ESTIMATION OF BEARING FORCES IN ROTATING MACHINERY: A PROBLEM REVISITED

Roger Kinns[1], John H McColl[2]

[1]School of Mechanical and Manufacturing Engineering
UNSW Australia, Sydney NSW 2052, Australia and RKAcoustics, Clynder, Helensburgh G84 0QQ, UK
Email: rogerkinns17@gmail.com

[2] School of Mathematics and Statistics, University of Glasgow
Glasgow G12 8QQ, UK
Email: John.McColl@glasgow.ac.uk

Abstract

Development of a new ship machinery installation may require combinations of prime movers, electric motors, generators, gearboxes and other items that have not been used previously. Large items of this type are expensive to develop, so that only existing production items are likely to be affordable. Modern quiet ship design requires careful attention to the source characteristics of the individual machinery items as well as the dynamic characteristics of any new combination, so that fluctuating forces transmitted to the hull via mounting systems and flexible connectors in the final installation are within acceptable limits. Almost periodic components, which arise at multiples of machine rotational frequency, are of particular concern.

Machine source properties are often known only in terms of the vibration characteristics of previous installations, not in terms of the disturbing forces that cause that vibration. The aim of the techniques described in this paper is to allow deduction of those disturbing forces from a matrix of transfer functions at each frequency of interest, measured with an existing machine in operation. The number of forces to be estimated must be less than the number of structural degrees of freedom. Those degrees of freedom arise from rigid body motions and machine flexural properties, which may change significantly when shafts are rotating. Also, the matrices must be redundant in order to allow estimation of the accuracy of derived force estimates. The larger the machine, the greater the number of degrees of freedom that are likely to arise at a given frequency.

A first use of the methodology was to establish the bearing forces in a marine turbo generator (TG) set with plain journal bearings. Measurements of transfer functions were made with the machine stationary and then with the machine in normal operation. Direct and reciprocal measurements in different directions were made for a large number of locations on the machine structure and bearings, covering the frequency range up to more than twice shaft rotational frequency. There were large differences in some frequency ranges between the static and operational conditions. Vibration due to machine operation was then measured to allow deduction of bearing forces using the transfer function matrices. Repeat measurements were made to establish whether machine source properties changed significantly with time, while statistical techniques were also used to identify and eliminate any suspect measurements. Those early experiments are described in this paper with a view to future application of similar techniques.

1. Introduction

Development of a new ship machinery installation may require combinations of prime movers, electric motors, generators, gearboxes and other items that have not been used previously. Large items of this type are expensive to develop, so that only existing production items are likely to be affordable. Modern quiet ship design requires careful attention to the source characteristics of the individual machinery items as well as the dynamic characteristics of any new combination, so that fluctuating forces transmitted to the hull via mounting systems and flexible connectors in the final installation are within acceptable limits. Almost periodic components, which arise at multiples of machine rotational frequency, are of particular concern.

1.1 Identification of machine source properties

Machine source properties are often known only in terms of the vibration characteristics of previous installations, not in terms of the disturbing forces that cause that vibration. The aim of the techniques described in this paper is to allow deduction of those disturbing forces from a matrix of transfer functions at each frequency of interest, measured with an existing machine in operation. The number of forces to be estimated must be less than the number of structural degrees of freedom. Those degrees of freedom arise from rigid body motions and machine flexural properties, which may change significantly when shafts are rotating. Also, the matrices must be redundant in order to allow estimation of the accuracy of derived force estimates. The larger the machine, the greater the number of degrees of freedom that are likely to arise at a given frequency.

1.1.1 Measurement of force transmission characteristics

In principle, trials on an existing machine in a shore establishment are more straightforward than trials at sea: there is better access to a running machine and trials can be conducted in a more benign environment. They are less likely to be curtailed by changes in operational requirements and seaway motion is absent. Also, the machine can be isolated from other items using flexible mounts and connectors, so there is usually less interference from noise and vibration due to adjacent machinery items. Accurate measurements of force transmission characteristics are still not easy to obtain. Individual machines often weigh tens of tonnes, while complete propulsion machinery installations can weigh hundreds of tonnes.

