Cyclic loading and fabric evolution in sand: a constitutive investigation

Sollicitation cyclique et évolution de tissue sur sable: une enquête de comportement

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ABSTRACT An anisotropic plasticity model is proposed to describe the effect of fabric and fabric evolution on the cyclic behaviour of sand within the framework of anisotropic critical state theory. The model employs a cone-shaped bounding surface in the deviatoric stress space and a yield cap perpendicular to the mean stress axis to describe sand behaviour in constant-mean-stress shear and constant-stress-ratio compression, respectively. The model considers a fabric tensor characterizing the internal structure of sand associated with the void space system which evolves with plastic deformation. The fabric evolution law is assumed to render the fabric tensor to become co-directional with the loading direction tensor and to reach a constant magnitude of unit at the critical state. In constant-stress-ratio compression, the final degree of anisotropy is proportional to a normalized stress ratio. An anisotropic variable defined by a joint invariant of the fabric tensor and the loading direction tensor is employed to describe the fabric effect on sand behaviour in constant-mean-stress monotonic and cyclic shear. Good comparison is found between the model simulations and test results on Toyoura sand in both monotonic and cyclic loadings with a single set of parameters.

RÉSUMÉ Un modèle anisotrope de plasticité est proposé pour décrire l'effet de fabrique et de l'évolution de fabrique sur le comportement cyclique de sable dans le cadre de la théorie d'état critique anisotrope. Le modèle utilise une surface de délimitation en forme de cône dans l'espace de contrainte déviatorique et une surface limite perpendiculaire à l'axe de la contrainte signifie pour décrire le comportement de sable dans contrainte moyenne constante et de rapport de contrainte constant, respectivement. Le modèle prend en compte un tenseur de fabrique caractérisation de la structure interne de sable associé au système d'indice des vides qui évolue avec déformation plastique. La loi d'évolution de fabrique est supposé rendre le tenseur de fabrique pour devenir co-directionnelle avec la direction de chargement tenseur et d'atteindre une amplitude constante de l'unité à l'état critique. Dans le rapport de contrainte constant en compression, le degré d'anisotropie final est proportionnelle à un rapport de contrainte normalisée. Une variable anisotrope définie par un invariant conjointe du tenseur de fabrique et la direction de tenseur de chargement est employé pour décrire l'effet de fabrique sur le comportement de sable dans contrainte moyenne constante monotone et cyclique. Bonne comparaison est trouvée entre les simulations du modèle et les résultats des essais sur le sable Toyoura dans les deux chargements monotones et cycliques avec les meme valeurs des paramètres.

1 INTRODUCTION

Natural and manmade sand deposits/samples are commonly cross-anisotropic due to gravitational forces and/or compaction. The anisotropic soil fabric (internal structure) plays an important role in affecting the overall behaviour of sand such as strength and dilatancy. For instance, Oda et al. (1978) demonstrated that the bearing capacity for the model with the load perpendicular to the bedding plane may be 34% higher than with the load parallel to the bedding plane. The observed difference in strength is apparently attributable to the effect of cross anisotropy. Meanwhile, the undrained shear strength and cyclic liquefaction resistance of sand, which are of great concern in earthquake engineering design, are also found to be strongly dependent on the degree of fabric anisotropy and the relative orientation between the loading direction and material fabric (e.g., Miura and Toki 1982). For instance, Miura and Toki (1982) found that sand deposits with a higher degree of anisotropy and a horizontal bedding plane show a higher undrained shear strength in monotonic triaxial compression tests but lower liquefaction resistance in undrained cyclic triaxial tests. This is mainly due to sand samples that are more anisotropic showing more contractive responses in the triaxial extension side in cyclic loading.

