Generalised tensor fluctuations and inflation

Dario Cannone[a,b,c], Gianmassimo Tasinato[c,d], David Wands[c]

[a] *Dipartimento di Fisica e Astronomia G. Galilei*
Università degli Studi di Padova, I-35131 Padova, Italy.

[b] *INFN, Sezione di Padova, I-35131 Padova, Italy.*

[c] *Institute of Cosmology & Gravitation,*
University of Portsmouth,
Dennis Sciama Building,
Burnaby Road, Portsmouth,
PO1 3FX, United Kingdom

[d] *Department of Physics, Swansea University,*
Swansea, SA2 8PP, UK

Using an effective field theory approach to inflation, we examine novel properties of the spectrum of inflationary tensor fluctuations, that arise when breaking some of the symmetries or requirements usually imposed on the dynamics of perturbations. During single-clock inflation, time-reparameterization invariance is broken by a time-dependent cosmological background. In order to explore more general scenarios, we consider the possibility that spatial diffeomorphism invariance is also broken by effective mass terms or by derivative operators for the metric fluctuations in the Lagrangian. We investigate the cosmological consequences of the breaking of spatial diffeomorphisms, focussing on operators that affect the power spectrum of fluctuations. We identify the operators for tensor fluctuations that can provide a blue spectrum without violating the null energy condition, and operators for scalar fluctuations that lead to non-conservation of the comoving curvature perturbation on superhorizon scales even in single-clock inflation. In the last part of our work, we also examine the consequences of operators containing more than two spatial derivatives, discussing how they affect the sound speed of tensor fluctuations, and showing that they can mimic some of the interesting effects of symmetry breaking operators, even in scenarios that preserve spatial diffeomorphism invariance.

I. INTRODUCTION

The recent results from the BICEP2 collaboration [1] suggest that CMB polarization measurements are reaching sufficient sensitivity to start detecting primordial B-modes, if foregrounds can be understood and the gravity wave amplitude is sufficiently large. In this optimistic situation, recent theoretical studies [2–4] suggest that if a sufficient delensing of the B-mode signal can be performed, then both the tensor-to-scalar ratio r, and the tilt of the tensor spectrum n_T might be measured with an accuracy sufficient to test the consistency relation

$$n_T = -r/8\,, \tag{1}$$

that holds for standard single clock inflation in Einstein gravity.

This motivates a general theoretical investigation of possible mechanisms for producing primordial tensor fluctuations during inflation, including scenarios that are more general than the ones studied so far. A generic prediction of standard single-field, slow-roll inflation is the production of a nearly scale invariant spectrum of tensor modes with an amplitude proportional to the Hubble parameter during inflation, a ratio $r < 1$ between the tensor and scalar power spectra, and a tilt $n_T < 0$ of the tensor spectrum related to r by eq (1): see e.g. [5] for a review. The single clock consistency relation (1) can be violated in multiple field models (see [6] for a review); however, in inflationary scenarios based on a slow-roll expansion, that do not violate the Null Energy Condition, n_T is generically negative. On the other hand, various specific examples have been proposed in the literature that are able to obtain a positive n_T in a controllable way. One can include to eq (1) contributions that are higher order slow-roll [7], or violate the Null Energy Condition in Galileon or Hordenski constructions [8]. Alternatively, one can consider particle production during inflation [9], or investigate specific non-standard scenarios as solid/elastic inflation [10, 11].

In this work, we take a more general perspective to the problem of characterizing tensor fluctuations. By implementing an effective field theory approach to inflation, we examine novel properties of the spectrum of inflationary tensor fluctuations, that arise when breaking some of the symmetries or requirements usually imposed on the dynamics of perturbations. During single-clock inflation, the time-diffeomorphism invariance is normally broken by the time dependent cosmological background configuration: the construction of the most general theory for fluctuations that preserves spatial diffeomorphisms, but breaks the time reparametrization invariance, leads to the effective theory of single field inflation initiated in [12], and developed by many groups over the past few years (see [13] for a recent review on this topic).

On the other hand, it might very well be possible that during inflation also the *spatial* diffeomorphism invariance is broken in the lagrangian for fluctuations. This possibility has not been much explored in the literature, apart from interesting specific set-ups as solid inflation [10]. Alternatively, operators with more than two spatial derivatives acting on the tensor perturbations – preserving or not spatial diffeomorphism invariance – could become important in situations where the leading order Einstein-Hilbert contributions to the tensor sector can be neglected, and provide interesting contributions to inflationary observables.

In this article, we explore these possibilities using an effective field theory approach. We consider the dynamics of metric fluctuations for single clock inflation in a unitary gauge in which the clock perturbations are set to zero, and for simplicity we concentrate on operators that are at most quadratic in fluctuations, since our main aim is to try to understand how they can affect observables such as r and n_T, that are directly associated with the tensor power spectrum.

In the first part of the work, we study contributions to the effective lagrangian for perturbations that break the spatial diffeomorphism invariance by effective mass terms, or by derivative operators for the metric fluctuations. In order not to induce spatial anisotropies, we limit our attention to contributions that do not break the Euclidean symmetry in the spatial sections, corresponding to an

$SO(3)$ rotational invariance. We study the conditions one has to satisfy to avoid ghost instabilities and to have well-behaved fluctuations; moreover we examine some cosmological consequences of our findings.

In the second part of this work, we study operators quadratic on the metric fluctuations that contain up to four spatial derivatives (but no more than two time derivatives), that can preserve spatial diffeomorphism invariance, and that can have interesting effects in regimes where they provide the dominant contribution to the tensor dynamics. We show that a non-trivial tensor sound speed can be generated, and the formula for the tilt of the tensor spectrum receives new contributions that depend on the coefficients of these higher derivative operators. In particular, we discuss a special case in which such operators can mimic the effect of a mass term in the tensor sector.

We do not wish to systematically investigate *all* possible operators with the properties we are interested in, but to study representative and promising examples that can be of some use to connect inflationary model building with observations, especially when focussing on the tensor sector. On the other hand, the tools that we develop can be further applied and generalized to study more general situations, for example in set-ups with broken isotropy in the effective action for fluctuations. Since we implement an effective field theory approach to the study of perturbations from inflation, we do not attempt to find actual theories or models whose cosmological fluctuations have the properties we investigate, although we will also comment on possible realizations for the operators we study. We limit our attention to operators that are quadratic in fluctuations. Given the fact that we break some of the symmetries such as spatial diffeomorphism invariance, many operators cubic or higher in fluctuations exist; this considerably complicates a systematic analysis of their effects, that we leave for future work.

II. BREAKING SPATIAL DIFFEOMORPHISM INVARIANCE

In this section we investigate an effective field theory for cosmological perturbations around quasi-de Sitter space, with broken spatial and time diffeomorphism invariance.

We take a conformal (FRW) ansatz for the background metric,

$$ds^2 \,=\, \bar{g}_{\mu\nu}\, dx^\mu dx^\nu \,=\, a^2(\eta)\, \left(-\eta_{\mu\nu}\, dx^\mu dx^\nu\right) \tag{2}$$

with $a^2(\eta)$ the conformal scale factor and $a(\eta) = 1/(-H\eta)$ for de Sitter space. We denote the metric fluctuations by $h_{\mu\nu} = g_{\mu\nu} - \bar{g}_{\mu\nu}$.

The time-reparameterization invariance for fluctuations is broken by the time dependence of the homogeneous background. In addition, we would like to study the effects of breaking spatial diffeomorphism invariance. The breaking of diffeomorphism invariance in the spatial sections is most easily achieved by mass terms, although derivative operators involving metric pertubations are also able to do so.

First we consider the effects of mass terms, before including diffeomorphism-breaking derivative operators in the next subsections. These operators corresponding to mass terms do not necessarily originate by a theory of massive gravity holding during inflation; they simply correspond to the most general way to express quadratic non-derivative operators in the fluctuations that break diffeomorphism invariance.