Shakers that are capable of sufficiently large force inputs to give measurable vibration are often large and heavy, but still have to be supported so that forces are applied at precise locations in chosen directions. Time and budgets may limit the number of possible measurements, while machines may have to be stopped unexpectedly. Errors may exist in individual measurements, which might arise, for example, from misalignment of shakers, wiring errors or faulty accelerometers and force gauges. That is why statistical techniques are potentially so important in protecting the analyst from gross errors, while providing a measure of accuracy of derived results.

1.1.2 Application of the reciprocity theorem

The reciprocity theorem allows measurement of transfer functions that might prove impossible without it. Its marine applications have been explored extensively by Ten Wolde and his colleagues in The Netherlands [1]. Application is straightforward for many ship trials, including hydrophone measurements as well as applied forces, noise and vibration, but limitations arise when there are significant gyroscopic effects, for example. The best test in any practical application is to make at least a limited number of direct and reciprocal measurements and check that they give similar results. Examples will be given for a running turbo-alternator, where the rotating masses represent a significant proportion of the total machine mass.

The basic process for direct and reciprocal measurements is shown in Figure 1. In the direct trial, the force F_1 is applied at the chosen frequency and the response acceleration is measured at a_1. In the reciprocal experiment, the force F_1' is applied at the location and in the direction of a_1 and the acceleration a_1' is measured at the location and in the direction of F_1. The forces and accelerations are complex numbers that contain amplitude and phase information. By reciprocity:

$$\frac{a_1}{F_1} = \frac{a_1'}{F_1'} \tag{1}$$

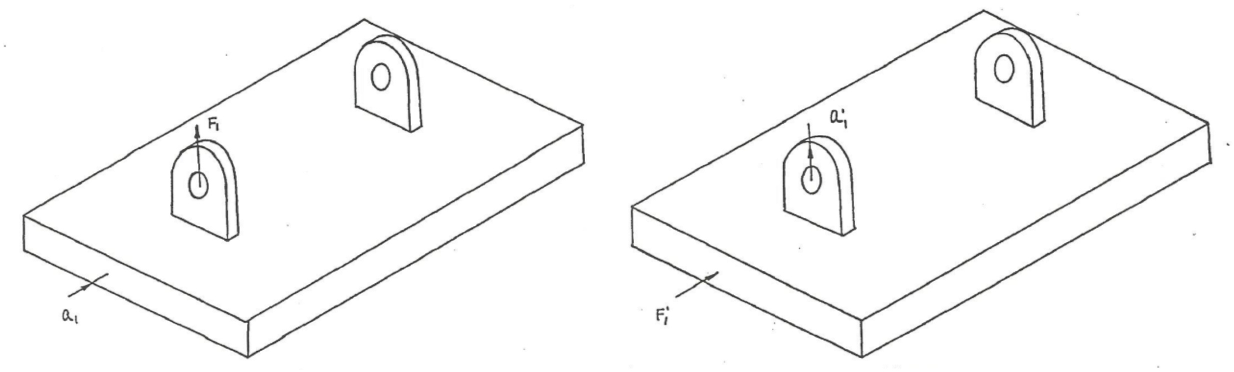

Figure 1. Direct and reciprocal measurements

2. Force Estimation Procedure

The basic procedure for force estimation is described in [2,3]. Suppose that a force F_i at the i^{th} possible position and direction of force application induces an acceleration a_{ij} at the j^{th} measurement position, and that:

$$a_{ij} = H_{ij} F_i \tag{2}$$

H_{ij} is a complex number, so that both the magnitude and phase of a_{ij} are defined relative to F_i. Although attention is restricted to the shaft rotational frequency, the frequency dependence of H_{ij} is to be understood.

In general, there will be p forces, F_i ($i = 1, 2, \ldots, p$) and n measured accelerations, A_j ($j = 1, 2, \ldots, n$). Each measured acceleration A_j is the sum of accelerations a_{ij}, given by:

$$A_j = \sum_{i=1}^{p} a_{ij} = \sum_{i=1}^{p} H_{ij} F_i \tag{3}$$

The set of equations for the (complex) accelerations can be written in the matrix form:

$$\mathbf{A} = \mathbf{H}^{\mathbf{T}} \mathbf{F} \tag{4}$$

Ideally, only p values of acceleration should be necessary to compute the set of exciting forces, but the force estimates will be strongly dependent on measurement errors and the matrix $\mathbf{H}^{\mathbf{T}}$ might be almost singular unless measurement locations are carefully selected.

