Forecasting Monthly Fisheries Prices: Model Comparison Using Data From Cornwall (UK)

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Abstract

Forecasting has a key role in applied economics and management of fisheries. In this paper we report the forecasting competition between Autoregressive AR(p), Moving Average MA(q) and ARMA(p,q) models of the monthly average fisheries prices. We consider twelve species landed into Cornwall: Anglerfish, Cod, Crabs, Dogfish, Haddock, Hake, Lemonsole, Mackerel, Plaice, Saithe, Sole and Whiting. In our evaluation of the out-of-sample forecasting accuracy of ten models, we show that simple ARMA(p,q) models generally prove to be the best forecasting models.

JEL Classification: C32, C53, Q22.

Keywords: Fisheries prices, Forecasting, Time series models, Cornwall (UK).

I. Introduction

Forecasting plays a key role in applied economics and management. Quantitative forecasting is important for policy makers because they model, analyse and forecast different events. It is an interesting subject because it is difficult to do accurately owing to the uncertainties confronting forecasters (Stergiou *et al.*, 1997). Time series forecasting methods are based on analysis of historical data (past quantitative information) and are able to explain the past, present and future. Fishery time series can be applied under such information. Modelling fisheries prices is important because it may give the performance of prices and may provide forecasts of its future levels.

Time series modelling have been useful in describing and forecasting fisheries dynamics (Yoo and Zhang, 1993; Park and Yoon, 1996; Park, 1998). Various techniques, from the simple OLS method to the Autoregressive Integrated Moving Average (ARIMA) models, have been used to explain the forecasting performance of prices in economics. However, a limited number of research papers have used time series models for forecasting fisheries catch prices. Recent investigations of forecasting fisheries catch prices include Accadia and Placenti (2001), Stergiou and Christou (1996), Stergiou, Christou and Petrakis (1997). Stergiou and Christou (1996) evaluate the performance of 11 forecasting

techniques using annual commercial landings of 16 species in the Hellenic marine waters. Stergiou *et al.* (1997) investigate the modelling and forecasting monthly fisheries catches using fisheries statistics from Hellenic waters and prove that ARIMA models show good performance.

We focus on modelling fisheries prices using monthly average prices of main species landed into Cornwall, UK. Our paper extends the previous work of Floros and Failler (2004). They examine the evidence for seasonal effects and cointegration between fisheries prices of main species landed into Cornwall. They report significant monthly effects in April and negative monthly effects in February, while they also find cointegration between prices.

In this paper we examine the evidence for forecasting of the following 12 species: Anglerfish, Cod, Crabs, Dogfish, Haddock, Hake, Lemonsole, Mackerel, Plaice, Saithe, Sole and Whiting. The main objective of this paper is to compare the out-of-sample forecasting accuracy of ten different Autoregressive (AR), Moving Average (MA) and Autoregressive Moving Average (ARMA) time series models using monthly fisheries prices. To the best of our knowledge, this is the first investigation for forecasting fisheries prices in England (Cornwall) using time series models. This paper is part of the EU funded project 'PECHDEV[1]' and gives important information on the modelling and forecasting fish prices under time series analysis.

The paper is organised as follows: Section II provides the methodology and data, while Section III presents the main empirical results from various econometric models. Section IV summarises our findings and concludes the paper.

II. Data & Methodology

The South West fishing industry is estimated to be worth £244 million and accounts for approximately 1,332 direct and 2,013 indirect jobs. Spend by direct and indirect employees helps to support a further 614 induced jobs in the local economy. A total of £72.4 million worth of fish was landed in South West ports in 2001. This is equivalent to 0.11% of the Gross Domestic Product of the region. In total, the fishing industry in the Objective 1 area is worth an estimated £99 million. This represents 2% of the total Objective 1 area GDP. The South West fishing fleet is made up of 1,149 vessels. Of these, 299 are over 10 metres and 850 are under 10 metres (70% of which are active).

2001 landings totalled 56,773 tonnes. Of this total, 13.8 million came from recorded landings by under 10m vessels and 57.6m from UK vessels over 10m. Landings by foreign vessels to SW ports in 2001 are estimated to £1.1m.

