,\mathbf{r} - \mathbf{r}_p\|}\,dF \;+\; \int_{B_i} 2\Omega_i\,\ln\frac{1}{\|\,\mathbf{r} - \mathbf{r}_p\|}\,dB_i \qquad (2.2.2)$$

The perturbation velocity field is obtained from (2.1.5):

$$2\pi\mathbf{u}_p \;=\; -\int_F \omega\,\frac{\mathbf{k}\times(\mathbf{r} - \mathbf{r}_p)}{\|\,\mathbf{r} - \mathbf{r}_p\|^2}\,dF \;-\; \int_{B_i} 2\Omega_i\,\frac{\mathbf{k}\times(\mathbf{r} - \mathbf{r}_p)}{\|\,\mathbf{r} - \mathbf{r}_p\|^2}\,dB_i \qquad (2.2.3)$$

and the total velocity is then: $\mathbf{U}_p = \mathbf{U}_\infty + \mathbf{u}_p$

Equation (2.2.3) is the representation of the *Biot-Savart* law, which enables the calculation of the velocity field from a known distribution of vorticity. It is appropriate to make the following comments regarding the vorticity distribution at this point:

(i) the fluid vorticity, ω, consists of that which has been *previously generated* (i.e. known) on the solid boundaries and subsequently convected and diffused into the fluid wake region, and that which has been *newly created* (i.e. unknown) to satisfy the surface boundary conditions (see section 2.2);

(ii) the body internal vorticity exists only when rotation is present, and the volume integration can be recast as the following equivalent integral around the bounding surface:

$$2\Omega\mathbf{k}\times\int_F \nabla_o\phi\, dB_i = 2\Omega\mathbf{k}\times\mathbf{n}\int_{S_i}\phi\, dS_i$$

where ∇_o is the differential operator in the "r" space rather than the "r_p" space. [Note: $\nabla_o\phi = -\nabla\phi$].

2.3 Boundary Conditions.

In the method presented herein the the newly created vorticity occupies boundary region F_b, and the remainder occupies region F_w. The indeterminacy of the new vorticity is removed by applying the *kinematic constraint* (2.1.9) on the velocity field:

$$2\pi\mathbf{u}_{S_i} = \int_{F_b}\omega\frac{\mathbf{k}\times(\mathbf{r}_{S_i}-\mathbf{r})}{\|\mathbf{r}_p-\mathbf{r}\|^2}dF_b + \int_{F_w}\omega\frac{\mathbf{k}\times(\mathbf{r}_{S_i}-\mathbf{r})}{\|\mathbf{r}_p-\mathbf{r}\|^2}dF_w + \int_{S_i}\mathbf{n}\times 2\Omega_i \ln\frac{1}{\|\mathbf{r}_{S_i}-\mathbf{r}\|}dS_i \qquad (2.3.1)$$

where $V_F = V_{F_b}\cup V_{F_w}$

Note that implementation of equation (2.3.1) directly would overspecify the problem. Only one component of the velocity should be taken; the other is implicit in the specification of the body interior vorticity. In this model the normal component is explicitly implemented, i.e. we have:

$\mathbf{n} \cdot \mathbf{u} = \mathbf{n}_i \cdot \mathbf{u}_i \quad$ on S_i.

2.4 Uniqueness of Solution.

The solution for the vorticity occupying region V_{F_b} is not unique until the total amount of new vorticity created on each S_i is known. The flow is initially vorticity free and irrotational, hence the circulation around a separate contour enclosing each body and moving with the fluid, Γ_{C_i}, is constant. Thus we have:

$$\int_{S_i} \mathbf{U}\cdot\mathbf{s}\, dS_i = \int_{S_i} \mathbf{k}\cdot(\mathbf{n} \times \mathbf{U})\, dS_i = 2\Omega_i A_i$$

$$\int_{S_i} \mathbf{U}\cdot\mathbf{s}\, dS_i = \int_{S_i+S_{C_i}} \mathbf{k}\cdot(\mathbf{n} \times \mathbf{U})\, d(S_i+S_{C_i}) - \int_{S_{C_i}} \mathbf{k}\cdot(\mathbf{n} \times \mathbf{U})\, dS_{C_i}$$