For these reasons, it is usual to assume that there are small errors in the measured accelerations and to obtain the estimated values of the exciting forces that give the closest overall agreement between the n measured acceleration levels ($n > p$) and the values derived from equation (2). Thus, A_j is written as:

$$A_j = \sum_{i=1}^{p} H_{ij} \hat{F}_i + r_j \qquad (5)$$

The estimates $\hat{F}_i$ are obtained to give the minimum possible values of r_j in a least-squares sense. That is, $\hat{F}_i$ are estimated to minimise

$$\sum_{j=1}^{n} \left| r_j \right|^2 = \sum_{j=1}^{n} r_j r_j^* \qquad (6)$$

where r_j^* is the complex conjugate of r_j and $\left| r_j \right|$ is the magnitude of r_j. Equation (4) can be written in the matrix form:

$$\mathbf{A} = \mathbf{H}^{\mathbf{T}} \hat{\mathbf{F}} + \mathbf{r} \qquad (7)$$

The objective, then, is to find force estimates $\hat{\mathbf{F}}$ to minimise:

$$\mathbf{r}^{\mathbf{T*}} \mathbf{r} \qquad (8)$$

The row vector $\mathbf{r}^{\mathbf{T*}}$ is the transpose of the conjugate of the column vector $\mathbf{r}$, so equation (8) is just a matrix representation of equation (6). Assuming that the matrix $\mathbf{H}^{\mathbf{T}}$ is of full column rank, *p*, then the solution of equation (7), subject to the condition given by equation (8), is given by:

$$\hat{\mathbf{F}} = (\mathbf{H}^{*} \mathbf{H}^{\mathbf{T}})^{-1} (\mathbf{H}^{*} \mathbf{A}) \qquad (9)$$

Once the set of forces has been estimated, the vector of residuals $\mathbf{r}$ can be computed using equation (7).

It is also possible to estimate the probable magnitudes of errors in the estimated forces, as long as certain key assumptions are made about the statistical properties of the measurement errors relating to the *n* accelerations. It is usual to assume that these all arise independently from a normal (or Gaussian) distribution with mean 0 and variance σ^2 (where σ^2 is unknown). With these assumptions, the least-squares estimate of the error variance is:

$$\hat{\sigma}^2 = \frac{1}{n-p} \left\{ (\mathbf{A}^{\mathbf{T*}} \mathbf{A}) - (\hat{\mathbf{F}}^{\mathbf{T}} \mathbf{H}^{*} \mathbf{A}) \right\} \qquad (10)$$

The estimated variance of the estimated force $\hat{F}_i$ is then given by:

$$\hat{f}_i = \hat{\sigma}^2 C_{ii} \qquad (11)$$

where C_{ii} is the *i*'th element in the i^{th} row of the matrix:

$$\mathbf{C} = (\mathbf{H}^{*} \mathbf{H}^{\mathbf{T}})^{-1} \qquad (12)$$

The normalised estimated standard error (ESE) of the i^{th} force estimate is then:

$$\frac{\sqrt{\hat{f}_i}}{\hat{F}_i}, \quad i = 1, 2, ..., p \qquad (13)$$

3. Bearing Forces

Figure 2 show a simplified representation of a rotor in journal bearings, where fluctuating forces are transmitted across oil films. There can be large differences between the dynamic properties of the machine when rotors are turning and those when rotors are stationary.

