The Emergent Universe: Inflationary cosmology with no singularity

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Observations indicate that the universe is effectively flat, but they do not rule out a closed universe. The role of positive curvature is negligible at late times, but can be crucial in the early universe. In particular, positive curvature allows for cosmologies that originate as Einstein static universes, and then inflate and later reheat to a hot big bang era. These cosmologies have no singularity, no "beginning of time", and no horizon problem. If the initial radius is chosen to be above the Planck scale, then they also have no quantum gravity era, and are described by classical general relativity throughout their history.

I. INTRODUCTION

The "standard" inflationary model is based on a flat ($K = 0 \Leftrightarrow \Omega_0 = 1$) Friedmann-Robertson-Walker (FRW) geometry, motivated by the fact that inflationary expansion rapidly wipes out any original spatial curvature. However, even though inflation drives the curvature term,

$$\Omega(t) - 1 = \frac{K}{a^2 H^2} \qquad (1)$$

towards zero, or equivalently, drives the total density parameter Ω towards 1, this does *not* imply that $K = 0$. Conditions leading to the open set of values $\Omega_0 > 1$ are far less fine-tuned than those corresponding precisely to $K = 0$ (although closed models have other fine-tuning aspects). But irrespective of any arguments about fine-tuning, the spatial curvature of the real universe is in principle determined by observations; theory will have to give way to data if the data clearly tell us that $\Omega_0 > 1$. Recent cosmic microwave background (CMB) and other data [1] are not conclusive, but include the possibility that $K = +1$, with

$$\Omega_0 = 1.02 \pm 0.02 \; . \qquad (2)$$

Future experiments such as PLANCK will reduce these error bars and give more accurate information about the curvature of the universe. (Note that the true global value of Ω_0 and its observed value in the local Hubble volume may differ due to cosmological perturbations seeded by inflation.) Current data mean that we should take closed universes, and therefore closed inflationary models, seriously. Inflation in a closed universe is sometimes considered to be ruled out by unrealistic fine-tuning [2]. If this were true, then observational confirmation of $\Omega_0 > 1$ would rule out inflation. But in fact there is a wide range of closed inflationary models, and it is not too difficult to construct simple and consistent models without excessive fine-tuning (see also Ref. [3]). It is also true that fine-tuned initial conditions cannot be ruled out by purely scientific arguments, as we discuss below. For the models that we consider, the value of $\Omega_0 - 1$ is set by choosing one parameter, while the magnitude of large-scale scalar perturbations fixes another parameter.

If Ω_0 is taken as 1.02, then the power spectra of CMB anisotropies and matter can show testable differences from the standard flat model [3]. If $\Omega_0 - 1$ is very small, then curvature is negligible as regards structure formation processes in the universe. The smaller that $\Omega_0 - 1$ is, the more closely the models approximate the standard flat model. But in the early inflationary universe, positive curvature can play a significant role and lead to novel features that do not arise when $K \le 0$, for example allowing a minimum in the scale factor $a(t)$ which is otherwise not possible, or putting limits on the number of possible e-foldings in the inflationary epoch [4, 5]. Primordial history is very different when $K = +1$ than in the case $\Omega_0 = 1$, regardless of how small $\Omega_0 - 1$ is, and a variety of possibilities arise, including universes with a minimum radius.

Singularity theorems have been devised that apply in the inflationary context, showing that the universe necessarily had a beginning (according to classical and semi-classical theory) [6]. In other words, according to these theorems the quantum gravity era cannot be avoided in the past even if inflation takes place. However, the models we present escape this conclusion, because they do not satisfy the geometrical assumptions of these theorems. Specifically, the theorems assume either (a) that the universe has open space sections, implying $K = 0$ or -1, or (b) that the Hubble expansion rate $H = \dot{a}/a$ is bounded away from zero in the past, $H > 0$. There are inflationary universes that evade these constraints and hence avoid the conclusions of the theorems (this was also noted in Ref. [7]). It is also possible to find counter-examples that are open, i.e., where assumption (a) is not violated, but this typically involves sophisticated constructions [8].