We use monthly observations of average prices (£/tonne) of main species landed into Cornwall covering the period January 1992 to December 2002 (we use the data of Floros and Failler, 2004). The data have been provided the Department for Environment, Food and Rural Affairs (Defra). The first 120 observations (January 1992 – December 2001) are used for parameter estimation, while the next 12 observations (January 2002 – December 2002) are used for forecast evaluation.

Table 1a and Table 1b give summary statistics for log-prices by species. Monthly log-prices are between 6.18 and 8.12. Negative (positive) values for skewness indicate that the series distributions are skewed to the left (right). Values for kurtosis are high (>3) for 5 out of 12 species. Hence, five species show excess kurtosis (leptokurtic pdf), implying fatter tails than a normal distribution.

The Jarque-Bera test rejects normality at the 5% level for two species only (crabs and saithe). The data are plotted in log-levels in Appendix 1.

Table 1a: Descriptive Statistics for log-prices by species

Species	Anglerfish	Cod	Crabs	Dogfish	Haddock	Hake
Mean	7.618289	7.217842	7.001934	6.570377	7.124778	8.066894
Median	7.641323	7.225468	6.966493	6.602543	7.153047	8.100917
Maximum	7.991254	7.711101	7.520776	7.377134	7.814803	9.040145
Minimum	7.253470	6.738152	6.210600	5.921578	6.431331	7.142827
Std. Dev.	0.136474	0.219179	0.162577	0.292059	0.289242	0.313628
Skewness	0.020803	0.039077	0.223700	0.131308	0.100913	0.366997
Kurtosis	2.802393	2.128884	6.470029	2.541025	2.954504	3.670488
Jarque-Bera	0.224289	4.207232	67.32697	1.537938	0.235418	5.435662
Probability	0.893915	0.122014	0.000000	0.463491	0.888955	0.066018
Observations	132	132	132	132	132	132

Table 1b: Descriptive Statistics

Species	Lemonsole	Mackerel	Plaice	Saithe	Sole	Whiting
Mean	8.120520	6.183595	7.374075	6.341004	8.779408	6.469075
sMedian	8.146564	6.167481	7.407014	6.357841	8.781095	6.504287
Maximum	8.587092	7.353082	7.939159	6.799056	9.059634	6.891626
Minimum	7.608374	5.147494	6.845880	4.941642	8.336151	5.937536
Std. Dev.	0.229629	0.435547	0.209110	0.241704	0.163355	0.205023
Skewness	-0.224134	0.062948	-0.062399	-2.114165	-0.325037	-0.265788
Kurtosis	2.328806	3.088459	3.503232	12.73401	2.482614	2.415963
Jarque-Bera	3.582953	0.130212	1.478496	619.4635	3.796557	3.430203
Probability	0.166714	0.936968	0.477473	0.000000	0.149826	0.179945
Observations	132	132	132	132	132	132

The following time series models are employed as forecast competitors:

- AR(p) model

    An Autoregressive AR(p) model is one where the current value of a variable Y_t depends on the values that the variable took in previous periods plus an error term. An AR(p) can be expressed as

    $$Y_t = c + a_1 Y_{t-1} + a_2 Y_{t-2} + ... + a_p Y_{t-p} + u_t$$

    where u_t is a white noise term.

- MA(q) model

    A moving average (MA) process is one in which the systematic component is a function of past innovations. An MA(q) model can be expressed as

    $$Y_t = c + b_1 \varepsilon_{t-1} + b_2 \varepsilon_{t-2} + ... + b_p \varepsilon_{t-p} + \varepsilon_t$$

- ARMA(p,q) model

    An ARMA(p,q) model combine AR(p) and MA(q) models, so that the current value depends linearly on its own previous values and on a combination of current and previous values of a white noise error term. The ARMA(p,q) specification has the form:

    $$Y_t = c + a_1 Y_{t-1} + a_2 Y_{t-2} + ... + a_p Y_{t-p} + \varepsilon_t + b_1 \varepsilon_{t-1} + b_2 \varepsilon_{t-2} + ... + b_q \varepsilon_{t-q}$$

In this paper, we compare the performance and measure the accuracy of different methods-techniques. Because all measures of accuracy suffer from advantages and disadvantages, we compare fits and forecasts under different measures-errors (see Stergiou and Christou, 1996). Following also Brailsford and Faff (1996), we compare the forecast performance of each model through the error statistics. Three error statistics are employed to measure the performance of the forecasting models.