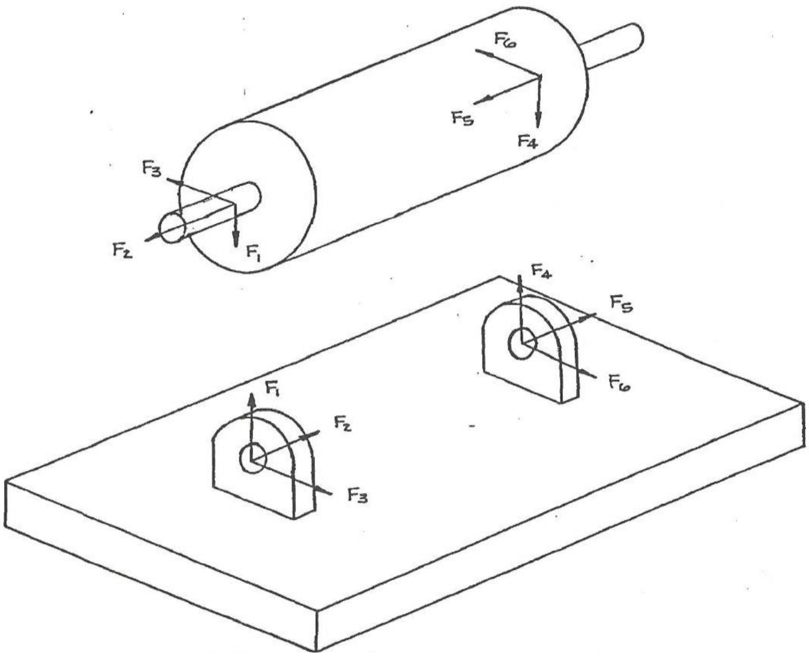

Figure 2. Forces at plain journal bearings

4. Trials on Marine Turbo Generator Sets

The effect of rotation on dynamic properties was explored for marine TG sets. The generator was driven directly by a steam turbine. Both rotors were supported by plain journal bearings.

4.1 Measurement locations

Figure 3 shows the layout of accelerometer and force application positions.

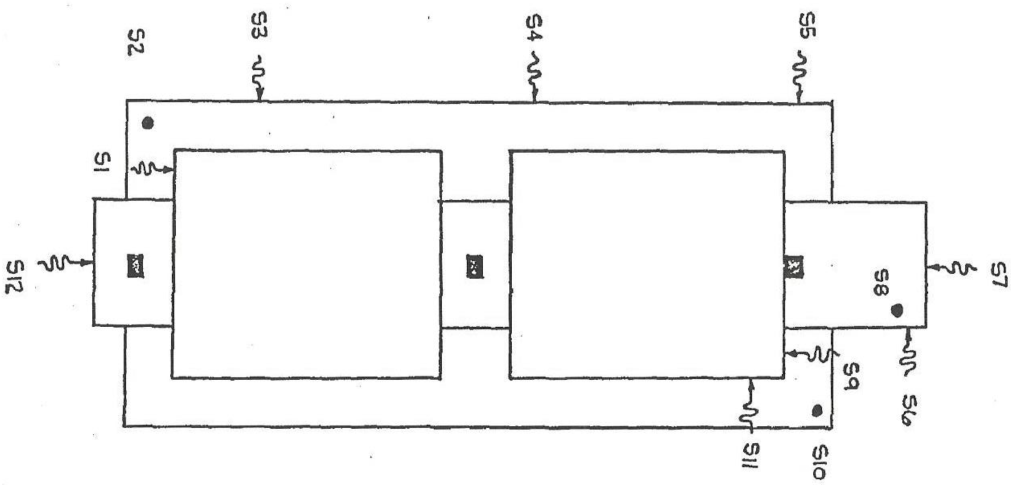

Figure 3. Layout of force input locations and directions

4.2 Trials results

Measurements were made on two production TG sets of the same type, to determine their dynamic transmission properties both when stopped and when running normally. Checks on reciprocal behaviour were carried out, and the TG sets were found to exhibit good reciprocity. An example is shown in Figure 4. The presence of higher background noise levels at one of the positions of acceleration measurement accounts for much of the apparent divergence between the two transfer functions at high frequencies. There was close agreement between the dynamic properties of the two sets when running. However, when the sets were stopped, substantial differences between them were discovered. It was found that the characteristics of each set were different when running from that observed when the set was stopped. This can be seen very clearly from Figure 5. It is believed this difference is due to the behaviour of the bearing oil films in both increasing the loss factor and in isolating the rotor masses from the static structure.