To characterize the fabric effect on sand behaviour, many theoretical attempts have been made during the past few decades. Various constitutive models have been developed to describe the effect of inherent anisotropy on sand responses (e.g., Li and Dafalias 2002). These models are shown to be able to characterize the stress-strain and strength behaviour of sand under certain loading conditions with varied degree of satisfaction. However, the assumption of a constant fabric during loading in these models may not be consistent with experimental and numerical observations where sand fabric has been found to change appreciably during loading in order to accommodate the applied stress (Li and Dafalias 2012; Zhao and Guo 2013). The evolution of sand fabric, if not properly accounted for, may result in some important features of sand behaviour not being modelled.

The main objective of this work is to present a comprehensive bounding surface model to describe the fabric effect on sand behaviour in both monotonic and cyclic loading based on the recent work by Gao et al. (2014) and the anisotropic critical state theory (Li and Dafalias 2012).

2 MODEL FORMULATIONS

2.1 Bounding surface and yield cap

The proposed model is based on the bounding surface concept originally described by Li (2002), with further adaption to be consistent with the anisotropic critical state theory recently developed by Li & Dafalias (2012) and materialized by Gao et al. (2014).

The bounding surface $\bar{f}_1$ is expressed as (Li 2002)

$$\bar{f}_1 = \bar{R} / g\left(\bar{\theta}\right) - \bar{H}_1 = 0 \tag{1}$$

where $\bar{R} = \sqrt{3/2 \bar{r}_{ij} \bar{r}_{ij}}$ with $\bar{r}_{ij}$ being the ‘image’ stress ratio tensor of the current stress ratio tensor $r_{ij} = s_{ij} / p = \left(\sigma_{ij} - p\delta_{ij}\right) / p$, in which σ_{ij} is the stress tensor, s_{ij} is the deviatoric stress tensor and δ_{ij} is the Kronecker delta; $\bar{H}_1$ is a function of the internal state variables associated with the loading history; $g\left(\bar{\theta}\right)$ is an interpolation function describing the variation of critical state stress ratio with Lode angle $\bar{\theta}$ (Li 2002)

$$g\left(\bar{\theta}\right) = \frac{\sqrt{\left(1+c^2\right)^2 + 4c\left(1-c^2\right)\sin 3\bar{\theta}} - \left(1+c^2\right)}{2\left(1-c\right)\sin 3\bar{\theta}} \tag{2}$$

where $c = M_e / M_c$ with M_e and M_c denoting the critical state stress ratio in triaxial extension and compression, respectively.

The condition of consistency for the cone is expressed as (Li and Dafalias 2002; Li 2002)

$$d\bar{f}_1 = p\bar{n}_{ij} d\bar{r}_{ij} - \langle L_1 \rangle \bar{K}_{p1} = p\bar{n}_{ij} dr_{ij} - \langle L_1 \rangle K_{p1} = 0 \tag{3}$$

where $\bar{n}_{ij}$ is the deviatoric unit loading direction tensor defined as the norm to $\bar{f}_1$ at the image stress ratio point $\bar{r}_{ij}$, $\bar{K}_{p1}$ and K_{p1} are respectively the plastic moduli for the reference and current stress state, L_1 is the loading index for constant-mean-stress shear and $\langle \ \rangle$ are the Macauley brackets. $\bar{r}_{ij}$ and $\bar{n}_{ij}$ are obtained by the radial mapping rule shown in Li (2002).

The cap yield surface is expressed as (Li, 2002)

$$f_2 = p - H_2 = 0 \tag{4}$$

where H_2 defines the location of the flat cap at the mean stress axis. The condition of consistency for this cap is (Li, 2002)

$$df_2 = dp - \langle L_2 \rangle K_{p2} = 0 \tag{5}$$

where L_2 is the loading index for constant-stress-ratio compression and K_{p2} is the plastic modulus for the yield cap. Following Gao et al. (2014) and Gao and Zhao (2013), a fabric dependent flow rule expressed as below is employed for constant-mean-stress shear