$$= \int_{V_{F_i}} -\mathbf{k}\cdot(\nabla \times \mathbf{U})\, dV_{F_i} - \int_{S_{C_i}} \mathbf{U}\cdot\mathbf{s}\, dS_{C_i}$$

i.e
$$\Gamma_{C_i} = \int_{V_{F_{b_i}}} \omega\, dV_{F_{b_i}} + \int_{V_{F_{w_i}}} \omega\, dV_{F_{w_i}} + 2\Omega_i A_i \qquad (2.4.1)$$

The unknown vorticity is, therefore, dependent on the value of Γ_{C_i}. Initially this is set to some arbitrary constant (e.g. zero).

2.5 Discrete Body and Vorticity Representation.

The vorticity in region V_{F_w}, i.e. the wake, is represented by a distribution of vortex "blobs" as described in section 1.2. These blobs provide a more accurate approximation to the vorticity field than point vortices, which in addition cause stability problems. Thus we have

$$\omega = \sum_k \Gamma_k\, g(\mathbf{r} - \mathbf{r}_k) \qquad (2.5.1)$$

where the core function g satisfies the condition $\int_\infty g\, dV = 1$.

Note that the point vortex method corresponds to the case where $g = \delta$. The corresponding velocity field is obtained by substituting (2.5.1) into (2.2.3), the form of which depends on the particular choice of core function.

The solid, generally curved, boundary is approximated by a series of linear panels as illustrated in Figs. 2a and 2b. The distribution of total vorticity in region V_{F_b}, i.e. adjacent to the solid boundary, is represented by piecewise linear and continuous functions over the panels, as illustrated in Fig.2c. That is:

$$\gamma = \left(1 - \frac{s}{l_j}\right)\gamma_j + \frac{s}{l_j}\gamma_{j+1} \quad \text{on } j^{th} \text{ panel.}$$

This continuous distribution is further discretised into vortex blobs of the form of equation (2.5.1), with separation distance d (Fig. 2d). That is:

Position of m^{th} vortex on panel - $s_m = \frac{d}{2}(2m - 1)$

Strength of m^{th} vortex on panel - $\Gamma_m = d\left\{\left(1 - \frac{s_m}{l_j}\right)\gamma_j + \frac{s_m}{l_j}\gamma_{j+1}\right\}$