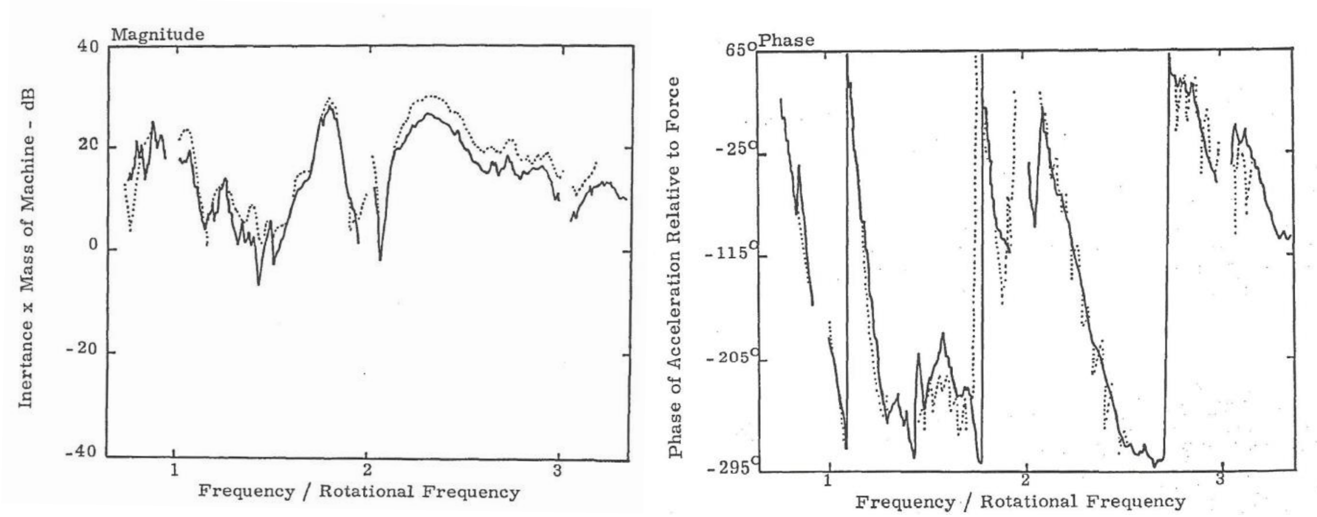

Figure 4. Direct and reciprocal measurements with the TG set running
—Acceleration at S2 (vertical); force at S6 (transverse)
….Acceleration at S6 (transverse); force at S2 (vertical)