Here we consider closed models in which $K = +1$ and H can become zero, so that both assumptions (a) and (b) of the inflationary singularity theorems are violated. The models are simple, obey general relativity, and contain only ordinary matter and (minimally coupled) scalar fields. Previous examples of closed inflationary models [4, 5, 7, 9] are, to our knowledge, either bouncing models or models in which inflation is preceded by deceleration (so that a singularity is not avoided). The $K = +1$ bouncing universe collapses from infinite size in the infinite past and then turns around at $t_{\rm i}$ to expand in an inflationary phase. The canonical model for such a bounce is the de Sitter universe in the $K = +1$ frame, with $a(t) = a_{\rm i} \cosh Ht$. These coordinates cover the whole spacetime, which is geodesically complete [10]. However, the bouncing models face serious difficulties as realistic cosmologies. The initial state is hard to motivate (collapsing from infinite size without causal interaction), and it is also difficult to avoid nonlinearities in the collapse that prevent a regular bounce.

II. THE EMERGENT UNIVERSE SCENARIO

We show here that when $K = +1$ there are closed inflationary models that do not bounce, but inflate from a static beginning, and then reheat in the usual way. The inflationary universe emerges from a small static state that has within it the seeds for the development of the macroscopic universe, and we call this the "Emergent Universe" scenario. (This can be seen as a modern version and extension of the Eddington universe.) The universe has a finite initial size, with a finite amount of inflation occurring over an infinite time in the past, and with inflation then coming to an end via reheating in the standard way. The redshift and the total number of e-folds remain bounded through the expansion of the universe until the present day, because the scale-factor is bounded away from zero in the past. There is *no* horizon problem, since the initial state is Einstein static. Since they start as Einstein static, they avoid a singularity. The initial static state can be chosen to have a radius above the Planck scale, so that these models can even avoid a quantum gravity regime, whatever the true quantum gravity theory may be.

Because they can undergo a large amount of inflation, these models can be effectively the same as the standard flat models as regards structure formation processes. Therefore they are not vulnerable to future reductions in the observed value of $\Omega_0 - 1$ below 0.02 (provided that it remains positive), and they can reproduce the successes of the standard inflationary cosmologies, but from very different primordial foundations.

We do not require exotic physics or matter. The early universe contains a standard scalar field ϕ with energy density $\rho_\phi = \frac{1}{2}\dot\phi^2 + V(\phi)$ and pressure $p_\phi = \frac{1}{2}\dot\phi^2 - V(\phi)$, and possibly also ordinary matter with energy density ρ and pressure $p = w\rho$, where $-\frac{1}{3} \le w \le 1$. The cosmological constant is absorbed into the potential V. There are no interactions between matter and the scalar field, so that they separately obey the energy conservation and Klein-Gordon equations,

$$\dot\rho + 3(1+w)H\rho \;=\; 0\,, \tag{3}$$

$$\ddot\phi + 3H\dot\phi + V'(\phi) \;=\; 0\,. \tag{4}$$

The Raychaudhuri field equation

$$\frac{\ddot a}{a} = -\frac{8\pi G}{3}\left[\frac{1}{2}(1+3w)\rho + \dot\phi^2 - V(\phi)\right], \tag{5}$$

has first integral the Friedmann equation,

$$H^2 = \frac{8\pi G}{3}\left[\rho + \frac{1}{2}\dot\phi^2 + V(\phi)\right] - \frac{K}{a^2}\,, \tag{6}$$

which together imply

$$\dot H = -4\pi G\left[\dot\phi^2 + (1+w)\rho\right] + \frac{K}{a^2}\,. \tag{7}$$

The Raychaudhuri equation gives the condition for inflation,

$$\ddot a > 0 \;\Leftrightarrow\; \dot\phi^2 + \frac{1}{2}(1+3w)\rho < V(\phi)\,. \tag{8}$$

For a positive minimum in the inflationary scale factor, $a_{\rm i} \equiv a(t_{\rm i}) > 0$,

$$H_{\rm i} = 0 \;\Leftrightarrow\; \frac{1}{2}\dot\phi_{\rm i}^2 + V_{\rm i} + \rho_{\rm i} = \frac{3K}{8\pi G a_{\rm i}^2}\,, \tag{9}$$

where the time $t_{\rm i}$ may be infinite. The only way to satisfy Eq. (9) with non-negative energy densities is if $K = +1$. Closed inflationary models admit a minimum scale factor if inflation occurs for long enough, since curvature will eventually win over a slow-rolling scalar field as we go back into the past (cf. [4]). The inflationary singularity theorems mentioned above exclude this case, since they either only consider $K \le 0$, or explicitly exclude the possibility $H_{\rm i} = 0$.