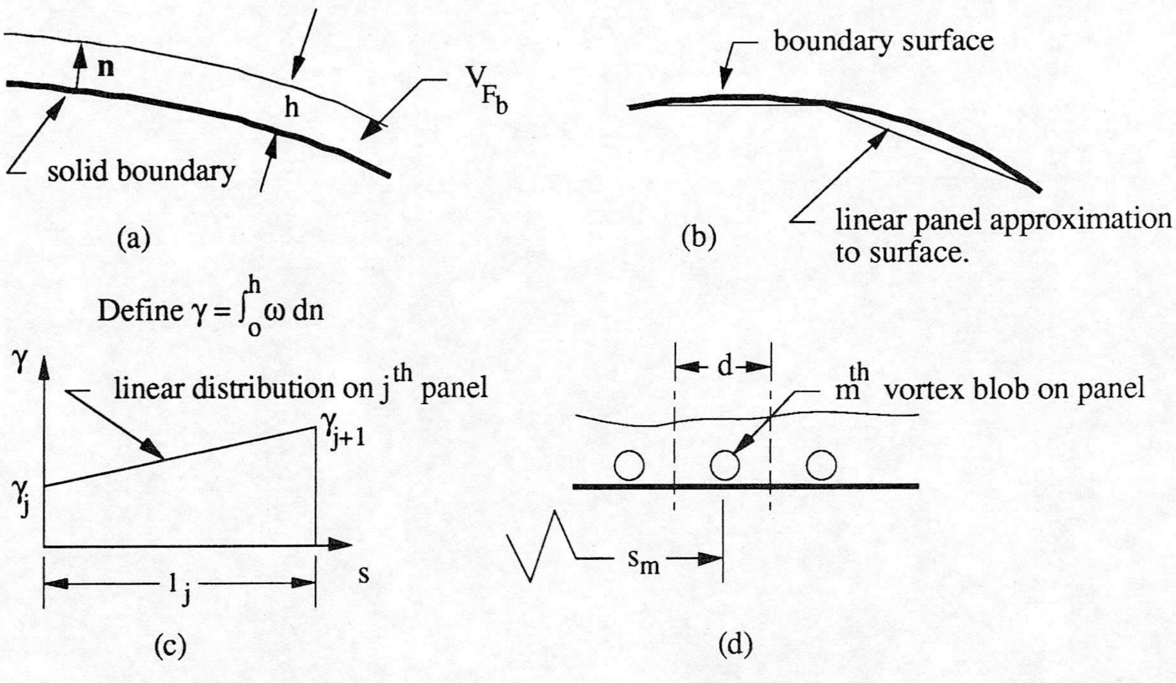

Fig. 2 Representation of solid boundary and total vorticity.

2.6 Vorticity Transport.

The evolution of the vorticity field is governed by the vorticity transport equation (2.1.7). This nonlinear equation describes both the convection and diffusion of the vortex field, and must be solved in addition to the equation of incompressibilty. A two stage process is adopted here, i.e. convection and diffusion are treated separately and the contributions added together.

Convection

The equation to be solved is: $$\frac{D\omega}{Dt} = 0 \qquad (2.6.1)$$

For a system of point vortices, i.e. (2.5.1) with g = δ, (2.6.1) is satisfied exactly by moving the vortices with the velocity of the flow:

$$\frac{D\mathbf{r}_k}{Dt} = \mathbf{u}(\mathbf{r}_k, t) \qquad (2.6.2)$$

where $\mathbf{r}_k$ is the position of the k^{th} vortex.

When vortex blobs are employed (2.6.2) provides an approximation to the vortex velocities, the accuracy depending on size and type of core (Beale and Majda,1982).

Diffusion

The governing equation is: $$\frac{\partial\omega}{\partial t} = \nu \nabla^2\omega \qquad (2.6.3)$$

The solution of (2.6.3) at time t for a unit vortex diffusing from the origin at t = 0 is:

$$\omega = \frac{1}{4\pi\nu t} e^{-\frac{(x^2+y^2)}{4\nu t}}$$

$$= \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{x^2}{2\sigma^2}} \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{y^2}{2\sigma^2}} \qquad (2.6.4)$$

where $\sigma = \sqrt{2\nu t}$.

Equation (2.6.4) is the form of the Gaussian probalility density function for the independent random variables x and y with zero mean and variance $2\nu t$, denoted $N(0,2\nu t)$. It is assumed that time t is obtained from a number of smaller time steps, $\beta\Delta t$ say. If x and y are obtained from a summation of Gaussian random variables x_i, y_i with distributions $N(0,2\nu\Delta t)$, then the distribution at time t of $x = \sum_{i=1}^{\beta} x_i$ and $y = \sum_{i=1}^{\beta} y_i$ is $N(0,2\nu t)$.

The Random Vortex Method simulates the diffusion process by incrementing the vortex positions as follows:

$$\Delta x_v = \eta_x$$
$$\Delta y_v = \eta_y$$

where η_x and η_y are $N(0,2\nu\Delta t)$.

2.7 Calculation of Pressure.

Applying the momentum equation (2.1.2) on the solid surface, an expression for pressure can be obtained. Taking $\mathbf{n} \times (2.1.2)$ on S_i, the pressure gradient along the surface is given by:

$$\frac{1}{\rho}\frac{\partial p}{\partial s} = -\mathbf{s}\cdot\frac{D\mathbf{U}_i}{Dt} + \nu\frac{\partial \omega}{\partial n} \qquad (2.7.1)$$

The last term represents the rate of creation of vorticity on the surface (with negative sign), and the total for the body is obtained by integrating (2.7.1) around the surface:

$$\int_{S_i}\frac{1}{\rho}\frac{\partial p}{\partial s}dS_i = 2A_i\frac{D\Omega_i}{Dt} - \int_{S_i}\nu\frac{\partial \omega}{\partial n}dS_i = 0 \qquad (2.7.2)$$

This is an expression of the fact that the pressure field must be single valued everywhere in the flow. It can be shown that (2.7.2) is equivalent to the uniqueness condition (2.4.1). Therefore, implementation of (2.4.1) ensures a single-valued pressure field (neglecting the effect of time discretisation). Surface vorticity creation is considered further in section 2.8.