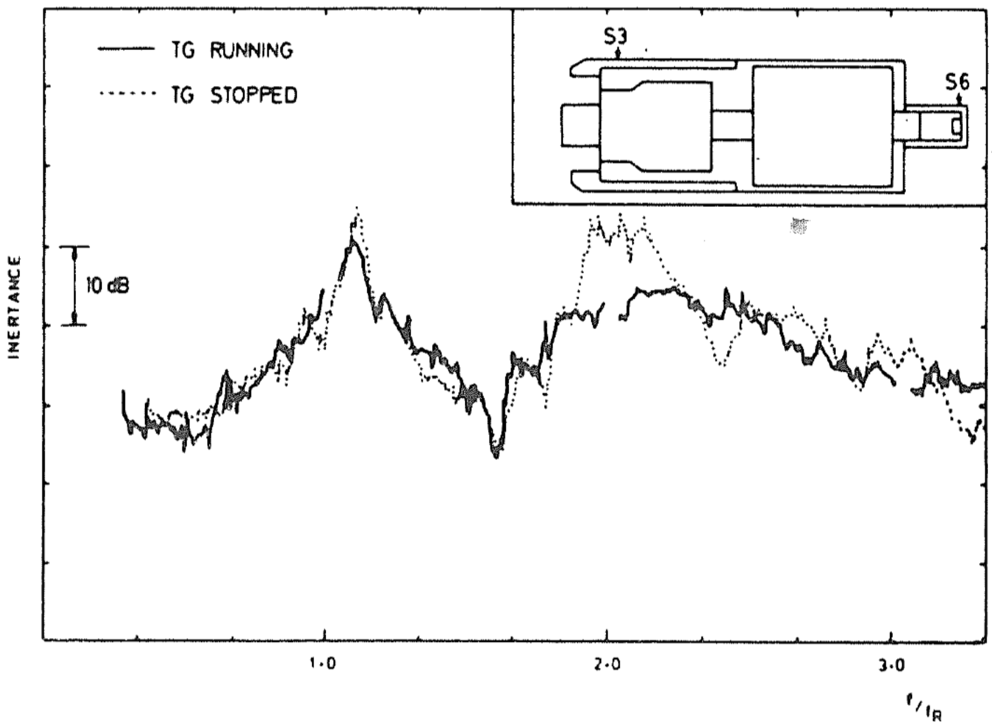

Figure 5. Transfer functions with the TG set running and stopped

4.3 Estimation of bearing forces at shaft rotational frequency

The results in this section are at shaft rotational frequency. This was the frequency of the principal tonal component under investigation. In order to obtain transfer function estimates, shaker trials were conducted at just below and just above rotational frequency when the machine was running. The vibration response to relatively weak shaker excitation could then be detected using high resolution analysis. The real and imaginary parts of these measurements were averaged separately to give the required transfer function data.

For the purpose of estimating bearing forces, two different sets of measurements were carried out, on one of the two TG sets, with an interval of eighteen months between them. It was expected that there would be nine forces operating at the bearings (one force in each of three orthogonal directions at each of three bearings), and a total of twelve positions of shaker input (S1 - S12) were used to provide for some degree of redundancy in the regression.

Table 1 shows the results obtained from regressions on the two sets of measurements. Also shown is the normalised estimated standard error of each resulting force estimate. There are obvious differences between the two sets of force estimates, both in magnitude and relative phase. However, examination of the standard errors indicate that there are greater uncertainties associated with the later of the two sets of trials data. By inspection, it was suggested that one or more of the vibration measurements obtained during the second trial were in error. This was later confirmed by statistical tests which indicated that data for S11 were responsible for reducing the confidence in the force estimates produced. Elimination of the suspect data led to the set of force estimates shown in Table 2. The standard errors are now similar to those obtained during the earlier of the two sets of trials, indicating a high degree of self-consistency for both sets of results.

Table 1. Initial bearing force estimates

Force magnitudes are in dB ref rotor weight		**Dec 1978 TFs Dec 1978 Acc data**			**Dec 1978 TFs Sep 1980 Acc data**			**Sep 1980 TFs Dec 1978 Acc data**			**Sep 1980 TFs Sep 1980 Acc data**		
Location	**Direction**	**Mag**	**Phase**	**Norm ESE**	**Mag**	**Phase**	**Norm ESE**	**Mag**	**Phase**	**Norm ESE**	**Mag**	**Phase**	**Norm ESE**
Forward turbine bearing	Vertical	-46	0	0.86	-32	0	1.14	-30	0	0.36	-56	0	18.3
	Axial	-59	-24	1.8	-48	-81	3.46	-46	-39	0.94	-42	-103	3.7
	Transverse	-39	-123	0.45	-34	-141	1.62	-59	-179	2.17	-46	-5	2.8
Forward generator bearing	Vertical	-29	-118	0.21	-33	-115	2.08	-29	5	0.27	-26	-131	1.3
	Axial	-35	4	0.36	-39	-115	3.53	-42	-89	1.07	-30	-67	1.8
	Transverse	-37	3	0.37	-25	-171	0.59	-37	133	0.48	-23	-84	0.6
Aft generator bearing	Vertical	-36	-68	0.23	-34	-136	1.17	-37	55	0.28	-35	-99	1.4
	Axial	-38	-142	0.13	-36	-166	0.64	-38	13	0.15	-36	-68	0.7
	Transverse	-35	54	0.15	-32	-172	0.65	-36	-143	0.18	-30	-75	0.6

Table 2. Initial and revised bearing force estimates, showing effect of data correction