Namely, the Root Mean Squared Error (*RMSE*), the Mean Absolute Error (*MAE*), and the Mean Absolute Percent Error (*MAPE*). We also use the *Theil* inequality coefficient.

Supposing that the forecast sample is $t = S, S+1,.., S+h$, we denote the actual and forecasted value in period t as y_t and $\hat{y}_t$, respectively. The reported forecast error statistics are then computed as follows:

$$RMSE = \sqrt{\frac{1}{h+1}\sum_{t=S}^{S+h}(\hat{y}_t - y_t)^2}$$

$$MAE = \frac{1}{h+1}\sum_{t=S}^{S+h}|\hat{y}_t - y_t|$$

$$MAPE = \frac{1}{h+1}\sum_{t=S}^{S+h}\left|\frac{\hat{y}_t - y_t}{y_t}\right|$$

$$Theil\ Inequality\ Coefficient = \frac{\sqrt{\frac{1}{h+1}\sum_{t=S}^{S+h}(\hat{y}_t - y_t)^2}}{\sqrt{\frac{1}{h+1}\sum_{t=S}^{S+h}\hat{y}_t^2} + \sqrt{\frac{1}{h+1}\sum_{t=S}^{S+h}y_t^2}}$$

The *RMSE* and *MAE* error statistics both depend on the scale of the dependent variable. We use them to compare forecasts for the same series and sample across different time series models. The better forecasting ability of the model, the smaller *RMSE* and *MAE* error statistics are. The *Theil* inequality coefficient lies between zero and one, where zero indicates a perfect fit to the data. When *Theil* coefficient is close to zero, then there is a good forecasting efficiency.

III. Empirical Results

Tables I-XII in Appendix 2 provide results of forecast error statistics (*RMSE, MAE* and *MAPE* and *Theil*) for each model by species. In the tables we highlight the forecasting models of fisheries prices for each of the twelve main species landed in Cornwall. Model selection is based on the forecast error statistics: *the smaller the error, the better the forecasting performance.*

In the case of *RMSE*, the selected error statistics by species vary from 0.063507 to 0.31387. Sole provides the smallest *RMSE* for MA(3) model, while the largest RMSE is from crabs for ARMA(1,2) model. Hence, in terms of *RMSE* selection by species, the MA(3) model is the best forecasting model, while ARMA(1,2) shows a good forecasting performance for crabs.

Using the *MAE* error measure, forecasting results show a minimum value of 0.046359 for sole, and a maximum value of 0.255996 for crabs. For sole, the smallest *MAE* value indicates that ARMA(1,1) model is superior than the other time series models. By contrast, the ARMA(1,2) model is the best forecasting model for crabs (even with a large *MAE* value).

In the case of *MAPE*, we find that sole has the smallest value (0.530974), while crabs has the largest value (3.687289). The results show that ARMA(1,1) model provides superior forecasts of fisheries prices.

Turning now to the *Theil* inequality coefficient, we find that sole provides the smallest coefficient with the value of 0.003623, while crabs show a large value of 0.022214. For sole, the ARMA(1,1) model is ranked one, while ARMA(1,2) model provide the forecasts of fisheries prices for crabs.

The tables in Appendix 2 present the forecasting performance of competing models. For anglerfish, we select ARMA(1,1) as the best forecast model because it provides a small *Theil* inequality coefficient. For cod, crabs, dogfish, haddock, mackerel, plaice and whiting we select ARMA(2,1), ARMA(1,2), ARMA(1,1), AR(2), ARMA(2,1), MA(1) and ARMA(2,1) respectively[2].

For hake, we select AR(1) as the best forecast model because it provides a small *Theil* inequality coefficient indicating a very good fit to the data. For the same reason, we select AR(2) model for lemonsole, ARMA(1,2) model for saithe and ARMA(1,1) model for sole. Table 2 presents the results for the selected models and the twelve main species landed into Cornwall.