We consider the Einstein-Hilbert action expanded to second order, and add generic operators with no derivatives, that are quadratic in the metric fluctuations $h_{\mu\nu}$

$$\begin{aligned} S \;=\; & \int \mathrm{d}^4x\sqrt{-g}M_{\rm Pl}^2\left[R-2\Lambda-2\,c\,g^{00}\right]+ \\ & \frac{1}{4}M_{\rm Pl}^2\int \mathrm{d}^4x\sqrt{-g}\left[m_0^2\,h_{00}^2+2\,m_1^2\,h_{0i}^2-m_2^2\,h_{ij}^2+m_3^2\,h_{ii}^2-2m_4^2\,h_{00}\,h_{ii}\right]. \end{aligned} \tag{3}$$

The terms in the first line are the ones that will give the homogeneous and isotropic background which we assume for inflation. They give a non-zero stress-energy tensor at background level,

$$T_{\mu\nu}^{(0)}=-\frac{2}{\sqrt{-g}}\frac{\delta S}{\delta g^{\mu\nu}}\Bigg|_{background} \tag{4}$$

and, using Friedmann equations, the parameters c and Λ can be expressed as functions of the Hubble parameter H and its time derivative $\dot{H}$ (that defines the slow-roll parameter $\epsilon=-\dot{H}/H^2$).

The quadratic terms in the second line of Eq. (3) break diffeomorphism invariance, yet they preserve a spatial $SO(3)$ invariance in order not to break spatial isotropy. The term proportional to m_0^2 breaks time reparameterization invariance, and is present also in the quadratic Lagrangian of [12]: the remaining terms in the second line of Eq. (3), instead, are absent in [12], and break spatial diffeomorphism invariance. They have the same structure of the Lorentz violating mass terms of [14], this time applied to the case of an expanding (quasi)-de Sitter universe. They were dubbed 'Lorentz violating' in [14, 15] since in the flat limit ($H\to 0$) they do break 4d Lorentz symmetry $SO(1,3)$ down to spatial rotational symmetry, $SO(3)$ [1]. Since the choice of operators we consider preserves isotropy at each point in space, they also preserve homogeneity in space. In the limit $m_i\to 0$ with $i\neq 0$, spatial diffeomorphisms are restored and, up to second order in perturbations, we recover the standard effective field theory of inflation [12] without extrinsic curvature terms, where only time diffeomorphisms are broken by powers of h_{00}.

We can consider the 'mass terms' in the second line of Eq. (3) as arising from couplings between the metric and fields acquiring a nontrivial time-dependent profile during inflation. We assume that their coefficients (as well as the ones that we will meet in the following) are effectively constant in space and time during inflation, while these coefficients go to zero after inflation, and hence are not constrained by present day observational limits. The constancy in space is not a strict requirement since effects of gradient terms are usually negligible at large scales during inflation. A (small) time dependence for these operators would instead be expected, proportional to slow-roll parameters quantifying the departure from an exact de Sitter phase during inflation: for simplicity we will neglect it.

We will not consider interactions in this paper, but we will limit our attention to terms quadratic in perturbations. Nevertheless, for the class of mass terms contained in action (3), general considerations show that the maximal cut-off is of order $\Lambda_c\simeq\sqrt{m\,M_{Pl}}$ [17], assuming that all the non-vanishing mass parameters are of the same magnitude m. In order to have a reliable theory, we must ensure that $\Lambda_c\geq H$, where H is the Hubble scale during inflation, so that

$$\frac{m}{H}\geq\frac{H}{M_{Pl}}\,. \tag{6}$$

Hence for inflation happening at high energy scales, the mass of the graviton must be quite large during the inflationary process (although it can be well below the Hubble scale). After inflation ends, we assume that the effective graviton mass becomes negligible, as we mentioned above.

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[1] For certain choices of the parameters, these mass terms (although breaking diffeomorphism invariance) can recover 4d Lorentz invariance in the flat limit $H\to 0$. The parameter choice one has to make is

$$m_0^2=\alpha+\beta\,,\qquad m_1^2=m_2^2=-\alpha\,,\qquad m_3^2=m_4^2=\beta\,. \tag{5}$$

and the Fierz-Pauli theory corresponds to $\alpha+\beta=0$. These arguments are reviewed in [16] .

Let us stress that in the spirit of our effective approach to cosmological fluctuations, only based on symmetry arguments, it is *not necessary* to specify the nature of the model (the 'UV completion') that leads to the fluctuation Lagrangian we are examining. Our theory appears as a version of (Lorentz violating) massive gravity because we are selecting a specific gauge – the unitary gauge – in which fluctuations of the field(s) driving inflation are set to zero: the dynamics of perturbations is entirely described by the sector of metric fluctuations. Depending on the set-up under consideration, other gauges could be chosen though, in which the graviton is massless, and other sectors play the role in determining the dynamics of fluctuations during inflation.

The UV completion of our scenario might be some specific version of massive gravity coupled to an inflaton field (for reviews of massive gravity, see e.g. [16, 18]), or some model of inflation making use of vectors (see [19] for a review), or sets of scalars obeying specific symmetries. For example, solid inflation [10] is a set-up with broken spatial diffeomorphisms (but preserving time-reparameterization); the dynamics of its fluctuations might be considered as a subclass of our general discussion.

A. Tensor-vector-scalar decomposition

It is helpful to rewrite the action (3) in terms of tensor, vector and scalar perturbations on spatial hypersurfaces, which evolve independently at linear order:

$$\begin{aligned} h_{00} &= \psi\,, \\ h_{0i} &= u_i + \partial_i v\,, && \text{with} \quad \partial_i u_i = 0\,, \\ h_{ij} &= \chi_{ij} + \partial_{(i} s_{j)} + \partial_i \partial_j \sigma + \delta_{ij}\tau\,, && \text{with} \quad \partial_i s_i = \partial_j \chi_{ij} = \delta_{ij}\chi_{ij} = 0\,. \end{aligned} \tag{7}$$

Under a diffeomorphism, $\eta \to \eta + \xi_0$, $x^i \to x^i + \xi^i$, these perturbations transform as

$$\begin{aligned} \chi_{ij} &\to \chi_{ij} \\ u_i &\to u_i + \partial_0 \xi_i^T \\ s_i &\to s_i + \xi_i^T \\ \psi &\to \psi + 2\partial_0 \xi_0 + 2aH\xi_0 \\ v &\to v + \partial_0 \xi^L + \xi_0 \\ \sigma &\to \sigma + 2\xi^L \\ \tau &\to \tau + 2aH\xi_0 \end{aligned} \tag{8}$$

where $\xi_i = \xi_i^T + \partial_i \xi^L$. Expanding (3) up to second order in these fluctuations, we find the following tensor-vector-scalar actions including the mass terms:

- Tensor action

$$S_m^{(T)} = \frac{1}{4} M_{\rm Pl}^2 \int {\rm d}^4 x a^2 \left[-\eta^{\mu\nu} \partial_\mu \chi_{ij} \partial_\nu \chi_{ij} - a^2 m_2^2 \chi_{ij}^2 \right]\,, \tag{9}$$

- Vector action

$$S_m^{(V)} = \frac{1}{2} M_{\rm Pl}^2 \int {\rm d}^4 x a^2 \left[-(u_i - s_i')\nabla^2 (u_i - s_i') + a^2 (m_1^2 u_i^2 + m_2^2 s_i \nabla^2 s_i) \right]\,, \tag{10}$$

- Scalar action

$$
\begin{aligned}
S_m^{(S)} &= \frac{1}{4}M_{\rm Pl}^2 \int \mathrm{d}^4x\, a^2 \Big\{ -6(\tau' + aH\psi)^2 + 2(2\psi-\tau)\nabla^2\tau + 4(\tau'+aH\psi)\nabla^2(2v-\sigma') \\
&\quad + a^2\Big[(m_0^2+2\epsilon H^2)\psi^2 - 2m_1^2 v\nabla^2 v - m_2^2(\sigma\nabla^4\sigma + 2\tau\nabla^2\sigma + 3\tau^2) \\
&\quad + m_3^2(\nabla^2\sigma+3\tau)^2 - 2m_4^2\psi(\nabla\sigma+3\tau)\Big]\Big\}
\end{aligned}
\tag{11}
$$

Since diffeomorphisms are broken, one would expect to find six propagating degrees of freedom, and one of these should generically be a ghost. Nevertheless, it has been shown that in a FRW background the theory can be ghost-free, and potential instabilities avoided, if the masses m_i satisfy certain conditions [20]. In the next subsections, we will generalize this analysis including also the effect of a selection of derivative operators that break diffeomorphism invariance, studying each sector of the theory and also discussing possible phenomenological consequences. To the operators considered so far we will add new quadratic operators that contain at most two space-time derivatives in $h_{\mu\nu}$. They potentially break spatial diffeomorphism invariance, although they preserve Euclidean invariance in the spatial sections. See Appendix A for a list of such operators. To conclude this section, let us point out that our analysis includes operators with higher spatial derivatives acting on the fields obtained after the tensor-vector-scalar decomposition of $h_{\mu\nu}$ (see for example the m_2^2 coefficient in eq. (11)) that have been removed by a parameter choice in [21]. See however [22] for a recent analysis including operators that are higher order in spatial derivatives.