Closed models with a minimum scale factor $a_{\rm i} > 0$ include both bouncing and ever-expanding cases. We do not proceed further with the bouncing models because of their acausal initial state and the difficulty of achieving a stable bounce. Ever-expanding models avoid these problems. A simple ever-inflating model is the closed model containing radiation ($w = \frac{1}{3}$) and cosmological constant $\Lambda = 8\pi G V$, with scale factor [11]

$$a(t) = a_{\rm i}\left[1 + \exp\left(\frac{\sqrt{2}\,t}{a_{\rm i}}\right)\right]^{1/2}. \tag{10}$$

In the infinite past, $t \to -\infty$, the model is asymptotically Einstein static, $a \to a_{\rm i}$. Inflation occurs for an infinite time to the past, but at any finite time $t_{\rm e}$, there is a finite number of e-folds,

$$N_{\rm e} = \ln\frac{a_{\rm e}}{a_{\rm i}} \approx \frac{t_{\rm e}}{\sqrt{2}a_{\rm i}}\,, \tag{11}$$

where the last equality holds for $t_{\rm e} \gg a_{\rm i}$. The curvature parameter at $t_{\rm e}$ is strongly suppressed by the de Sitter-like expansion:

$$\Omega_{\rm e} - 1 \approx 2e^{-N_{\rm e}}\,. \tag{12}$$

The exact model Eq. (10) is a simple example of Eddington-type solutions. There are trajectories of this type in the classical phase space [12] satisfying Einstein's equations. They are past-asymptotically Einstein static and ever-expanding, and the fact that they exist already shows there are inflationary universes that evade the conditions of the singularity theorems presented in [6]. However, in these models inflation does not end. Below we discuss more realistic models, based on inflationary potentials, that do exit from inflation. These universes are singularity-free, without particle horizons, and ever-expanding ($H \geq 0$). Even though Emergent Universes admit closed trapped surfaces, these do not lead to a singularity, since $K = +1$ and the weak energy condition is violated in the past [13].

The Einstein static universe is characterized by $K = +1$ and $a = a_{\rm i} =$const. Equations (3)–(7) then imply that

$$\frac{1}{2}(1 - w_{\rm i})\rho_{\rm i} + V_{\rm i} \;=\; \frac{1}{4\pi G a_{\rm i}^2}\,, \qquad (13)$$

$$(1 + w_{\rm i})\rho_{\rm i} + \dot{\phi}_{\rm i}^2 \;=\; \frac{1}{4\pi G a_{\rm i}^2}\,, \qquad (14)$$

where $\dot{\rho}_{\rm i} = 0 = \ddot{\phi}_{\rm i}$ and $V_{\rm i} = \Lambda_{\rm i}/8\pi G$ is the primordial vacuum energy. If the scalar field kinetic energy vanishes, i.e. if $\dot{\phi}_{\rm i} = 0$, then $(1 + w_{\rm i})\rho_{\rm i} > 0$, so that there must be matter to keep the universe static. If the static universe has only a scalar field, i.e. if $\rho_{\rm i} = 0$, then the field must have nonzero (but constant) kinetic energy, so that it rolls at constant speed along the flat potential. (Dynamically, this case is equivalent to a stiff fluid, $w_{\rm i} = 1$, plus cosmological constant $\Lambda_{\rm i}$ [14].)

The radius $a_{\rm i}$ of the initial static universe can be chosen to be above the Planck scale,

$$a_{\rm i} > M_{\rm p}^{-1}\,, \qquad (15)$$

by suitable choice of $V_{\rm i}$, $\dot{\phi}_{\rm i}^2$ and $\rho_{\rm i}$ (with all of them $\ll M_{\rm p}^4$). Thus these models can in principle avoid the quantum gravity era.

A simple way to realize the scenario of the Emergent Universe is the following.