Table 2: Summary Forecasting Results for Twelve Main Species Landed into Cornwall

Species	Model	
Anglerfish	ARMA(1,1)	$Y = 0.6057 + 0.9205 Y_{t-1} - 0.5368 \varepsilon_{t-1}$
Cod	ARMA(2,1)	$Y = 0.0516 + 1.2131 Y_{t-1} - 0.2199 Y_{t-2} - 0.8198 \varepsilon_{t-1}$
Crabs	ARMA(1,2)	$Y = 1.4119 + 0.7980 Y_{t-1} - 0.4248 \varepsilon_{t-1} - 0.0675 \varepsilon_{t-2}$
Dogfish	ARMA(1,1)	$Y = 0.7420 + 0.8869 Y_{t-1} - 0.6878 \varepsilon_{t-1}$
Haddock	AR(2)	$Y = 1.2256 + 0.5453 Y_{t-1} + 0.2831 Y_{t-2}$
Hake	AR(1)	$Y = 2.1288 + 0.7362 Y_{t-1}$
Lemonsole	AR(2)	$Y = 1.4872 + 0.8771 Y_{t-1} - 0.0921 Y_{t-2}$
Mackerel	ARMA(2,1)	$Y = 1.2931 + 1.3538 Y_{t-1} - 0.5618 Y_{t-2} - 0.5684 \varepsilon_{t-1}$
Plaice	MA(1)	$Y = 7.3731 + 0.5816 \varepsilon_{t-1}$
Saithe	ARMA(1,2)	$Y = 5.6994 + 0.1014 Y_{t-1} + 0.4663 \varepsilon_{t-1} + 0.2762 \varepsilon_{t-2}$
Sole	ARMA(1,1)	$Y = 2.1739 + 0.7527 Y_{t-1} + 0.0498 \varepsilon_{t-1}$
Whiting	ARMA(2,1)	$Y = 0.2374 + 1.4764 Y_{t-1} - 0.5132 Y_{t-2} - 0.8806 \varepsilon_{t-1}$

IV. Summary & Conclusions

Modelling and forecasting has an important role in management of fisheries. In this paper we report the forecasting competition between Autoregressive (AR), Moving Average (MA) and ARMA models of the monthly average fisheries prices. We consider twelve species landed into Cornwall: Anglerfish, Cod, Crabs, Dogfish, Haddock, Hake, Lemonsole, Mackerel, Plaice, Saithe, Sole and Whiting.

We compare the forecasting techniques based on symmetric error statistics: Root Mean Square Error (*RMSE*), Mean Absolute Error (*MAE*) and Mean Absolute Percent Error (*MAPE*). In addition we report the Theil inequality coefficient.

The results show that ARMA(p,q) models perform well in terms of forecasting monthly average prices. We find that eight species can be modelled using ARMA(1,1), ARMA(2,1) and ARMA(1,2) as they give superior forecast results, while the simple AR(1), AR(2) and MA(1) models provide superior forecasts of monthly fisheries prices for four species.

We compare forecasting techniques based on symmetric error statistics (*RMSE, MAE, MAPE* and *Theil* inequality coefficient). In the case of *RMSE*, the ARMA(2,1) model provides the smaller error statistics measure in three species. According to *RMSE*, ARMA(2,1) model tends to be preferred.

In the case of *MAE*, the ARMA(2,1) and ARMA(1,1) models clearly produce the most accurate forecasts for six out of the twelve species. In terms of *MAPE*, these models also provide the best forecasts for the same species.

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2   These models generally prove to be the best forecasting models (they are ranked number one)

Turning our attention to the *Theil* inequality coefficient, the ARMA(1,1) provides the best forecasts for four out of twelve species. The ARMA(2,1) model provides the second best price forecast.

In our evaluation of the out-of-sample forecasting accuracy of ten models for monthly fisheries prices of twelve species landed into Cornwall, we show that simple ARMA(p,q) models generally prove to be the best forecasting models. Future research is needed to investigate the forecasting ability of several time series models (ARIMA, ARFIMA and GARCH) using monthly/weekly fisheries catch prices.