B. Tensor Fluctuations

Let us start by discussing the tensor fluctuations, since this is the sector we are most interested in. We see from the action $S_m^{(T)}$ in Eq. (9) that tensors acquire a mass only in the case $m_2^2 \neq 0$ and no instabilities arise if $m_2^2 \geq 0$. Hence only the operator proportional to m_2^2 in Eq. (3) influences the tensor spectrum by giving an effective mass to the tensors. On the other hand, we can add to the mass term additional operators that contain up to two space-time derivatives and preserve isotropy: they can change speed of sound for tensor perturbations in eq. (9). In particular, the only allowed operators that can contribute to the tensor sound speed are the ones in eqs. (A15), (A17) in appendix A.

We may add to the action (9) two derivative operators [2], with dimensionless coefficients b_1 and d_1:

$$
S_d^{(T)} \equiv \frac{1}{4}M_{\rm Pl}^2\left[b_1(\partial_0 h_{ij})^2 + d_1(\partial_i h_{jk})^2\right] \ . \tag{12}
$$

It is important to notice that these two derivative operators do not necessarily originate from contributions that break the 3-dimensional diffeomorphism invariance *per se*. In particular these terms can arise from the diffeomorphism invariant combination $b_1\delta K_{ij}\delta K^{ij} - d_1{}^{(3)}R$, where $\delta K_{ij}\delta K^{ij}$ is the perturbed extrinsic curvature and ${}^{(3)}\delta R$ is the three-dimensional Ricci scalar [12, 23]. These specific combinations, on the other hand, contain specific additional vector and scalar contributions that have to be taken into account. We will consider them in the next subsections, but for the moment we do not need to restrict to any special case; we can focus on (12) regardless of its origin.

[2] Notice that also a parity violating, one derivative operator could be included, $\epsilon^{ijk}\,(\partial_i\, h_{jm})\, h_{km}$, with ϵ^{ijk} the totally antisymmetric operator in three spatial dimensions. On the other hand, in this work we concentrate on operators that preserve parity, so we do not consider its effects. We thank Azadeh Maleknejad for discussions on this point.

The complete action for tensor fluctuations becomes

$$S^{(T)} = S^{(T)}_m + S^{(T)}_d = \frac{1}{4} M^2_{\rm Pl} \int {\rm d}^4 x a^2 \left\{ (1+b_1) \left[(\dot{\chi}_{ij})^2 - c_T^2 (\partial_i \chi_{jk})^2 \right] - a^2 m_2^2 \chi_{ij}^2 \right\} , \tag{13}$$

where the speed of sound for tensors is

$$c_T^2 = \frac{1+d_1}{1+b_1} . \tag{14}$$

In this case, in order to avoid ghosts one should also require $b_1 > -1$, $d_1 \geq -1$; moreover we could also demand $d_1 \leq b_1$ not to have superluminal propagation.

Taking the action (13), it is easy to derive the expression for the tensor power spectrum, quantizing the tensor fluctuations starting from the usual Bunch-Davies vacuum. Upon canonical normalization and neglecting for simplicity time dependencies of c_T and m_2, the equation of motion for tensors has the usual Mukhanov–Sasaki form. It can be solved to give the following expression for the power spectrum and its scale dependence:

$$\mathcal{P}_T = \frac{2H^2}{\pi^2 M^2_{\rm Pl} c_T} \left(\frac{k}{k_*} \right)^{n_T} , \qquad n_T = -2\epsilon + \frac{2}{3} \frac{m_2^2}{(1+b_1)^2 H^2} \left(1 + \frac{4}{3} \epsilon \right) , \tag{15}$$

at leading order in slow-roll and in an expansion in $m_2/H \ll 1$. Notice that the mass term can render the tensor spectrum blue if m_2/H is sufficiently large and positive so that the second term in n_T wins out over the negative contribution from the first term. In this sense, a blue spectrum for tensors can be obtained without violating the Null Energy Condition or exploiting the time-dependence of parameters: it is the effect of the mass term proportional to m_2^2 and is not depending on the sign of $\dot{H}$.

It would be interesting to explore whether if we choose different initial conditions that do not preserve isotropy, then the operators that we consider would lead to an anisotropic signal during inflation, as happens in the particular set-up of solid inflation [24, 25], both in the tensor and in the scalar and vector sectors. This will be the subject of future work [26].

The amplitude of the tensor power spectrum is enhanced by the inverse of the sound speed c_T. On the other hand, it has been recently shown in [27] that, when focussing on operators containing at most two derivatives – as we do in this section – there exists a disformal redefinition of the metric which converts the theory with a speed of sound $c_T \neq 1$ into a theory (in the Einstein frame) with unit speed of sound. Thus, in the Einstein frame, during inflation the sound speed is equal to one. Hence – neglecting the scale dependence of $\mathcal{P}_T$ – the amplitude of the tensor power spectrum is directly linked to the scale of inflation. Notice that in our scenario we do have an additional source of scale-dependence though, associated with the mass term m_2 that breaks the spatial diffeomorphism invariance. The disformal transformation of [27] does not involve spatial coordinates hence does not modify our predictions for the scale dependence of the tensor spectrum, whose sign is still controlled by m_2^2/H^2 versus ϵ.

It has been discussed in Ref. [27] that terms involving higher derivatives can actually change the situation and induce a non-trivial sound speed. While in [27] three-derivative terms were included, we will extend this possibility and study healthy four derivative terms (with at most two time derivatives) in Section III.

C. Vector Fluctuations

We now discuss the propagation of vector fluctuations in our set-up. In this and in the next subsection (where we will discuss the dynamics of scalars) we do not pretend to be exhaustive in our analysis, but only to investigate simple and interesting cases among the many possibilities allowed within our large parameter space. In particular, aiming for simplicity, our purpose is to reduce as much as we can the number of propagating degrees of freedom in our scenario, and choose parameters which can eliminate the vector degrees of freedom. We will study the general case in [26].

In principle we have two vector degrees of freedom, u_i and s_i, from the decomposition in eq (7). Examining the action (10) for vector perturbations including mass terms, and in absence of additional derivative operators, it is straightforward to show that the field u_i is not dynamical, since we obtain

$$\nabla^2(u_i - s_i') - a^2 m_1^2 u_i = 0\,. \tag{16}$$

Hence u_i can be integrated out to give the effective action

$$S_m^{(V)} = \frac{1}{2}M_{\rm Pl}^2 \int \mathrm{d}^4x\, a^4 \left[m_1^2 s_i' \frac{\nabla^2}{\nabla^2 - a^2 m_1^2} s_i' + m_2^2 s_i \nabla^2 s_i \right]. \tag{17}$$

The action is free of instabilities for $m_1^2 \geq 0$ and $m_2^2 \geq 0$. The case $m_1^2 = 0$ is particularly interesting as there are no propagating vector modes, since the coefficient of the s_i kinetic term in (17) vanishes. Hence in order to eliminate vector degrees of freedom, we make the choice $m_1 = 0$.

On the other hand, the situation can drastically change if also other possible derivative contributions are included in the action, choosing from the list of allowed operators in Appendix A. There are six possible terms with up to two derivatives that contribute to the vector sector, that contribute to an effective Lagrangian that we dub $\mathcal{L}_d^{(V)}$:

$$\begin{aligned}\mathcal{L}_d^{(V)} = &\frac{1}{4}M_{\rm Pl}^2 \Big[b_1(\partial_0 h_{ij})^2 + b_2(\partial_i h_{0j})^2 + b_3(\partial_j h_{0i}\partial_0 h_{ij}) \\ &+ d_1(\partial_i h_{jk})^2 + d_2(\partial_i h_{ij})^2 \Big] \\ &+ \frac{1}{4}M_{\rm Pl}^3\, \alpha_4\, (h_{ij}\partial_i h_{0j})\,,\end{aligned} \tag{18}$$

where b_i, d_i and α_4 are arbitrary constant coefficients. Notice that also a single derivative term is allowed in the last line of eq (18).

These derivative contributions in $S_d^{(V)}$ in general switch on a non-trivial dynamics for s_i even if $m_1^2 = 0$. On the other hand, it can be shown (c.f., appendix A) that if one chooses the particular values

$$b_1 = \frac{1}{2}b_2 = -\frac{1}{4}b_3\,, \tag{19}$$

then the structure of the action (10) would be unaltered and the vector s_i, when $m_1^2 = 0$, would still be non-dynamical. This corresponds to a combination of the operators forming the spatial diffeomorphism invariant quantity $(\delta K_{ij})^2$. Provided this condition (19) is satisfied, adding the operators proportional to d_1, d_2 and α_4 in eq. (18) does not change the conclusion such that s_i not dynamical.