$$de_{ij}^{p1} = \langle L_1 \rangle \bar{m}_{ij} = \langle L_1 \rangle \frac{\partial \bar{g}/\partial \bar{r}_{ij} - (\partial \bar{g}/\partial \bar{r}_{mn})\delta_{mn}\delta_{ij}/3}{\left\| \partial \bar{g}/\partial \bar{r}_{ij} - (\partial \bar{g}/\partial \bar{r}_{mn})\delta_{mn}\delta_{ij}/3 \right\|} \quad (6)$$

where de_{ij}^{p1} is the plastic deviatoric strain increment associated with the loading index L_1. The plastic potential function $\bar{g}$ is expressed as

$$\bar{g} = \bar{R}/g(\bar{\theta}) - \bar{H}_g e^{-k(\bar{A}-1)^2} = 0 \quad (7)$$

where k is a positive model parameter with default value of 0.03; $\bar{A}$ is an anisotropic variable and $\bar{H}_g$ should be adjusted to make $\bar{g} = 0$ based on current $\bar{r}_{ij}$ and F_{ij}.

In constant-stress-ratio compression, the plastic deviatoric strain increment is assumed to align in the same direction of r_{ij} as follows (Li 2002)

$$de_{ij}^{p2} = \langle L_2 \rangle l_{ij} = \langle L_2 \rangle r_{ij} / \| r_{ij} \| \quad (8)$$

where de_{ij}^{p2} is the plastic shear strain increment associated with the loading index L_2.

Assuming that the plastic deviatoric and volumetric strain increments (de_{ij}^{p} and $d\varepsilon_v^p$) can be decomposed into two parts associated with L_1 and L_2, respectively, one has

$$de_{ij}^{p} = de_{ij}^{p1} + de_{ij}^{p2} = \langle L_1 \rangle \bar{m}_{ij} + \langle L_2 \rangle l_{ij} \quad (9)$$

$$\begin{aligned} d\varepsilon_v^p &= d\varepsilon_v^{p1} + d\varepsilon_v^{p2} \\ &= \sqrt{2/3}\left(D_1 \sqrt{de_{ij}^{p1} de_{ij}^{p1}} + D_2 \sqrt{de_{ij}^{p2} de_{ij}^{p2}} \right) \quad (10) \\ &= \sqrt{2/3}\left(\langle L_1 \rangle D_1 + \langle L_2 \rangle D_2 \right) \end{aligned}$$

where D_1 ($= d\varepsilon_v^{p1} \big/ \sqrt{2de_{ij}^{p1} de_{ij}^{p1}/3}$) and D_2 ($= d\varepsilon_v^{p2} \big/ \sqrt{2de_{ij}^{p2} de_{ij}^{p2}/3}$) denote the dilatancy relations for constant-mean-stress shear and constant-stress-ratio compression, respectively.

2.2 Anisotropic variable and dilatancy state parameter

The following anisotropic variable $\bar{A}$ and dilatancy state parameter ζ (Li and Dafalias 2012) will be used to characterize the fabric effect on the dilatancy and plastic hardening of sand in constant-mean-stress shear

$$\bar{A} = F_{ij}\bar{n}_{ij} \quad (11)$$

$$\zeta = \psi - e_A(\bar{A}-1) = \psi - e_r\left[2 - (\rho/\bar{\rho})^x\right](\bar{A}-1) \quad (12)$$

where e_r is a positive model parameter, ψ is the state parameter and x=50 is a default model constant which makes the term $(\rho/\bar{\rho})^x \approx 0$ unless ρ is very close to $\bar{\rho}$. $\bar{\rho}$ and ρ denote respectively the distances of the 'image' and current stress ratio point from the projection centre (Li, 2002).

In the present model, the critical state line in the $e-p$ plane is given by (Li and Wang 1998)

$$e_c = e_\Gamma - \lambda_c (p/p_a)^\xi \quad (13)$$

where e_c is the critical stat void ratio; e_Γ, λ_c and ξ are material constants and p_a (=101 kPa) is the atmospheric pressure.