Acknowledgements

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Appendix 1: Logarithms of prices for the twelve main species landed into Cornwall (1992-2002)

LEMONSOLE

MACKEREL

PLAICE

SAITHE

SOLE

WHITING
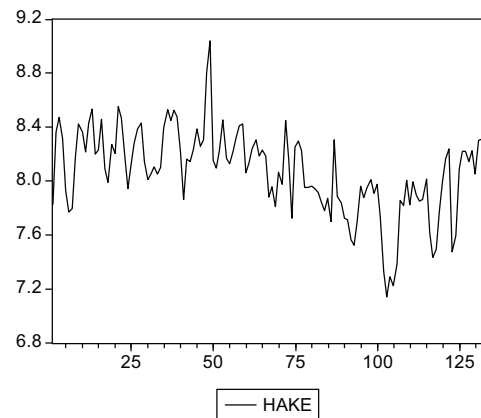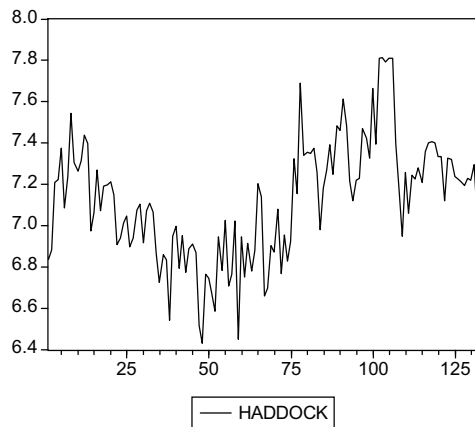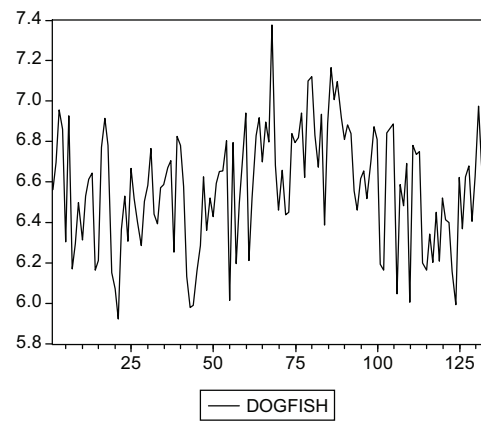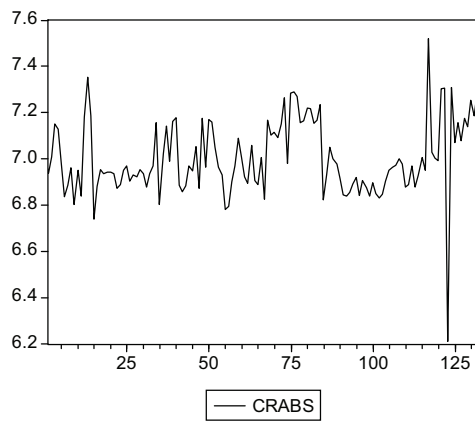