Hence, the condition $m_1^2 = 0$ is appealing since we can still ensure that no vectors propagate. As we will see, this condition also gives only one propagating mode in the scalar sector, since extrinsic curvature terms do not render a second scalar mode dynamical. Of course, other cases can be considered (with propagating vector modes) and our approach will allow us to study them in future [26].

Fine-tuning relations on mass parameters, such as $m_1^2 = 0$ can be motivated and protected by residual gauge symmetries [14]. Indeed, this is the case for $m_1^2 = 0$; if we require invariance under time-dependent diffeomorphisms,

$$x^i \to x^i + \xi^i(t)\,, \tag{20}$$

then the operator h_{0i}, associated with m_1^2, is forbidden in the action.

D. Scalar Fluctuations

Not surprisingly, the scalar sector is the most tricky to analyze due to the number of fields involved and their mixings. We separate the discussion in two parts. First we study the case in which only scalar masses are included, and no derivative operators are added to eq. (11). We show that an important physical consequence of our construction is that the curvature perturbation ζ is generally not conserved on super-horizon scales. We then proceed, including derivative operators in the second part of this section.

The main aim is to find the conditions required to propagate at most one (healthy) scalar degree of freedom in our system.

1. *Only masses are included*

When only scalar masses are switched on, the action we are working with is Eq. (11). This action potentially propagates two degrees of freedom, σ and τ. It can be shown that even in the case where all the masses are different from zero, the theory has no ghosts nor other instabilities provided that $m_1^2 > 0$, $6H^2 \geq m_0^2 - 2\dot{H} > 0$ and $\dot{H} < 0$ [20].

Here we focus instead on the case $m_1^2 = 0$ that, besides having no vectors, it also has only one propagating scalar, as we are going to discuss. From eq. (11) with $m_1^2 = 0$ one can obtain the equations of motion for the auxiliary fields ψ and v,

$$\begin{aligned} \psi &= -\frac{\tau'}{\mathcal{H}}\,, \\ \nabla^2 v &= \frac{a^2}{4\mathcal{H}}\left[(m_0^2 - 2\dot{H})\tau' - \frac{2}{a^2}\nabla^2\tau + \frac{2\mathcal{H}}{a^2}\nabla^2\sigma' + m_4^2(\nabla^2\sigma + 3\tau)\right]\,, \end{aligned} \tag{21}$$

and substitute them back into the action obtaining (where we write $\mathcal{H} = aH$ and $\dot{H} = -\epsilon H^2$)

$$\begin{aligned} S = \frac{1}{4}M_{\rm Pl}^2 \int \mathrm{d}^4x\ a^2 \Bigg[&-2\left(\frac{\tau'}{aH} + \tau\right)\nabla^2\tau + a^2(m_0^2 + 2\epsilon H^2)\left(\frac{\tau'}{aH}\right)^2 - a^2 m_2^2(\sigma\nabla^4\sigma + 2\tau\nabla^2\sigma + 3\tau^2) \\ &+ m_3^2(\nabla^2\sigma + 3\tau)^2 + \frac{2m_4^2 a^2}{aH}\tau'(\nabla^2\sigma + 3\tau)\Bigg]\,. \end{aligned} \tag{22}$$

This shows that σ is also an auxiliary field:

$$aH(m_2^2 - m_3^2)\nabla^2\sigma = m_4^2\tau' - aH(m_2^2 - 3m_3^2)\tau\,. \tag{23}$$

The action becomes

$$S = M_{\rm Pl}^2 \int \mathrm{d}^4 x \;\; \frac{a^2}{H^2}\Bigg[\frac{(m_0^2+2\epsilon H^2)(m_2^2-m_3^2)+m_4^2}{2(m_2^2-m_3^2)}\tau'^2+\epsilon H^2\tau\nabla^2\tau \\ -\frac{m_2^2a^2H^2(m_2^2-3m_3^2+(3+\epsilon)m_4^2)}{m_2^2-m_3^2}\tau^2\Bigg] \tag{24}$$

.

After canonical normalization of τ, the action finally is given by

$$S = \int \mathrm{d}^4 x a^2\left[\hat\tau'^2 + c_s^2(\hat\tau\nabla^2\hat\tau)+a^2M^2\hat\tau^2\right] , \tag{25}$$

where effective mass and speed of sound are

$$c_s^2 = \frac{2\epsilon H^2(m_3^2-m_2^2)}{m_0^2(m_2^2-m_3^2)+m_4^2} , \tag{26}$$

$$M^2 = -\frac{2H^2m_2^2\left(m_2^2-3m_3^2+3m_4^2\right)}{m_0^2(m_2^2-m_3^2)+m_4^4} , \tag{27}$$

at leading order in slow-roll.

An exhaustive analysys of all the possibilities for the scalar action is beyond the scope of this work. Other cases besides the one considered here could be interesting. For example, when $m_1^2=0$ and $m_2^2=m_3^2$, case that is not included in (24), it can be shown that no scalar degrees of freedom propagate [20]. However this is true only if no derivative operators for h_{ij} are considered. When all the other combinations of h and derivatives are considered, they can provide kinetic terms for scalars, changing the previous conclusions. We will return to this later.

2. *Non-conservation of $\mathcal{R}$ and ζ at super-horizon scales*

Reconsidering the action (24), some interesting points can be made. There is only one scalar perturbation, τ, which is related to the comoving curvature perturbation $\mathcal{R}$. In an arbitrary gauge we define

$$\mathcal{R} = \tau - \frac{\mathcal{H}(\tau'-\mathcal{H}\psi)}{\mathcal{H}'-\mathcal{H}^2} . \tag{28}$$

However in the unitary gauge the equation of motion of the auxiliary field ψ, eq. (21), requires $\tau'=\mathcal{H}\psi$ and we have $\mathcal{R}=\tau$, even when diffeomorphisms are broken by the masses. In the limit where all masses go to zero, the scalar action (24) reduces to the standard slow-roll action for $\mathcal{R}$.

Since $\mathcal{R}$ coincides with the (massive) scalar fluctuation τ, $\mathcal{R}$ (before canonical normalization) has a non-vanishing mass given by

$$M_{\mathcal{R}}^2 = \frac{m_2^2(m_2^2-3m_3^2+(3+\epsilon)m_4^2)}{m_2^2-m_3^2} . \tag{29}$$

Notice that this mass is present only if $m_2^2\neq 0$, exactly as for tensor perturbations. A profound implication of this result is that $\mathcal{R}$ is in general *not* constant after horizon exit, as it is in standard single-field models of inflation. For $M_{\mathcal{R}}^2>0$ the solution of the Mukhanov-Sasaki equation for $\mathcal{R}$ will decay after horizon exit.

The standard picture of different super-horizon patches of the universe evolving as separate universes with constant $\mathcal{R}$ [28] is not valid anymore. A simple physical interpretation is that, given that diffeomorphism invariance is broken in our set-up, very long wavelength fluctuations can no longer be considered as a gauge mode in the zero momentum limit, and there is actually a preferred frame (the unperturbed background, $\mathcal{R} = 0$) towards which the fluctuation dynamics is attracted for $M_{\mathcal{R}}^2 > 0$. This is analogous to what happens in the specific set-up of solid inflation [10], whose consequences can be considered as a special case of our general discussion.

Notice that, phenomenologically, in order for the perturbations to remain over-damped on super-horizon scales (not to oscillate and decay rapidly), we require $M_{\mathcal{R}}^2 \ll H^2$, which gives a constraint on $M_{\mathcal{R}}^2$. On the other hand, given that the mass of the tensor depends only on m_2^2 while the mass of the scalar also on m_3^2 and m_4^2, there is still enough freedom to have a blue tilt for the tensor spectrum and a nearly constant $\mathcal{R}$ outside the horizon. Actually, making the particular choice $m_2^2 = 3m_3^2 - (3+\epsilon)m_4^2$ one finds that $\mathcal{R}$ is massless and conserved outside the horizon.