III. A SIMPLE EMERGENT POTENTIAL

Consider a potential that is asymptotically flat in the infinite past,

$$V(\phi) \to V_{\rm i} \text{ as } \phi \to -\infty\,,\; t \to -\infty\,, \qquad (16)$$

but drops towards a minimum at a finite value $\phi_{\rm f}$. The scalar field kinetic energy density is asymptotic to the constant Einstein static value,

$$\frac{1}{2}\dot{\phi}^2 \to \frac{1}{2}V_{\rm i} = \frac{1}{8\pi G a_{\rm i}^2} \text{ as } \phi \to -\infty\,,\; t \to -\infty\,, \qquad (17)$$

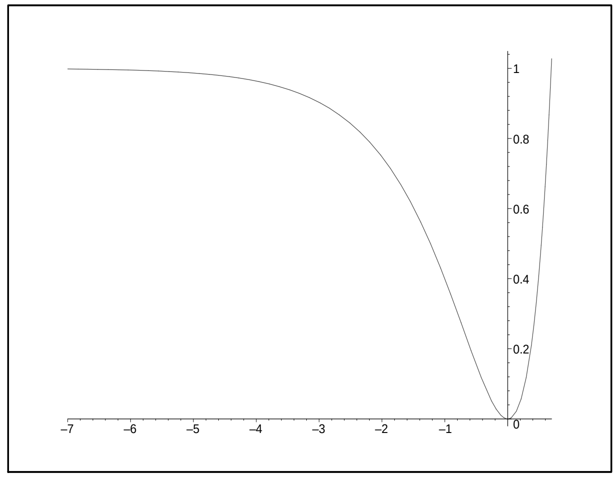

FIG. 1: Schematic of a potential for an Emergent Universe, with $V - V_{\rm f}$ plotted against $\phi - \phi_{\rm f}$.

where we used Eqs. (13) and (14) with $\rho_{\rm i} = 0$. Because $\dot{\phi}_{\rm i} \neq 0$, no matter is needed to achieve the initial static state. The field rolls from the Einstein static state at $-\infty$ and the potential slowly drops from its Einstein static original value. Provided that $\dot{\phi}^2$ decreases more rapidly than the potential, we have $V - \dot{\phi}^2 > 0$, so that the universe accelerates, by Eq. (8). Since $\ddot{\phi} < 0$ and $V' < 0$, while $\dot{\phi} > 0$, the Klein-Gordon equation (4) shows that the universe is expanding ($H > 0$).

Inflation ends at time $t_{\rm e}$, where $V_{\rm e} = \dot{\phi}_{\rm e}^2$. Then reheating takes place as the field oscillates about the minimum at $\phi_{\rm f}$. In the asymptotic past, $V \to V_{\rm i}$, the primordial cosmological constant $\Lambda_{\rm i} = 8\pi G V_{\rm i}$ is given by Eq. (17) as

$$\Lambda_{\rm i} = \frac{2}{a_{\rm i}^2}\,, \qquad (18)$$

so that $\Lambda_{\rm i}$ is large for a small initial radius. At the minimum, $V_{\rm f}$ defines the cosmological constant that dominates the late universe,

$$\Lambda = 8\pi G V_{\rm f} \;\ll\; \Lambda_{\rm i}\,. \qquad (19)$$

A typical example of a potential is shown in Fig. 1, which is based on the potential presented in Ref. [15]:

$$V - V_{\rm f} = (V_{\rm i} - V_{\rm f})\left[\exp\left(\frac{\phi - \phi_{\rm f}}{\alpha}\right) - 1\right]^2, \qquad (20)$$

where α is a constant energy scale.

The infinite time of inflation, from $t = -\infty$ to $t = t_{\rm e}$ ($< t_{\rm f}$), produces a finite amount of inflation (and a finite redshift to the initial Einstein static state). The potential produces expansion that is initially qualitatively similar to the exact solution in Eq. (10), so that the total number of e-folds from the initial expansion can be estimated, following Eq. (11), as

$$N_{\rm e} = O\left(\frac{t_{\rm e}}{a_{\rm i}}\right)\,. \qquad (21)$$

Provided that a_i is chosen small enough and t_e large enough, a very large number of e-folds can be produced. The parameters in the potential in Eq. (20) can be chosen so that the primordial universe is consistent with current observations [15].

IV. FINE-TUNING

As with standard inflationary models, fine-tuning is necessary to produce density perturbations at the $O(10^{-5})$ level, and to fix Λ so that $\Omega_{\Lambda 0} \sim 0.7$. These issues are faced by all inflationary universe models [16]. The specific geometrical fine-tuning problem in the Emergent models is the requirement of a particular choice of the initial radius a_i, or equivalently a specific unique choice of the primordial cosmological constant, Eq. (18). This choice must then be supplemented by a further fine-tuning – a choice of initial kinetic energy such that Eq. (17) holds. Both conditions are required to attain an asymptotic Einstein static state, the one ensuring asymptotic validity of the Friedmann equation, and the other, that of the Raychaudhuri equation.