Force magnitudes are in dB ref rotor weight		Sep 1980 TFs			Sep 1980 TFs		
		Sep 1980 Acc data, before correction			Sep 1980 Acc data, after correction		
Location	**Direction**	**Mag.**	**Phase**	**Norm ESE**	**Mag.**	**Phase**	**Norm ESE**
Forward turbine bearing	Vertical	-56	0	18.3	-40	0	0.38
	Axial	-42	-103	3.7	-44	-52	0.64
	Transverse	-46	-5	2.8	-62	-82	2.46
Forward generator bearing	Vertical	-26	-131	1.3	-32	-14	0.36
	Axial	-30	-67	1.8	-49	-132	2.2
	Transverse	-23	-84	0.6	-49	105	2.38
Aft generator bearing	Vertical	-35	-99	1.4	-35	11	0.2
	Axial	-36	-68	0.7	-41	11	0.19
	Transverse	-30	-75	0.6	-38	-115	0.32

A formal statistical test was carried out to test the hypothesis that the detailed distribution of machine forces had remained unaltered in the interval between the two sets of trials. The results indicated that such a hypothesis must be rejected. It is concluded, therefore, that the detailed distribution of bearing forces within the machine had changed through time though the overall level of forces at each bearing had remained similar.

4.4 Discussion

The procedure described above is likely to be useful for estimating bearing forces in rotating machinery, but key assumptions must hold before it can be applied to give reliable results.

First of all, the matrix of transfer function values, $\mathbf{H^T}$, must be of full column rank *p*. Otherwise, the matrix inverse $(\mathbf{H^*H^T})^{-1}$ is not uniquely defined and the forces cannot be estimated using equation (8). This problem, sometimes known as multi-collinearity or ill-conditioning, will arise in the present context whenever there are fewer than *p* structural degrees of freedom which, for any machinery installation, will be a problem at low enough frequencies. It can be the case that, due to measurement errors in the transfer function values, the matrix will be arithmetically of full rank but 'almost singular' in the sense that small changes in any of the entries of $\mathbf{H^T}$ would cause a massive change to some of the values in $(\mathbf{H^*H^T})^{-1}$ and hence $\hat{\mathbf{F}}$. The detection of this problem is discussed at length in Belsley et al. [4].

Secondly, the force estimates obtained by least squares are only optimal, and the estimated standard errors only valid, when the measurement errors are all independent observations from a common normal (Gaussian) distribution. These assumptions are commonly checked using plots of residuals from the fitted model, see for example Faraway [5].

5. Conclusions

It has been demonstrated that transfer function relating input forces to vibration response can vary markedly between trials with a machine stationary and running. These differences are likely to arise from the properties of journal bearings in isolating rotational masses from the static structure. It has also been shown that direct and reciprocal trials can be valuable in allowing estimates of fluctuating forces within a machine. These derived forces can then be used to establish the vibration properties of a new installation, where the foundation structure is different or the machine is combined with others in a large machinery raft.

It has been shown that estimates of fluctuating bearing forces in rotating machines can be obtained using a multiple regression technique. By the use of standard statistical tests, the quality of the data used in the estimation can be checked for self-consistency and poor data eliminated.

Application of the technique to the case of a marine turbo generator indicates that, after discarding inconsistent vibration data, force estimates of high self-consistency can be obtained. Examination of two sets of such estimates for the same machine, but from measurements made eighteen months apart, showed that the distribution of forces within the set had altered within that time.

References

[1] Ten Wolde, T. "Reciprocity experiments on the transmission of sound in ships", *Drukkerij Hoogland en Waltman N.V.*, Delft, 1973.

[2] Kinns, R. "The deduction of bearing forces in rotating machinery", *Proceedings of EUROMECH 122* (edited by Armand, J.L. and Bishop, R.E.D.), ATMA, pp. 345-361, Paris, 1979.

[3] Kinns, R., McColl, J.H. and McKinstry, J. "Use of forced response measurements for the estimation of machine source properties", *Proceedings of Internoise 83,* pp. 475-478, Edinburgh, 1983.

[4] Belsley, D.A., Kuh, E. and Welsch, R.E. *Regression diagnostics*, John Wiley & Sons, New Jersey, 1980.

[5] Faraway, J.J. *Linear models with R*, second edition, Taylor & Francis Group, Boca Raton, 2015.