2.3 Plastic modulus and dilatancy relation

The following plastic modulus is employed in constant-mean-stress shear

$$K_{p1} = \frac{Gh}{\bar{R}}\left[M_c g(\bar{\theta}) \mathrm{e}^{-n\zeta} \left(\frac{\bar{\rho}}{\rho}\right)^2 - \bar{R} \right] \quad (14)$$

where G is the elastic shear modulus, the expression of which will be shown in the subsequent sections, n is a positive model parameter, h is a scaling factor for the plastic modulus When $(\rho/\bar{\rho})^x = 1$, $K_{p1} = \bar{K}_{p1}$. In the present model, the following form of h is used

$$h = (1 - c_h e)\mathrm{e}^{\bar{A}} h_c \quad (15)$$

where

$$h_c = (\rho/\bar{\rho})^x + \left[1 - (\rho/\bar{\rho})^x\right] h_1 / (1+F)^2 \quad (16)$$

where c_h and h_1 are two positive model parameters.

The following dilatancy relation in constant-mean-stress shear is proposed based on the work by Li (2002), Li and Dafalias (2012) and Gao et al. (2014),

$$D_1 = \frac{d}{M_c g(\bar{\theta})}\left[M_c g(\bar{\theta}) \mathrm{e}^{m\zeta} \left(\frac{\bar{\rho}}{\rho}\right) - \bar{R} \right] \quad (17)$$

where

$$d = d_1 \left\{ (\rho/\bar{\rho})^x + \left[1 - (\rho/\bar{\rho})^x\right] d_c \right\} \quad (18)$$

$$d_c = \frac{e^{\omega\int\langle -d\varepsilon_v^{p1}\rangle}}{1+d_r e^{\omega\int\langle -d\varepsilon_v^{p1}\rangle}} \tag{19}$$

where m, d_1, ω and d_r are positive model parameters. ω is a model constant with default value of 5000. d_r is a relatively small number with default value of 0.1.

We propose the following plastic modulus under constant stress ratio loading

$$K_{p2} = \sqrt{\frac{2}{3}} d_2 \frac{M_c g(\theta)}{R r_2} \tag{20}$$

where θ is the Lode angle for r_{ij}, $R = \sqrt{3 r_{ij} r_{ij}/2}$ is the current stress ratio, d_2 is a positive model parameter. The expression for r_2 describes the *e-p* relation in constant-stress-ratio compression which is always greater than zero. The term $\sqrt{2/3}$ is added to offer a simpler relation between $d\varepsilon_v^{p2}$ and dp [see Eq. (22) below], dp is the increment of mean effective stress.

The dilatancy in constant-stress-ratio loading is expressed as follows

$$D_2 = d_2 \frac{M_c g(\theta)}{R} \left\langle 1 - \left[R / M_c g(\theta)\right]^x \right\rangle \tag{21}$$

Based on Eqs. (5), (10), (20) and (21), the compressive behaviour of sand under constant-stress-ratio loading [$R < M_c g(\theta)$] can be obtained as below

$$d\varepsilon_v^{p2} \approx r_2 dp \tag{22}$$

In the present model, the expression for r_2 is proposed based on Taiebat and Dafalias (2008)

$$r_2 = \frac{e}{1+e}\left[\rho_c - \frac{(p/p_a)^{1/3}}{K_0}\right]\frac{1}{p}\left(1 - \operatorname{sgn}\delta\,|\delta|^{\beta}\right) \tag{23}$$

where K_0 is a model parameter for the elastic modulus of sand, β is a parameter which controls the curvature of the predicted *e-p* relation in constant-stress-ratio compression, ρ_c is the slope of the limit compression curve (LCC) for isotropic compression in the $\log e - \log p$ space (Taiebat and Dafalias, 2008) and

$$\delta = 1 - \frac{p}{p_b}\left[1 + 2\left(\frac{R}{M_c g(\theta)}\right)^2\right] \tag{24}$$

where p_b is the 'image' mean stress on the LCC for isotropic compression corresponding to the current void ratio e. The expression for the LCC in isotropic compression is $\log e = \rho_c \log(p_r/p)$, where p_r is the means stress corresponding to $e = 1$ on the LCC.