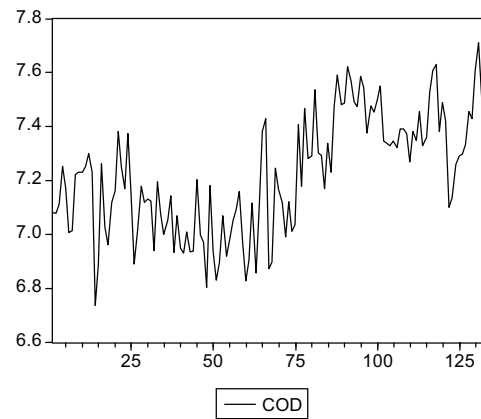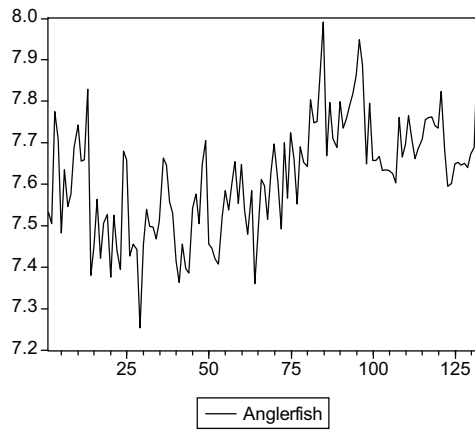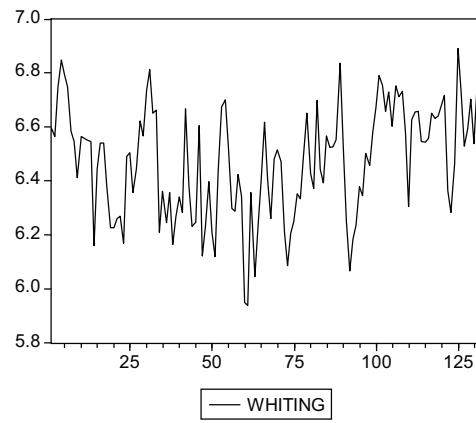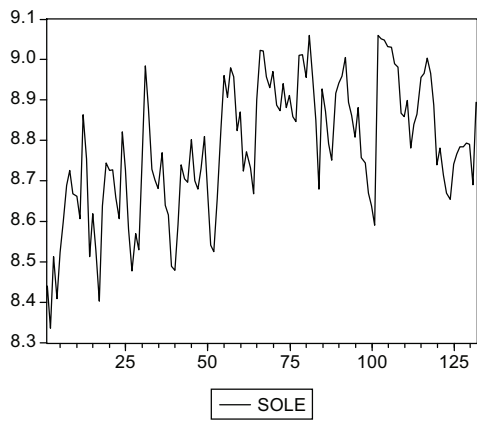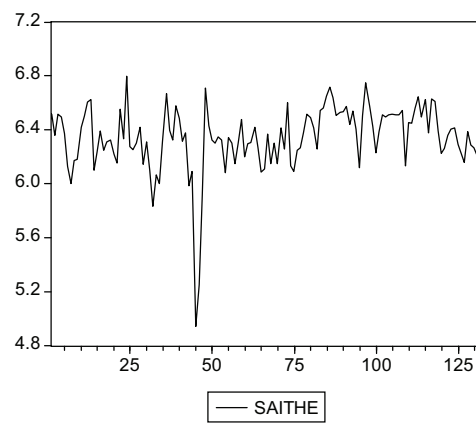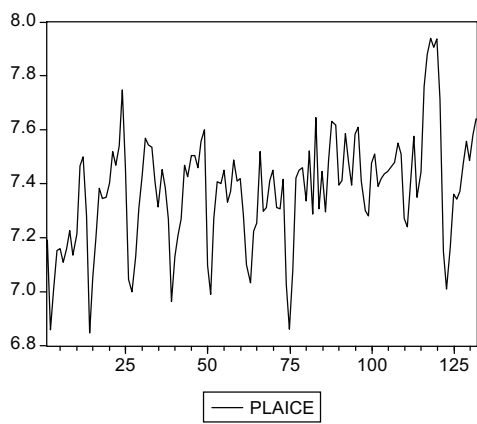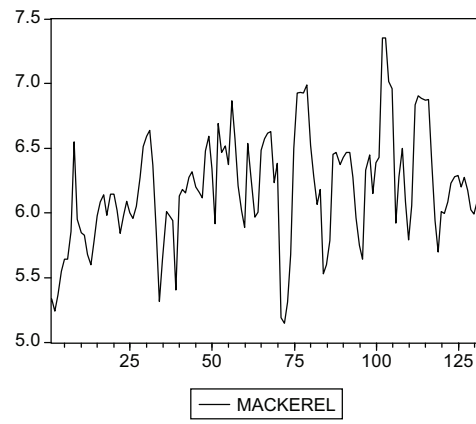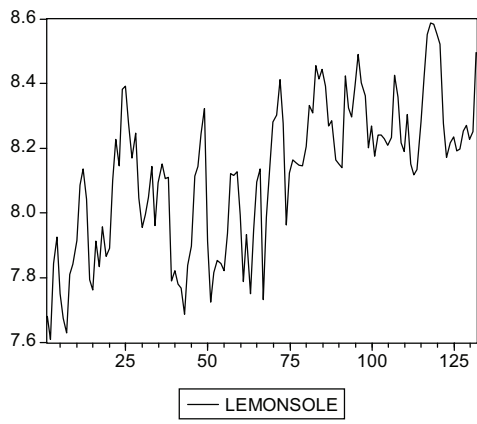
Appendix 2: Forecasting Performance of Competing Models