2.8 Vorticity Creation and Absorption.

An element of the boundary region V_{F_b} is illustrated in Fig. 3.

Vorticity is generated in this region by the processes of creation at the surface and absorption from the external flow (V_{F_w}).

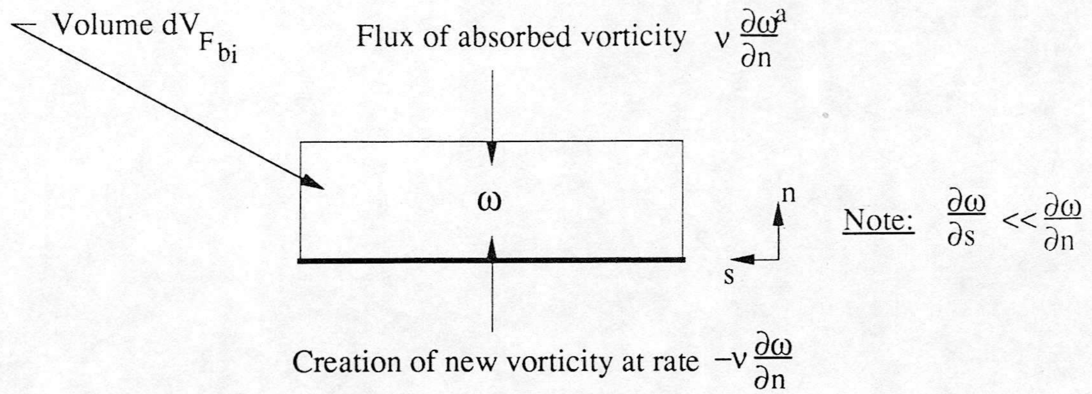

Fig. 3 Vorticity creation and absorption in elemental boundary zone.

In time dt, the total amount of vorticity in elemental region $dV_{F_{b_i}}$ is given by:

$$\omega \, dV_{F_{b_i}} = -\nu \frac{\partial \omega}{\partial n} dt \, dS_i + \nu \frac{\partial \omega^a}{\partial n} dt \, dS_i$$

Hence,

$$-\nu \frac{\partial \omega}{\partial n} dt = \gamma - \nu \frac{\partial \omega^a}{\partial n} dt \qquad (2.8.1)$$

In the formulation presented herein vorticity is aborbed whenever vortex blobs transgress the boundary zone. Therefore we must have:

$$\int_{S_i} \nu \frac{\partial \omega^a}{\partial n} dt \, dS_i = \left(\sum_k \Gamma_k^a \right)_i$$

We can identify a quantity of total vorticity equivalent to that absorbed in time dt. That is:

$$\gamma^a = \nu \frac{\partial \omega^a}{\partial n} dt$$

(2.8.2)

and therefore,

$$\int_{S_i} \gamma^a \, dS_i = \left(\sum_k \Gamma_k^a \right)_i \qquad (2.8.3)$$

Hence from (2.8.1) and (2.8.2), the rate of vorticity creation term in the pressure equation (2.7.1) can be written equivalently as:

$$\nu \frac{\partial \omega}{\partial n} = -\frac{(\gamma - \gamma^a)}{dt} \qquad (2.8.4)$$

The condition for a single valued pressure field, equation (2.7.2), can thus be written:

$$2A_i \frac{D\Omega_i}{Dt} + \frac{1}{dt} \int_{S_i} (\gamma - \gamma^a)\, dS_i = 0 \qquad (2.8.5)$$

Equation (2.8.5) is consistent with the uniqueness condition (2.4.1).