In our framework, analogously to solid inflation, the comoving curvature perturbations $\mathcal{R}$ and the curvature perturbations on uniform density slices ζ *do not coincide* in the large scale limit, as they do in standard single-field inflation. Indeed, taking the definition of the function ζ,

$$\zeta = \tau - H\frac{\delta\rho}{\dot{\rho}} \,, \tag{30}$$

and computing the density ρ and its perturbation from the energy-momentum tensor, one finds at leading order in gradients a contribution that does not vanish at large scales:

$$\zeta = \tau + \frac{(1-\epsilon)m_4^2}{m_0^2 + 2\epsilon H^2}\tau + \mathcal{O}(\nabla^2) \neq \mathcal{R} \,. \tag{31}$$

Also ζ is not conserved and evolves after horizon exit. Following [28],

$$\dot{\zeta} = -\frac{H}{\rho + p}\delta p_{\rm nad} + \mathcal{O}(\nabla^2) \,, \tag{32}$$

it can be understood that the reason for this non-conservation is the existence of a non-adiabatic stress induced by the presence of the masses. While in the standard case one finds that $\delta p_{\rm nad}$ is proportional only to gradient terms, here there is a non-trivial contribution in the perturbed (spatial) energy-momentum tensor even on super-horizon scales, given by

$$\mathrm{Tr}\,[\delta T_{ij}] = (m_2^2 - 3m_3^2)\,\mathrm{Tr}[h_{ij}] + 3(\epsilon H^2 + \frac{1}{2}m_4^2)h_{00} \,. \tag{33}$$

When diffeomorphisms are preserved, this trace is proportional only to $h_{00} = \psi$, which can then be substituted using the constraints (21) to see that indeed only gradients remain. When diffeomorphisms are broken by the masses, the use of the equation of motion (21) and (23) does not allow us to get rid of all the terms and we are left with

$$\mathrm{Tr}\,[\delta T_{ij}] = m_2^2 f(m_i)\tau + \mathcal{O}(\nabla^2) \,. \tag{34}$$

where $f(m_i)$ is a (complicated) function of all the mass parameters. This term will not vanish on large scales, making ζ evolve also after the horizon exit. The cause of the non conservation of ζ and $\mathcal{R}$ has to be understood in terms of the contribution m_2^2. Indeed if $m_2^2 = 0$ curvature perturbations are constant beyond the horizon. The operator proportional to m_2^2 is the only one that gives a non-trivial off-diagonal contribution to the energy-momentum tensor,

$$T_{ij} \sim m_2^2 h_{ij} \,, \tag{35}$$

and hence an anisotropic stress, that is sourced by the very same operator that gives an effective mass to the graviton (although we will see next that diffeomorphism breaking derivative operators can also play a role). This is coherent and very similar with what was found in [10], where it is shown that a non-vanishing anisotropic stress with certain characteristic on large scale violate some technical assumptions of Weinberg's theorem on the conservation of curvature perturbations [29].

3. Adding derivative operators

Let us now add derivative operators. We by adding the combination $(\delta K_{ij})^2$ [corresponding to the first line of eq (18) with the condition (19) for the operators $(\partial_0 h_{ij})^2$, $(\partial_i h_{0j})^2$ and $(\partial_j h_{0i}\partial_0 h_{ij})$], that as we have seen has the nice feature of avoiding the propagation of vectors. We then subtract $(\delta K_{ii})^2$ [including the operators $(\partial_0 h_{ii})^2$, $(\partial_i h_{0i})^2$ and $(\partial_i h_{0i}\partial_0 h_{jj}))$] in order to avoid the propagation of a second (ghostly) scalar mode.

After this choice is made, we are free to add other derivative operators and write the Lagrangian density as

$$\begin{aligned}\mathcal{L}_d^{(s)} &= M_{\rm Pl}^2\, b\,\left[(\delta K_{ij})^2 - (\delta K_{ii})^2\right]\\ &+\frac{1}{4}M_{\rm Pl}^2\Big[d_1(\partial_i h_{jk})^2 + d_2(\partial_i h_{ij}) + d_3(\partial_i h_{jj})^2 + d_4(\partial_i h_{jj}\partial_k h_{ik})\\ &\qquad +c_1(\partial_i h_{00}\partial_j h_{ij}) + c_2(\partial_i h_{0i}\partial_0 h_{jj}) + c_3(\partial_i h_{00})^2\Big]+\\ &+\frac{1}{4}a M_{\rm Pl}^3\left[\alpha_1(h_{00}\partial_0 h_{ii}) + \alpha_2(h_{00}\partial_i h_{0i}) + \alpha_3(h_{ii}\partial_j h_{0j}) + \alpha_4(h_{ij}\partial_i h_{0j})\right]\,. \end{aligned} \tag{36}$$

Interestingly, also first derivative terms can be added, however the condition $\alpha_1 = 2\alpha_2$ in the single derivative sector has to be imposed, in order to avoid the propagation of a second (ghostly) scalar mode.

Collecting these pieces together, the new action for the scalars will then be

$$\begin{aligned}S^{(S)} = \frac{1}{4}M_{\rm Pl}^2\int {\rm d}^4x\, a^2 \Big\{ &-6\left(\tau' + aH\psi\right)^2 + 2\left(2\psi-\tau\right)\nabla^2\tau + 4\left(\tau'+aH\psi\right)\nabla^2\left(2v-\sigma'\right)\\ &+a^2\left[\left(m_0^2+2\epsilon H^2\right)\psi^2 - 2m_1^2 v\nabla^2 v - m_2^2\left(\sigma\nabla^4\sigma + 2\tau\nabla^2\sigma + 3\tau^2\right)\right.\\ &\left.+\, m_3^2\left(\nabla^2\sigma+3\tau\right)^2 - 2m_4^2\psi\left(\nabla\sigma + 3\tau\right)\right]\\ &+b\left(8\tau'\nabla^2 v - 4\tau'\nabla^2\sigma' - 6\tau'^2\right) - c_1\nabla^2\psi\left(\nabla^2\sigma+\tau\right)\\ &-c_2\nabla^2\psi\left(\nabla^2\sigma + 3\tau\right) - c_3\psi\nabla^2\psi - \left(d_1+d_2+d_3+d_4\right)\nabla^2\sigma\nabla^4\sigma\\ &-2\left(d_1+d_2+3d_3+2d_4\right)\tau\nabla^4\sigma - \left(3d_1+d_2+9d_3+3d_4\right)\tau\nabla^2\tau\\ &+aM_{\rm Pl}^3\left[\alpha_1\psi(\nabla^2\sigma'+3\tau') + 2\alpha_1\psi\nabla^2 v + \alpha_3\nabla^2 v(\nabla^2\sigma+3\tau) + \alpha_4\nabla^2 v(\nabla^2\sigma+\tau)\right]\Big\}\end{aligned} \tag{37}$$

where the parameter b is associated to the combination $(\delta K_{ij})^2 - (\delta K_{ii})^2$ expanded at quadratic order in fluctuations. As we said, the fields v and ψ are again auxiliary and their equations of motion can be solved algebraically. The main point is that the action resulting from their substitution does not contain any time derivative term σ', which means that the dangerous 'sixth-mode' σ is not dynamical and can be integrated away. The action for the only remaining dynamical scalar has the following simple structure:

$$S = M_{\rm Pl}^2\int {\rm d}^4x a^2\left[A_1\tau'^2 + A_2\tau\tau' + A_3\tau^2 + A_4\sigma^2 + A_5\sigma\tau + A_6\sigma\tau'\right]\,, \tag{38}$$

where the A_i are functions of all the parameters and the gradient ∇^2 (see Appendix B). The field σ can then be integrated out to give (after some integrations by parts)

$$S = M_{\rm Pl}^2 \int \mathrm{d}^4 x a^2 \left[B_1 \tau'^2 + B_2 \tau^2 \right] \,, \tag{39}$$

At this point, one can canonically normalize $\hat{\tau} = \sqrt{B_1}\tau$ and symbolically expand in ∇^2 (which can be understood in Fourier space as an expansion in the momentum k), so that one can read the mass and the speed of sound of the scalar mode:

$$S = \int \mathrm{d}^4 x \, a^2 \left[\hat{\tau}'^2 + \hat{c}_s^2 \hat{\tau} \nabla^2 \hat{\tau} + a^2 \hat{M}^2 \hat{\tau}^2 + \mathcal{O}(\nabla^4) \right] \,. \tag{40}$$

The expression of $\hat{c}_s^2$ and $\hat{M}^2$ are complicated functions of all the parameters. It can be checked that in the limit where all the parameters of the modified kinetic terms b, c_i, d_i, δ_i, α_i vanish, we recover the expressions of the previous section where c_s is given by Eq. (26) and mass is given by Eq. (27), while higher-order derivative terms correctly drop to zero. As an example, we write here the effective mass and speed of sound at leading order in slow roll in the case where all the parameters are zero except for masses and α_1:

$$\hat{c}_s^2 = \frac{\alpha_1 \Lambda (m_2^2 - m_3^2)(\alpha_1 \Lambda - 4H)}{(m_2^2 - m_3^2)\left(3\alpha_1\Lambda(\alpha_1\Lambda - 8H) + 8m_0^2\right) + 8m_4^4} \,, \tag{41}$$

$$\hat{M}^2 = -\frac{m_2^2 (4H - \alpha_1 \Lambda)\left(4H\left(m_2^2 - 3m_3^2 + 3m_4^2\right) - \alpha_1 \Lambda \left(m_2^2 - 3m_3^2\right)\right)}{(m_2^2 - m_3^2)\left(3\alpha_1\Lambda(\alpha_1\Lambda - 8H) + 8m_0^2\right) + 8m_4^4} \,. \tag{42}$$

One can see that 'kinetic operators' like the one proportional to α_1 can also affect the effective mass. A natural question to ask is whether, by exploiting this fact, effective mass contributions can be generated even in the absence of explicit non-derivative terms in the action. This will be the subject of the next section.