The model can be criticized because of its fine-tuned initial state. There are two basic responses to this criticism.

- The criticism does not rule out these models as valid physical models, but rather claims they are not likely to occur in reality because they are improbable. But the inflationary singularity theorems [6] are not based on probabilities, and the Emergent models do indeed show that the geometric conditions of those theorems need not be satisfied.

- The force of the fine-tuning criticism is based partly in philosophy not only in physics. There is no scientifically based proof that the unique physical universe has to be probable.

This points to a tension in cosmology between two viable but opposed views on how to explain the current state of the universe.

- The one view is that the present state of the universe is highly probable because physical processes make it very likely to have occurred [17].

- The opposite view is that Nature prefers symmetry, and the universe is likely to have originated in a highly symmetric, and necessarily fine-tuned, state [18].

The key point is that there is no scientific proof that the one or the other of these approaches is the correct approach to use in cosmology. A quantum gravity theory may resolve this and other issues relating to the origin of the universe, but in the absence of such a theory, it seems worthwhile to pursue the implications of both approaches.

The underlying problem within classical and semi-classical cosmology is the uniqueness of the universe, and all the scientific and philosophical difficulties that this entails [19]. One cannot straightforwardly apply statistics or probability to a unique object. There may be a scientific basis for the use of probability in the context of an ensemble of universes – a multiverse. But this is a complex and controversial proposal, and there is no evidence that it is correct; indeed it may not be scientifically testable [20]. Consequently the choice between these two fundamentally different approaches to cosmological origins remains of necessity a partly philosophical one, while we lack a more complete quantum gravity theory that explains what actually underlies the existence of the real physical universe.

Furthermore, it is not simply a case of fine-tuning versus non-fine-tuning. The standard $K = 0$ inflationary models, which avoid a special initial state, are not free of fine-tuning, in particular, the fine-tuning entailed in the choice of $K = 0$. There is no mechanism to attain $K = 0$ (as opposed to $\Omega \to 1$), and no obvious way to prove observationally that $\Omega = 1$ to the infinite accuracy required to establish that in reality $K = 0$. This standard assumption would appear to involve the same kind of fine-tuning as occurs in the Emergent Universe, because of the exact balance required to set $K = 0$.

The standard inflationary models necessarily have a singularity in the past [6]. Although it is possible to evade the singularity when $K = 0$ [8], this requires complicated modifications of the spacetime. It also appears to invoke a fine-tuned initial state, not unlike the fine-tuning involved in the Emergent models. The choice can therefore also be seen as between a singularity and a fine-tuned non-singular initial state. There are arguments for each choice. The singular inflationary models have the strength of generality of initial conditions (at some time after the classical singularity, i.e., effectively after the Planck era). They have a disadvantage of starting at a spacetime singularity where all of physics breaks down and spacetime itself comes to an end (though quantum gravity effects may be able to avoid this). The Emergent Universe has the advantage of avoiding a singularity (with or without a Planck era). There are also arguments that a highly symmetric start to the universe is necessary for the thermodynamic arrow of time to function as it does [21].

The Emergent Universe scenario gives a framework for investigating what are the implications if whatever process caused the universe to come into being, preferred the high-symmetry state of the Einstein static universe to any less ordered state. There are various arguments in support of the Einstein static model as a preferred initial state.

- It is neutrally stable against inhomogeneous linear perturbations when $\rho_i = 0$ (as in the simple Emergent model above), or when $\rho_i > 0$ and the sound speed of matter obeys $c_s^2 > \frac{1}{5}$, as shown in Ref. [14], extending the initial results of Refs. [22, 23]. It is of course unstable to *homogeneous* perturbations, which break the balance between curvature and energy density. This instability is crucial for producing an inflationary era.

- It has no horizon problem.
- It maximizes the entropy within the family of FRW radiation models [23].
- It is the unique highest symmetry non-empty FRW model, being invariant under a 7-dimensional group of isometries [24].

The Einstein static model has a well-defined vacuum, and Casimir effects (see, e.g., Ref. [25]) could play an important role in determining the primordial parameters of the Emergent Universe.

