Modelling 3D voxel slope for MRI partial volume quantification

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Introduction

Use of slope or gradient magnitude information can assist in the identification of Regions Of Interest (ROIs) in image data [1] and in the detection of Partial Volume (PV) voxels for image quantification [2]. Previously, the gradient distribution has been modelled in 2D as a Rayleigh distribution [1]:

$$p_{Rayleigh}(z)=\frac{z}{\sigma^2}\exp\left(-\frac{z^2}{2\sigma^2}\right). \quad (1)$$

Williamson et al [1] developed this idea further for modelling PV voxels and generalised the slope as a Rician distribution [3,4]:

$$p_{Rice}(z)=\frac{z}{\sigma^2}\exp\left(-\frac{z^2+A^2}{2\sigma^2}\right)I_0\left(\frac{A\cdot z}{\sigma^2}\right). \quad (2)$$

A Rician distribution tends towards a Rayleigh distribution as $A\rightarrow 0$, but tends towards a Gaussian distribution as $A\rightarrow\infty$. This enables the slope of regions composed of a single Gaussian distributed (intensity) compartment such as Grey Matter (GM) or White Matter (WM) to be modelled as a Rayleigh distribution, while the slope of regions composed of PV voxels would be modelled as Gaussian distributions.

The authors in [1] and [2] thus developed models for 2D slope. However, the MRI acquisition process is 3D in terms of the down sampling process. Therefore, classification and segmentation algorithms that utilise 3D slope should possess superior performance over algorithms that use 2D slope (see fig. 1). This paper therefore extends slope modelling to three dimensions with the principle aim of providing a tool to improve MRI PV voxel quantification.

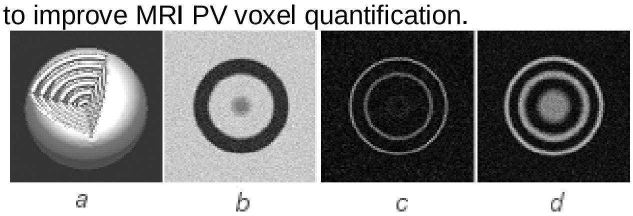

Fig.1. **Demonstration of the effect of 3D vs. 2D slope estimation.(a)surface rendering of a 3D synthetic PV data set (two tissue model in concentric spheres) rendered using MRIcro software [5]. (b) slice through the synthetic data in (a). (c) 2D slope image of (b). (d) 3D slope of (b) demonstrating superior detection of PV voxels, compared to (c).**

Method

The slope at a 3D point, $\boldsymbol{\omega}$, can be calculated using a kernel of the form $\Delta G_l(\boldsymbol{\omega})=(+1,0,-1)$ where G represents the data volume and $l=i, j \text{ or } k$ (the axis of the kernel). The slope is then the magnitude of these values:

$$\left|\Delta G_{ijk}(\boldsymbol{\omega})\right|=\sqrt{(\Delta G_i(\boldsymbol{\omega}))^2+(\Delta G_j(\boldsymbol{\omega}))^2+(\Delta G_k(\boldsymbol{\omega}))^2}\ . \quad (3)$$

In this new work we demonstrate that the 3D slope Probability Distribution Function (PDF) for a single tissue ROI is of the form, (after some mathematical derivation):

$$p_{Zpure}(z)=\frac{2z^2}{\sigma^3\sqrt{2\pi}}\exp\left(-\frac{z^2}{2\sigma^2}\right). \quad (4)$$

An extension of this is to consider the conditions where the kernel approaches the edge of a tissue compartment: the PDF can then be shown, after some further mathematical derivation, to be:

$$p_{Zmixed}(z)=\frac{K_\Gamma z^{\frac{3}{2}}}{\sigma^2\sqrt{\pi\mu}}\exp\left(-\frac{z^2+\mu^2}{2\sigma^2}\right).I_{\frac{1}{2}}\left(-\frac{\mu z}{\sigma^2}\right). \quad (5)$$

Results

Fig. 2 compares the new derived distributions with data points from actual 3D data.

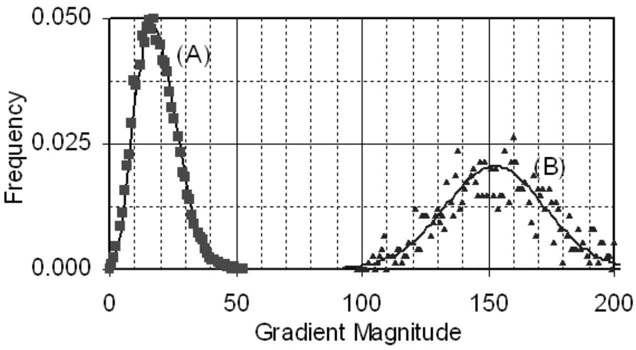

Fig.2. **Comparison of derived parametric models with actual data points for 3D slope demonstrating goodness of fit. Curve and points labelled(A)are for a pure tissue (eq.4) while(B) corresponds to PV voxels (eq.5).**

Conclusions

Equations for 3D slope PDFs have been derived and preliminary results demonstrate good correspondence. This new 3D parameterisation will allow development of a new 3D PV Bayesian classifier.

References

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