Also after adding derivative contributions, the curvature perturbation is again not conserved and decays after horizon exit. As previously, this can be seen also from the trace of the spatial part of the energy-momentum tensor, which, in the simple example we do, now reads

$$\mathrm{Tr}\left[\delta T_{ij}\right] = m_2^2 \tau + \frac{1}{2} \alpha_1 M_{\rm Pl} (a\psi)' + \mathcal{O}(\nabla^2) \,, \tag{43}$$

hence it does not vanish at superhorizon scales, due to the contributions proportional to m_2^2 and α_1. One might use the constraint equation (21) to express ψ' in terms of τ, the only propagating scalar degree of freedom in the system. It would be interesting to analyze how the curvature perturbation ζ evolves at superhorizon scales when α_1 or other diffeomorphism-breaking kinetic terms are included.

III. GENERATING A MASS WITHOUT MASS: FOUR DERIVATIVE OPERATORS

We have learned in the previous section that by breaking spatial diffeomorphism invariance of the action for metric perturbations, by means of mass terms or derivative operators, we can change some of the properties of the tensor spectrum with respect to the standard inflationary predictions, in particular its tilt n_T and the value of the tensor sound speed c_T.

It is natural to ask whether it is really necessary to explicitly break spatial diffeomorphism invariance to do so. The aim of this section is to show that the answer is no, provided that we allow for higher spatial derivative operators in the quadratic action for fluctuations. An effective field theory approach to inflation that takes into account of higher derivative operators has also been proposed in [30]. Adding such operators, one can avoid the argument [27] (based on operators with at most two space-time derivatives) and find genuine contributions to the tensor sound speed c_T, that cannot be eliminated by disformal transformations. This has interesting implications since the tensor sound speed enters in the amplitude of the tensor power spectrum (see eq (15)) in a way that enhances the amplitude of $\mathcal{P}_T$ that scales as c_T^{-1}. It would be interesting to find explicit models able to avoid the Lyth bound using this fact, but would also need to consider the effect on the scalar modes and hence the observed tensor-to-scalar ratio r.

In particular, we will explore the effect of 4-derivative contributions to the action for fluctuations, organized in such a way as not to break the spatial diffeomorphism invariance, and not to introduce instabilities. The starting point is to consider the quantities

$$\partial_0\partial_l\, h_{ij} = \partial_l\, \chi'_{ij} + \partial_l\partial_{(i}s'_{j)} + \partial_l\partial_i\partial_j\, \sigma' + \delta_{ij}\, \partial_l\tau'\,, \tag{44}$$

$$\partial_0\partial_i\, h_{ij} = \nabla^2 s'_j + \partial_j\, \nabla^2\, \sigma' + \partial_j\tau'\,, \tag{45}$$

$$\partial_0\partial_j\, h_{ii} = \partial_j\, \nabla^2\, \sigma' + 3\partial_j\tau'\,, \tag{46}$$

that we can use to build quadratic operators with four derivatives, that we can potentially add to the action for metric perturbations

$$L_1 = (\partial_l\, \partial_0\, h_{ij})^2 \;=\; \left(\partial_l\, \chi'_{ij}\right)^2 + 2\left(\nabla^2 s'_j\right)^2 - \nabla^2\, \sigma'\, \nabla^2\nabla^2\, \sigma' - 3\tau'\, \nabla^2\tau' - 2\, \nabla^2\, \sigma'\, \nabla^2\tau'\,, \tag{47}$$

$$L_2 = (\partial_0\partial_i\, h_{ij})^2 \;=\; \left(\nabla^2 s'_j\right)^2 - \nabla^2\, \sigma'\, \nabla^2\nabla^2\, \sigma' - \tau'\nabla^2\tau' - 2\, \nabla^2\, \sigma'\, \nabla^2\tau'\,, \tag{48}$$

$$L_3 = (\partial_0\partial_j\, h_{ii})^2 \;=\; -\nabla^2\, \sigma'\, \nabla^2\nabla^2\, \sigma' - 9\tau'\, \nabla^2\tau' - 6\, \nabla^2\, \sigma'\, \nabla^2\tau'\,, \tag{49}$$

$$L_4 = \partial_0\partial_i\, h_{ij}\partial_0\partial_j\, h_{ii} \;=\; -\nabla^2\, \sigma'\, \nabla^2\nabla^2\, \sigma' - 3\tau'\, \nabla^2\tau' - 4\, \nabla^2\, \sigma'\, \nabla^2\tau'\,, \tag{50}$$

where integrations by parts have been performed. We would like to build a combination of L_i such that only contributions associated with $\chi'_{ij}\, \nabla^2\, \chi'_{ij}$ and $\tau'\nabla^2\tau'$ are non-vanishing, while the vectors and the remaining scalars do not appear. If such combination can be found, it is invariant under spatial diffeomorphisms, since χ_{ij} and τ do not transform under this symmetry (see eq (8), noticing that τ transforms but only under time-reparameterization). The combination with the desired properties is

$$L_{\omega_1} = \omega_1(L_1 - 2L_2 - L_3 + 2L_4) \tag{51}$$

$$= -\omega_1\chi'_{ij}\nabla^2\chi'_{ij} + 2\omega_1\tau'\nabla^2\tau'\,. \tag{52}$$

In analogy to what happens for the two derivatives operators, see the comment after Eq. (12), this combination (51) corresponds to a particular combination of the extrinsic curvature perturbation,

$$(\partial_i\delta K_{jk})^2 - (\partial_i\delta K)^2 - 2(\partial_i\delta K_{ij})^2 - 2\partial_i\delta K\partial_j\delta K_{ij}\,, \tag{53}$$

expanded at quadratic order in perturbations.

Analogously, one can consider four derivative operators that lead only to combinations involving four spatial derivatives acting on the tensors $\nabla^2\chi_{ij}\, \nabla^2\, \chi_{ij}$. The following Lagrangians arise from all possible contractions of two spatial derivatives and h_{ij} (once integrations by parts are taken into account):

$$L_1 = (\nabla^2 h_{ij})^2 = (\nabla^2\chi_{ij})^2 - 2s_i\nabla^4 s_i + (\nabla^4\sigma)^2 + 3(\nabla^2\tau)^2 + 2\nabla^2\tau\nabla^4\sigma\,, \tag{54}$$

$$L_2 = (\partial_i\partial_j h_{ij}) = (\nabla^4\sigma + \nabla^2\tau)^2\,, \tag{55}$$

$$L_3 = (\nabla^2 h_{ii})^2 = (\nabla^4\sigma + 3\nabla^2\tau)^2\,, \tag{56}$$

$$L_4 = (\partial_k\partial_i h_{ij})^2 = -s_i\nabla^4 s_i + (\nabla^4\sigma + \nabla^2\tau)^2\,, \tag{57}$$

$$L_5 = (\nabla^2 h_{kk}\partial_i\partial_j h_{ij}) = (\nabla^4\sigma + \nabla^2\tau)(\nabla^4\sigma + 3\nabla^2\tau)\,. \tag{58}$$

There exist combinations of these operators which allow us to avoid contributions from all vectors and scalars:

$$L_{\omega_2} = \omega_2 (L_1 + \frac{1}{2} L_2 - \frac{1}{2} L_3 - 2 L_4 + L_5) = \tag{59}$$

$$= \omega_2 (\nabla^2 \chi_{ij})^2 \,, \tag{60}$$

hence this combination preserves full four dimensional diffeomorphism invariance.