I. Anglerfish

Model	Rmse	Mae	Mape	Theil
AR(1)	0.100198	0.065676	0.844702	0.006541
AR(2)	0.096395	0.057410	0.737809	0.006285
AR(3)	0.092814	0.065578	0.846904	0.006038
MA(1)	0.107117	0.073100	0.940104	0.007000
MA(2)	0.106381	0.072310	0.929891	0.006952
MA(3)	0.106306	0.073449	0.945200	0.006946
ARMA(1,1)	0.092104	0.066755	0.862717	0.005990
ARMA(1,2)	0.093584	0.080576	1.045035	0.006077
ARMA(2,1)	0.093447	0.079842	1.035368	0.006068
ARMA(2,2)	0.093032	0.076442	0.990728	0.006043

II. Cod

Model	Rmse	Mae	Mape	Theil
AR(1)	0.237332	0.186107	2.495609	0.016200
AR(2)	0.228264	0.177106	2.378086	0.015555
AR(3)	0.203370	0.173314	2.347394	0.013769
MA(1)	0.239258	0.191488	2.557209	0.016391
MA(2)	0.242378	0.197004	2.635020	0.016598
MA(3)	0.243364	0.199684	2.672273	0.016664
ARMA(1,1)	0.196928	0.162187	2.218925	0.013260
ARMA(1,2)	0.195441	0.162101	2.218686	0.013155
ARMA(2,1)	0.194033	0.159755	2.186248	0.013064
ARMA(2,2)	0.196868	0.164236	2.247977	0.013248

* We highlight the smaller forecast error statistics values.

III. Crabs

Model	Rmse	Mae	Mape	Theil
AR(1)	0.316590	0.262693	3.777038	0.022422
AR(2)	0.316665	0.262866	3.779506	0.022427
AR(3)	0.317538	0.264623	3.803730	0.022492
MA(1)	0.317687	0.264244	3.798215	0.022502
MA(2)	0.315521	0.262108	3.768971	0.022345
MA(3)	0.321299	0.268334	3.853842	0.022765
ARMA(1,1)	0.315439	0.260225	3.744133	0.022335
ARMA(1,2)	0.313870	0.255996	3.687289	0.022214
ARMA(2,1)	0.314422	0.256757	3.697847	0.022255
ARMA(2,2)	0.314921	0.257941	3.713869	0.022292

IV. Dogfish

Model	Rmse	Mae	Mape	Theil
AR(1)	0.261669	0.203186	3.188460	0.020013
AR(2)	0.256174	0.197874	3.102999	0.019606
AR(3)	0.255775	0.198689	3.113629	0.019581
MA(1)	0.264498	0.207018	3.248584	0.020223
MA(2)	6.111944	5.955573	91.57883	0.771546
MA(3)	0.262294	0.203491	3.193111	0.020061
ARMA(1,1)	0.234639	0.187665	2.911426	0.018052
ARMA(1,2)	0.241706	0.197491	3.057095	0.018610
ARMA(2,1)	0.240066	0.195318	3.026889	0.018475
ARMA(2,2)	0.240252	0.195723	3.032098	0.018492

* We highlight the smaller forecast error statistics values.

V. Haddock

Model	Rmse	Mae	Mape	Theil
AR(1)	0.094300	0.086996	1.204290	0.006544
AR(2)	0.080088	0.054895	0.765138	0.005537
AR(3)	0.081850	0.055755	0.778277	0.005654
MA(1)	0.135985	0.124034	1.709147	0.009478
MA(2)	0.135230	0.124393	1.715215	0.009422
MA(3)	0.135329	0.126210	1.740623	0.009429
ARMA(1,1)	0.086808	0.059797	0.835882	0.005990
ARMA(1,2)	0.084944	0.057215	0.799654	0.005864
ARMA(2,1)	0.085562	0.058076	0.811755	0.005905
ARMA(2,2)	0.101046	0.072853	1.019390	0.006960

VI. Hake

Model	Rmse	Mae	Mape	Theil
AR(1)	0.255828	0.204305	2.577723	0.015847
AR(2)	0.258906	0.206995	2.610422	0.016042
AR(3)	0.260535	0.226272	2.835795	0.016181
MA(1)	0.259631	0.203080	2.565229	0.016077
MA(2)	0.258269	0.201904	2.549586	0.015995
MA(3)	0.259305	0.203469	2.568752	0.016060
ARMA(1,1)	0.256935	0.199898	2.526712	0.015906
ARMA(1,2)	0.343546	0.334110	4.122409	0.021554
ARMA(2,1)	0.259140	0.207589	2.617469	0.016057
ARMA(2,2)	0.465557	0.439044	5.380154	0.029478

We highlight the smaller forecast error statistics values.