2.4 Fabric evolution

By neglecting potential fabric change due to pure elastic deformation, the following fabric evolution is assumed in the present model

$$dF_{ij} = \sqrt{\frac{2}{3}} k_f \left\{ \langle L_1 \rangle \left(\bar{n}_{ij} - F_{ij}\right) + \langle L_2 \rangle D_2 \left[\frac{R l_{ij}}{M_c g(\theta)} - F_{ij}\right] \right\} \tag{25}$$

where k_f is a model parameter describing the rate of fabric evolution with plastic strain increment associated with $d\varepsilon_q^{p1}$ ($= \sqrt{2 de_{ij}^{p1} de_{ij}^{p1}/3}$) and $d\varepsilon_v^{p2}$. Eq. (25) indicates that F_{ij} will eventually become co-directional with $\bar{n}_{ij}$ and reach a constant magnitude of F=1 when at the critical state (Li and Dafalias 2012). In a pure constant-stress-ratio compression, Eq. (25) will not lead F_{ij} to critical state but give a material fabric which is co-directional with l_{ij} and has a constant magnitude $F = R / M_c g(\theta)$ when ε_v^{p2} ($= \int d\varepsilon_v^{p2}$) is large enough.

2.5 Elastic stress strain relations

Hypo-elastic stress-strain relations are used in this model. The elastic shear modulus G is expressed as a function of e and p as below

$$G = G_0 \frac{(2.97 - e)^2}{1+e} \sqrt{p p_a} \tag{26}$$

where G_0 is a model parameter.

Following Taiebat and Dafalias (2008), the elastic bulk modulus K expressed below is used for the present model

$$K = K_0 p_a \frac{1+e}{e}\left(\frac{p}{p_a}\right)^{2/3} \quad (27)$$

3 MODEL SIMULATION

Figure 1 compares the model simulations against test data for Toyoura sand in undrained cyclic simple shear tests ($F_0 = 0.5$ is used in the simulations and the model parameters are shown in Table 1). For the test shown here, the sample was first isotropically consolidated to $p = 100\text{kPa}$ and cyclic undrained simple shear was then applied with constant amplitude of shear stress τ (Chiaro et al. 2009). Evidently, the model gives good predictions for the effective stress path and shear stress-strain relation. In Figure 1, $\tau_{\max}$ and $\tau_{\min}$ respectively denote the maximum and minimum shear stresses in each cycle and γ is the shear strain.

Table 1. Model parameters

Parameter	Value
G_0	125
K_0	150
M_c	1.25
c	0.75
e_Γ	0.934
λ_c	0.019
ξ	0.7
c_h	0.90
e_r	0.09
n	4.0
d_1	0.4
m	5.3
h_1	7.6
ω	5000
d_r	0.1
ρ_c	0.37
p_r (kPa)	5500.0
β	0.18
d_2	1
k_f	7.35

Figures. 2a and 2b show the undrained cyclic triaxial test results on Toyoura sand prepared by two different methods (Miura and Toki, 1982). The monotonic triaxial test results indicate that the sample prepared by wet rodding method is approximately isotropic (Miura and Toki, 1982), and thus $F_0 = 0$ is used in the simulations (Figure 2c). The sample prepared by the multiple sieving pluviation method is found to be initially anisotropic (Miura and Toki, 1982), its initial degree of anisotropy is set to be $F_0 = 0.22$ based on best fitting of the effective stress path shown in Figures 2b and d. Note that the model parameters listed in Table 1 are used for these two samples.