2.9 Solution Procedure.

The time evolution of the vorticity field is obtained via a solution, at each time step, of the boundary equation (2.3.1) in discretised form. The surface flux conditions are implemented in integral form to improve the robustness and stability of the algorithm, i.e.

$$\int_{\Delta S_i} (\mathbf{u} - \mathbf{u}_i).\mathbf{n}_i \; dS_i = 0$$

The resulting matrix equation for the new total vorticity is represented by:

$$\mathbf{K}\gamma = \mathbf{R} - \mathbf{R}^a \qquad (2.9.1)$$

where $\mathbf{R}^a$ represents the influence of vortices to be removed by absorption.

The solution for γ is made unique by implementing the following condition on the circulation for each body:

$$\int_{S_i} \gamma\, dS_i + \left(\sum_k \Gamma_k\right)_i - \left(\sum_k \Gamma_k^a\right)_i + 2\Omega_i A_i = 0 \qquad (2.9.2)$$

A similar set of equations are developed for the absorbed total vorticity:

$$\mathbf{K}\gamma^a = \mathbf{R}^a \qquad (2.9.3)$$

$$\int_{S_i} \gamma^a\, dS_i - \left(\sum_k \Gamma_k^a\right)_i = 0 \qquad (2.9.4)$$

Once equations (2.9.1) to (2.9.4) have been solved the pressure gradient, and hence aerodynamic loads, can be calculated from (2.7.1) and (2.8.4). Time can then be advanced and the procedure repeated for the incremented flow field.

The numerical details of the implementation of the above algorithm will be presented in a future report, along with a number of test cases to illustrate the behaviour of the model.

3. CONCLUSIONS.

Details of the theoretical development of a mathematical model of viscous, incompressible, unsteady flow around closed bodies have been presented in the previous section. The vortex methodology employed has been aimed at maximising the stability, accuracy and robustness of the algorithm, often a problem with models of this kind. A number of features, e.g. the vortex absorption scheme and the integral formulation, have been designed with this aim firmly in mind.

A future report will be produced which will provide details of the numerical implementation of the method. A series of test cases will be presented illustrating the behavior of the various features incorporated into the algorithm. The extent to which the above aims have been realised will then be able to be assessed.

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GLOSSARY

Roman symbols.

A	interior area of solid body.
B	region inside solid body.
d	separation of surface vortices.
F	fluid region.
g	vortex core function.
h	extent of surface boundary zone.
i, j, k	cartesian base vectors.
K	matrix of influence coefficients.
l	length of body panel.
n, **n**	direction normal to boundary surface.
p	static pressure.
R	vortex influence vector.
r	position vector.
S, s, **s**	distance along boundary surface.
t, dt	time, time step.
U, u	total/perturbation velocity.
V	volume.
x, y, z	cartesian coordinates.

Roman subscripts/superscripts.

a	absorbed vorticity.
b	boundary region adjacent to body.
C	contour enclosing body.
i	solid body counter.
j	body panel counter.
k	vortex counter.
m	panel vortex counter.
o	coordinate system origin.
p	point in flowfield.
v	vortex.
w	fluid wake.
x, y	cartesian coordinate directions.

Greek symbols.

*	solid body side of boundary surface.
β	number of time steps.
δ	Kronecker delta function.
ϕ	fundamental solution.
Γ	circuit circulation , vortex strength
γ, γ	total vorticity in boundary zone.
η	random variables.
ν	kinematic viscosity.
ρ	fluid density.
σ	standard deviation.
Ω, Ω	rotational velocity of body.
ω, ω	fluid vorticity.
Ψ, ψ, Ψ	total/perturbation stream function, vector potential.

Greek subscripts/superscripts.

∞	far field.

Mathematical operators.

$\frac{D}{Dt}$	Lagrangian derivative.
e	exponential function.
ln	natural logarithm.
N	normal distribution function.
$\|\ \|$	norm or magnitude.
$\times$	vector cross product.
∇	gradient operator.
∇^2	Laplace operator.
$\nabla\times$	curl.
$\nabla\cdot$	divergence.

$\cdot$	scalar product.
Σ	summation function.