VII. Lemonsole

Model	Rmse	Mae	Mape	Theil
AR(1)	0.129695	0.098530	1.187641	0.007823
AR(2)	0.125042	0.095420	1.148245	0.007554
AR(3)	0.128664	0.097910	1.179953	0.007762
MA(1)	0.179909	0.159111	1.910908	0.010974
MA(2)	0.170267	0.149885	1.802070	0.010380
MA(3)	0.169636	0.146140	1.757179	0.010339
ARMA(1,1)	0.127426	0.097680	1.176291	0.007694
ARMA(1,2)	0.129001	0.098020	1.181396	0.007782
ARMA(2,1)	0.125520	0.096024	1.155944	0.007580
ARMA(2,2)	0.125044	0.094971	1.142420	0.007557

VIII. Mackerel

Model	Rmse	Mae	Mape	Theil
AR(1)	0.105918	0.086564	1.413013	0.008595
AR(2)	0.110864	0.088497	1.454053	0.008969
AR(3)	0.110459	0.086827	1.428449	0.008928
MA(1)	0.114125	0.094747	1.551661	0.009250
MA(2)	0.110186	0.096410	1.575436	0.008938
MA(3)	0.112810	0.098704	1.612250	0.009152
ARMA(1,1)	0.105385	0.086644	1.418656	0.008541
ARMA(1,2)	0.104920	0.086408	1.415975	0.008500
ARMA(2,1)	0.102758	0.078571	1.292955	0.008308
ARMA(2,2)	0.105987	0.082660	1.359408	0.008570

We highlight the smaller forecast error statistics values.

IX. Plaice

Model	Rmse	Mae	Mape	Theil
AR(1)	0.267818	0.207672	2.855096	0.017974
AR(2)	0.221361	0.169443	2.322887	0.014908
AR(3)	0.234793	0.182355	2.502204	0.015794
MA(1)	0.185820	0.152970	2.079393	0.012556
MA(2)	0.196907	0.154731	2.112433	0.013288
MA(3)	0.194850	0.154189	2.103998	0.013151
ARMA(1,1)	0.242957	0.188803	2.589618	0.016345
ARMA(1,2)	0.216404	0.165972	2.270374	0.014592
ARMA(2,1)	0.224737	0.172793	2.369152	0.015132
ARMA(2,2)	0.216617	0.166442	2.279448	0.014596

X. Saithe

Model	Rmse	Mae	Mape	Theil
AR(1)	0.094820	0.083515	1.333317	0.007513
AR(2)	0.095465	0.083602	1.335064	0.007563
AR(3)	0.097573	0.087074	1.389675	0.007733
MA(1)	0.095510	0.081824	1.308657	0.007560
MA(2)	0.096182	0.082835	1.324222	0.007617
MA(3)	0.096841	0.084255	1.346617	0.007670
ARMA(1,1)	0.094818	0.083220	1.328753	0.007512
ARMA(1,2)	0.094750	0.083153	1.328373	0.007506
ARMA(2,1)	0.095673	0.083565	1.334425	0.007580
ARMA(2,2)	0.095798	0.083828	1.339094	0.007588

* We highlight the smaller forecast error statistics values.

XI. Sole

Model	Rmse	Mae	Mape	Theil
AR(1)	0.064213	0.046642	0.534318	0.003662
AR(2)	0.065586	0.048893	0.560157	0.003740
AR(3)	0.067650	0.050415	0.577692	0.003857
MA(1)	0.069496	0.053669	0.614419	0.003964
MA(2)	0.066936	0.051751	0.592081	0.003820
MA(3)	0.063507	0.048695	0.556659	0.003625
ARMA(1,1)	0.063522	0.046359	0.530974	0.003623
ARMA(1,2)	0.071250	0.053450	0.612618	0.004061
ARMA(2,1)	0.065399	0.048528	0.555959	0.003729
ARMA(2,2)	0.064755	0.047855	0.548204	0.003693