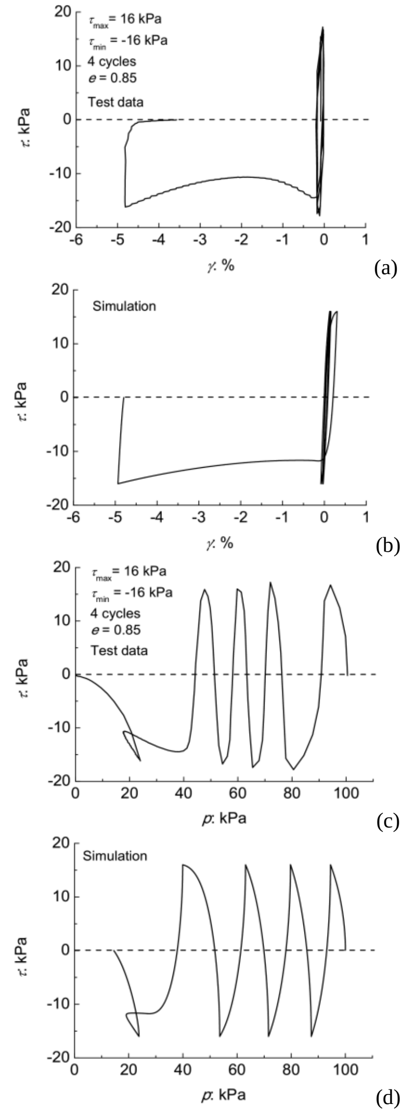
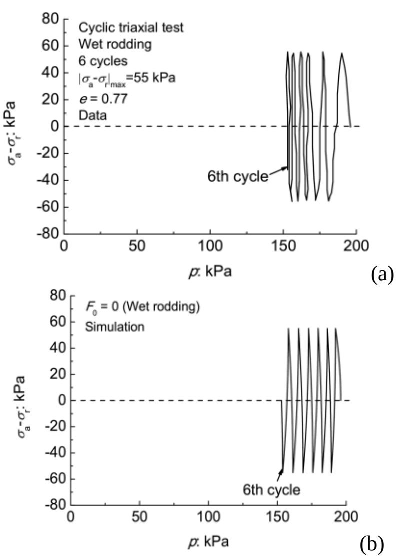


Figure 1 Model simulation for sand behaviour in undrained cyclic simple shear test

Though the predicted effective stress path shows a relatively large deviation from the measured one for the sample prepared by multiple sieving pluviation method, the model does offer reasonable characterizations of the fabric effect on the sand behaviour in cyclic loading. The more isotropic sample shows higher liquefaction resistance in undrained cyclic triaxial tests (p decreases with the number of cycles more slowly). For example, at the end of the 6th cycle, p for the sample prepared by multiple sieving pluviation is around 110 kPa, which is lower than that for the wet-rodded sample (155 kPa).

4 CONCLUSION

The paper presented a comprehensive bounding surface model to characterize the fabric effect on the behaviour of sand in both monotonic and cyclic loading conditions within the framework of the anisotropic critical state theory (Li and Dafalias 2012). It assumes that the fabric evolves with both plastic shear and volumetric strains. An anisotropic variable defined by the joint invariant of the deviatoric fabric tensor and the loading direction tensor is used to model the fabric effect on sand behaviour in constant-mean-stress shear. The model offers a unified description for the effect of fabric and fabric evolution in both monotonic and cyclic loading. The model predictions of sand behaviour for a series tests on Toyoura sand compare well with the test data.



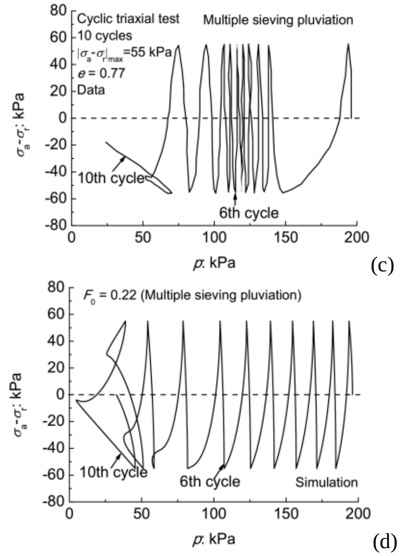


Figure 2 Model simulation for effect of initial degree of anisotropy on sand behaviour in undrained cyclic triaxial tests

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