By adding the Lagrangians L_{ω_1} and L_{ω_2} to the quadratic EH Lagrangian plus the two derivative contribution (12) – that can preserve diffeomorphism invariance if it originates from a combination of δK_{ij}^2 and ${}^{(3)}R$ (see the comment after Eq. (12)) – one obtains the effective Lagrangian for tensor modes[3]

$$\mathcal{L}^{(T)} \,=\, \frac{M_{Pl}^2}{4}\, a^2\, \left[(1+b)(\chi_{ij}')^2 - \frac{\omega_1}{a^2 \Lambda^2}\, \chi_{ij}'\, \nabla^2 \chi_{ij}' + (1+d) \chi_{ij}\, \nabla^2\, \chi_{ij} + \frac{\omega_2}{a^2\, \Lambda^2}\, \chi_{ij}\, \nabla^2\, \nabla^2\, \chi_{ij} \right] \tag{61}$$

with $\omega_{1,2}$ arbitrary parameters, and Λ some cut-off energy scale, that will depend on the UV completion, and that to be safe we take larger than the Hubble scale during inflation. Let us emphasize that we constructed the Lagrangians L_{ω_1} and L_{ω_2} as space diffeomorphism invariant combinations, with the specific aim to analyze the phenomenological consequences of higher order derivative operators in the tensor sector. These Lagrangians are characterized by a specific choice of parameters among their terms: it would be interesting to investigate whether such combinations can be enforced by some symmetry principle.

To canonically normalize the tensor field appearing in the Lagragian $\mathcal{L}^{(T)}$ of eq. (61), we pass for simplicity to Fourier space, and define the quantity

$$\chi_{ij} \,=\, \frac{\sqrt{2}\, \tilde{\chi}_{ij}}{M_{Pl}\, a\, \sqrt{1+b+\omega_1\, k^2/(a^2\, \Lambda^2)}} \,. \tag{62}$$

Using this tilde quantity $\tilde{\chi}_{ij}$, the Lagrangian, after an integration by parts, acquires a relatively simple form in a quasi-de Sitter universe

$$\mathcal{L}^{(T)} \,=\, \frac{1}{2}\, \left[(\tilde{\chi}_{ij}')^2 - F(k,\eta)\, \tilde{\chi}_{ij}^2 \right] \tag{63}$$

with

$$F(k,\, \eta) = \frac{1}{\left(1+b+\frac{\omega_1\, k^2}{a^2\, \Lambda^2}\right)^2} \Big[-\, (1+b)^2\, (2-\epsilon)\, a^2\, H^2 + k^2\, (1+b) \left(1 + d - (3-\epsilon)\, \frac{\omega_1 H^2}{\Lambda^2} \right)$$
$$+ \frac{k^4}{a^2\, \Lambda^2}\, (\omega_1 + d\, \omega_1 + \omega_2 + b\, \omega_2) + \omega_1 \omega_2\, \frac{k^6}{a^4\, \Lambda^4} \Big] \,. \tag{64}$$

We can now work out some consequences of these results:

• By making the choice $b = -1$, the quadratic terms containing two time derivatives cancel from the action (61), and the dynamics is driven by the four derivative operator proportional to ω_1. In a certain sense, the situation can be seen as analogous to what happens in ghost inflation [31], where the leading terms in the gradients of the ghost field vanish, and the next-to-leading contributions in gradients become dominant.

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[3] The same operators will also modify the scalar sector. Considering for simplicity only the Einstein-Hilbert part plus these four-derivative operators, it can be easily seen that the action for the scalar has the same form of the action for the tensors (61) and that the arguments that can be developed for the scalar sector are very similar to the ones we are carring on for the tensors.

The expression for the function F above simplifies considerably:

$$F(k,\,\eta) = \frac{\omega_2}{\omega_1}\,k^2 + \frac{(1+d)\,\Lambda^2}{\omega_1}\,a^2\,, \tag{65}$$

$$= \frac{\omega_2}{\omega_1}\,k^2 - 2\,H^2\,a^2 + \frac{(1+d)\,\Lambda^2 + 2\,H^2\,\omega_1}{\omega_1}\,a^2\,. \tag{66}$$

The first term in the right hand side of (66) can be recognized as the usual first contribution to the dispersion relation associated with $\tilde{\chi}_{ij}$, characterized by an effective sound speed $c_T^2 = \omega_1/\omega_2$. The second piece is the effective 'mass term' that usually arises in a quasi-de Sitter universe. Then, we have the third contribution, that mimics exactly a mass term with

$$m^2_{\tilde{\chi}} = \frac{(1+d)\,\Lambda^2 + 2\,H^2\,\omega_1}{\omega_1}\,. \tag{67}$$

Interestingly this effective mass arises only from the higher derivative terms, with no need to break diffeomorphism invariance! In this sense, 4-derivative contributions can be interpreted as being able to generate mass without an explicit mass parameter. On the other hand, notice that in this case the relation between the canonically normalized tensor field $\tilde{\chi}_{ij}$ and original one χ_{ij} scales as the inverse of the momentum: $\chi_{ij} \propto \tilde{\chi}_{ij}/k$: see eq. (62). This typically implies – by the arguments outlined around eq (6) – a low cut-off scale when focussing at large scales; on the other hand, this crucially depends on the tensor interactions during inflation, that might conspire in such a way to raise the cut-off. This is an interesting question that we intend to pursue in the future.

• Let us now consider the more general situation with $b \neq -1$, focusing on the large and small scale limits for the function F:

$$F(k,\,\eta) \overset{k\to 0}{\simeq} (-2+\epsilon)a^2H^2 + \mathcal{O}(k^2)\,, \tag{68}$$

$$F(k,\,\eta) \overset{k\to +\infty}{\simeq} \frac{\omega_2}{\omega_1}k^2 + \frac{a^2\Lambda^2}{\omega_1^2}\left[(1+d)\omega_1 - (1+b)\omega_2\right] + \mathcal{O}(k^{-2})\,. \tag{69}$$

No major differences with respect to the standard case arise, apart from the presence of a non-trivial sound speed c_T: the system can be quantized selecting a Bunch-Davies vacuum at very small scales, while at large scales the tensors behave as in a standard quasi-de Sitter universe, with no mass.

This preliminary analysis of the role of operators with higher spatial derivatives shows their possible relevance for characterizing tensor modes, and can find some motivation for example (but not only) in the context of Horava-Lifshitz cosmology (see [32] for a review). It shows that in this set-up a non-unity tensor sound speed c_T can be generated, and that it cannot in general be set to one by a set of transformations of the metric [27].

IV. CONCLUSIONS

By implementing an effective field theory approach to single clock inflation, we have examined interesting properties of the spectrum of inflationary tensor fluctuations, that arise when breaking some of the symmetries or requirements usually imposed on the dynamics of inflationary perturbations.

In the first part of the paper we considered the possibility that, besides time-reparameterization, spatial diffeomorphisms are also broken in the quadratic Lagrangian controlling fluctuations during inflation. We do so considering quadratic operators that break spatial diffeomorphisms, maintaining spatial isotropy and homogeneity, that contain at most two space-time derivatives. Such operators can be motivated by a modification of gravity during the inflationary era, or by some particular behavior of the fields that drive inflation. We identified the single operator that contributes at leading order to the tensor spectral tilt n_T, and that can change its sign leading to a positive n_T without necessarily violating the null energy condition. We have then shown that this operator has important consequences in the scalar sector. It generically leads to superhorizon non-conservation of the curvature perturbation ζ on uniform energy density slices, even in single clock inflation – since ζ acquires an effective mass – although additional allowed operators can render the mass of ζ (and its non-conservation after horizon exit) arbitrary small.

In the second part of the paper, we returned to the case of spatial diffeomorphism invariant Lagrangians, including quadratic operators with more than two spatial derivatives (but no more than two time derivatives) acting on the tensor perturbations. We showed that also in this case, by a judicious choice of the operators, one can obtain properties for the fluctuations that are very similar to the ones of a diffeomorphism breaking set-up. In particular, a non-trivial tensor sound speed can be generated, and the formula for n_T receives new contributions that depend on the coefficients of these higher derivative operators. We also discussed a special case in which such operators can mimic the effect of a mass term in the tensor sector.

The power of our approach is the use of effective field theory of inflation [12], that relies on symmetry principles only, and encompasses various scenarios in a model independent way. In a companion work [26], using again an effective field theory approach, we will examine model independent consequences of breaking isotropy and homogeneity in the Lagrangian for cosmological fluctuations.