V. EMERGENT MODELS WITH A FINITE TIME OF INFLATION

The realization of the Emergent Universe outlined above and elaborated in Ref. [15], illustrated qualitatively in Fig. 1, has the advantage of simplicity. But the fact that the initial Einstein static state is only achieved asymptotically in the infinite past could be seen as a disadvantage. It is possible to find other realizations of the scenario in which the universe starts expanding from an Einstein static state at a finite time in the past.

For example, one can consider potentials with a critical value $V(\phi_{\rm i})$, where $V'(\phi_{\rm i}) = 0$, above a stable minimum at $\phi_{\rm f}$, i.e.,

$$V(\phi_{\rm f}) < V(\phi_{\rm i})\,,\ V'(\phi_{\rm f}) = 0 < V''(\phi_{\rm f})\,. \qquad (22)$$

The scalar field is initially at the critical position $\phi = \phi_{\rm i}$; since there is no kinetic energy ($\dot{\phi}_{\rm i} = 0$), matter is necessary ($\rho_{\rm i} > 0$) to provide an initial static state. Specifically, by Eqs. (13) and (14), we have

$$V(\phi_{\rm i}) = \frac{1}{2}(1 + 3w_{\rm i})\rho_{\rm i}\,. \qquad (23)$$

If the initial position is an unstable (tachyonic) maximum, i.e., $V''(\phi_{\rm i}) < 0$, then the field rolls down to the true minimum, similar to natural inflation potentials. If the initial position is a false vacuum minimum, then the field needs to tunnel towards the true minimum.

Another alternative is an inflationary potential that depends on temperature in much the same way as in the first inflationary universe models. In the initial Einstein static state, with matter at some temperature, the field starts at a minimum, which becomes unstable after a perturbation. This leads to expansion of the universe, and consequently the temperature falls, which results in the appearance of a lower minimum.

VI. CONCLUSIONS

We have shown that inflationary cosmologies exist in which the horizon problem is solved before inflation begins, there is no singularity, no exotic physics is involved, and the quantum gravity regime can even be avoided. These Emergent Universe models can be constructed with simple potentials (illustrated schematically in Fig. 1), giving past-infinite inflation from an asymptotically initial Einstein static state, with a bounded number of e-folds and redshift, followed by reheating in the usual way. Explicit and simple forms of the potential can be found that are consistent with present cosmological observations for suitable choice of parameters in the potential [15]. Other realizations exist in which the universe starts inflating from an Einstein static state at a finite time in the past. The Emergent models illustrate the potentially strong primordial effects of positive spatial curvature, leading to a very different early universe than the standard models, while producing a late universe that can be observationally distinguished from the standard case only by high-precision observations.

If one requires, within classical and semi-classical general relativity theory, a non-singular universe with standard fields and matter, then this is only possible if $K = +1$. If one further rules out bouncing models because of problems in achieving a regular bounce, then one is led to the Emergent Universe scenario, with its fine-tuned initial Einstein static state. This state is preferred by its appealing stability, entropy and geometric properties, and provides, because of its compactness, an interesting arena for investigating the Casimir and other effects.

The Emergent Universe scenario with $K = +1$ provides an interesting alternative to the standard inflationary scenario with $K = 0$, in the case that Ω_0 is close to, but above, 1. This may be seen as a choice between a fine-tuned non-singular initial state, and a singularity that precedes generic initial conditions. We do not claim that the Emergent scenario is superior to the standard scenario, but simply that it is worthwhile investigating, since neither it nor the standard scenario are yet ruled out by scientific arguments. In the absence of a quantum gravity theory that can explain the true nature of the origin of the universe, both approaches remain valid to investigate and test. The two approaches lead to observationally viable models, at least until there is firm observational evidence determining the curvature parameter K of the universe.

If future observations turn out to provide strong evidence for positive curvature, then the standard models will be ruled out, and the Emergent models and other closed models with a singularity and deceleration preceding inflation [3], would be contenders to describe the universe. Of course if $\Omega_0 - 1$ is shown to be negative, then all closed models and the standard models will be ruled out. However, if the Universe is in fact exactly flat, observations will not be able to rule out a very tiny positive curvature (and the existence of perturbations prevents a very accurate measurement). Thus if the standard is model is correct, the Emergent Universe with perturbatively small curvature cannot be falsified by observations, although it does produce a very different early universe.

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