In this work, for simplicity we focussed on a quadratic action for fluctuations since when we break symmetries such as spatial diffeomorphism invariance, operators cubic or higher in fluctuations exist in large number. It would be interesting to extend our analysis to higher order in perturbations, to study the consequences for non-linearity and non-Gaussianity of the primordial metric perturbations from inflation.

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Appendix A: Combinations of h and derivatives

Combinations up to second order in h and up to two derivatives, avoiding time derivatives on N or N^i (some integrations by parts have already been performed).

$$h_{00}\partial_0 h_{ii} = \psi(\nabla^2\sigma' + 3\tau') \tag{A1}$$

$$h_{00}\partial_i h_{0i} = \psi\nabla^2 v \tag{A2}$$

$$h_{ii}\partial_j h_{0j} = \nabla^2 v(\nabla^2\sigma + 3\tau) \tag{A3}$$

$$h_{ij}\partial_i h_{0j} = \nabla^2 v(\nabla^2\sigma + \tau) - u_i\nabla^2 s_i \tag{A4}$$

$$(\partial_i h_{00})^2 = -\psi\nabla^2\psi \tag{A5}$$

$$(\partial_0 h_{ii})^2 = (\nabla^2\sigma' + 3\tau')^2 \tag{A6}$$

$$(\partial_i h_{0i})^2 = (\nabla^2 v)^2 \tag{A7}$$

$$\partial_i h_{0i}\partial_0 h_{jj} = \nabla^2 v(\nabla\sigma' + 3\tau') \tag{A8}$$

$$(\partial_i h_{jj})^2 = -(\nabla^2\sigma + 3\tau)\nabla^2(\nabla^2\sigma + 3\tau) \tag{A9}$$

$$(\partial_i h_{ij})^2 = -(\nabla^2\sigma + \tau)\nabla^2(\nabla^2\sigma + \tau) + (\nabla^2 s_j)^2 \tag{A10}$$

$$\partial_i h_{jj}\partial_k h_{ik} = -(\nabla^2\sigma + 3\tau)\nabla^2(\nabla^2\sigma + \tau) \tag{A11}$$

$$\partial_i h_{00}\partial_i h_{jj} = -\nabla^2\psi(\nabla^2\sigma + 3\tau) \tag{A12}$$

$$\partial_i h_{00}\partial_j h_{ij} = -\nabla^2\psi(\nabla^2\sigma + \tau) \tag{A13}$$

$$\partial_j h_{0i}\partial_0 h_{ij} = \nabla^2 v(\nabla^2\sigma' + \tau') - u_i\nabla^2 s_i' \tag{A14}$$

$$(\partial_0 h_{ij})^2 = (\chi_{ij}')^2 + (\nabla^2\sigma')^2 + 2\tau'\nabla^2\sigma' + 3{\tau'}^2 - 2s_j'\nabla^2 s_j' \tag{A15}$$

$$(\partial_i h_{0j})^2 = (\partial_i u_j)^2 + (\nabla^2 v)^2 \tag{A16}$$

$$(\partial_i h_{jk})^2 = (\partial_i\chi_{jk})^2 + (\partial_i\partial_j\partial_k\sigma)^2 - 2\nabla^2\sigma\nabla^2\tau - 3\tau\nabla^2\tau + 2(\nabla^2 s_i)^2 \tag{A17}$$

Appendix B: Speed of sound and mass

Coefficients A_i for the scalar action (38)

$$A_1 = -\frac{M_{\rm Pl}^2(1+b)}{2(\alpha_1\Lambda-4H)^2}\left[-8(1+b)\left(c_3k^2+(m_0^2+2\epsilon H^2)\right)+48baH-3a^2\alpha_1\Lambda(\alpha_1\Lambda-8H)\right] \tag{B1}$$

$$\begin{aligned} A_2 =& \frac{aM_{\rm Pl}^2(1+b)}{(\alpha_1\Lambda-4H)^2}\Big\{\Big[(3c_2+c_1-4)(\alpha_1\Lambda-4H)+c_3(3\alpha_3+\alpha_4)\Lambda\Big]k^2+ \\ &+\left[\left(m_0^2+2\epsilon H^2-\frac{6bH^2}{1+b}\right)(3\alpha_3+\alpha_4)\Lambda-6m_4^2(\alpha_1\Lambda-4H)\right]\Big\}+ \\ &+\frac{a^3M_{\rm Pl}^2\alpha_1(3\alpha_3+\alpha_4)(\alpha_1\Lambda-8H)\Lambda^2}{8(\alpha_1\Lambda-4H)} \end{aligned} \tag{B2}$$

$$\begin{aligned} A_3 =& \frac{a^2M_{\rm Pl}^2k^2}{(\alpha_1\Lambda-4H)^2}\Big[4(2+3d_1+d_2+9d_3+d_4)(\alpha_1\Lambda-4H)^2 \\ &+(3\alpha_3+\alpha_4)(2(3c_2+c_1-4)(\alpha_1\Lambda-4H)+c_3(3\alpha_3+\alpha_4))\Lambda\Big]+ \\ &+\frac{a^4M_{\rm Pl}^2}{(\alpha_1\Lambda-4H)^2}\left[6H(m_2^2-3m_3^2)(\alpha_1-2H)+3(3\alpha+\alpha_4)m_4^2H\Lambda\right]+ \\ &-\frac{a^4M_{\rm Pl}^2\Lambda^2}{16(\alpha_1\Lambda-4H)^2}\left[12\alpha_1^2(m_2^2-m_3^2)+(3\alpha_3+\alpha_4)(12\alpha_1m_4^2-(3\alpha_3+\alpha_4)(m_0^2+2\epsilon H^2-6H^2))\right] \end{aligned} \tag{B3}$$

$$\begin{aligned} A_4 =& \frac{a^2M_{\rm Pl}^2k^6}{16(\alpha_1\Lambda-4H)^2}\Big[4(d_1+d_2+d_3+d_4)(\alpha_1\Lambda-4H)^2+2(\alpha_3+\alpha_4)(c_1+c_2)(\alpha_1\Lambda-4H)\Lambda+c_3(\alpha_3+\alpha_4)^2\Lambda^2\Big] \\ &-\frac{a^4M_{\rm Pl}^2k^4}{16(\alpha_1\Lambda-4H)^2}\Big[4(m_2^2-m_3^2)(\alpha_1\Lambda-4H)^2+4m_4^2(\alpha_1\Lambda-4H)(\alpha_3+\alpha_4)\Lambda+ \\ &+(m_0^2+2\epsilon H^2-6H^2)(\alpha_3+\alpha_4)^2\Lambda^2\Big] \end{aligned} \tag{B4}$$

$$\begin{aligned} A_5 =& -\frac{a^2M_{\rm Pl}^2k^4}{8(\alpha_1\Lambda-4H)^2}\Big[4(d_1+d_2+5d_4)(\alpha_1\Lambda-4H)^2-c_3(\alpha_3+\alpha_4)(3\alpha_3+2\alpha_4)\Lambda^2 \\ &-2\alpha_3(\alpha_1\Lambda-4H)(3c_2+2c_1-2)\Lambda-2\alpha_4(\alpha_1\Lambda-4H)(2c_2+c_1-1)\Lambda\Big] \\ &+\frac{a^4M_{\rm Pl}^2k^2}{16(\alpha_1\Lambda-4H)^2}\Big[4(m_2^2-3m_3^2)(\alpha_1\Lambda-4H)^2 \\ &-(m_0^2+2\epsilon H^2-6H^2)(\alpha_3+\alpha_4)(3\alpha_3+2\alpha_4)\Lambda^2+4m_4^2(\alpha_1\Lambda-4H)(3\alpha_3+2\alpha_4)\Lambda\Big] \end{aligned} \tag{B5}$$

$$\begin{aligned} A_6 =& -\frac{a^2M_{\rm Pl}^2k^4}{8(\alpha_1\Lambda-4H)}\Big[(c_1+c_2)(\alpha_1\Lambda-4H)-c_3(\alpha_3+\alpha_4)\Lambda\Big]+ \\ &+\frac{a^3M_{\rm Pl}^2k^2}{8(\alpha_1\Lambda-4H)}\big[16m_4^2(1+b)(\alpha_1\Lambda-4H)-8(1+b)(m_0^2+2\epsilon H^2)(\alpha_3+\alpha_4)\Lambda+ \\ &+3(16bH^2-\alpha_1\Lambda(\alpha_1\Lambda-8H))(\alpha_3+\alpha_4)\Lambda\big] \end{aligned} \tag{B6}$$

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